CMSSM WITH GENERALIZED YUKAWA QUASI-UNIFICATION: AN UPDATE

C. Pallis

DEPARTAMENT DE FÍSICA TÉORICA – IFIC
UNIVERSITY OF VALÉNCIA – CSIC

Based On

• N. Karagiannakis, G. Lazarides and C. P., to Appear.

Outlook

• Embedding The Minimal Supersymmetric (SUSY) Standard Model (MSSM) in A Pati-Salam (PS) SUSY Grand Unified Theory (GUT)
• Confronting the Resulting Constrained MSSM (CMSSM) With LHC, PLANCK & LUX
• Results: Stau-Coannihilation Vs Focus-Point Region
• Conclusions

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## The Initial Superpotential

We focus on the PS and PQ-invariant superpotential

\[ W_1 = W_{\text{MSSM}} + W_{\text{PQ}} + W_{\text{HPS}} \]

where

- \( W_{\text{MSSM}} = y_{ij} F_i h F_j^c = y_{ij} F_i \left( \tilde{H} \right) F_j^c = y_{ij} \left( H^T \varepsilon L_i e_j^c - \tilde{H}^T \varepsilon L_i \nu_j^c + H^T \varepsilon Q_{ia} d_j^c - \tilde{H}^T \varepsilon Q_{ia} u_j^c \right), \]

with \( \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \), \( Q_{ia} = \begin{pmatrix} u_{ia} \\ d_{ia} \end{pmatrix} \) and \( L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \).
### The Initial Superpotential

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where

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with $$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ Q_{ia} = \begin{pmatrix} u_{ia} \\ d_{ia} \end{pmatrix} \text{ and } L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}. $$

- $$W_{\text{MSSM}} \text{ leads to Yukawa unification (YU), i.e., }$$

\[
h_t(M_{\text{GUT}}) = h_b(M_{\text{GUT}}) = h_{\tau}(M_{\text{GUT}}) = y_{33}
\]

### Embedding MSSM in a Pati-Salam SUSY GUT

**Confronting the Resulting CMSSM With LHC, PLANCK & LUX Results: Stau-Coannihilation Vs Focus-Point Region**

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**CMSSM With Generalized Yukawa Quasi-Unification: An Update**
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where

\[ W_{\text{MSSM}} = y_{ij} F_i h F_j^c = y_{ij} F_i \begin{pmatrix} \tilde{H} \\ H \end{pmatrix} F_j^c \]

\[ y_{ij} \begin{pmatrix} H^T \epsilon L_i e_j^c - \bar{H}^T \epsilon L_i \nu_i^c + H^T \epsilon Q_{ia} d_{ja}^c - \bar{H}^T \epsilon Q_{ia} u_{ja}^c \end{pmatrix}, \]

with \( \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, Q_{ia} = \begin{pmatrix} u_{ia} \\ d_{ia} \end{pmatrix} \) and \( L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \).

- \( W_{\text{MSSM}} \) leads to Yukawa unification (YU), i.e.,

\[ h_i(M_{\text{GUT}}) = h_b(M_{\text{GUT}}) = h_r(M_{\text{GUT}}) = y_{33} \]

- \( W_{\text{PQ}} = \lambda_{pq} \frac{P^2 \bar{P}^2}{M_S} - \lambda_{\mu} \frac{P^2}{2M_S} \text{Tr}(h\epsilon h^T \epsilon), \)

  to generate \( \mu = \frac{\lambda_{\mu} f_a^2}{M_S} \sim 1 \text{ TeV} \)

- \( W_{\text{HPS}} = \lambda_S \left( \bar{H}^c H^c - M^2 \right) + \lambda_{ivc} \frac{(\bar{H}^c F_i^c)^2}{M_S}, \)

  to generate masses for RHNs

### Table: Superfields

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\[ W_1 = W_{\text{MSSM}} + W_{\text{PQ}} + W_{\text{HPS}} \]

**Where**

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with \( \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \), \( Q_{ia} = \begin{pmatrix} u_{ia} \\ d_{ia} \end{pmatrix} \), and \( L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \).

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\[ h_i(M_{\text{GUT}}) = h_b(M_{\text{GUT}}) = h_\tau(M_{\text{GUT}}) = y_{33} \]

- **\( W_{\text{PQ}} = \alpha_{\text{PQ}} \frac{p^2 \bar{p}^2}{M_S} - \lambda_\mu \frac{p^2}{2M_S} \text{Tr}(h e h^T \epsilon), \)**

to generate \( \mu = \lambda_\mu f_a^2 / M_S \sim 1 \text{ TeV} \)

- **\( W_{\text{HPS}} = \lambda_S \left( \tilde{H}^c H^c - M^2 \right) + \lambda_{iv} \frac{\left( \tilde{H}^c F_i^c \right)^2}{M_S} \)**

to generate masses for RHNs

\[ + \left( \lambda_H H^c H^c + \lambda_{\tilde{H}} \tilde{H}^c \tilde{H}^c \right) \]

to generate masses for \( d_H^c, \tilde{d}_H^c \)

*with* \( G = \begin{pmatrix} \epsilon_{abc} g_{c}^c & \bar{g}_{a}^c \\ -\bar{g}_{a}^c & 0 \end{pmatrix} \) \( \Rightarrow \bar{G} = \begin{pmatrix} \epsilon_{abc} \bar{g}_{c}^c & g_{a}^c \\ -g_{a}^c & 0 \end{pmatrix} \)
SOFT SUSY BREAKING TERMS

WORKING IN THE CONTEXT OF CMSSM, WE ADOPT UNIVERSAL INITIAL CONDITIONS AT $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV:

- A COMMON MASS $M_{1/2}$ FOR GAUGINOS:

  $$M_1(M_{\text{GUT}}) = M_2(M_{\text{GUT}}) = M_3(M_{\text{GUT}}) = M_{1/2}$$

  $$\Rightarrow M_1(M_{\text{SUSY}}) < M_2(M_{\text{SUSY}}) < M_3(M_{\text{SUSY}}).$$

- A COMMON MASS $m_0$ FOR SCALARS:
  - SLEPTONS, $m_L(M_{\text{GUT}}) = m_E(M_{\text{GUT}}) = m_0$
  - SQUARKS,

  $$m_Q(M_{\text{GUT}}) = m_U(M_{\text{GUT}}) = m_D(M_{\text{GUT}}) = m_0,$$

- HIGGS, $m_H(M_{\text{GUT}}) = m_{\tilde{H}}(M_{\text{GUT}}) = m_0$.

- A COMMON TRILINEAR COUPLING CONSTANT, $A_0$:

  $$A_t(M_{\text{GUT}}) = A_b(M_{\text{GUT}}) = A_\tau(M_{\text{GUT}}) = A_0.$$  

THE MINIMIZATION OF THE TREE LEVEL EFFECTIVE POTENTIAL AT AN OPTIMAL SCALE $M_{\text{SUSY}} = \sqrt{m_{11} m_{12}}$ GIVES:

$$\mu^2(M_{\text{SUSY}}) \approx \frac{m_H^2(M_{\text{SUSY}}) - m_{\tilde{H}}^2(M_{\text{SUSY}}) \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2. \quad (\text{NATURALITY ENTAILS: } \mu \sim m_{H,\tilde{H}} \sim M_Z)$$

FREE PARAMETERS OF CMSSM: $M_{1/2}$, $m_0$, $A_0$, $\tan \beta = \langle \tilde{H} \rangle / \langle H \rangle$, sign$\mu$.  

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1 B.C. Allanach (2002); G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov, http://lapth.in2p3.fr/micromegas
Soft SUSY Breaking Terms

Working in the Context of CMSSM, We Adopt Universal Initial Conditions At $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV:

- **A Common Mass $M_{1/2}$ For Gauginos:**
  
  $$M_1(M_{\text{GUT}}) = M_2(M_{\text{GUT}}) = M_3(M_{\text{GUT}}) = M_{1/2}$$

  \[ \Rightarrow M_1(M_{\text{SU}}) < M_2(M_{\text{SU}}) < M_3(M_{\text{SU}}). \]

- **A Common Mass $m_0$ For Scalars:**
  
  - **Sleptons,** $m_L(M_{\text{GUT}}) = m_E(M_{\text{GUT}}) = m_0$
  - **Squarks,**
    
    $$m_Q(M_{\text{GUT}}) = m_U(M_{\text{GUT}}) = m_D(M_{\text{GUT}}) = m_0,$$
  
  - **Higgs,** $m_H(M_{\text{GUT}}) = m_H(M_{\text{GUT}}) = m_0$.

- **A Common Trilinear Coupling Constant,** $A_0$:
  
  $$A_t(M_{\text{GUT}}) = A_b(M_{\text{GUT}}) = A_\tau(M_{\text{GUT}}) = A_0.$$

The minimization of the tree level effective potential at an optimal scale $M_{\text{SU}} = \sqrt{m_1 m_2}$ gives:

$$\mu^2(M_{\text{SU}}) \approx \frac{m_H^2(M_{\text{SU}}) - m_H^2(M_{\text{SU}}) \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2. \quad \text{(Naturality entails: $\mu \sim m_{H,R} \sim M_Z$)}$$

Free Parameters Of **CMSSM:** $M_{1/2}, m_0, A_0, \tan \beta = \langle \tilde{H} \rangle / \langle H \rangle, \text{sign}\mu.$

Software used for the Analysis of The Parameter Space of CMSSM

- Mathematica For Solving the RGEs (Gauge and Yukawa Constants and Soft SUSY Masses and Trilinears);

- SOFTSUSY$^1$ For the Calculation of SUSY Spectrum;

- micrOMEGAs2.4.5$^1$ For the Computation of the Various Cosmological Constraints.

$^1$B.C. Allanach (2002); G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov, http://lapth.in2p3.fr/micromegas
**SUSY Corrections to the Fermion Masses**

- **(a) Sbottom-Gluino** and **(b) Stop-Higgsino** loops give rise to sizable (20%) **SUSY corrections**\(^2\) to \(b\)-quark mass:

\[
\Delta m_b \approx \frac{g_3^2}{6\pi^2} m_g \mu \tan \beta \left( m_Z, m_{b_1}, m_{b_2} \right) + \frac{h_t^2}{16\pi^2} A_t \mu \tan \beta \left( \mu, m_{t_1}, m_{t_2} \right)
\]

Where \(I(x, y, z)\) is a **dimensionless function** which arises from the loop computation.

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\(^2\) L. Hall, R. Rattazzi and U. Sarid (1994); M. Carena et al. (1994); D. Pierce et al. (1997); S.F. King and M. Oliveira (2000)
**SUSY Corrections to the Fermion Masses**

- **(a) Sbottom-Gluino** and **(b) Stop-Higgsino Loops** give rise to sizable (20%) **SUSY corrections** to $b$-quark mass:

$$\Delta m_b \approx \frac{g_s^2}{6\pi^2} m_\tilde{g} \mu \tan\beta I(m_\tilde{g}, m_{\tilde{b}_1}, m_{\tilde{b}_2}) + \frac{h_t^2}{16\pi^2} A_t \mu \tan\beta I(\mu, m_{\tilde{t}_1}, m_{\tilde{t}_2})$$

Where $I(x, y, z)$ is a dimensionless function which arises from the loop computation.

- **We require** $m_t(\text{physical}) = 173$ GeV $\Leftrightarrow m_t(m_t) = 164.6$ GeV, $m_{\tilde{t}}(M_Z) = m_t(M_Z)(1 + \Delta m_t) = 1.748$ GeV

With given $m_t(m_t)$ and $m_{\tilde{t}}(M_Z)$ YU leads to an unacceptable $m_{\tilde{g}}(M_Z) = m_b(M_Z)(1 + \Delta m_b)$ in the context of CMSSM.

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2 L. Hall, R. Rattazzi and U. Sarid (1994); M. Carena et al. (1994); D. Pierce et al. (1997); S.F. King and M. Oliveira (2000).
**Superfields** | **Representations** | **Transformations** | **Global Charges** | **Global Charges** | **Z^mp**
---|---|---|---|---|
Under $G_{PS}$ | Under $G_{PS}$ | $R$ | $PQ$ | $Z^mp$

### Extra Higgs Fields

- $h'$ (15, 2, 2) $U_c^* U_L h' U_R^T U_c^T$ 0 1 0
- $h'$ (15, 2, 2) $U_c^* U_L h' U_R^T U_c^T$ 1 -1 0
- $\phi$ (15, 1, 3) $U_c U_R \phi U_R^T U_c^T$ 0 0 0
- $\bar{\phi}$ (15, 1, 3) $U_c U_R \bar{\phi} U_R^T U_c^T$ 0 0 0
- $\phi'$ (15, 1, 1) $U_c \phi' U_c^T$ 0 0 0
- $\bar{\phi}'$ (15, 1, 1) $U_c \bar{\phi}' U_c^T$ 0 0 0

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#### The Additional Superpotential Terms

- **We Add the Pati-Salam and Peccei-Quinn (PQ) Invariant Superpotential Terms**

$$W_2 = W_H + W_m + W_{MSSM'}$$ **Where**

- $W_H = m \phi \bar{\phi} + m' \phi' \bar{\phi}' - S \left( \beta \phi^2 + \beta' \phi'^2 \right)$
- $+ (\lambda \phi + \lambda' \phi') H^c \bar{H}^c$,

- **From the Scalar Potential We Find That The SUSY Vacuum Lies At**

$$\langle H^c \bar{H}^c \rangle \sim \langle \phi \rangle^2 \sim \langle \phi' \rangle^2 \sim M_{GUT}^2, \langle S \rangle = \langle \phi \rangle = \langle \phi' \rangle = 0$$

- **$W_m$ Is The Part Of $W$ Which Is Responsible For The Mixing of the Doublets in $h$ and $h'$**

$$W_m = M_{\bar{h}} h' h' + \lambda_3 \phi \bar{h}^* h + \lambda_1 \phi' \bar{h}' h = M_{\bar{h}} h_1 h_1^T \epsilon \left( h_2^2 + \alpha_2 h_2 \right) + M_{\bar{h}} \left( h_1^T + \alpha_1 h_1^T \right) \epsilon \bar{h}'_2 + \cdots \text{, with } \alpha_{1,2} = F (\langle \phi \rangle, \langle \phi' \rangle) \sim 1$$

- **Models With Only $\phi$ and $\bar{\phi}$ ($\alpha_1 = -\alpha_2$) Or $\phi'$ and $\bar{\phi}'$ ($\alpha_1 = \alpha_2$) Have Been Excluded** within CMSSM.

- **We Obtain Two Pairs Of Superheavy Doublets With Mass $M_{\bar{h}}$ and One Pair Which Remains Massless At The GUT Scale:**

$$h_1', h_2' \text{ and } H_1', \bar{h}'_2, \text{ where } H_r' = \frac{h_r' + \alpha_r h_r}{\sqrt{1 + |\alpha_r|^2}}, \text{ for } r = 1, 2 \Rightarrow H_r = \frac{-\alpha_r h_r' + h_r}{\sqrt{1 + |\alpha_r|^2}} \text{, } (1)$$

Which can be identified with the Electroweak Doublets $H_1 := H$ and $H_2 := \bar{H}$ and are orthogonal to the $H_r'$.

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**THE ADDITIONAL SUPERPOENTIAL TERMS**

- **We Add the Pati-Salam and Peccei-Quinn (PQ) Invariant Superpotential Terms**

\[
W_2 = W_H + W_m + W_{MSSM'} \quad \text{Where}
\]

\[
W_H = m\phi\bar{\phi} + m'\phi'\bar{\phi}' - S \left( \beta\phi^2 + \beta'\phi'^2 \right)
\]

\[
+ \left( \lambda\phi + \lambda'\phi' \right) H^c\bar{H}^c,
\]

- **From the Scalar Potential We Find That The SUSY Vacuum Lies At**

\[
\langle H^c\bar{H}^c \rangle \sim \langle \phi \rangle^2 \sim \langle \phi' \rangle^2 \sim M_{GUT}^2, \quad \langle S \rangle = \langle \phi \rangle = \langle \phi' \rangle = 0
\]

- **$W_m$ is the Part of $W$ Which is Responsible For the Mixing of the Doublets in $h$ and $h'$**

\[
W_m = M_h h' h' + \lambda_3 \phi \bar{h}' h + \lambda_1 \phi' \bar{h}' h = M_h \bar{h}'_1 \epsilon \left( h'_2 + \alpha_2 h_2 \right) + M_h \left( h'_1 + \alpha_1 h_1 \right) \epsilon h'_2 + \cdots, \quad \text{With} \quad \alpha_{1,2} = F \left( \langle \phi \rangle, \langle \phi' \rangle \right) \sim 1
\]

- **Models With Only $\phi$ and $\bar{\phi}$ ($\alpha_1 = -\alpha_2$) Or $\phi'$ and $\bar{\phi}'$ ($\alpha_1 = \alpha_2$) Have Been Excluded**\(^3\) **Within CMSSM.**

- **We Obtain Two Pairs Of Superheavy Doublets With Mass $M_h$ and One Pair Which Remains Massless At The GUT Scale:**

\[
\bar{h}'_1, \ H'_2 \quad \text{and} \quad H'_1, \ \bar{h}'_2, \quad \text{Where} \quad H'_r = \frac{h'_r + \alpha_r h_r}{\sqrt{1 + |\alpha_r|^2}}, \ r = 1, 2 \Rightarrow H_r = \frac{-\alpha^* r h'_r + h_r}{\sqrt{1 + |\alpha_r|^2}}, \quad (1)
\]

Which can be identified with the **Electroweak Doublets** $H_1 := H$ and $H_2 := \bar{H}$ and are orthogonal to the $H'_r$.

- **The Yukawa Interactions** Of the 3rd family of fermions are described by the following **Two** Superpotential Terms

\[
W_{MSSM} + W_{MSSM'} = y_{33} F_3 h F_3^c + 2 y_{33}' F_3 h' F_3'^c = y_{33} F_3 \left( h_2 + 2\rho h'_2 \right) - h_2 + 2\rho h'_2 \right) F_3^c
\]

With $\rho := y_{33}'/y_{33}$ and $h' \sim T_c^{15}$ and solving Eq. (1) w.r.t. $h_r$ and $h'_r$, we obtain

\[
h_r = \left( H_r + \alpha^*_r H'_r \right) / \left( \sqrt{1 + |\alpha_r|^2} \right) \quad \text{and} \quad h'_r = \left( -\alpha_r H_r + H'_r \right) / \left( \sqrt{1 + |\alpha_r|^2} \right).
\]

YUKAWA QUASI-UNIFICATION CONDITIONS

- We can readily derive the superpotential terms of the MSSM — except for the \( \mu \) term:

\[
W_{\text{MSSM}} + W_{\text{MSSM}'} = -h_t \bar{H}^T \epsilon Q^c u^c_3 + h_b \bar{H}^T \epsilon Q^c d^c_3 + h_t \bar{H}^T \epsilon L^c_3 e^c_3 - h_{\nu t} \bar{H}^T \epsilon L^c_3 \nu^c_3
\]
YUKAWA QUASI-UNIFICATION CONDITIONS

- Where the $h_t$, $h_b$, and $h_\tau$ must obey the following set of generalized Yukawa quasi-unification conditions:

$$h_t(M_{GUT}) : h_b(M_{GUT}) : h_\tau(M_{GUT}) = \frac{1 - \tan^2 \beta}{\sqrt{1 + |\alpha|^2}} : \frac{1 - \tan^2 \beta}{\sqrt{1 + |\alpha|^2}} : \frac{1 + V_3 \rho}{\sqrt{1 + |\alpha|^2}}$$

- We can readily derive the superpotential terms of the MSSM except for the $\mu$ term:

$$W_{\text{MSSM}} + W_{\text{eff}} = -h_\tau H^T e_3 d_3 + h_\tau H^T e_L e_3 - h_t H^T T e_L \bar{\nu}_3$$

$W_{\text{MSSM}}$ side.

$W_{\text{MSSM}}$ side.
**Yukawa Quasi-Unification Conditions**

- **We can readily derive the superpotential terms of the MSSM — except for the \(\mu\) term:**

\[
W_{\text{MSSM}} + W_{\text{MSSM}'} = -h_t \tilde{H}^T \varepsilon Q_3 u_3^c + h_b \tilde{H}^T \varepsilon Q_3 d_3^c + h_t \tilde{H}^T \varepsilon L_3 e_3^c - h_{\nu_t} \tilde{H}^T \varepsilon L_3 \nu_3^c
\]

- **Where the \(h_t, h_b\) and \(h_\tau\) must obey the following set of generalized Yukawa quasi-unification conditions:**

\[
h_t(M_{\text{GUT}}) : h_b(M_{\text{GUT}}) : h_\tau(M_{\text{GUT}}) = \left| \frac{1 - \rho \alpha_2 / \sqrt{3}}{\sqrt{1 + |\alpha_2|^2}} \right| : \left| \frac{1 - \rho \alpha_1 / \sqrt{3}}{\sqrt{1 + |\alpha_1|^2}} \right| : \left| \frac{1 + \sqrt{3} \rho \alpha_1}{\sqrt{1 + |\alpha_1|^2}} \right|
\]

- **For \(\rho, \alpha_1\) and \(\alpha_2 \sim 1\), we expect that \(h_m / h_n \sim 1\) with \(m, n = t, b, \tau\).**

**Stau-Coannihilation Region**

- E.g. For \(\tan \beta = 48\), \(\Delta \tilde{\tau}_2 \approx 0\), \(A_0/M_{1/2} = -1.4\) and \(M_{1/2} = 2.2\) TeV we find \(h_t/h_\tau(M_{\text{GUT}}) = 1.117\), \(h_b/h_\tau(M_{\text{GUT}}) = 0.623\) and \(h_t/h_b(M_{\text{GUT}}) = 1.792\).
**Yukawa Quasi-Unification Conditions**

- **We Can Readily Derive The Superpotential Terms of the MSSM – Except for the \( \mu \) Term:**

\[
W_{\text{MSSM}} + W_{\text{MSSM}'} = -h_t \tilde{H}^T \epsilon Q_3 u^c_3 + h_b H^T \epsilon Q_3 d^c_3 + h_\tau H^T \epsilon L_3 e^c_3 - h_{\nu_\tau} \tilde{H}^T \epsilon L_3 \nu^c_3
\]

- **Where The \( h_t, h_b \) and \( h_\tau \) Must Obey The Following Set of Generalized Yukawa Quasi-Unification Conditions:**

\[
h_t(M_{\text{GUT}}) : h_b(M_{\text{GUT}}) : h_\tau(M_{\text{GUT}}) = \left| \frac{1 - \rho \alpha_2 / \sqrt{3}}{\sqrt{1 + |\alpha_2|^2}} \right| : \left| \frac{1 - \rho \alpha_1 / \sqrt{3}}{\sqrt{1 + |\alpha_1|^2}} \right| : \left| \frac{1 + \sqrt{3} \rho \alpha_1}{\sqrt{1 + |\alpha_1|^2}} \right|
\]

- **For \( \rho, \alpha_1 \) and \( \alpha_2 \sim 1 \), We Expect That \( h_m / h_n \sim 1 \) with \( m, n = t, b, \tau \).

**Stau-Coannihilation Region**

**Focus-Point Region**

- E.g. For \( \tan \beta = 48 \), \( \Delta \tilde{t}_2 \approx 0 \), \( A_0/M_{1/2} = -1.4 \) and \( M_{1/2} = 2.2 \text{ TeV} \) we find \( h_t / h_\tau(M_{\text{GUT}}) = 1.117 \)
  
  \( h_b / h_\tau(M_{\text{GUT}}) = 0.623 \) and \( h_t / h_b(M_{\text{GUT}}) = 1.792 \).

- E.g. For \( \tan \beta = 48 \), \( M_{1/2} = 2.9 \text{ TeV} \), \( A_0/M_{1/2} = -1.5 \) and \( m_0 = 8.8 \text{ TeV} \) we find \( h_t / h_\tau(M_{\text{GUT}}) = 1.107 \)
  
  \( h_b / h_\tau(M_{\text{GUT}}) = 0.763 \) and \( h_t / h_b(M_{\text{GUT}}) = 1.45 \).
Cosmo-Phenomenological Requirements: Preliminaries

Phenomenological Requirements

1. Mass Of the Lighter CP- Even Higgs Boson:
   
   \[ 122 \lesssim m_h/\text{GeV} \lesssim 129.2. \]

2. The Branching Ratios of the Rare B decays:
   
   - \( \text{BR} (B_s \rightarrow \mu^+\mu^-) \leq 4.2 \times 10^{-9} \) or
     \[ 1.1 \leq \text{BR} (B_s \rightarrow \mu^+\mu^-)/10^{-9} \lesssim 6.4; \]
   - \( 2.84 \times 10^{-4} \leq \text{BR} (b \rightarrow s\gamma) \leq 4.2 \times 10^{-4}; \)
   - \( 0.52 \leq \text{R} (B_u \rightarrow \tau\nu) \leq 2.04. \)

3. The Bounds on the Masses of Sparticles;
   Most Notably \( m_{\chi^\pm} \geq 103.5 \text{ GeV} \) and \( m_{\tilde{g}} \geq 1.4 \text{ TeV}. \)

4. Muon Anomalous Magnetic Moment of \( \mu, a_\mu: \)
   
   \[ 7.5 \times 10^{-10} \leq \delta a_\mu = a_\mu^{\exp} - a_\mu^{\text{SM}} \leq 42.3 \times 10^{-10}. \]
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Cosmological Considerations

- To avoid the Proton Decay via terms
  \[ UDD, Q^T \varepsilon LD, L^T \varepsilon LE, L^T \varepsilon H, \]
  We require that \( \mathcal{L}_{\text{MSSM}} \) is invariant under a multiplicative quantum number defined as follows:
  \[ R = \begin{cases} +1 & \text{for SM particles and Higgs} \\ -1 & \text{for sparticles} \end{cases} \]
  with the following consequences:
  - In every term of \( \mathcal{L}_{\text{MSSM}} \) there is even number of sparticles.
  - The sparticles can be produced in pairs;
  - The heavier sparticles decay to the lighter;
  - The lightest SUSY particle (LSP) is stable.
  - The LSP is weakly interacting (i.e., it interacts via the exchange of a heavier sparticle)
  - The LSP can act as a viable CDM candidate if
    \[ \Omega_{\text{LSP}} h^2 \leq 0.12 \] at 95% c.l.
  - The spin-independent LSP-proton cross section is to be lower than the LUX results.
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  - The spin-independent LSP-proton cross section is to be lower than the LUX results.

\[ ?????? \lesssim m_{\text{LSP}} / \text{GeV} \lesssim ?????? \]
1. MASS OF THE LIGHTER CP-EVEN (SM-LIKE) HIGGS BOSON

- At Tree Level we obtain \( m_h \leq M_Z \cos 2\beta \leq M_Z \).
- SUSY Corrections Arise From Sfermions \( \tilde{f} \to \tilde{t}, \tilde{b} \) Loops Which INCREASE \( m_h \) AS FOLLOWS

\[
\Delta m_h^2 \approx \ln \frac{M_{SUSY}^2}{m_t^2} + \frac{X_t^2}{M_{SUSY}^2} \left( 1 - \frac{X_t^2}{12M_{SUSY}^2} \right)
\]

With \( X_t = A_t - \mu \cot \beta \) and \( M_{SUSY} \approx \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}} \).

- The LHC \(^5\) DISCOVERED A Bosonic Particle Which Fits The Profile Of The SM Higgs with Mass

\[
m_h = \begin{cases} 
125.3 \pm 0.4 \text{ (stat)} \pm 0.5 \text{ (sys)} \text{ GeV} & \text{CMS} \\
126.0 \pm 0.4 \text{ (stat)} \pm 0.4 \text{ (sys)} \text{ GeV} & \text{ATLAS}
\end{cases}
\]

\( \Rightarrow \) \( 122 \leq m_h/\text{GeV} \leq 129.2 \).

\(^4\) Allowing for a Theoretical Uncertainty of \( \pm 1.5 \) GeV.

- We Present \( m_h \) Vs \( m_{\text{LSP}} \) for Various \( A_0/M_{1/2} \)'s.
- A LOWER Bound on \( m_{\text{LSP}} \) Can Be Inferred from THE LOWER Bound on \( m_h \)
- We REMARK THAT \( m_h \) INCREASES With \( m_{\text{LSP}} \) and AS \( A_0/M_{1/2} \) Decreases To VALUES Lower Than Zero. This Occurs since \( X_t \) is MAXIMIZED FOR \( A_0/M_{1/2} < 0 \).
- As A Consequence the bound on \( m_{\text{LSP}} \) for \( A_0/M_{1/2} < 0 \) is Less Restrictive.


---


2. The Branching Ratio of $B_s \rightarrow \mu^+\mu^-$

- The SUSY Contributions Originate$^6$ from Neutral Higgs Bosons in Chargino-, $H^{\pm}$-, and $W^{\pm}$-Mediated Penguins which are Particularly Important For Large $\tan\beta$'s:

- We Impose the Following Very Stringent Constraint $\text{BR}(B_s \rightarrow \mu^+\mu^-) \sim |A_t \tan^3 \beta/m_A|^2 \lesssim 4.2 \times 10^{-9}$ (95% c.l.$^7$) (i)
  Although There is Recently a Novel Less Restrictive Bound $1.1 \lesssim \text{BR}(B_s \rightarrow \mu^+\mu^-)/10^{-9} \lesssim 6.4$ (95% c.l.).

- We Present $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ Vs $m_{\text{LSP}}$ for Various $A_0/M_{1/2}$'s.
- Eq. (i) Imposes a Lower bound on $m_{\text{LSP}}$
- We Remark that $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ Decreases As $m_{\text{LSP}}$ and $A_0/M_{1/2}$ Increases.
- As A Consequence the Bound on $m_{\text{LSP}}$ for $A_0/M_{1/2} < 0$, Favored by $m_h$ Data, is More Restrictive.
- The Inclusion Of $\phi, \bar{\phi}$ AND $\phi' \bar{\phi}'$ Assists us to Decrease $\tan\beta$ Below 50, Reducing Thereby $\text{BR}(B_s \rightarrow \mu^+\mu^-)$.

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$^7$LHCb (2012).
3. **Muon Anomalous Magnetic Moment of $\mu$, $a_\mu$**

- **There is a 2.9 - $\sigma$ Discrepancy Between the Experimental Value of $a_\mu$, $a_\mu^{\text{exp}}$, and the one Predicted by SM, $a_\mu^{\text{SM}}$, Which Can Be Attributed to SUSY Corrections**\(^8\) Arising From (a) Muon - Neutralino and (b) Sneutrino - Chargino Loops:

\[ \delta a_\mu \]

\[ 7.5 \times 10^{-10} \leq \delta a_\mu \leq 42.3 \times 10^{-10}, \]

Where: $\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$

- **We Present $\delta a_\mu$ Vs $m_{\text{LSP}}$ for $A_0/M_{1/2} = 0$ and Both signs of $\mu$.**
- **Eq. (ii) Imposes a Lower Bound on $m_{\text{LSP}}$ and Eq. (i) Imposes an Upper Bound on $m_{\text{LSP}}$, Not Compatible With the Bound Inferred by $\Omega_{\text{LSP}} h^2$ as We will See.**
- **The $\mu < 0$ Case is More Disfavored Than the $\mu > 0$ Case From the $\delta a_\mu$ Considerations. Therefore, We Decide To Use $\mu > 0$.**

- **Are These Lower Limits on $m_{\text{LSP}}$ Compatible With The Candidacy of LSP as CDM Particle?**
- **A Reply to This Dilemma Can be given the Cosmological Consideration of LSP as a CDM Candidate.**

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9 G.W. Bennett et al. (2006); K. Hagiwara et al. (2011); T. Aoyama et al. (2012).
**The Standard Cosmological Scenario (SC)**

Assumptions of SC for the Decoupling of a Weekly Interacting Massive Particle (WIMP) $\chi$ from the Cosmic Bath:

- **The $\chi$’s Achieve Kinetic and Chemical Equilibrium.** Therefore, the Thermal Averaged Cross Section Times The Relative Velocity of $\chi$’s is to be $\langle \sigma v \rangle \gtrsim 10^{-20}$ GeV$^{-2}$. Within a Specific Particle Model $\langle \sigma v \rangle$ can be Derive Self-Consistently from the (s)particle Spectrum of the Theory.

- **The $\chi$’s Are Produced by Thermal Scatterings.** The Number Density of $\chi$, $n_\chi$, Obey the Boltzmann Equation:

  $$ n_\chi + 3Hn_\chi + \langle \sigma v \rangle \left( n_\chi^2 - n_\chi^{eq2} \right) = 0 \quad \text{With Initial Condition: } n_\chi(T = m_\chi) = n_\chi^{eq}(x = 1) \quad \text{(Note: } \rho_\chi = m_\chi n_\chi)$$

  **Where** $n_\chi^{eq}(x) = \frac{g}{(2\pi)^3/2} m_\chi^3 x^{3/2} e^{-1/x} P_2(1/x)$, With $x = T/m_\chi$, $P_n(z) = 1 + (4n^2 - 1)/8z$, $g = 2$.

- **The $\chi$’s Decouple During a Radiation Dominated Era**, $H = \sqrt{\rho_R}/\sqrt{3}m_p \sim T^2$ And $sR^3 = \text{cst} \Rightarrow TR = \text{cst}$.

![Graph showing $\rho_\chi / s$ vs $T$ (GeV)](image)

- **The Relic Abundance of $\chi$ is Given By** $\Omega_\chi = \rho_\chi / \rho_\text{c0} = m_\chi s_0 Y_\chi / \rho_\text{c0} \Rightarrow \Omega_\chi h^2 = 2.748 \cdot 10^8 Y_{\chi0} m_\chi / \text{GeV}$

  Where $m_\chi$, the Mass of WIMP $\chi$ and $Y_\chi = n_\chi / s$. Therefore $\Omega_\chi h^2 = f \left( m_\chi, \langle \sigma v \rangle \right)$. Mostly $\Omega_\chi h^2 \sim m_\chi / \langle \sigma v \rangle$. 

![Graph showing $\log P_R$ vs $T$ (GeV)](image)
The Relic Abundance of The Neutralino LSP, $\tilde{\chi}$, & the CMSSM Parameter Space

- Focusing on the Case of CMSSM, The Computation of $\langle \sigma v \rangle$ Includes Annihilation and Coannihilation Processes:

$$\langle \sigma v \rangle \sim \sigma_{\text{eff}} \sim \sigma_{\text{LSP-LSP}} + e^{-\Delta_{\text{LSP}}/x_F} \sigma_{\text{LSP-NLSP}} + e^{-2\Delta_{\text{LSP}}/x_F} \sigma_{\text{LSP-NLSP}(*)}, \quad \text{With } x_F \sim \frac{1}{25} \quad \text{and} \quad \Delta_{\text{LSP}} = \frac{m_{\text{NLSP}} - m_{\text{LSP}}}{m_{\text{LSP}}}$$

- $\Omega_{\tilde{\chi}} h^2$ Can Be Consistent With Observations Only Into Well Localized Portions Of the CMSSM Parameter Space.
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- Focusing on the case of CMSSM, the computation of $\langle \sigma v \rangle$ includes **annihilation** and **coannihilation** processes:

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with $x_{F} \sim \frac{1}{25}$ and $\Delta_{\text{NLSP}} = \frac{m_{\text{NLSP}} - m_{\text{LSP}}}{m_{\text{LSP}}}$

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### 1. Annihilation Processes ($\tilde{\chi} \tilde{\chi}$) - $\Delta_{\text{NLSP}} \gg 0.25$

<table>
<thead>
<tr>
<th>States</th>
<th>Channels</th>
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</tr>
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<tbody>
<tr>
<td>$\tilde{\chi} \tilde{\chi}$</td>
<td>$f \tilde{f}$</td>
<td>$s(h), s(H), s(A), s(Z)$</td>
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<td>(f := t, $\tau$, b)</td>
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<td>$t(\tilde{f}<em>{1[2]}), u(\tilde{f}</em>{1[2]})$</td>
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With $x_F \sim \frac{1}{25}$ and $\Delta_{\text{NLSP}} = \frac{m_{\text{NLSP}} - m_{\text{LSP}}}{m_{\text{LSP}}}$

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- Bulk Region With $m_0, M_{1/2} \ll 500$ GeV Where $\tilde{\chi}$ is Bino-like And The Dominant Processes Are

\[
\begin{align*}
\chi &\rightarrow \tau^- [e^+] \\
\tilde{\chi} &\rightarrow \tau^+ [e^+] \\
\tilde{\chi} &\rightarrow \tau^+ [e^+] \\
\end{align*}
\]

This Region Is Excluded by the LHC bound on $m_h$. 

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- **The A-Funnel Region** where the process $\tilde{\chi}\tilde{\chi} \rightarrow b\bar{b}$ is enhanced if $\Delta_A = (m_A - 2m_{\tilde{\chi}})/2m_{\tilde{\chi}} \approx 0$ for $\tan\beta > 40$. This region is also almost excluded by the LHC bound on $m_h$.

- **Bulk Region** with $m_0, M_{1/2} \ll 500 \text{ GeV}$ where $\tilde{\chi}$ is bino-like and the dominant processes are

\[\tilde{\chi} \rightarrow \tau^- [e^+]\]
\[\tilde{\chi}_{1[2]} \rightarrow \tilde{\tau}_{1[2]} [\tilde{e}_R]\]

This region is excluded by the LHC bound on $m_h$. 

- C. Pallis

**CMSSM With Generalized Yukawa Quasi-Unification: An Update**
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<td>$AA, ZA$</td>
</tr>
<tr>
<td></td>
<td>$aH, HA$</td>
</tr>
<tr>
<td></td>
<td>$W^+W^-, H^+H^-$</td>
</tr>
<tr>
<td></td>
<td>$W_\pm W_\mp$</td>
</tr>
<tr>
<td></td>
<td>$Z_{\pm}$</td>
</tr>
<tr>
<td></td>
<td>$Zh, ZH$</td>
</tr>
</tbody>
</table>

- **Bulk Region** With $m_0, M_{1/2} < 500$ GeV Where $\tilde{\chi}$ is Bino-like And The Dominant Processes Are

- **The A-Funnel Region** Where The Process $\tilde{\chi}\tilde{\chi} \rightarrow b\bar{b}$ Is Enhanced If $\Delta_A = (m_A - 2m_{\tilde{\chi}})/2m_{\tilde{\chi}} \approx 0$ For $\tan\beta > 40$. This Region Is Also Almost Excluded by the LHC bound on $m_h$.

- **The Hyperbolic Branch/Focus Point Region** at Large $m_0 > 5$ TeV, Where $|\mu|$ Becomes Small, and $\tilde{\chi}$ Develops A Significant Higgsino Component. The Dominant Process Is:
\section{Coannihilation Processes - $\Delta_{NLSP} < 0.25$}

- These processes can be activated\textsuperscript{10} for every $\tan \beta$ if $\Delta_{NLSP} \ll 0.25$ since $\sigma_{LSP-NLSP} + \sigma_{LSP-NLSP^*} \gg \sigma_{LSP-LSP}$. Can dominate the $\langle \sigma v \rangle$ computation. For given $m_{LSP}$, $\Omega_{LSP} h^2$ decreases with $\Delta_{NLSP}$.

**Focus-Point (FP) Coannihilations**

(In the limit $m_{H,A,H^\pm} \gg m_{\tilde{\chi}}$, $m_{\tilde{\chi}\pm}$)

<table>
<thead>
<tr>
<th>States</th>
<th>Final</th>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Interactions</td>
</tr>
<tr>
<td>$\tilde{\chi} \tilde{\chi}^+$</td>
<td>$W^+ h$</td>
<td>$t(\tilde{\chi}_k^0)$, $u(\tilde{\chi}_k^0)$, $s(H)$, $s(W)$</td>
</tr>
<tr>
<td></td>
<td>$W^+ Z$</td>
<td>$t(\tilde{\chi}_k^0)$, $u(\tilde{\chi}_k^0)$, $s(W)$</td>
</tr>
<tr>
<td></td>
<td>$\gamma W^+$</td>
<td>$t(\tilde{\chi}_j^0)$, $s(W)$</td>
</tr>
<tr>
<td></td>
<td>$u\bar{d}$</td>
<td>$t(\tilde{d}<em>{1,2})$, $u(\tilde{u}</em>{1,2})$, $s(H)$, $s(W)$</td>
</tr>
<tr>
<td></td>
<td>$\nu\bar{\ell}$</td>
<td>$t(\tilde{\ell}_{L,R})$, $u(\tilde{\nu}_L)$, $s(H)$, $s(W)$</td>
</tr>
<tr>
<td>$\tilde{\chi}_i^+ \tilde{\chi}_j^+$</td>
<td>$W^+ W^+$</td>
<td>$t(\tilde{\chi}_k^0)$, $u(\tilde{\chi}_k^0)$</td>
</tr>
<tr>
<td>$\tilde{\chi}_i^+ \tilde{\chi}_j^-$</td>
<td>$ZZ$</td>
<td>$t(\tilde{\chi}_k^0)$, $u(\tilde{\chi}_k^0)$, $s(h,H)$</td>
</tr>
<tr>
<td></td>
<td>$W^+ W^-$</td>
<td>$t(\tilde{\chi}_k^0)$, $s(h,H)$, $s(Z,\gamma)$</td>
</tr>
<tr>
<td></td>
<td>$\gamma \gamma$ (for $i = j$)</td>
<td>$t(\tilde{\chi}_i^0)$, $u(\tilde{\chi}_i^0)$</td>
</tr>
<tr>
<td></td>
<td>$Z\gamma$</td>
<td>$t(\tilde{\chi}_j^0)$, $u(\tilde{\chi}_j^0)$</td>
</tr>
<tr>
<td></td>
<td>$u\bar{u}$</td>
<td>$t(\tilde{d}_{L,R})$, $s(h,H,A)$, $s(Z,\gamma)$</td>
</tr>
<tr>
<td></td>
<td>$\nu\bar{\nu}$</td>
<td>$t(\tilde{\ell}_{L,R})$, $s(Z)$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{d}\bar{d}$</td>
<td>$t(\tilde{u}_{L,R})$, $s(h,H,A)$, $s(Z,\gamma)$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\ell}\bar{\ell}$</td>
<td>$t(\tilde{\nu}_L)$, $s(h,H,A)$, $s(Z,\gamma)$</td>
</tr>
</tbody>
</table>

\textsuperscript{10} J. Edsjo & P. Gondolo (1997); J. Ellis et al. (1999); M.E. Gómez, G. Lazarides and C.P. (2000, 2002); G. Bélanger et al. (micromegas) (2001).
2. Coannihilation Processes - $\Delta_{NLSP} < 0.25$

- These processes can be activated\(^{10}\) for every $\tan \beta$ if $\Delta_{NLSP} \ll 0.25$ since $\sigma_{LSP-\text{NLSP}} + \sigma_{LSP-\text{NLSP}(\ast)} \gg \sigma_{LSP-LSP}$. Can dominate the $\langle \sigma v \rangle$ computation. For given $m_{LSP}$, $\Omega_{LSP} h^2$ decreases with $\Delta_{NLSP}$.

**Focus-Point (FP) Coannihilations**

(In the limit $m_{H,A,H^\pm} \gg m_{\tilde{\chi}}, m_{\tilde{\chi}_{i}^{\pm}}$)

<table>
<thead>
<tr>
<th>States</th>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\chi}\tilde{\chi}$</td>
<td>$W^{+}h$, $t(\tilde{\chi}<em>{j}^{0}), u(\tilde{\chi}</em>{j}^{+}), s(H^{+}), s(W^{+})$</td>
</tr>
<tr>
<td>$\tilde{\chi}\tilde{\chi}$</td>
<td>$W^{+}Z$, $t(\tilde{\chi}<em>{k}^{0}), u(\tilde{\chi}</em>{k}^{+}), s(W^{+})$</td>
</tr>
<tr>
<td>$\gamma W^{+}$</td>
<td>$\gamma W^{+}$, $t(\tilde{\chi}_{j}^{0}), s(W^{+})$</td>
</tr>
<tr>
<td>$u\bar{d}$</td>
<td>$t(\tilde{a}<em>{1,2}), u(\tilde{a}</em>{1,2}), s(H^{+}), s(W^{+})$</td>
</tr>
<tr>
<td>$\nu \bar{\ell}$</td>
<td>$t(\tilde{\ell}<em>{L,R}), u(\tilde{\nu}</em>{L}), s(H^{+}), s(W^{+})$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>States</th>
<th>Final Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\chi}<em>{i}^{+}\tilde{\chi}</em>{j}^{+}$</td>
<td>$W^{+}W^{+}$, $t(\tilde{\chi}<em>{j}^{0}), u(\tilde{\chi}</em>{k}^{0})$</td>
</tr>
</tbody>
</table>

$\chi - \tilde{\tau}_{2}$ Coannihilations ($\tilde{\tau}_{2}$-CAs)

(At low $m_{0}$ and almost any $M_{1/2}$)

<table>
<thead>
<tr>
<th>States</th>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\chi}\tilde{\tau}_{2}$</td>
<td>$\tau h, \tau H, \tau Z$</td>
</tr>
<tr>
<td></td>
<td>$\tau A$</td>
</tr>
<tr>
<td></td>
<td>$\tau \gamma$</td>
</tr>
<tr>
<td></td>
<td>$\tau \bar{\tau}$</td>
</tr>
<tr>
<td>$\tilde{\tau}<em>{2}\tilde{\tau}</em>{2}$</td>
<td>$\tau \bar{\tau}$</td>
</tr>
<tr>
<td></td>
<td>$t(\tilde{\chi}), u(\tilde{\chi})$</td>
</tr>
<tr>
<td>$\tilde{\tau}<em>{2}\tilde{\tau}</em>{2}^{\ast}$</td>
<td>$hh, hH, HH, ZZ$</td>
</tr>
<tr>
<td></td>
<td>$AA$</td>
</tr>
<tr>
<td></td>
<td>$hZ, HZ$</td>
</tr>
<tr>
<td></td>
<td>$h\gamma, H\gamma$</td>
</tr>
<tr>
<td></td>
<td>$hA, HA$</td>
</tr>
<tr>
<td></td>
<td>$AZ$</td>
</tr>
<tr>
<td></td>
<td>$H^{+}H^{-}, W^{+}W^{-}$</td>
</tr>
<tr>
<td></td>
<td>$H^{+}W^{-}$</td>
</tr>
<tr>
<td></td>
<td>$\gamma\gamma, \gamma Z$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{u}, b\tilde{b}$</td>
</tr>
<tr>
<td></td>
<td>$\tau\bar{\tau}$</td>
</tr>
<tr>
<td></td>
<td>$u\bar{u}, d\bar{d}, e\bar{e}$</td>
</tr>
<tr>
<td></td>
<td>$\nu\nu$</td>
</tr>
</tbody>
</table>

\(^{10}\) J. Edsjo & P. Gondolo (1997); J. Ellis et al. (1999); M.E. Gómez, G. Lazarides and C.P. (2000, 2002); G. Bélanger et al. (micrOMEGAs) (2001)
**Neutralino–Proton Spin Independent (SI) Cross Section** $\sigma_{\tilde{\chi} p}^{SI}$

- **The Lagrangian** $L_{\text{eff}} = \alpha_{q}^{SI} (\tilde{\chi} \tilde{\chi}) (\bar{q} q)$ for the $\tilde{\chi}–$Quark Elastic Scattering Takes Contributions\(^\text{11}\) From the Processes:

  \[
  \tilde{\chi} \rightarrow \tilde{q}_1, \tilde{q}_2 \rightarrow \tilde{\chi} q
  \]

  \[
  \tilde{\chi} \rightarrow h, H \rightarrow \tilde{\chi} q q
  \]

  \[
  \tilde{\chi} \rightarrow Z \rightarrow \tilde{\chi} q q
  \]

- $L_{\text{eff}}$ Gives Rise to $\sigma_{\tilde{\chi} p}^{SI} = \frac{4}{\pi} \mu_{\tilde{\chi} p}^{2} f_{p}^{2}$, Where $\mu_{\tilde{\chi} p} = \frac{m_{\tilde{\chi}} m_{p}}{m_{\tilde{\chi}} + m_{p}}$ and $f_{p} = \sum_{q=u,d,s} m_{q} f_{T_{q}}^{p} \alpha_{q}^{SI} + \frac{2}{27} f_{T_{G}}^{p} \sum_{q=c,b,t} m_{q} \alpha_{q}^{SI}$

Where the **Hadronic Inputs** $f_{T_{q}}^{p}$ and $f_{T_{G}}^{p}$ Encode The Transition From The Quark To Nucleon Level.

- In The FP Region, Data Coming From LUX Provide **Strict Bounds** On The CMSSM Parameters Since

  \[
  \alpha_{q}^{SI} = \frac{g_{H} m_{H}^{2}}{2 m_{H}^{2}} \tan^{2} \beta \sim \frac{\tan^{2} \beta}{m_{H}^{4}} |N_{11}|^{2} |N_{13}|^{2}
  \]

  With $N$ the Matrix Which Diagonalizes The Neutralino Mass Matrix.

- In Order To Reliably Compare data From LUX With $\sigma_{\tilde{\chi} p}^{SI}$ When $\Omega_{\tilde{\chi}} h^{2} < 0.12$, We Use The **Scaled Quantity**:

  \[
  \sigma_{\text{CDM–p}}^{SI} = \xi \sigma_{\tilde{\chi} p}^{SI}, \quad \xi = \Omega_{\tilde{\chi}} h^{2}/0.12.
  \]

\(^{11}\) M.W. Goodman and E. Witten (1985); J. Ellis and R. Flores (1988); K. Griest (1988).
Restrictions to the CMSSM Parameter Space

The Free Parameters of CMSSM are \( M_{1/2}, m_0, A_0, \tan \beta, \text{sign}\mu \Rightarrow M_{1/2}, A_0/M_{1/2}, \tan \beta, \Delta \tilde{\tau}_2 \) \( \left( \Delta \tilde{\tau}_2 = \frac{m_{\tilde{\tau}_2} - m_{\tilde{\chi}}}{m_{\tilde{\chi}}} \right) \)

We Present Our Results:

• In the \( M_{1/2} - m_0 \) Plane for Various \( A_0/M_{1/2}'s \)

We Find \( 0.9 \leq m_{\tilde{\chi}}/\text{TeV} \leq 1.4 \) & \( 123.7 \leq m_h/\text{GeV} \leq 125.9 \)
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The free parameters of CMSSM are $M_{1/2}$, $m_0$, $A_0$, $\tan \beta$, $\text{sign}\mu \implies M_{1/2}, A_0/M_{1/2}, \tan \beta, \Delta \bar{\tau}_2 \left( \Delta \bar{\tau}_2 = \frac{m_{\bar{\tau}^2} - m_{\tilde{\chi}}} {m_{\tilde{\chi}}} \right)$

We present our results:

- In the $M_{1/2} - m_0$ plane for various $A_0/M_{1/2}$'s

We find $0.9 \leq m_{\tilde{\chi}} / \text{TeV} \leq 1.4$ & $123.7 \leq m_h / \text{GeV} \leq 125.9$

- In the $M_{1/2} - A_0/M_{1/2}$ plane for various $\tan \beta$'s

We find $0.75 \leq m_{\tilde{\chi}} / \text{TeV} \leq 1.4$ & $122 \leq m_h / \text{GeV} \leq 127$
Restrictions to the CMSSM Parameter Space

The Free Parameters of CMSSM are $M_{1/2}$, $m_0$, $A_0$, $\tan \beta$, sign$\mu$ \[ \mu > 0 \] \[ M_{1/2}, A_0/M_{1/2}, \tan \beta, \Delta \bar{\tau}_2 \] \[ \Delta \bar{\tau}_2 = \frac{m_{\tilde{\tau}_2} - m_{\tilde{\chi}}}{m_{\tilde{\chi}}} \]

We present our results:

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We find $0.9 \lesssim m_{\tilde{\chi}}/\text{TeV} \lesssim 1.4$ & $123.7 \lesssim m_h/\text{GeV} \lesssim 125.9$

- We obtain a fulfillment of all the restrictions but this from the lower bound of $\delta a_\mu$.
- The interplay of BR ($B_s \to \mu^+\mu^-$) and $\Omega_\chi h^2$ constraints determine the allowed $m_{\tilde{\chi}}$'s and $\tan \beta$'s i.e., $43.8 \leq \tan \beta \leq 52$
- Since $m_{\tilde{g}}, m_{\tilde{t}_{1,2}}$ and $m_{\tilde{b}_{1,2}} \geq 3 \text{ TeV}$, their discovery is very difficult.
- Since $\mu \geq 1 \text{ TeV}$, the SSB requires some tuning named "Little Hierarchy".
- In the allowed region we obtain $h_t/h_\tau \approx 0.98 - 1.29, h_b/h_\tau \approx 0.60 - 0.65$, and $h_t/h_b \approx 1.62 - 2.00$.
- These values can be motivated by the embedding of CMSSM in our SUSY Pati-Salam model.
Restrictions to the CMSSM Parameter Space

The free parameters of CMSSM are $M_{1/2}$, $m_0$, $A_0$, $\tan\beta$, $\text{sign} \mu \quad \mu > 0 \rightarrow m_h, m_{\text{LSP}}, A_0/M_{1/2}, \tan\beta$.

We present our results for $\tan\beta = 48$:

- In the $M_{1/2} - m_0$ plane for various $A_0/M_{1/2}$'s

We find $0.1 \lesssim m_{\text{LSP}}/\text{TeV} \lesssim 1.1$ & $122 \lesssim m_h/\text{GeV} \lesssim 127$. 
Restrictions to the CMSSM Parameter Space

The free parameters of CMSSM are $M_{1/2}, m_0, A_0, \tan \beta, \text{sign} \mu \implies m_h, m_{\text{LSP}}, A_0/M_{1/2}, \tan \beta$.

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- In the $m_{\text{LSP}} - A_0/M_{1/2}$ plane for $m_h = 125.5$ GeV

We find $0.1 \lesssim m_{\text{LSP}}/\text{TeV} \lesssim 1.08$ & $-11 \lesssim A_0/M_{1/2} \lesssim 14.1$. 
Restrictions to the CMSSM Parameter Space

The free parameters of CMSSM are $M_{1/2}$, $m_0$, $A_0$, $\tan\beta$, $\text{sign}\mu \Rightarrow m_h$, $m_{\text{LSP}}$, $A_0/M_{1/2}$, $\tan\beta$.

We present our results for $\tan\beta = 48$:

- In the $M_{1/2} - m_0$ plane for various $A_0/M_{1/2}$’s

We find $0.1 \leq m_{\text{LSP}}/\text{TeV} \leq 1.1$ & $122 \leq m_h/\text{GeV} \leq 127$.

- Varying $40 \leq \tan\beta \leq 51$, the restrictions in the $m_{\text{LSP}} - A_0/M_{1/2}$ plan remain almost intact.
- We obtain a fulfillment of all the restrictions but this from the lower bound of $\delta a_{\mu}$.
- The bound $m_{\tilde{\chi}_i}^\pm \geq 103.5$ GeV implies $m_{\text{LSP}} \geq 99$ GeV whereas $\text{BR}(B_s \to \mu^+\mu^-)$ remains close to its SM value.
- Since $m_{\tilde{g}}, m_{\tilde{t}_{1,2}}$ and $m_{\tilde{b}_{1,2}} \gtrsim 5$ TeV, their discovery is very difficult.
- For $\Omega_{\tilde{\chi}}h^2 \approx 0.11$ the little hierarchy problem insists, but for $\Omega_{\tilde{\chi}}h^2 \ll 0.11$, we obtain $\mu \approx 0.1$ TeV
- For $40 \leq \tan\beta \leq 51$ we obtain the $h_t/h_T \approx 1 - 1.5$, $h_b/h_T \approx 0.75 - 0.79$, and $h_t/h_b \approx 1.2 - 2.00$. These ratios can be motivated by the embedding of CMSSM in our SUSY Pati-Salam model.
FP Region and Direct Detection of CDM

- In the $\tilde{\tau}_2$-CA Region, $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ implies $m_{\text{LSP}} \gtrsim 746.5$ GeV and therefore the detection of LSP is unlikely.
FP Region and Direct Detection of CDM

- In the $\tilde{\tau}_2$-CA Region, $\text{BR} (B_s \rightarrow \mu^+ \mu^-)$ implies $m_{\text{LSP}} \gtrsim 746.5$ GeV and therefore the detection of LSP is unlikely.
- In the FP Region we take constraints from LUX and achieve predictions for $\sigma_{\tilde{\chi}p}^{\text{SI}}$.
  - For $A_0/M_{1/2} < 0$
  - For $A_0/M_{1/2} > 0$

- The upper boundary curve comes from the LUX data.
- The remaining $\sigma_{\tilde{\chi}p}^{\text{SI}}$'s are predictions of our scheme.
- The entire allowed region may be probed by XENON1T experiment.
- The present bounds on the spin-dependent $\tilde{\chi} - p$ cross section are not as restrictive as bounds from LUX.
FP REGION AND DIRECT DETECTION OF CDM

• In the $\tilde{\tau}_2$-CA REGION, $\text{BR} \left( B_s \rightarrow \mu^+\mu^- \right)$ implies $m_{\text{LSP}} \gtrsim 746.5$ GeV and therefore the detection of LSP is unlikely.

• In the FP REGION we take constraints from LUX and achieve predictions for $\sigma_{\tilde{\chi}p}^{\text{SI}}$.

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The upper boundary curve comes from the LUX data.
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The entire allowed region may be probed by XENON1T experiment.
The present bounds on the spin-dependent $\tilde{\chi} - p$ cross section are not as restrictive as bounds from LUX.

As a bottom line we could say that the FP region is more attractive or even natural than the $\tilde{\tau}_2$-CA REGION.

C. PALLIS  CMSSM WITH GENERALIZED YUKAWA QUASI-UNIFICATION: AN UPDATE
## Summary

CMSSM can become consistent with the data (except for that of $\delta\alpha_\mu$) within two regions of its parameter space:

### Comparison of the allowed regions

<table>
<thead>
<tr>
<th></th>
<th>$\tilde{\tau}_2$-CA Region</th>
<th>FP Region</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{1/2}$/TeV</td>
<td>1.6 – 3</td>
<td>2 – 4</td>
</tr>
<tr>
<td>$m_0$/TeV</td>
<td>1.6 – 3</td>
<td>4 – 16</td>
</tr>
<tr>
<td>$A_0/M_{1/2}$</td>
<td>0.2 – (−3)</td>
<td>(−11) – 15</td>
</tr>
<tr>
<td>$\tan\beta$</td>
<td>43.8 – 52</td>
<td>40 – 50</td>
</tr>
<tr>
<td>$\mu$/TeV</td>
<td>$\geq$ 3</td>
<td>0.1 – 1</td>
</tr>
<tr>
<td>$h_t/h_\tau$</td>
<td>0.98 – 1.29</td>
<td>1 – 1.5</td>
</tr>
<tr>
<td>$h_b/h_\tau$</td>
<td>0.6 – 0.65</td>
<td>0.75 – 0.79</td>
</tr>
<tr>
<td>$h_t/h_b$</td>
<td>1.62 – 2</td>
<td>1.2 – 2</td>
</tr>
<tr>
<td><strong>Spectroscopy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LSP:</td>
<td>Bino</td>
<td>Bino-Higgsino</td>
</tr>
<tr>
<td>NLSP:</td>
<td>$\tilde{\tau}_2$</td>
<td>$\tilde{\chi}_i^\pm$, $\tilde{\chi}_i^0$</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}}$/TeV</td>
<td>0.75 – 1.4</td>
<td>0.1 – 1.08</td>
</tr>
<tr>
<td>$m_{\tilde{t}_{1,2}}$/TeV</td>
<td>3 – 4</td>
<td>6 – 10</td>
</tr>
<tr>
<td>$m_{\tilde{u},\tilde{d}}$/TeV</td>
<td>4 – 6</td>
<td>10 – 12</td>
</tr>
<tr>
<td>$m_{H^+,H^0}$/TeV</td>
<td>2.1 – 2.6</td>
<td>2 – 6</td>
</tr>
<tr>
<td>$m_{\tilde{g}}$/TeV</td>
<td>5 – 6</td>
<td>5 – 8</td>
</tr>
<tr>
<td><strong>Predictions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_\chi h^2$ Tunings</td>
<td>$\Delta_{\tilde{\tau}_2} \approx 0$, $\Delta_H \approx 0$</td>
<td>$\Delta_{\tilde{\chi}_i^\pm} \approx 0$</td>
</tr>
<tr>
<td>$\sigma^{SI}_{\tilde{\chi}_i^0}$/pb</td>
<td>$10^{-12}$</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>$\delta a_\mu/10^{-10}$</td>
<td>(0.35 – 2.76)</td>
<td>(0.04 – 0.27)</td>
</tr>
</tbody>
</table>

## Final Remarks

1. For $\Omega_\chi h^2 \ll 0.12$, the naturality of the FP region increases, since $\mu \sim (100 \pm 20)$ GeV.
2. The sparticle spectra – besides $\tilde{\chi}$ – in the FP region are more heavy than the ones in the $\tilde{\tau}_2$-CA region.
3. The allowed FP region is more ample and natural than the $\tilde{\tau}_2$-CA region.
4. The obtained $h_t/h_\tau$’s for $m, n = t, b, \tau$ can be motivated by the embedding of CMSSM in our SUSY Pati-Salam model.
### Summary

CMSSM can become consistent with the data (except for that of $\delta \alpha_{\mu}$) within two regions of its parameter space:

#### Comparison of the Allowed Regions

<table>
<thead>
<tr>
<th></th>
<th>$\tilde{\tau}_2$-CA Region</th>
<th>FP Region</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model Parameters</strong></td>
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<td></td>
</tr>
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<td>$M_{1/2}$/TeV</td>
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<td>(−11) – 15</td>
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<td>40 – 50</td>
</tr>
<tr>
<td>$\mu$/TeV</td>
<td>≥ 3</td>
<td>0.1 – 1</td>
</tr>
<tr>
<td>$h_t/h_\tau$</td>
<td>0.98 – 1.29</td>
<td>1 – 1.5</td>
</tr>
<tr>
<td>$h_b/h_\tau$</td>
<td>0.6 – 0.65</td>
<td>0.75 – 0.79</td>
</tr>
<tr>
<td>$h_t/h_b$</td>
<td>1.62 – 2</td>
<td>1.2 – 2</td>
</tr>
<tr>
<td><strong>Spectroscopy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LSP:</td>
<td>Bino</td>
<td>Bino-Higgsino</td>
</tr>
<tr>
<td>NLSP:</td>
<td>$\tilde{\tau}_2$</td>
<td>$\tilde{\chi}_i^\pm$, $\tilde{\chi}_i^0$</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}}$/TeV</td>
<td>0.75 – 1.4</td>
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</tr>
<tr>
<td>$m_{\tilde{\tau}_{1,2}}$/TeV</td>
<td>3 – 4</td>
<td>6 – 10</td>
</tr>
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<td>4 – 6</td>
<td>10 – 12</td>
</tr>
<tr>
<td>$m_{H,A,H\pm}$/TeV</td>
<td>2.1 – 2.6</td>
<td>2 – 6</td>
</tr>
<tr>
<td>$m_{\tilde{\gamma}}$/TeV</td>
<td>5 – 6</td>
<td>5 – 8</td>
</tr>
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</table>

#### Predictions

<table>
<thead>
<tr>
<th>$\Omega_\chi h^2$ Tunings</th>
<th>$\Delta \tilde{\tau}_2 \approx 0$, $\Delta H \approx 0$</th>
<th>$\Delta \tilde{\chi}_i^\pm \approx 0$</th>
</tr>
</thead>
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<tr>
<td>$\sigma^{SI}_{\tilde{\chi}P}$/pb ~</td>
<td>10$^{-12}$</td>
<td>10$^{-9}$</td>
</tr>
<tr>
<td>$\delta a_\mu/10^{-10}$ ≥ 7.5</td>
<td>(0.35 – 2.76)</td>
<td>(0.04 – 0.27)</td>
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### Final Remarks

1. For $\Omega_\chi h^2 \ll 0.12$, the **naturalness** of the FP region increases, since $\mu \sim (100 \pm 20)$ GeV.
2. The sparticle spectra – besides $\tilde{\chi}$ – in the FP region are **more heavy** than the ones in the $\tilde{\tau}_2$-CA region.
3. The allowed FP region is more **ample & natural** than the $\tilde{\tau}_2$-CA region.
4. The obtained $h_t/h_\tau$’s for $m, n = t, b, \tau$ can be **motivated** by the embedding of CMSSM in our SUSY Pati-Salam model.
5. Although the deviation of $\nu$ is not so small, the obtained $h_t/h_\tau$’s are much **closer to unity** than in generic models with lower values of $\tan \beta$ – e.g. we obtain $h_t/h_b = 10$ (!) for $\tan \beta = 10$.  

C. Pallis

**CMSSM with Generalized Yukawa Quasi-Unification: An Update**
SUMMARY

CMSSM CAN BECOME CONSISTENT WITH THE DATA (EXCEPT FOR THAT OF $\delta \alpha_\mu$) WITHIN TWO REGIONS OF ITS PARAMETER SPACE:

**COMPARISON OF THE ALLOWED REGIONS**

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<th>$\tilde{\tau}_2$-CA Region</th>
<th>FP Region</th>
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<tr>
<td>$M_{1/2}$/TeV</td>
<td>1.6 – 3</td>
<td>2 – 4</td>
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<td>$m_0$/TeV</td>
<td>1.6 – 3</td>
<td>4 – 16</td>
</tr>
<tr>
<td>$A_0/M_{1/2}$</td>
<td>0.2 – (–3)</td>
<td>(–11) – 15</td>
</tr>
<tr>
<td>$\tan \beta$</td>
<td>43.8 – 52</td>
<td>40 – 50</td>
</tr>
<tr>
<td>$\mu$/TeV</td>
<td>≥ 3</td>
<td>0.1 – 1</td>
</tr>
<tr>
<td>$h_t/h_\tau$</td>
<td>0.98 – 1.29</td>
<td>1 – 1.5</td>
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6. To Obtain a Fulfilment of the Constraints Above An Amount of Tuning ($\mu$ Determination, $\Delta_{\tilde{\tau}_2} = 0$ and $\Delta_H = 0$ or $\Delta_{\tilde{\chi}_1}^{\pm} \approx 0$) Is Required Which, Though, Is Significantly Lower Than The Amount Of Tuning Which Is Removed By SUSY.

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$^a$S. Antusch et al. (2012).
BACK-UP: TESTING YUKAWA QUASI-UNIFICATION

We present the complex parameters $\alpha_1$, $\alpha_2$ for various $\rho$'s corresponding to $h_b/h_\tau = 0.618$ and $h_t/h_\tau = 1.079$.

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