# String Theory and Double Field Theory

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- W. Siegel, hep-th/9305073, O. H., C. Hull, B. Zwiebach  arXiv: 1003.5027, 1006.4823
- O. H., W. Siegel, B. Zwiebach  arXiv: 1306.2970
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# Plan of the talks:

## Part I: Duality covariant Geometry of DFT

- Efficient reformulation of supergravity ('generalized geometry')
- Gauge structure of DFT:
  generalized diffeomorphisms, duality-covariantized Courant bracket
- extension: heterotic, type II, Romans mass deformation,
  generalized Scherk-Schwarz, non-geometric fluxes, etc.

## Part II: Higher-derivative $\alpha'$ deformations

- exact deformation of gauge structure
- physical interpretation on physical subspace
  $\Rightarrow$ Green-Schwarz mechanism and $\alpha'$-deformed Courant bracket
- further deformations from bosonic closed SFT
- Conclusions and Outlook

# Part I: Duality covariant Geometry of DFT

String theory: consistent quantum gravity in $D = 10$ (or $D = 26$)

$$\text{massless fields:} \qquad g_{ij}\ , \quad b_{ij} = -b_{ji}\ , \quad \phi$$

Spacetime action for massless string fields:

$$S = \int d^D x \sqrt{-g} e^{-2\phi} \left[ R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} \right]$$

where

$$H_{ijk} \;=\; \partial_i b_{jk} + \partial_j b_{ki} + \partial_k b_{ij}$$

Two gauge symmetries: 1) general coordinate invariance,

$$2) \quad \delta_{\tilde{\xi}} b_{ij} \;=\; \partial_i \tilde{\xi}_j - \partial_j \tilde{\xi}_i$$

action not uniquely determined by bosonic symmetries (only SUSY) infinite number of higher-derivative $\alpha'$ corrections

$\Rightarrow$ spacetime action for massless string fields:

$$S = \int d^D x \sqrt{-g} e^{-2\phi} \Big[ R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk}$$

$$+ \alpha' \Big( \frac{1}{4} R^{ijkl} R_{ijkl} + R\, H\, H + H^4 + \cdots \Big) + \mathcal{O}(\alpha'^2) \Big]$$

largely ambiguous, not determined by symmetries

in string theory couplings uniquely determined, compatible with T-duality

T-duality group $O(D,D)$ : $\eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ , $M,N = 1,\ldots,2D$

suggests that fundamental field in string theory

$$\mathcal{E}_{ij} = g_{ij} + b_{ij} \qquad \text{or} \qquad \mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix}$$

$\Rightarrow$ Double Field Theory based on doubled coordinates $X^M = (\tilde{x}_i, x^i)$,

# Two-derivative Double Field Theory

Reformulation (Extension?) of spacetime action for massless string fields:

$$S_{\text{NS}} = \int d^D x \sqrt{-g} e^{-2\phi} \left[ R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} + \frac{1}{4}\alpha' R^{ijkl} R_{ijkl} + \cdots \right]$$

generalized metric and doubled coordinates $X^M = (\tilde{x}_i, x^i)$,

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix} \in O(D,D)$$

DFT Action (dilaton density $e^{-2d} = e^{-2\phi}\sqrt{-g}$):

$$S_{\text{DFT}} = \int d^{2D} X \, e^{-2d} \mathcal{R}(\mathcal{H}, d) \quad \xrightarrow{\tilde{\partial}^i = 0} \quad S_{\text{NS}}\big|_{\alpha'=0}$$

generalized curvature scalar

$$\begin{aligned} \mathcal{R} \; \equiv \;& 4\,\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\,\mathcal{H}^{MN}\partial_M d\,\partial_N d + 4\partial_M\mathcal{H}^{MN}\,\partial_N d \\ & + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_K\mathcal{H}_{NL} \end{aligned}$$

## Gauge transformations and generalized Lie derivatives

In DFT gauge invariance governed by generalized Lie derivatives

$$\widehat{\mathcal{L}}_\xi \mathcal{H}_{MN} = \xi^P \partial_P \mathcal{H}_{MN} + \left(\partial_M \xi^P - \partial^P \xi_M\right) \mathcal{H}_{PN} + \left(\partial_N \xi^P - \partial^P \xi_N\right) \mathcal{H}_{MP}$$

$$\widehat{\mathcal{L}}_\xi \left(e^{-2d}\right) = \partial_M \left(\xi^M e^{-2d}\right)$$

Invariance and closure, $[\widehat{\mathcal{L}}_{\xi_1}, \widehat{\mathcal{L}}_{\xi_2}] = \widehat{\mathcal{L}}_{[\xi_1,\xi_2]_C}$,

$$\left[\, \xi_1 \,,\, \xi_2 \,\right]_C^M \;=\; \xi_1^N \partial_N \xi_2^M - \xi_2^N \partial_N \xi_1^M - \frac{1}{2}\xi_{1N}\partial^M \xi_2^N + \frac{1}{2}\xi_{2N}\partial^M \xi_1^N$$

modulo strong constraint

$$\eta^{MN} \partial_M \partial_N = 2\tilde{\partial}^i \partial_i = 0 \qquad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

solved by

$$\partial_M = \begin{cases} \partial_i & \text{if } M = i \\ 0 & \text{else} \end{cases}$$

$O(D,D)$ covariant, captures IIA/M-theory & IIB simultaneously

## Conventional gauge transformations and Courant bracket

Setting $\tilde{\partial}^i = 0$ gauge transformations imply for $\xi^M = (\tilde{\xi}_i, \xi^i)$

$$\delta g \;=\; \mathcal{L}_\xi g \;, \qquad \delta b \;=\; d\tilde{\xi} + \mathcal{L}_\xi b$$

Viewing $\xi + \tilde{\xi}$ as section in $T \oplus T^*$ ('generalized geometry')

C-bracket reduces to Courant bracket

$$\Big[\,\xi_1 + \tilde{\xi}_1, \xi_2 + \tilde{\xi}_2\,\Big] \;=\; \big[\,\xi_1\,, \xi_2\,\big] \,+\, \mathcal{L}_{\xi_1}\tilde{\xi}_2 - \mathcal{L}_{\xi_2}\tilde{\xi}_1 - \tfrac{1}{2}\mathrm{d}\big(i_{\xi_1}\tilde{\xi}_2 - i_{\xi_2}\tilde{\xi}_1\big)$$

exact term not fixed by closure but by gauge covariance of C-bracket

or 'B automorphism' of Courant bracket

# Large Gauge Transformations and Non-Geometric Spaces

Generalized g.c.t. that reproduce this infinitesimally:

$$S'(X') = S(X) \qquad A'_M(X') = \mathcal{F}_M{}^N A_N(X)$$

and analogously on higher tensors, where [O.H., Zwiebach, 1207.4198]

$$\mathcal{F}_M{}^N \equiv \frac{1}{2}\left(\frac{\partial X^P}{\partial X'^M}\frac{\partial X'_P}{\partial X_N} + \frac{\partial X'_M}{\partial X_P}\frac{\partial X^N}{\partial X'^P}\right) \in O(D,D)$$

Setting $X'^M = X^M - \xi^M(X)$ we get $\delta_\xi = \widehat{\mathcal{L}}_\xi$ .

- $x^{i\prime} = x^{i\prime}(x)$ , $\tilde{x}'_i = \tilde{x}_i$ leads to usual g.c.t.,

  $\tilde{x}'_i = \tilde{x}_i - \tilde{\xi}_i(x)$ , $x^{i\prime} = x^i$ leads to $b_{ij} \to b_{ij} + \partial_i\tilde{\xi}_j - \partial_j\tilde{\xi}_i$
- composition according to BCH of C-bracket,

  equivalent to $\exp(\widehat{\mathcal{L}}_\xi)$ [Berman, Cederwall, Perry (2014)]
- truly non-geometric spaces [O.H., D. Lüst & B. Zwiebach (2013)]

# Supersymmetric and Heterotic Extensions

(Generalized) vielbein formalism required [Siegel (1993), O.H. & Ki Kwak (2010)]

$$\mathcal{H}^{MN} = \widehat{\eta}^{AB} E_A{}^M E_B{}^N , \qquad \widehat{\eta}_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & \eta_{\bar{a}\bar{b}} \end{pmatrix}$$

local $SO(1,9)_L \times SO(1,9)_R$ Lorentz symmetry

Gauge fixing to diagonal subgroup

$$E_A{}^M = \begin{pmatrix} E_{ai} & E_a{}^i \\ E_{\bar{a}i} & E_{\bar{a}}{}^i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e_{ia} + b_{ij} e_a{}^j & e_a{}^i \\ -e_{i\bar{a}} + b_{ij} e_{\bar{a}}{}^j & e_{\bar{a}}{}^i \end{pmatrix}$$

Fermions: singlets under $O(10,10)$ and $\widehat{\mathcal{L}}_\xi$

[Coimbra, Strickland-Constable, Waldram, 1107.1733; O.H., S. Ki Kwak, 1111.7293]

$\Psi_a$ : vector of $SO(1,9)_L$ , spinor of $SO(1,9)_R$

$\rho$ : spinor of $SO(1,9)_R$ ,

$\epsilon$ : spinor of $SO(1,9)_R$

$\mathcal{N} = 1$ supersymmetric Lagrangian

$$\mathcal{L} = e^{-2d}\Big(\mathcal{R}(E,d) - \bar{\Psi}^a \gamma^{\bar{b}} \nabla_{\bar{b}} \Psi_a + \bar{\rho}\gamma^{\bar{a}} \nabla_{\bar{a}}\rho + 2\bar{\Psi}^a \nabla_a \rho\Big)$$

$\mathcal{N} = 1$ supersymmetry transformations

$$E_{\bar{b}}{}^{M} \delta_\epsilon E_{aM} = \frac{1}{2}\bar{\epsilon}\gamma_{\bar{b}}\,\Psi_a \qquad \delta_\epsilon d = -\frac{1}{4}\bar{\epsilon}\rho \qquad \delta_\epsilon \Psi_a = \nabla_a \epsilon \qquad \delta_\epsilon \rho = \gamma^{\bar{a}} \nabla_{\bar{a}} \epsilon$$

Proof of supersymmetric invariance: variation of bosonic term

$$e^{2d}\,\delta_\epsilon \mathcal{L}_{\mathsf{B}} \;=\; \frac{1}{2}\bar{\epsilon}\rho\mathcal{R} + \bar{\epsilon}\gamma^{\bar{b}}\Psi^a \mathcal{R}_{a\bar{b}}$$

variation of fermionic terms

$$\begin{aligned} e^{2d}\,\delta_\epsilon \mathcal{L}_{\mathsf{F}} &= -2\bar{\Psi}^a \gamma^{\bar{b}} \nabla_{\bar{b}} \nabla_a \epsilon + 2\bar{\rho}\gamma^{\bar{a}}\nabla_{\bar{a}}\big(\gamma^{\bar{b}}\nabla_{\bar{b}}\epsilon\big) + 2\nabla^a \bar{\epsilon}\,\nabla_a \rho + 2\bar{\Psi}^a \nabla_a \big(\gamma^{\bar{b}}\nabla_{\bar{b}}\epsilon\big) \\ &= -2\bar{\Psi}^a \Big[\gamma^{\bar{b}}\nabla_{\bar{b}}, \nabla_a\Big]\epsilon + 2\bar{\rho}\left(\gamma^{\bar{a}}\nabla_{\bar{a}}\gamma^{\bar{b}}\nabla_{\bar{b}} - \nabla^a\nabla_a\right)\epsilon \\ &= \bar{\Psi}^a \gamma^{\bar{b}} \mathcal{R}_{a\bar{b}}\epsilon - \frac{1}{2}\bar{\rho}\mathcal{R}\epsilon = -\frac{1}{2}\bar{\epsilon}\rho\mathcal{R} - \bar{\epsilon}\gamma^{\bar{b}}\Psi^a\mathcal{R}_{a\bar{b}} \end{aligned}$$

Thus: $\quad \delta_\epsilon\big(S_{\mathsf{B}} + S_{\mathsf{F}}\big) = 0$

Add vector multiplets: $SO(1,9+n)\times SO(1,9)\subset O(10+n,10)$

($n=16$: heterotic string truncated to Cartan of $E_8\times E_8$ or $SO(32)$)

Frame field: $A=(a\,,\;\bar{a})=(\underline{a}\,,\;\underline{\alpha}\,,\;\bar{a})\;,\;\underline{a}=0,\ldots,9\;,\;\underline{\alpha}=1,\ldots,n$

$$E_A{}^M=\frac{1}{\sqrt{2}}\begin{pmatrix} e_{i\underline{a}}-e_{\underline{a}}{}^k c_{ki} & -e_{\underline{a}}{}^k A_k{}^\beta & e_{\underline{a}}{}^i \\ \sqrt{2}A_{i\underline{\alpha}} & \sqrt{2}\delta_{\underline{\alpha}}{}^\beta & 0 \\ -e_{i\bar{a}}-e_{\bar{a}}{}^k c_{ki} & -e_{\bar{a}}{}^k A_k{}^\beta & e_{\bar{a}}{}^i \end{pmatrix}$$

where $c_{ij}=b_{ij}+\frac{1}{2}A_i{}^\alpha A_{j\alpha}$

Additional gauginos $\chi_\alpha$ encoded in

$$\Psi_a\;=\;\big(\Psi_{\underline{a}}\,,\;\Psi_{\underline{\alpha}}\big)\;\equiv\;\big(e_{\underline{a}}{}^i\Psi_i\,,\;\tfrac{1}{\sqrt{2}}\chi_{\underline{\alpha}}\big)$$

Formally same Lagrangian and supersymmetry variations as above!

$\rightarrow$ reduces to standard action and SUSY rules setting $\tilde{\partial}^i=0$

# Comparison: standard $\mathcal{N} = 1$ supergravity action

$$
\begin{aligned}
S \;=\; \int d^{10}x\, e\, e^{-2\phi}\Big[ & \Big(R + 4\partial^i\phi\,\partial_i\phi - \frac{1}{12}\widehat{H}^{ijk}\widehat{H}_{ijk} - \frac{1}{4}F_{ij}F^{ij}\Big) \\
& - \bar{\psi}_i\gamma^{ijk}D_j\psi_k - 2\bar{\lambda}\gamma^i D_i\lambda - \frac{1}{2}\bar{\chi}^\alpha \not{D}\chi_\alpha \\
& + 2\bar{\psi}^i(\partial_i\phi)\gamma^j\psi_j - \bar{\psi}_i(\not{\partial}\phi)\gamma^i\lambda - \frac{1}{4}\bar{\chi}_\alpha\gamma^i\gamma^{jk}F_{jk}{}^\alpha\big(\psi_i + \frac{1}{6}\gamma_i\lambda\big) \\
& + \frac{1}{24}\widehat{H}_{ijk}\Big(\bar{\psi}_m\gamma^{mijkn}\psi_n + 6\bar{\psi}^i\gamma^j\psi^k - 2\bar{\psi}_m\gamma^{ijk}\gamma^m\lambda + \frac{1}{2}\bar{\chi}^\alpha\gamma^{ijk}\chi_\alpha\Big) \\
& + \text{quartic fermions}\Big]
\end{aligned}
$$

where

$$
\widehat{H}_{ijk} \;=\; 3\left(\partial_{[i}b_{jk]} - A_{[i}{}^\alpha\partial_j A_{k]\alpha}\right)
$$

## Comparison: standard $\mathcal{N} = 1$ supersymmetry rules

$$
\begin{aligned}
\delta_\epsilon e_i{}^a &= \frac{1}{2}\bar{\epsilon}\,\gamma^a\psi_i - \frac{1}{4}\bar{\epsilon}\,\lambda\, e_i{}^a\ , \\
\delta_\epsilon \phi &= -\,\bar{\epsilon}\lambda \quad , \quad \delta_\epsilon A_i{}^\alpha \ = \ \frac{1}{2}\bar{\epsilon}\,\gamma_i \chi^\alpha \quad , \\
\delta_\epsilon \chi^\alpha &= -\,\frac{1}{4}\gamma^{ij}F_{ij}{}^\alpha \epsilon \\
\delta_\epsilon \psi_i &= D_i\epsilon - \frac{1}{8}\gamma_i(\partial\!\!\!/\phi)\epsilon + \frac{1}{96}(\gamma_i{}^{klm} - 9\delta_i{}^k\gamma^{lm})\widehat{H}_{klm}\epsilon\ , \\
\delta_\epsilon \lambda &= -\,\frac{1}{4}(\partial\!\!\!/\phi)\epsilon + \frac{1}{48}\gamma^{ijk}\widehat{H}_{ijk}\epsilon\ , \\
\delta_\epsilon b_{ij} &= \frac{1}{2}(\bar{\epsilon}\,\gamma_i\psi_j - \bar{\epsilon}\,\gamma_j\psi_i) - \frac{1}{2}\bar{\epsilon}\,\gamma_{ij}\lambda + \frac{1}{2}\bar{\epsilon}\gamma_{[i}\chi^\alpha A_{j]\alpha}\ .
\end{aligned}
$$

# Type II Double Field Theory

NS-NS: dilaton $d$, lift of $\mathcal{H} \in O(10,10)$ to $\mathbb{S} \in Spin(10,10)$

RR: Majorana-Weyl spinor $\chi$ of $O(10,10)$

Action:

$$S = \int dx d\tilde{x} \left( e^{-2d} \mathcal{R} + \frac{1}{4} (\partial\!\!\!/ \chi)^\dagger \, \mathbb{S} \, \partial\!\!\!/ \chi \right)$$

Dirac operator in terms of raising and lowering operators $\psi_i$, $\psi^i$ of $O(10,10)$

$$\partial\!\!\!/ \equiv \psi^i \partial_i + \psi_i \tilde{\partial}^i \quad \Rightarrow \quad \partial\!\!\!/^2 = \frac{1}{2} \eta^{MN} \partial_M \partial_N = 0$$

(Self-)duality constraint ( $C$: charge conjugation matrix )

$$\partial\!\!\!/ \chi = -\mathcal{K} \, \partial\!\!\!/ \chi \qquad \mathcal{K} \equiv C^{-1} \mathbb{S}$$

Reduces to democratic type IIA (or IIB) supergravity for $\tilde{\partial}^i = 0$,
where conventional RR p-forms $C^{(p)}$ encoded as

$$\chi = \sum_p \frac{1}{p!} C_{i_1 \ldots i_p} \, \psi^{i_1} \ldots \psi^{i_p} |0\rangle$$

# Unification of IIA/IIB and relation to generalized geometry

Type II DFT encodes both IIA and IIB for different solutions of constraint

$$\tilde{\partial}^i = 0\ , \quad \partial_i \neq 0\ : \quad \xi_M \;=\; \left(\xi^i, \tilde{\xi}_i\right) \;\cong\; \xi + \tilde{\xi} \;\in\; T(M) \oplus T^*(M)$$

$$S_{\mathsf{DFT}_{II}}\Big|_{\tilde{\partial}=0} \;=\; S_{\text{type IIA}}$$

For different solution T-dual theory:

$$\tilde{\partial}^i \neq 0\ , \quad \partial_i = 0\ : \quad \xi_M \;\cong\; \xi + \tilde{\xi} \;\in\; T^*(M) \oplus T(M)$$

$$S_{\mathsf{DFT}_{II}}\Big|_{\partial=0} \;=\; S_{\text{type IIA}^*}$$

timelike T-duality: type IIA⋆ and IIB⋆ [Hull, hep-th/9806146]

intermediate frames: $S_{\mathsf{DFT}_{II}}\Big| = S_{\text{type IIB}}$

More intriguing in ExFT; different bundles for different solutions

$$\mathsf{E}_{6(6)} \supset \mathsf{SL}(6) \times \mathsf{SL}(2)\ : \qquad T(M) \oplus \Lambda^2 T^*(M) \oplus \cdots$$

Advantage of DFT/EFT: universal (covariant) formulation for all theories

# Massive Type IIA: Romans theory

Massive type IIA obtained for

$$C^{(1)}(x,\tilde{x}) \; = \; C_i(x)dx^i + m\tilde{x}_1 dx^1$$

Ansatz consistent because gauge transformations can be re-written

$$\delta_\xi \chi \; = \; \not{\xi}\,\not{\partial}\chi$$

so that linear $\tilde{x}$ dependence drops out.
General field strengths

$$F = \not{\partial}\chi = (\psi^i\partial_i + \psi_i\tilde{\partial}^i)\chi = F_{m=0} + \psi_i\tilde{\partial}^i(m\tilde{x}_1)\psi^1|0\rangle$$

lead to non-trivial 0-form field strength

$$F^{(0)} \; = \; m$$

$\Rightarrow$ '$(-1)$-form' $\equiv$ 1-form depending on $\tilde{x}$ [Lavrinenko, Lu, Pope, Stelle (1999)]

$\Rightarrow$ Type II DFT reduces to (democratc formulation of) massive Type IIA

# Generalized Scherk-Schwarz compactification

Scherk-Schwarz Reduction of DFT in generalized metric form.

[ Aldazabal, Baron, Marques & Nunez; Geissbuhler (2011) ]

$$\mathcal{H}_{MN}(x, \mathbb{Y}) = U^A{}_M(\mathbb{Y})\mathcal{H}_{AB}(x)U^B{}_N(\mathbb{Y}) \,, \qquad U \in O(D, D)$$

Flux components in lower-dimensional (4D) theory directly given by

$$F_{ABC} = 3\eta_{D[A}(U^{-1})^M{}_B(U^{-1})^N{}_{C]}\partial_M U^D{}_N$$

[see also: Andriot, O.H., Larfors, Lüst, Patalong & Blumenhagen, Deser, Plauschinn, Rennecke]

yields gauged supergravities with ‘non-geometric fluxes’

however, not all gaugings obtained because of strong constraint

$\Rightarrow$ relaxation of strong constraint? [ Geissbuhler, Marques, Nunez & Penas (2013) ]

Intriguing first steps, but complete picture still elusive

# Summary

## Most conservatively:

- Strong constraint solved by

$$\partial_M = \begin{cases} \partial_i & \text{if } M = i \\ 0 & \text{else} \end{cases} \quad .$$

  but technically, $\partial_i$, $g$, $b$ and $\phi$ never used!

- (very economic!) *reformulation* of low-energy action for string theory
  $\Rightarrow$ geometry can be thought of as 'generalized geometry' [Hitchin, Gualtieri]
  (to the extent it had been developed)

## Concrete reasons for more:

- Full closed string field theory *is* a truly doubled field theory

- mild relaxations of strong constraint possible
  $\rightarrow$ massive IIA & gauged supergravity

- potentially: geometry of $\alpha'$ corrections!

# String Theory and Double Field Theory II:
# $\alpha'$ Corrections

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# Motivation:

- $\alpha'$ corrections encode truly stringy effects beyond supergravity
- usually written with higher powers of $R_{mnkl}$ and $H = \mathrm{d}b$,
  e.g. determined by string S-matrix calculations
- *very* messy. [Metsaev &Tseytlin (1987), Gross & Sloan (1987), Hull & Townsend (1987)]
  Is there some principle? T-dualty/U-duality invariance?
  [A. Sen (1991), K. Meissner (1996)]
- Use double field theory to make T-duality manifest
  $\Rightarrow$ novel (duality-covariant) gauge principle

# T-duality of $\alpha'$ corrections

Obstacle to writing $(\text{Riem})^2$ in $O(d,d)$ covariant form:

Is there $O(d,d)$ scalar $\mathcal{I}(\mathcal{H},d)$ s.t. $\mathcal{I}(\mathcal{H},d)\Big|_{\tilde{\partial}=0,b_{ij}=0} = R_{ijkl}R^{ijkl}$ ?

No! $\rightarrow$ problematic tensor structure in $(\text{Riem})^2$ [O. H., B. Zwiebach (2012)]

$$S = \int dx\,\sqrt{g}\left(R+\frac{1}{4}\alpha' R_{ijkl}R^{ijkl}\right) = \int dx\,\sqrt{g}\,R + \frac{1}{4}\alpha'\int dx\,\partial^k h^{lp}\,\partial^i h_{pq}\,\partial_i\partial_k h^q{}_l + \cdots$$

$$= \int dx\,\sqrt{g}\,R - \frac{1}{8}\,\alpha'\int dx\,\Box h_{pq}\,\partial^k h^{lp}\,\partial_k h_l{}^q + \cdots$$

Using $R_{ij}\sim\Box h_{ij}+\cdots$ problematic structure can be redefined away:

$$h'_{ij} \;=\; h_{ij} - \frac{1}{4}\,\alpha'\,\partial_k h_i{}^p\,\partial^k h_{jp} + \cdots$$

<u>Note:</u> cannot be lifted to covariant redefinition $\delta g_{ij}\sim R_{ij}+\cdots$

but compatible with Meissner (1997)! $\rightarrow$ $\alpha'$ deformed diffeomorphisms!

# Doubled $\alpha'$ Geometry

Geometrical structures for generalized vector $\Xi \equiv (\xi^M)$ in $\alpha' = 0$ DFT:

$$\langle \Xi_1 | \Xi_2 \rangle \;=\; \xi_1^M \xi_2^N \eta_{MN}\,, \qquad [\Xi_1, \Xi_2]_C^M \;=\; \xi_{[1}^N \partial_N \xi_{2]}^M - \frac{1}{2}\xi_1^K \overleftrightarrow{\partial}^M \xi_{2K}$$

$$\widehat{\mathcal{L}}_\Xi V^M \;=\; \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P$$

All receive non-trivial higher-derivative $\alpha'$ corrections:

$$\langle \Xi_1 | \Xi_2 \rangle \;=\; \xi_1^M \xi_2^N \eta_{MN} - (\partial_N \xi_1^M)(\partial_M \xi_2^N)$$

$$[\Xi_1, \Xi_2]_C^M \;=\; \xi_{[1}^N \partial_N \xi_{2]}^M - \frac{1}{2}\xi_1^K \overleftrightarrow{\partial}^M \xi_{2K} + \frac{1}{2}(\partial_K \xi_1^L) \overleftrightarrow{\partial}^M (\partial_L \xi_2^K)$$

$$\mathbf{L}_\Xi V^M \;=\; \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P - (\partial^M \partial_K \xi^L)\partial_L V^K$$

Closure and gauge invariance exact! ( $\mathbf{L}_\Xi \langle V, W \rangle = \xi^N \partial_N \langle V, W \rangle$, etc. )

Not removable by $O(D,D)$ covariant redefinitions

Non-vanishing for $\tilde{x} = 0 \;\Rightarrow\;$ deformation of Courant bracket, etc.

$\alpha' = 0$ DFT relations for $\mathcal{H} \in O(D,D)$

$$(\mathcal{H}^2)_{MN} \;\equiv\; \mathcal{H}_{MK}\mathcal{H}^K{}_N \;=\; \eta_{MN} \qquad\qquad \mathrm{Tr}\,\mathcal{H} \;\equiv\; \eta^{MN}\mathcal{H}_{MN} \;=\; 0$$

get $\alpha'$ deformed $\;\Rightarrow\;$ dynamical equations!

$$(\mathcal{M}\star\mathcal{M})_{MN} \;\equiv\; 2(\mathcal{M}^2)_{MN} - \frac{1}{2}\partial_M\mathcal{M}^{PQ}\partial_N\mathcal{M}_{PQ} + \cdots \;=\; 2\eta_{MN}$$

$$\mathrm{tr}\,\mathcal{M} \;\equiv\; \eta^{MN}\mathcal{M}_{MN} - 3\partial_M\partial_N\mathcal{M}^{MN} + \cdots \;=\; 0$$

In derivative expansion:

$$\mathcal{O}(\alpha'^0)\;:\quad \mathcal{M}_{MN} \;=\; \mathcal{H}_{MN}\,, \qquad \mathcal{H}^2 = \eta$$

$$\mathcal{O}(\alpha'^1)\;:\quad \mathcal{M}_{MN} \;=\; \mathcal{H}_{MN} + \frac{1}{2}\{\mathcal{H},\mathcal{V}^{(2)}\}_{MN}$$

Then

$0 = \mathrm{tr}\mathcal{M} = 3\mathcal{R}(\mathcal{H},\phi)$ [dilaton eq.] $\qquad \mathcal{V}^{(2)}\mathcal{H} - \mathcal{H}\mathcal{V}^{(2)} = 0$ [gravity eq.]

plus infinite tower of higher-derivative $\alpha'$ corrections!

# CFT Derivation and Action

doubled world-sheet scalars $X^M(z)$ , $M = 1, \ldots, 2D$,
chirality condition: $\quad P^M = X'^M \equiv Z^M \quad [\ ' = \frac{\partial}{\partial z}\ ]$

postulate the (two) Virasoro generators

$$\mathcal{S} \equiv \frac{1}{2}(Z^2 - \phi'') \qquad \mathcal{T} \equiv \tfrac{1}{2}\mathcal{M}^{MN} Z_M Z_N - \tfrac{1}{2}(\widehat{\mathcal{M}}^M Z_M)'$$

OPE defines (various) 'quantum products'. OPE yields Virasoro$^2$

$$\mathcal{S}(z_1)\mathcal{S}(z_2) = \frac{D}{z_{12}^4} + \frac{2\mathcal{S}(z_2)}{z_{12}^2} + \frac{\mathcal{S}'(z_2)}{z_{12}} + \text{finite}\,, \quad \text{same for } \mathcal{T}$$

$$\mathcal{S}(z_1)\mathcal{T}(z_2) = \frac{2\mathcal{T}(z_2)}{z_{12}^2} + \frac{\mathcal{T}'(z_2)}{z_{12}} + \text{finite}$$

*provided* dilaton and gravity equations hold!

Gauge invariant action

$$S = \int e^{\phi}\big(\langle\mathcal{T}|\mathcal{S}\rangle - \tfrac{1}{6}\langle\mathcal{T}|\mathcal{T}\star\mathcal{T}\rangle\big) = \int e^{\phi}\,\mathrm{Tr}\big(\mathcal{M} - \frac{1}{3}\mathcal{M}^3 + \cdots\big)$$

# Interpretation on physical subspace?

(Perturbative) analysis shows that $b$-field transforms as

$$\delta_{\xi+\tilde{\xi}}\, b \;=\; \mathsf{d}\tilde{\xi} \;+\; \mathcal{L}_{\xi} b \;+\; \tfrac{1}{2}\,\mathsf{tr}\big(\mathsf{d}(\partial\xi) \wedge \Gamma\big)$$

with (Christoffel) connection 1-form $(\Gamma)^k{}_l \equiv \Gamma^{\,k}_{\,il}\, dx^i$

deformed gauge invariant 3-form curvature

$$\widehat{H}(b,\Gamma) \;=\; \mathsf{d}b + \tfrac{1}{2}\,\Omega(\Gamma)\,, \quad \Omega(\Gamma) \;=\; \mathsf{tr}\big(\Gamma \wedge \mathsf{d}\Gamma + \tfrac{2}{3}\,\Gamma \wedge \Gamma \wedge \Gamma\big)$$

$\Rightarrow$ Green-Schwarz anomaly cancellation mechanism of heterotic string but with deformed diffeomorphisms rather than deformed Lorentz

# Deformation of Courant bracket

Deformed gauge transformations close according to bracket

$$\Big[\,\xi_1+\tilde{\xi}_1,\xi_2+\tilde{\xi}_2\,\Big]' \;=\; \big[\,\xi_1\,,\xi_2\,\big] \;+\; \mathcal{L}_{\xi_1}\tilde{\xi}_2-\mathcal{L}_{\xi_2}\tilde{\xi}_1-\tfrac{1}{2}\mathsf{d}\big(i_{\xi_1}\tilde{\xi}_2-i_{\xi_2}\tilde{\xi}_1\big)$$
$$-\,\tfrac{1}{2}\big(\tilde{\varphi}(\xi_1,\xi_2)-\tilde{\varphi}(\xi_2,\xi_1)\big)$$

with the map $\tilde{\varphi}$ that produces a ‘one-form’ from 2 vectors

$$\tilde{\varphi}(V,W)\;\equiv\;\mathsf{tr}\big(\mathsf{d}(\partial V)\partial W\big)\;\equiv\;\partial_i\partial_k V^l\partial_l W^k dx^i$$

*not* genuine 1-form $\;\Rightarrow\;$ anomalous transformation under diffeomorphisms

Bracket covariant under *deformed* diffeomorphisms

$$\delta_{\xi+\tilde{\xi}}\tilde{V}\;\equiv\;\mathcal{L}_\xi\tilde{V}-i_V\mathsf{d}\tilde{\xi}\;-\;\tilde{\varphi}(\,\xi,V)$$

# $\alpha'$ Corrections for Bosonic Strings and Closed SFT

$\alpha'$ corrections for bosonic string (Riemann-sq.) ? ($\mathbb{Z}_2$ invariant $b \to -b$)

Closed bosonic SFT $\Rightarrow$ deformed gauge algebra for *cubic* theory

$$\big[\,\xi_1,\,\xi_2\,\big]_+^M \;=\; \big[\,\xi_1,\,\xi_2\,\big]_C^M \;+\; \tfrac{1}{2}\,\bar{\mathcal{H}}^{KL}\,K_{[1\,K}{}^{P}\,\partial^M K_{2]\,LP}$$

with $K_{MN} = 2\partial_{[M}\xi_{N]}$ and background generalized metric $\bar{\mathcal{H}}_{MN}$

$\Rightarrow$ $\alpha'$-deformed diffeomorpisms as implied by (perturbative) redefinition

$$h'_{ij} \;=\; h_{ij} - \tfrac{1}{4}\,\alpha'\,\partial_k h_i{}^p\,\partial^k h_{jp} + \cdots\;,$$

agrees with earlier results on duality-invariant Riemann-sq.

[Meissner (1996), Hohm & Zwiebach (2011)]

More general $\mathbb{Z}_2$ even/odd deformations (with parameters $\gamma^\pm$)

$$\big[\,\xi_1,\,\xi_2\,\big]_{\alpha'}^M \;=\; \big[\,\xi_1,\,\xi_2\,\big]_C^M \;+\; \tfrac{1}{2}\big(\gamma^+\bar{\mathcal{H}}^{KL} - \gamma^-\eta^{KL}\big)\,K_{[1\,K}{}^{P}\,\partial^M K_{2]\,LP}$$

# Cubic Action

$$
\begin{aligned}
S \;=\;& S^{(2,2)} + S^{(3,2)} \\
&+ \tfrac{1}{4}\,\mathcal{R}^{\underline{M}\,\underline{N}\bar{K}\bar{L}}\,\mathcal{R}_{\underline{M}\,\underline{N}\bar{K}\bar{L}} + \tfrac{1}{4}\,\phi\,\mathcal{R}^{\underline{M}\,\underline{N}\bar{K}\bar{L}}\,\mathcal{R}_{\underline{M}\,\underline{N}\bar{K}\bar{L}} \\
&- \tfrac{1}{8}\Big(\Gamma^{\underline{P}\bar{M}\bar{N}}\,\Gamma_{\bar{M}}{}^{\underline{K}\,\underline{L}}\,\partial_{\underline{P}}\Gamma_{\bar{N}\,\underline{K}\,\underline{L}} \;-\; \Gamma^{\bar{P}\,\underline{M}\,\underline{N}}\,\Gamma_{\underline{M}}{}^{\bar{K}\bar{L}}\,\partial_{\bar{P}}\Gamma_{\underline{N}\bar{K}\bar{L}} \\
&\qquad - \Gamma^{\bar{M}}{}_{\underline{K}\,\underline{L}}\,\Gamma^{\bar{N}\,\underline{K}\,\underline{L}}\,\partial_{\bar{M}}\Gamma_{\bar{N}} \;+\; \Gamma^{\underline{M}}{}_{\bar{K}\bar{L}}\,\Gamma^{\underline{N}\bar{K}\bar{L}}\,\partial_{\underline{M}}\Gamma_{\underline{N}}\Big) \\
&- \tfrac{1}{2}\,\mathcal{R}_{\underline{M}\,\underline{N}\bar{K}\bar{L}}\,\Gamma^{\bar{K}\,\underline{M}\,\underline{P}}\,\Gamma^{\bar{L}\,\underline{N}}{}_{\underline{P}} + \tfrac{1}{2}\,\mathcal{R}_{\underline{K}\,\underline{L}\bar{M}\bar{N}}\,\Gamma^{\underline{K}\bar{M}\bar{P}}\,\Gamma^{\underline{L}\bar{N}}{}_{\bar{P}} \\
&- \tfrac{1}{2}m_{\underline{M}\bar{N}}\,\mathcal{R}^{\underline{M}\,\underline{K}\bar{P}\bar{Q}}\,\partial^{\bar{N}}\Gamma_{\underline{K}\bar{P}\bar{Q}} + \tfrac{1}{2}m_{\underline{M}\bar{N}}\,\mathcal{R}^{\underline{P}\,\underline{Q}\bar{N}\bar{K}}\,\partial^{\underline{M}}\Gamma_{\bar{K}\,\underline{P}\,\underline{Q}} \\
&+ \tfrac{1}{2}\,\mathcal{R}_{\underline{M}\,\underline{N}\bar{K}\bar{L}}\,\partial^{\underline{P}}m^{\underline{M}\bar{K}}\,\partial_{\underline{P}}m^{\underline{N}\bar{L}}\;.
\end{aligned}
$$

# Alternative heterotic construction?

$\mathcal{O}(\alpha')$ corrections to heterotic string theory:

define torsionful spin connection [Bergshoeff & de Roo (1989)]

$$\omega_{\mu ab}^{(\pm)}(e,b) \equiv \omega_{\mu ab}(e) \pm \frac{1}{2} H_{\mu ab}$$

then

$$\left(\omega_{\mu ab}^{(-)},\ \psi_{ab}\right), \qquad \psi_{ab} \equiv D_a^+ \psi_b - D_b^+ \psi_a$$

transforms as $SO(1,9)$ vector multiplet under SUSY!

$\rightarrow$ super-Yang-Mills action gives Riemann-squared & LCS modifications

Use heterotic DFT for $O(10, 10+n)$, $n = \mathrm{dim}(SO(1,9))$,
identify gauge fields with $\omega^{(-)}$ [Bedoya, Marques, Nunez (2014)]

drawback: compatibility with $O(d,d)$ not manifest

# Summary & Outlook

- DFT provides strikingly economic reformulation of supergravity
- Beyond supergravity (non-zero $\alpha'$): duality covariance requires novel field variables with *non-standard* diffeomorphisms
- However, usual diffeomorphism covariance replaced by duality-covariant gauge principle
- so far only partial results:
  background-independent extension for bosonic strings?
  Field-dependent gauge algebra? Higher order in $\alpha'$?
  Type II Strings and M-theory extensions?
- Extension to 'Exceptional Field Theory'
  with exceptional duality groups $E_{6(6)}$, $E_{7(7)}$, $E_{8(8)}$, ... ?