Extension of *Chern-Simons* Forms

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Overview

1. Anomalies (abelian-non abelian)
2. Zumino's reduction method
3. Tensor Gauge Theory (TGT)
4. Chern characters in TGT
5. Chern Simons forms in TGT
6. Anomalies in TGT

Abelian anomalies

- anomalies: classical symmetries which are violated by radiative corrections (loops).
- For *Yang-Mills Lagrangian* (massless spinor):

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}(\partial_{\mu} - iT^{a}A_{\mu}^{a})\psi$$

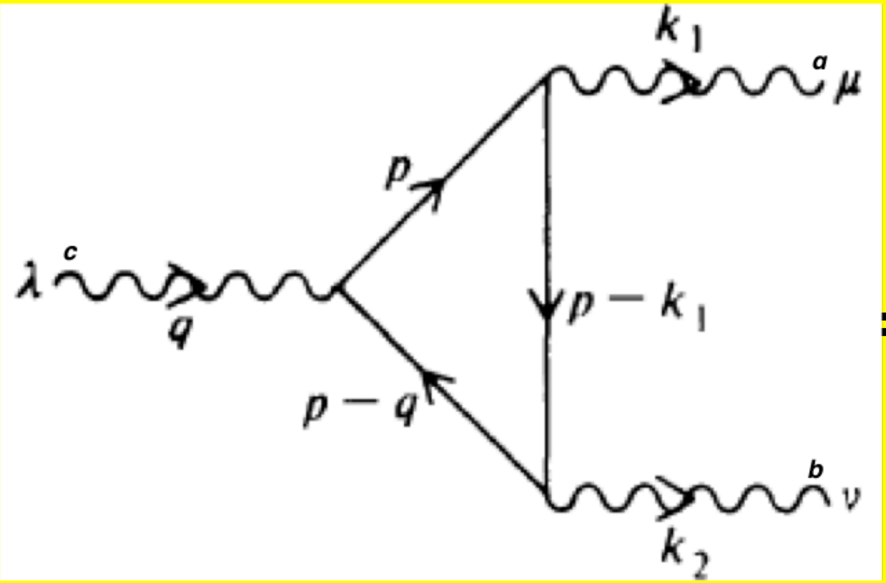
Chiral current: $J_{\mu}^{5,a} = \bar{\psi}\gamma_{\mu}\gamma_{5}\psi$

is conserved at classical level ($\partial^{\mu}J_{\mu}^{5} = 0$) .

In one loop approximation the symmetry breaks down to:

$$\partial^{\mu}J_{\mu}^{5} = -\frac{1}{16\pi^{2}}\varepsilon^{\mu\nu\lambda\rho}Tr(F_{\mu\nu}F_{\lambda\rho}) \neq 0$$

Abelian anomalies



$$= T^{abc}_{\mu\nu\lambda}(k_1, k_2, q)$$

$$q^{\lambda} T^{abc}_{\mu\nu\lambda} = -\frac{1}{2\pi^2}\varepsilon_{\mu\nu\lambda\rho} k_1^{\lambda} k_2^{\rho} D^{abc}, \quad D^{abc} = \frac{1}{2} tr(\{T^a, T^b\} T^c)$$

Non Abelian anomalies

- For left (right) handed currents:

$$J^{\alpha}_{H,\mu} = \bar{\psi}_H \gamma_\mu T^\alpha \psi_H$$

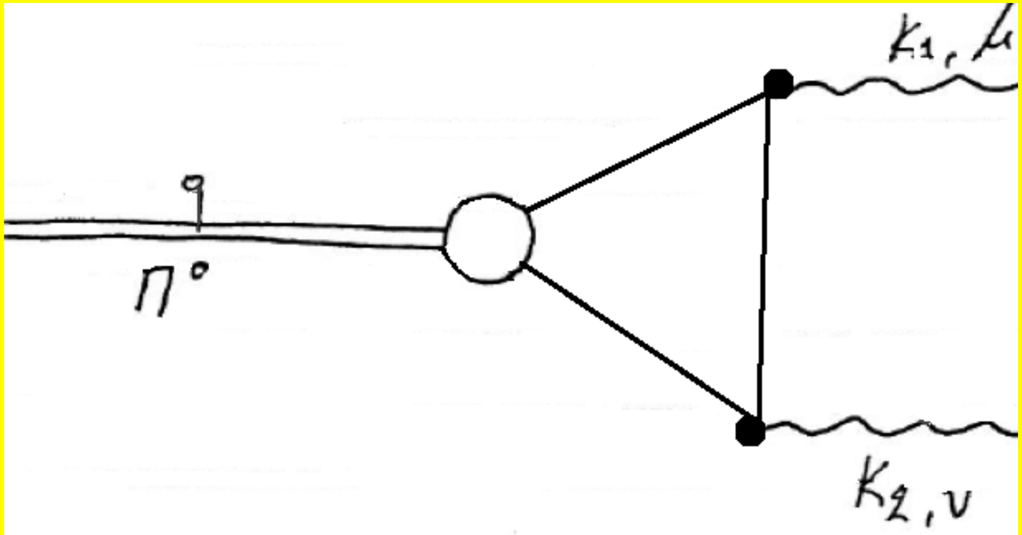
We have the non abelian anomaly:

$$D^\mu J^a_{H,\mu} = \eta_H \frac{1}{24\pi^2} \varepsilon_{\mu\nu\lambda\rho} Tr[T^a \partial^\mu (A^\nu_H \partial^\lambda A^\rho_H + \frac{1}{2} A^\nu_H A^\lambda_H A^\rho_H)]$$

To be compared with the abelian anomaly:

$$\partial^\mu J^5_\mu = -\frac{1}{4\pi^2} \varepsilon_{\mu\nu\lambda\rho} Tr[\partial^\mu (A^\nu \partial^\lambda A^\rho + \frac{2}{3} A^\nu A^\lambda A^\rho)]$$

Soft $\pi^0 \to \gamma\gamma$



$$\lim_{q\to 0}\Gamma_{\mu\nu}(k_1,k_2,q)\sim D, \qquad D=\frac{1}{6}\times 3$$

$$\Gamma_{theory}(0)=0.0369\,m_\pi^{-1}$$

$$\Gamma_{\exp}(0)=0.0375\,m_\pi^{-1}$$

$$\frac{|\Delta\Gamma|}{\Gamma_{\exp}}\approx 1.6\%$$

Zumino's connection

- The close resemblance between the previous expressions led to the discovery of a connection between abelian and non-abelian anomalies.
- Non abelian anomalies in 2n dimensions can be obtained from abelian anomalies in 2n+2 dimensions by a reduction method (without having to evaluate Feynman diagrams).
- What we need is differential geometry.

Differential Forms

- Expressing the field strength tensor as a 2nd rank form: $F = dA + A^2$

the previous expressions (abelian/non abelian anomaly) can be written as total divergencies:

$$d * J^5 \propto d\, tr[(AdA + \frac{2}{3} A^3)]$$

$$D * J^a_{L,R} \propto d\, tr[T^a (AdA + \frac{1}{2} A^3)]$$

The reduction method

- Starting from the primary form (Chern character) $\Omega_{2n+2}(A)$ which is closed, metric independent, gauge invariant, and represents the abelian anomaly, we can write:

$$\Omega_{2n+2} = d\,\omega^0_{2n+1}$$

- Because it is gauge invariant we get from Poincare lemma:

$$\delta_\xi \Omega_{2n+2} = 0 = d\,(\delta_\xi \omega^0_{2n+1}) \Rightarrow$$
$$\delta_\xi \omega^0_{2n+1} = d\,\omega^1_{2n}$$

- ω^1_{2n} is the non abelian anomaly

Anomalies

↑

Secondary Forms

↑

Primary Forms

Tensor Gauge Theory

- New gauge fields are introduced. Rank – (s+1) tensors, symmetric over the λ indices: $A_{\mu\lambda_1\lambda_2...\lambda_s}$

- <u>Extended gauge transformations</u>:

$$\delta_\xi A_\mu = \partial_\mu \xi - ig[A_\mu, \xi]$$

$$\delta_\xi A_{\mu\lambda} = \partial_\mu \xi_\lambda - ig[A_\mu, \xi_\lambda] - ig[A_{\mu\lambda}, \xi]$$

$$\delta_\xi A_{\mu\lambda_1\lambda_2} = \partial_\mu \xi_{\lambda_1\lambda_2} - ig[A_\mu, \xi_{\lambda_1\lambda_2}] - ig[A_{\mu\lambda_1}, \xi_{\lambda_2}] - ig[A_{\mu\lambda_2}, \xi_{\lambda_1}] - ig[A_{\mu\lambda_1\lambda_2}, \xi]$$

form a closed algebraic structure:

$$[\delta_{\xi_1}, \delta_{\xi_2}] A_{\mu\lambda_1\lambda_2...\lambda_s} = -ig\delta_{\xi_3} A_{\mu\lambda_1\lambda_2...\lambda_s}$$

$$\xi_3 = [\xi_1, \xi_2]$$

- Extended Field Strength Tensors:

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

$$G_{\mu\nu,\lambda} = \partial_\mu A_{\nu\lambda} - \partial_\nu A_{\mu\lambda} - ig([A_\mu, A_{\nu\lambda}] + [A_{\mu\lambda}, A_\nu])$$

$$G_{\mu\nu,\lambda\rho} = \partial_\mu A_{\nu\lambda\rho} - \partial_\nu A_{\mu\lambda\rho} - ig([A_\mu, A_{\nu\lambda\rho}] + [A_{\mu\lambda}, A_{\nu\rho}] + [A_{\mu\rho}, A_{\nu\lambda}] + [A_{\mu\lambda\rho}, A_\nu])$$

- Gauge invariant Lagrangians:

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

$$-\frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\nu,\lambda} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu,\lambda\lambda} +$$

$$+\frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\lambda,\nu} + \frac{1}{4} G^a_{\mu\nu,\nu} G^a_{\mu\lambda,\lambda} + \frac{1}{2} G^a_{\mu\nu} G^a_{\mu\lambda,\nu\lambda} + \ldots$$

Tensor Gauge Theory with mixed symmetries

- New set of fields with different symmetry properties: $A_{\mu\hat{\sigma}_1\hat{\sigma}_2\ldots\hat{\sigma}_s}$
- and gauge transformations:

$$\delta A_\mu = \partial_\mu \zeta - ig[A_\mu, \zeta]$$

$$\delta A_{\mu\hat{\sigma}_1} = \partial_\mu \zeta_{\hat{\sigma}_1} - ig[A_\mu, \zeta_{\hat{\sigma}_1}] - ig[A_{\mu\hat{\sigma}_1}, \zeta]$$

$$\delta A_{\mu\hat{\sigma}_1\hat{\sigma}_2} = \partial_\mu \zeta_{\hat{\sigma}_1\hat{\sigma}_2} - ig[A_\mu, \zeta_{\hat{\sigma}_1\hat{\sigma}_2}] - ig[A_{\mu\hat{\sigma}_1}, \zeta_{\hat{\sigma}_2}] - ig[A_{\mu\hat{\sigma}_2}, \zeta_{\hat{\sigma}_1}] - ig[A_{\mu\hat{\sigma}_1\hat{\sigma}_2}, \zeta]$$

- which form a closed algebraic structure:

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] A_{\mu\nu\lambda} = \delta_{\zeta_3} A_{\mu\nu\lambda}$$

$$\zeta_3 = [\zeta_1, \zeta_2], \quad \zeta_{3\sigma_1\sigma_2} = [\zeta_1, \zeta_{2\sigma_1\sigma_2}] + [\zeta_{1\sigma_1\sigma_2}, \zeta_2]$$

- Field strength tensors:

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

$$G_{\mu\nu,\hat{\sigma}_1} = \partial_\mu A_{\nu\hat{\sigma}_1} - \partial_\nu A_{\mu\hat{\sigma}_1} - ig([A_\mu, A_{\nu\hat{\sigma}_1}] + [A_{\mu\hat{\sigma}_1}, A_\nu])$$

$$G_{\mu\nu,\hat{\sigma}_1\hat{\sigma}_2} = \partial_\mu A_{\nu\hat{\sigma}_1\hat{\sigma}_2} - \partial_\nu A_{\mu\hat{\sigma}_1\hat{\sigma}_2} - ig([A_\mu, A_{\nu\hat{\sigma}_1\hat{\sigma}_2}] + [A_{\mu\hat{\sigma}_1}, A_{\nu\hat{\sigma}_2}] + [A_{\mu\hat{\sigma}_2}, A_{\nu\hat{\sigma}_1}] + [A_{\mu\hat{\sigma}_1\hat{\sigma}_2}, A_\nu])$$

- Can be written as differential forms

$$G = dA + A^2$$

$$G_4 = dA_3 + \{A, A_3\}$$

$$G_6 = dA_5 + \{A, A_5\} + \{A_3, A_3\}$$

Georgiou G., Savvidy G., “Non-Abelian tensor gauge fields and new topological invariants”, arxiv: 1212.5228

Closed, metric independent, gauge invariant forms (Chern characters) in higher dimensions

$$P_{2n} = Tr(G^n)$$

$$\Phi_{2n+4} = Str(G_4, G^n)$$

$$\Xi_{2n+6} = Str(G_6, G^n) + nStr(G_4^2, G^{n-1})$$

$$Y_{2n+8} = Str(G_8, G^n) + 3nStr(G_4, G_6, G^{n-1}) + n(n-1)Str(G_4^3, G^{n-2})$$

From Poincare lemma, these closed forms can be locally written as exterior derivatives of secondary forms. Thus, they can be viewed as abelian anomalies (Chern characters).

$$P_{2n} = d\omega^0_{2n-1}, \quad \Phi_{2n+4} = d\psi^0_{2n+3},$$

$$\Xi_{2n+6} = d\varphi^0_{2n+5}, \quad Y_{2n+8} = d\rho^0_{2n+7}$$

The simplest representative of the cohomology class

- Two forms that differ by and exact form are cohomologous.

$$\tilde{\psi}^{0}_{2n+3} \sim \psi^{0}_{2n+3} + d\,a_{2n+2}$$

$$\tilde{\varphi}^{0}_{2n+5} \sim \varphi^{0}_{2n+5} + d\,\beta_{2n+4}$$

$$\tilde{\rho}^{0}_{2n+7} \sim \rho^{0}_{2n+7} + d\,\gamma_{2n+6}$$

- The challenge is to find the simplest representatives for the secondary forms

The simplest secondary forms

- $\psi^0_{2n+3} = Str(A_3, G^n)$

- $\varphi^0_{2n+5} = Str(A_5, G^n) + nStr(A_3, G_4, G^{n-1})$

- $$\rho^0_{2n+7} = Str(A_7, G^n) + n(n-1)Str(G_4^2, A_3, G^{n-2}) + \\ + nStr(G_6, A_3, G^{n-1}) + 2nStr(G_4, A_5, G^{n-1})$$

δ(secondary forms) = *d*(potential anomalies)

$$* \quad \delta_{\xi}\psi^{0}_{2n+3} = 0,$$
$$\delta_{\zeta_2}\psi^{0}_{2n+3} = d\psi^{1}_{2n+2}(\zeta_2, A)$$

$$* \quad \delta_{\xi}\varphi^{0}_{2n+5} = 0,$$
$$\delta_{\zeta_2}\varphi^{0}_{2n+5} = d\varphi^{1}_{2n+4}(\zeta_2, A, A_3),$$
$$\delta_{\zeta_4}\varphi^{0}_{2n+5} = d\varphi^{1}_{2n+4}(\zeta_4, A)$$

$$* \quad \delta_{\xi}\rho^{0}_{2n+7} = 0,$$
$$\delta_{\zeta_2}\rho^{0}_{2n+7} = d\rho^{1}_{2n+6}(\zeta_2, A, A_3, A_5),$$
$$\delta_{\zeta_4}\rho^{0}_{2n+7} = d\rho^{1}_{2n+6}(\zeta_4, A, A_3),$$
$$\delta_{\zeta_6}\rho^{0}_{2n+7} = d\rho^{1}_{2n+6}(\zeta_6, A)$$

- There are no anomalies with respect to the standard gauge parameter!!

Anomalies

- $\psi^{1}_{2n+2}(\zeta_2, A) = Str(\zeta_2, G^{n})$

- $\varphi^{1}_{2n+4}(\zeta_4, A) = Str(\zeta_4, G^{n})$

  $\varphi^{1}_{2n+4}(\zeta_2, A, A_3) = nStr(\zeta_2, G_4, G^{n-1})$

- $\rho^{1}_{2n+6}(\zeta_6, A) = Str(\zeta_6, G^{n})$

  $\rho^{1}_{2n+6}(\zeta_4, A, A_3) = 2nStr(\zeta_4, G_4 G^{n-1})$

  $\rho^{1}_{2n+6}(\zeta_2, A, A_3, A_5) = nStr(\zeta_2, G_6, G^{n-1}) + n(n-1)Str(\zeta_2, G_4^{2}, G^{n-2})$

S. Konitopoulos, G. Savvidy “Extension of Chern-Simons forms”,
J. Math. Phys. 55, 062304 (2014)