Induced-Gravity Inflation in Sugra Confronted With Planck13 & Bicep2

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Based on:

- C.P., *J. Cosmol. Astropart. Phys.* **04**, *024 (2014)* [arXiv:1312.3623];
- C.P., *J. Cosmol. Astropart. Phys.* **08**, *057 (2014)* [arXiv:1403.5486];
- C.P., arXiv:1407.8522.

Outline

Coupling non-Minimally the Inflaton to Gravity

• Our Starting Point is The Action in the **Jordan Frame** Of A Scalar Field ϕ with Potential $V(\phi)$ non-Minimally Coupled to the Ricci Scalar Curvature, $\mathcal{R}$, Through A Frame Function $f_{\mathcal{R}}(\phi)$ (JF). This is:

$$\mathcal{S} = \int d^4x\sqrt{-\mathfrak{g}}\left(-\frac{1}{2}m_{\rm P}^2 f_{\mathcal{R}}(\phi)\mathcal{R} + \frac{f_{\rm K}(\phi)}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right),$$

where $\mathfrak{g}$ is the Determinant Of The Background Friedmann-Robertson-Walker Metric and $f_{\mathcal{R}}(\langle\phi\rangle) \simeq 1$ to Guarantee the Ordinary **Einstein Gravity** At Low Energy. We Allow Also for a Kinetic Mixing Through the Function $f_{\rm K}(\phi)$.

[1] K. Maeda (1989), D.S. Salopek, J.R. Bond and J.M. Bardeen (1989); D.I. Kaiser (1995); T. Chiba and M. Yamaguchi (2008).

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• We can Write $\mathcal{S}$ in the **Einstein Frame** (EF) as follows

$$\mathcal{S}=\int d^4x\sqrt{-\widehat{\mathfrak{g}}}\left(-\frac{1}{2}m_{\rm P}^2\widehat{\mathcal{R}}+\frac{1}{2}\widehat{g}^{\mu\nu}\partial_\mu\widehat{\phi}\partial_\nu\widehat{\phi}-\widehat{V}\left(\widehat{\phi}\right)\right)$$

Performing a **Conformal Transformation**[1] According Which We Define the EF Metric:

$$\widehat{g}_{\mu\nu}=f_{\mathcal{R}}\,g_{\mu\nu}\quad\Rightarrow\quad\begin{cases}\sqrt{-\widehat{\mathfrak{g}}}=f_{\mathcal{R}}^2\sqrt{-\mathfrak{g}}\quad\text{and}\quad\widehat{g}^{\mu\nu}=g^{\mu\nu}/f_{\mathcal{R}},\\ \widehat{\mathcal{R}}=\left(\mathcal{R}+3\Box\ln f_{\mathcal{R}}+3g^{\mu\nu}\partial_\mu f_{\mathcal{R}}\partial_\nu f_{\mathcal{R}}/2f_{\mathcal{R}}^2\right)/f_{\mathcal{R}}\end{cases}$$

and Introduce the **EF Canonically Normalized Field**, $\widehat{\phi}$, and Potential, $\widehat{V}$, Defined As Follows:

$$\left(\frac{d\widehat{\phi}}{d\phi}\right)^2=J^2=\frac{f_{\rm K}}{f_{\mathcal{R}}}+\frac{3}{2}m_{\rm P}^2\left(\frac{f_{\mathcal{R},\phi}}{f_{\mathcal{R}}}\right)^2\quad\text{and}\quad\widehat{V}(\widehat{\phi})=\frac{V\left(\widehat{\phi}(\phi)\right)}{f_{\mathcal{R}}\left(\widehat{\phi}(\phi)\right)^2}.$$

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- For Convenient Choices of $V(\phi)$ and $f_{\mathcal{R}}(\phi)$ we can Obtain $\widehat{V}(\phi)$ **Suitable** for Observationally Compatible Inflation.
- The Analysis of IGI in **the EF** Using The Standard Slow-Roll Approximation is Equivalent With **The Analysis in JF**.

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The Idea of Induced Gravity

The Functional forms of $f_{\mathcal{R}}$ Can be Distinguished into **two Categories**:

$$f_{\mathcal{R}} = \begin{cases} 1 + \tilde{f}_{\mathcal{R}}, & \text{with } \tilde{f}_{\mathcal{R}}(\langle\phi\rangle) \simeq 0 \Leftrightarrow \langle\phi\rangle \simeq 0 \quad : \text{Non-Minimal Inflation (nMI)}; \\ \tilde{f}_{\mathcal{R}}, & \text{with } \tilde{f}_{\mathcal{R}}(\langle\phi\rangle) = 1 \Leftrightarrow \langle\phi\rangle \neq 0 \quad : \text{Induced-Gravity Inflation (IGI)}; \end{cases}$$

In the **IGI**, Einstein Gravity is Generated at the SUSY Vacuum by $\langle\phi\rangle$. As A Consequence The Theory is **Unitarity-Safe !!!**

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The Ultraviolet (UV) Cut-off Scale

- If we Use $\tilde{f}_{\mathcal{R}}=c_{\mathcal{R}}\phi^2/m_{\rm P}^2$ and Expand $g_{\mu\nu}$ About The Flat Spacetime Metric $\eta_{\mu\nu}$ and ϕ Abound Its V.E.V As Follows

$$g_{\mu\nu}\simeq\eta_{\mu\nu}+h_{\mu\nu}/m_{\rm P}\quad\text{and}\quad\phi=\langle\phi\rangle+\delta\phi\quad\text{Where}\quad\langle\phi\rangle\simeq 0\ \ [m_{\rm P}/\sqrt{c_{\mathcal{R}}}]\quad\text{for nMI [IGI]}.$$

The Lagrangian Corresponding To The Two First Terms In The Right-Hand Side Of $\mathcal{S}$ Takes The Form

$$\delta\mathcal{L}=-\frac{\langle f_{\mathcal{R}}\rangle}{4}F_{\rm EH}\left(h^{\mu\nu}\right)+\frac{1}{2}\partial_\mu\delta\phi\partial^\mu\delta\phi+\left(m_{\rm P}\langle f_{\mathcal{R},\phi}\rangle+c_{\mathcal{R}}\frac{\delta\phi}{m_{\rm P}}\right)F_{\mathcal{R}}\delta\phi+\cdots=-\frac{1}{8}F_{\rm EH}\left(\bar{h}^{\mu\nu}\right)+\frac{1}{2}\partial_\mu\overline{\delta\phi}\partial^\mu\overline{\delta\phi}+\frac{c_{\mathcal{R}}}{\sqrt{2}m_{\rm P}}\frac{\sqrt{\langle f_{\mathcal{R}}\rangle}}{\langle\bar{f}_{\mathcal{R}}\rangle}\overline{\delta\phi}^2\,\Box\bar{h}+\cdots$$

Where The Functions $F_{\rm EH}$ and $F_{\mathcal{R}}$ Read $F_{\rm EH}=h^{\mu\nu}\Box h_{\mu\nu}-h\Box h+2\partial_\rho h^{\mu\rho}\partial^\nu h_{\mu\nu}-2\partial_\nu h^{\mu\nu}\partial_\mu h$ and $F_{\mathcal{R}}=\Box h-\partial_\mu\partial_\nu h^{\mu\nu}$

And The **JF Canonically Normalized Fields** $\bar{h}_{\mu\nu}$ and $\overline{\delta\phi}$ Are Defined By The Relations

$$\overline{\delta\phi}=\sqrt{\frac{\langle\bar{f}_{\mathcal{R}}\rangle}{\langle f_{\mathcal{R}}\rangle}}\delta\phi\quad\text{and}\quad\frac{\bar{h}_{\mu\nu}}{\sqrt{2}}=\sqrt{\langle f_{\mathcal{R}}\rangle}\,h_{\mu\nu}+\frac{m_{\rm P}\langle f_{\mathcal{R},\phi}\rangle}{\sqrt{\langle f_{\mathcal{R}}\rangle}}\eta_{\mu\nu}\delta\phi\quad\text{with}\quad\bar{f}_{\mathcal{R}}=f_{\mathcal{R}}+\frac{3}{2}m_{\rm P}^2 f_{\mathcal{R},\phi}^2.$$

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Where The Functions $F_{\rm EH}$ and $F_{\mathcal{R}}$ Read $F_{\rm EH}=h^{\mu\nu}\Box h_{\mu\nu}-h\Box h+2\partial_\rho h^{\mu\rho}\partial^\nu h_{\mu\nu}-2\partial_\nu h^{\mu\nu}\partial_\mu h$ and $F_{\mathcal{R}}=\Box h-\partial_\mu\partial_\nu h^{\mu\nu}$

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- The **Problematic Scattering Amplitude** Is Written In Terms Of The Center-Of-Mass Energy E As Follows

$$\mathcal{A}\sim\left(\frac{E}{\Lambda_{\rm UV}}\right)^2 \quad \text{With} \quad \Lambda_{\rm UV}=\frac{2m_{\rm P}}{c_{\mathcal{R}}}\frac{\langle\bar f_{\mathcal{R}}\rangle}{\sqrt{\langle f_{\mathcal{R}}\rangle}}=\frac{m_{\rm P}}{c_{\mathcal{R}}}\left(\frac{1}{\sqrt2}+3\sqrt2 m_{\rm P}^2\langle f_{\mathcal{R},\phi}\rangle^2\right)\sim\begin{cases}m_{\rm P}/c_{\mathcal{R}}, & \text{for nMI, Since } \langle f_{\mathcal{R},\phi}\rangle\simeq 0;\\ m_{\rm P}, & \text{for IGI, Since } \langle f_{\mathcal{R},\phi}\rangle^2\simeq c_{\mathcal{R}};\end{cases}$$

Recall that $\mathcal{A}$ Remains Within The Validity Of The Perturbation Theory Provided That $E<\Lambda_{\rm UV}$.

- The Same Results Can Be Obtained in the **EF** Expanding $J\dot\phi$ and $\widehat V$ In terms of $\dot{\widehat{\delta\phi}}$ and $\widehat{\delta\phi}$.

Inflationary Observables - Requirements

- The **Number of e-foldings**, $\widehat{N}_\star$, that the Scale $k_\star = 0.05/\mathrm{Mpc}$ Suffers During nMI has to be Sufficient to Resolve the Horizon and Flatness Problems of Standard Big Bang:

$$\widehat{N}_\star = \frac{1}{m_\mathrm{P}^2}\int_{\widehat{\phi}_\mathrm{f}}^{\widehat{\phi}_\star} d\widehat{\phi}\,\frac{\widehat{V}}{\widehat{V}_{,\widehat{\phi}}} = \frac{1}{m_\mathrm{P}^2}\int_{\phi_\mathrm{f}}^{\phi_\star} d\phi\, J^2\,\frac{\widehat{V}}{\widehat{V}_{,\phi}} \simeq 19.4 + 2\ln\frac{V(\phi_\star)^{1/4}}{1\ \mathrm{GeV}} - \frac{4}{3}\ln\frac{V(\phi_\mathrm{f})^{1/4}}{1\ \mathrm{GeV}} + \frac{1}{3}\ln\frac{T_\mathrm{rh}}{1\ \mathrm{GeV}} + \frac{1}{2}\ln\frac{f_\mathcal{R}(\phi_\mathrm{f})}{f_\mathcal{R}(\phi_\star)},$$

Where $\phi_\star$ $[\widehat{\phi}_\star]$ is The Value of ϕ $[\widehat{\phi}]$ When $k_\star$ Crosses Outside The Inflationary Horizon;
ϕ_f $[\widehat{\phi}_\mathrm{f}]$ is the Value of ϕ $[\widehat{\phi}]$ at the end of IGI Which Can Be Found From The Condition

$$\max\{\widehat{\epsilon}(\phi_\mathrm{f}), |\widehat{\eta}(\phi_\mathrm{f})|\} = 1, \quad \text{With } \widehat{\epsilon} = \frac{m_\mathrm{P}^2}{2}\left(\frac{\widehat{V}_{,\widehat{\phi}}}{\widehat{V}}\right)^2 = \frac{m_\mathrm{P}^2}{2J^2}\left(\frac{\widehat{V}_{,\phi}}{\widehat{V}}\right)^2 \quad \text{and} \quad \widehat{\eta} = m_\mathrm{P}^2\,\frac{\widehat{V}_{,\widehat{\phi}\widehat{\phi}}}{\widehat{V}} = \frac{m_\mathrm{P}^2}{J^2}\left(\frac{\widehat{V}_{,\phi\phi}}{\widehat{V}} - \frac{\widehat{V}_{,\phi}}{\widehat{V}}\frac{J_{,\phi}}{J}\right).$$

[2] *B. Audren, D.G. Figueroa and T. Tram, (2014); H. Liu, P. Mertsch and S. Sarkar (2014); R. Flauger, J.C. Hill and D.N. Spergel (2014).*

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- The **Amplitude of the Power Spectrum** $A_{\rm s}$ of the Curvature Perturbations is To Be Consistent with PLANCK Data:

$$A_{\rm s}^{1/2} = \frac{1}{2\sqrt{3}\,\pi m_{\rm P}^3}\,\frac{\widehat V(\widehat\phi_\star)^{3/2}}{|\widehat V_{,\widehat\phi}(\widehat\phi_\star)|} = \frac{|J(\phi_\star)|}{2\sqrt{3}\,\pi m_{\rm P}^3}\,\frac{\widehat V(\phi_\star)^{3/2}}{|\widehat V_{,\phi}(\phi_\star)|} = 4.685\cdot 10^{-5}$$

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- The (Scalar) Spectral Index, $n_{\rm s}$, Its Running, $a_{\rm s}$, And The Scalar-To-Tensor Ratio r are to be Consistent With the Fitting of the **PLANCK** Results by the **ΛCDM** Model (at 95% c.l.):

$$n_{\rm s}=1-6\widehat{\epsilon}_\star+2\widehat{\eta}_\star=0.96\pm0.014,\quad -0.0314\leq\alpha_{\rm s}=2\left(4\widehat{\eta}_\star^2-(n_{\rm s}-1)^2\right)/3-2\widehat{\xi}_\star\leq0.0046\quad \text{and}\quad r=16\widehat{\epsilon}_\star<0.135,$$

Where $\widehat{\xi}=m_{\rm P}^4\widehat{V}_{,\widehat{\phi}}\widehat{V}_{,\widehat{\phi}\widehat{\phi}\widehat{\phi}}/\widehat{V}^2=m_{\rm P}^2\,\widehat{V}_{,\phi}\,\widehat{\eta}_{,\phi}/\widehat{V}\,J^2+2\widehat{\eta}\widehat{\epsilon}$ And The Variables With Subscript $\star$ Are Evaluated at $\phi=\phi_\star$.

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- If The Detected **B-Mode** In The Polarization Of The Cosmic Microwave Background Radiation Observed By **Bicep2** Is Attributed To **Gravity Waves** Predicted By IGI, We Can Find A Simultaneously Compatible Region[2]

$$0.06 \lesssim r \lesssim 0.135 \ \ \text{at 95\%c.l.} - \text{Bicep2 Only Requires:}\ \ r = 0.16^{+0.06}_{-0.05}.$$

[2] B. Audren, D.G. Figueroa and T. Tram, (2014); H. Liu, P. Mertsch and S. Sarkar (2014); R. Flauger, J.C. Hill and D.N. Spergel (2014).

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The Quartic Model Can Be **Revitalized** In the Context of Induced Gravity

The Quartic Potential, $V = \lambda\phi^4/4$ – Or Even $V = \lambda\left(\phi^2 - M^2\right)^2/4$ with $M \simeq 10^{16}$ GeV $\ll m_{\rm P}$

- If We Adopt $f_{\mathcal{R}}(\phi) = 1 + c_{\mathcal{R}}(\phi/m_{\rm P})^2$, **Recovering Einstein Gravity** at the Vacuum Implies $f_{\mathcal{R}}(\langle\phi\rangle) = 1 \Rightarrow \langle\phi\rangle = M \simeq 0.$

For $c_{\mathcal{R}} \gg 1$, $\widehat{V} = \frac{\lambda\phi^4}{4f_{\mathcal{R}}^2} \simeq \widehat{V}_0 = \frac{\lambda m_{\rm P}^4}{4c_{\mathcal{R}}^2}$, $\widehat{\epsilon} \simeq \frac{4m_{\rm P}^4}{3c_{\mathcal{R}}^2\phi^4}$ and $\widehat{\eta} \simeq -\frac{4m_{\rm P}^2}{3c_{\mathcal{R}}\phi^2}$

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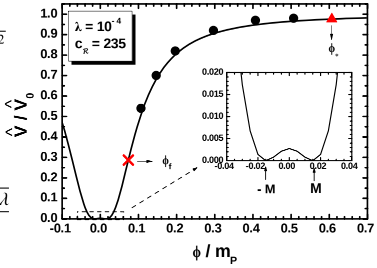
$\widehat{N}_\star \simeq 3c_{\mathcal{R}}\phi_\star^2/4m_{\rm P}^2 \Rightarrow \phi_\star = 2m_{\rm P}\sqrt{\widehat{N}_\star/3c_{\mathcal{R}}}$ $< m_{\rm P}$ for $c_{\mathcal{R}} > 80$.

For $\widehat{N}_\star \simeq 52$, $A_{\rm s}^{1/2} \simeq \frac{\sqrt{\lambda}\widehat{N}_\star}{6\sqrt{2}\pi c_{\mathcal{R}}} = 4.685\cdot10^{-5} \Rightarrow \underline{c_{\mathcal{R}} \simeq 41850\sqrt{\lambda}}$

$n_{\rm s} \simeq 1 - 2/\widehat{N}_\star \simeq 0.965, \quad \alpha_{\rm s} \simeq -2/\widehat{N}_\star^2 \simeq -6.4\cdot10^{-4}$

Also $r \simeq 12/\widehat{N}_\star^2 \simeq 4\cdot10^{-3}$ **In Agreement With PLANCK.**



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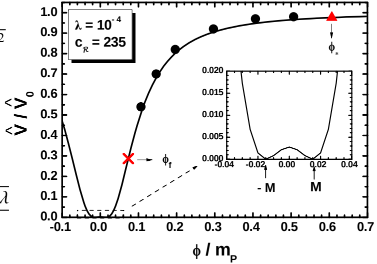

- Analyzing The Small-Field ($\delta\phi = \phi - M \simeq 0$) Behavior Of $\mathcal S$ We Find $\Lambda_{\rm UV} = m_{\rm P}/c_{\mathcal R}$ Since $\widehat{\delta\phi}/\delta\phi = \sqrt{1 + 6c_{\mathcal R}^2M^2/m_{\rm P}^2}$,

$$J^2\dot\phi = \left(1 + \frac{12c_{\mathcal R}^2M\widehat{\delta\phi}}{m_{\rm P}^2} + \frac{6c_{\mathcal R}^2\widehat{\delta\phi}^2}{m_{\rm P}^2} - \frac{48c_{\mathcal R}^3M\widehat{\delta\phi}^3}{m_{\rm P}^4} + \cdots\right)\dot{\widehat{\delta\phi}} \text{ and } \widehat V = \lambda M^2\widehat{\delta\phi}^2\left(1 + \frac{\widehat{\delta\phi}}{M} - 6\frac{c_{\mathcal R}\widehat{\delta\phi}^2}{m_{\rm P}^2} - 3\frac{c_{\mathcal R}^2\widehat{\delta\phi}^2}{m_{\rm P}^2}\right)$$

- Imposing $\phi \leq m_{\rm P}$ & $\widehat V(\phi_\star)^{1/4} \leq \Lambda_{\rm UV}$ We End Up With $74 \simeq 4\widehat N/3 \lesssim c_{\mathcal R} \lesssim 300$ and $0.3 \lesssim \lambda/10^{-5} \lesssim 4.6$.
- The Mass of the Inflaton at the Vacuum is: $\widehat m = \left\langle\widehat V_{,\widehat{\phi}\widehat{\phi}}\right\rangle^{1/2} = \left\langle\widehat V_{,\phi\phi}/J^2\right\rangle^{1/2} = \sqrt{\lambda}m_{\rm P}/\sqrt{3}c_{\mathcal R} \simeq 3\cdot10^{13}$ GeV.

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- If $f_{\mathcal{R}}(\phi) = c_{\mathcal{R}}(\phi/m_{\rm P})^2$, **Recovering Einstein Gravity** at the Vacuum Implies[3] $f_{\mathcal{R}}(\langle\phi\rangle) = 1 \Rightarrow \langle\phi\rangle = M = m_{\rm P}/\sqrt{c_{\mathcal{R}}}$.
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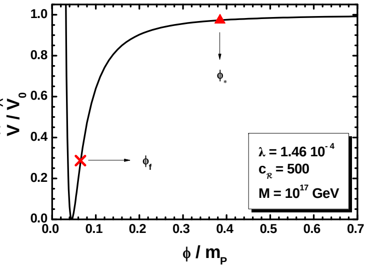
$n_{\rm s} \simeq 1-2/\widehat{N}_\star \simeq 0.962, \alpha_{\rm s} \simeq -2/\widehat{N}_\star^2 \simeq -7\cdot 10^{-4}, r \simeq 12/\widehat{N}_\star^2 \simeq 4\cdot 10^{-3}$



3 A. Zee (1979); R. Fakir and W.G. Unruh (1990); N. Kaloper, L. Sorbo and J. Yokoyama (2008); C.P. (2013); G.Giudice and H.M. Lee (2014).

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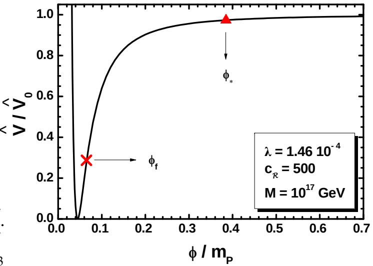

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Is Possible To Obtain A Realization Of IGI Within SUGRA? Are We Able to Approach Bicep2 Results?

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Therefore, Implementing IGI Within SUGRA Requires The Appropriate Selection of W and K

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- We Observe that Ω Enters The Kinetic Terms of the Φ^α's too. $\mathcal{S}$ **Can Exhibit Non-Minimal Couplings** of Φ^α's to $\mathcal{R}$ If
  - $\mathcal{A}_\mu=0$ Where $\mathcal{A}_\mu=-i(1+m)m_{\rm P}^2\left(\partial_\mu\Phi^\alpha\Omega_\alpha-\partial_\mu\Phi^{*\bar\alpha}\Omega_{\bar\alpha}\right)/2\Omega$. This Happens When $\Phi^\alpha=|\Phi^\alpha|$ or $\Phi^\alpha=0$ During IGI;
  - Ω Consists of an **Holomorphic** $\Omega_{\rm H}$ and a **Kinetic** $\Omega_{\rm K}$ Part, With $\Omega_{\rm H}\gg\Omega_{\rm K}\simeq\delta_{\alpha\beta}\Phi^\alpha\Phi^\beta/m_{\rm P}^2$:

$$\Omega=\Omega_{\rm K}-3(1+m)\left(\Omega_{\rm H}(\Phi^\alpha)+\Omega^*_{\rm H}(\Phi^{*\bar\alpha})\right)\Rightarrow K=-3(1+m)m_{\rm P}^2\ln\left(\Omega_{\rm H}(\Phi^\alpha)+\Omega^*_{\rm H}(\Phi^{*\bar\alpha})-\Omega_{\rm K}/3(1+m)\right),\quad \text{Where}$$

$$\Omega_{\rm K}\left(|\Phi^\alpha|^2\right)=|\Phi^\alpha|^2/m_{\rm P}^2+k_{\Phi^\alpha\Phi^\beta}|\Phi^\alpha|^2|\Phi^\beta|^2/m_{\rm P}^4\quad(\text{Terms }\Phi^*_\alpha\Phi^\beta\text{ With }\alpha\neq\beta\text{ Can Be Forbidden})$$

  With $k_{\Phi^\alpha\Phi^\beta}\sim1$. The terms $|\Phi^\alpha|^2|\Phi^\beta|^2/m_{\rm P}^4$ Are Included In Order To Evade A **Tachyonic Instability** Occurring Along This Direction of The "Stabilized" (Non-Inflaton) Field.
  - For $m=0$ Canonical Terms for Φ_α's are Obtained But IGI can be also Obtained for $m\neq0$.

[4] M.B. Einhorn and D.R.T. Jones (2010); S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen (2010, 2011); H.M. Lee (2010).

SELECTING CONVENIENTLY THE SUPERPOTENTIAL AND FRAME FUNCTION

- **RECOVERING EINSTEIN GRAVITY** AT LOW ENERGIES DICTATES $\langle\Omega_{\rm H}\rangle + \langle\Omega^*_{\rm H}\rangle = 1 \Rightarrow \langle\Omega_{\rm H}\rangle = 1/2$ FOR $\langle\Omega_{\rm H}\rangle = \langle\Omega^*_{\rm H}\rangle$ AND $\langle\Omega_{\rm K}\rangle \sim 0$.
- WE USE 2 SUPERFIELDS $\Phi^1 = \Phi$ (**INFLATON**) AND $\Phi^2 = S$ (**"STABILIZED" FIELD**) AND ADOPT THE FOLLOWING SUPERPOTENTIAL:

$$W = \frac{\lambda m_{\rm P}^2}{c_{\mathcal R}} S\left(\Omega_{\rm H} - 1/2\right) \quad \text{WITH} \quad \Omega_{\rm H}(\Phi) = c_{\mathcal R}\frac{\Phi^2}{m_{\rm P}^2} + \sum_{k=1}^{\infty}\lambda_k\frac{\Phi^{4k}}{m_{\rm P}^{4k}} \simeq c_{\mathcal R}\frac{\Phi^2}{m_{\rm P}^2} \quad \text{FOR} \quad \Phi < m_{\rm P} \quad \text{AND} \quad \lambda_k \ll 1$$

- W IS INVARIANT UNDER A GLOBAL $\mathbb{Z}_2$ SYMMETRY AND RESPECTS A CONTINUOUS R SYMMETRY, I.E.,

$$W \to W \quad \text{FOR} \quad \Phi \to -\Phi, \quad \text{AND} \quad W \to e^{i\varphi}\, W \quad \text{FOR} \quad S \to e^{i\varphi}\, S \quad \text{AND} \quad \Omega_{\rm H} \to \Omega_{\rm H}\,.$$

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In The Following We Show Details on the Realization of These Three Models.

Definition of the Model

- The Model is Determined by Imposing the $U(1)_R$ and $\mathbb{Z}_2$ Symmetries and a **No-Scale-Type Symmetry** on the Kähler Manifold.

$$\text{i.e.,}\quad W = \lambda m_{\rm P}^2 S(\Omega_{\rm H} - 1/2)/c_{\mathcal{R}} \quad \text{and} \quad K = -3m_{\rm P}^2 \ln\left(\Omega_{\rm H}(\Phi) + \Omega_{\rm H}^*(\Phi^*) - \frac{|S|^2}{3m_{\rm P}^2} + k_S\frac{|S|^4}{3m_{\rm P}^4}\right)$$

- The Adoption of a **No-Scale-Type Symmetry** Assists Us to Avoid $|\Phi|^2$ and $|\Phi|^2|S|^2$ Terms, i.e. $\Omega_{\rm K} = |S|^2/3m_{\rm P}^2 - k_S|S|^4/3m_{\rm P}^4$.
- This Model has 2 Essential Modifications Respecting the **Cecotti Model**[5] Which is Equivalent to $\mathcal{R}+\mathcal{R}^2$ SUGRA:

$$W = \lambda m_{\rm P} S(\Phi - m_{\rm P}) \quad \text{and} \quad K = -3m_{\rm P}^2 \ln\left(\frac{\Phi+\Phi^*}{m_{\rm P}} - \frac{|S|^2}{3m_{\rm P}^2} + k_S\frac{|S|^4}{3m_{\rm P}^4}\right)$$

  - We Use Φ^2 and not Φ. No Symmetry Can Prohibit Φ^2 from W and K;
  - Thanks to $c_{\mathcal{R}}$ Involved We Achieve IGI Even With **Sub-Planckian** Φ's (Which Though Result to $\hat\Phi \gg m_{\rm P}$).

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[6] J.R. Ellis, C. Kounnas and D.V. Nanopoulos (1984); J. Ellis, D.V. Nanopoulos and K.A. Olive (2013).

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Connection to the no-Scale SUGRA

- If we Perform an Analytic Transformation[6]:

$$\Omega_{\rm H}(\Phi) = \frac{1 - y_0/\sqrt{3}m_{\rm P}}{1 + y_0/\sqrt{3}m_{\rm P}} \quad \text{And} \quad S = \frac{y_1/m_{\rm P}}{1 + y_0/\sqrt{3}m_{\rm P}}, \quad K \text{ Becomes: } K \simeq \underbrace{-3m_{\rm P}^2 \ln\left(1 - \frac{|y_0|^2 + |y_1|^2)}{3m_{\rm P}^2}\right)}_{K_0} + 3m_{\rm P}^2 \ln\left|1 + \frac{y_0}{\sqrt{3}m_{\rm P}}\right|^2.$$

- The Choices (K, W) and (K_0, W_0) Are Equivalent Since the SUGRA Lagrangian Only Depends On The Combination

$$\mathcal{G} = K/m_{\rm P}^2 + \ln|W/m_{\rm P}^3| = K_0/m_{\rm P}^2 + \ln|W_0/m_{\rm P}^3| \quad \text{With} \quad W_0 = (1 + y_0/\sqrt{3}m_{\rm P})^3 W.$$

- Therefore, K_0 Corresponds to a $SU(2,1)/SU(2)\times U(1)_R\times\mathbb{Z}_2$ Globally Symmetric Kähler Manifold.

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The F-term SUGRA Potential

- For $S = 0, \quad \theta = \arg\Phi = 0$ and $c_\Phi \gg 1$, $\widehat{V} = \widehat{V}_{\rm I0}$ and the Corresponding Hubble Parameter $\widehat{H}_{\rm I}$ Become (Almost Constant):

$$\widehat{V}_{\rm I0} = \frac{\lambda^2 m_{\rm P}^4 (1-2\Omega_{\rm H})^2}{4c_{\mathcal{R}}^2 x_\phi^4} = \frac{\lambda^2 m_{\rm P}^4 f_\Phi^2}{4c_{\mathcal{R}}^4 x_\phi^4} \quad \text{and} \quad \widehat{H}_{\rm I} = \frac{\widehat{V}_{\rm I0}^{1/2}}{\sqrt{3}m_{\rm P}} \simeq \frac{\lambda m_{\rm P}}{2\sqrt{3}c_{\mathcal{R}}^2}, \quad \text{With} \;\; f_\Phi = 1 - c_{\mathcal{R}} x_\phi^2 \;\; \text{and} \;\; x_\phi = \phi/m_{\rm P}.$$

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$$\widehat{V}_{\rm I0} = \frac{\lambda^2 m_{\rm P}^4 (1-2\Omega_{\rm H})^2}{4c_{\mathcal{R}}^2 x_\phi^4} = \frac{\lambda^2 m_{\rm P}^4 f_\Phi^2}{4c_{\mathcal{R}}^4 x_\phi^4} \quad \text{and} \quad \widehat{H}_{\rm I} = \frac{\widehat{V}_{\rm I0}^{1/2}}{\sqrt{3}m_{\rm P}} \simeq \frac{\lambda m_{\rm P}}{2\sqrt{3}c_{\mathcal{R}}^2}, \quad \text{With } f_\Phi = 1 - c_{\mathcal{R}} x_\phi^2 \text{ and } x_\phi = \phi/m_{\rm P}.$$

- Expanding Φ and S as Follows:

$$\Phi = \frac{\phi e^{i\theta/m_{\rm P}}}{\sqrt{2}} \quad \text{and} \quad S = \frac{s_1 + is_2}{\sqrt{2}},$$

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$$\frac{d\widehat{\phi}}{d\phi} = J = \sqrt{6}/x_\phi \;\Rightarrow\; \widehat{\phi} = \widehat{\phi}_{\rm c} + \sqrt{6}m_{\rm P}\ln\frac{\phi}{\langle\phi\rangle} \quad \text{with} \quad \langle\phi\rangle = \frac{\sqrt{2}m_{\rm P}}{\sqrt{2c_{\mathcal{R}}}}, \quad \widehat{\theta} \simeq \sqrt{6}\theta \quad \text{and} \quad \widehat{s_i} \simeq s_i/\sqrt{c_{\mathcal{R}}} \quad \text{with} \quad i = 1,2$$

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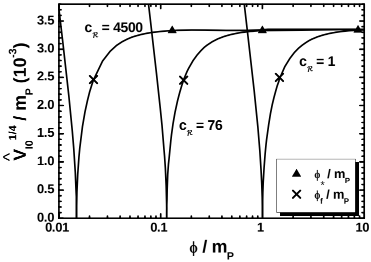
The Mass Spectrum Along The Inflationary Trajectory

Fields	Eingestates	Masses Squared
1 real scalar	$\widehat{\theta}$	$\widehat{m}_\theta^2 = \lambda^2 m_{\rm P}^2 (f_\Phi + 2c_{\mathcal{R}}^2 x_\phi^2))/3c_{\mathcal{R}}^4 x_\phi^4 \simeq 4\widehat{H}_{\rm I}^2$
2 real scalars	$\widehat{s}_1,\ \widehat{s}_2$	$\widehat{m}_s^2 = \lambda^2 m_{\rm P}^2 (1 + c_{\mathcal{R}} x_\phi^2 (2 - c_{\mathcal{R}} x_\phi^2 + 6k_S f_\Phi^2))/6c_{\mathcal{R}}^4 x_\phi^4$
2 Weyl spinors	$\widehat{\psi}_\pm = \frac{\widehat{\psi}_\Phi \pm \widehat{\psi}_S}{\sqrt{2}}$	$\widehat{m}_{\psi\pm}^2 \simeq \lambda^2 m_{\rm P}^2/3c_{\mathcal{R}}^4 x_\phi^4$

- All mass$^2 > 0$. Especially $\widehat{m}_S^2 > 0 \;\Leftrightarrow\; k_S > (2 - c_{\mathcal{R}} x_\phi^2)/6f_\Phi$;
- All mass$^2 > \widehat{H}_{\rm I}^2$ and So Any Inflationary Perturbations Of The Fields Other Than The Inflaton Are Safely Eliminated;
- The One-Loop **Radiative Corrections** (RCs) Have **No Significant Effect** On The Inflationary Dynamics And Predictions, Since The Slope Of The Inflationary Path Is Generated At The Classical Level and $m_I \propto \lambda$ With $I = \widehat{\theta}, \widehat{s}$ and $\widehat{\psi}_\pm$.

Testing Against Observations

- The Results Are Similar to the Original **non-SUSY** IGI With $\sqrt{\lambda} \to \lambda$.
- The 1 Extra Variable ($c_{\mathcal{R}}$) Compared To The Cecotti Model Assists Us in Reducing $\phi_\star$ Below $m_{\rm P}$.
- The **EF Values Are TransPlanckian** ($\widehat{\phi}_\star = 5.3 m_{\rm P}$ and $\widehat{\phi}_{\rm f} = 0.94 m_{\rm P}$) for any $c_{\mathcal{R}}$. However, The Theory is Defined in Terms of ϕ And Therefore, **Possible Corrections** from non-Renormalisable Terms Including ϕ **Remain Subdominant**.



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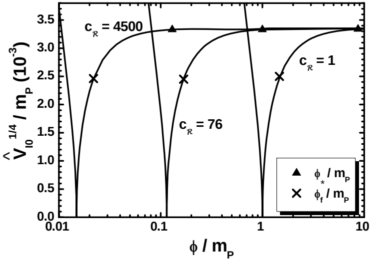

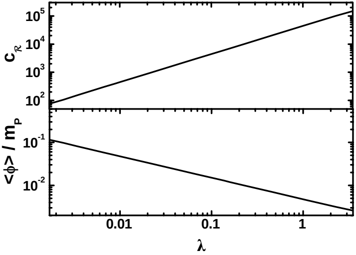

- Imposing the Observational Constraints we Obtain the Following **Allowed Regions**:

$$76 \lesssim c_{\mathcal{R}} \lesssim 1.5\cdot 10^5,\quad 0.11 \gtrsim \langle\phi\rangle/m_{\rm P} \gtrsim 0.002 \quad\text{and}\quad 1.7\cdot 10^{-3} \lesssim \lambda \lesssim 3.54 \quad\text{for}\quad \widehat{N}_\star \simeq 52.$$

- Contrary to The Cases of IGI Without No-Scale-Type Symmetry the Resulting $n_{\rm s}$ is **A Prediction And Not Adjustable**.

$$0.961 \lesssim n_{\rm s} \lesssim 0.963,\quad -7.4 \lesssim \alpha_{\rm s}/10^{-4} \lesssim -6.7 \quad\text{and}\quad 4.2 \gtrsim r/10^{-3} \gtrsim 3.8.$$

- The Predicted $n_{\rm s}$ is Close to Its Central Observational Value, While $\alpha_{\rm s}$ and r Are Consistent PLANCK. Therefore, IGI with No-Scale-Type Symmetry Is **A Predictive** Model But **not Compatible With Bicep2**.

Definition of the Model

If We Lift **Partially** ($m = 0$) The Assumption Of no-Scale SUGRA, W and K take the form:

$$W = \lambda m_{\rm P}^2 S(\Omega_{\rm H} - 1/2)/c_{\mathcal{R}} \quad \text{and} \quad K = -3m_{\rm P}^2 \ln\left(\Omega_{\rm H}(\Phi) + \Omega_{\rm H}^*(\Phi^*) - \frac{|S|^2}{3m_{\rm P}^2} - \frac{|\Phi|^2}{3m_{\rm P}^2} + k_S\frac{|S|^4}{3m_{\rm P}^4} + 2k_\Phi\frac{|\Phi|^4}{3m_{\rm P}^4} + 2k_{S\Phi}\frac{|S|^2|\Phi|^2}{3m_{\rm P}^4}\right),$$

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Stability of the Inflationary Trajectory

If we Perform An Expansion For Small x_ϕ's And Keep The Lowest Order Terms, The Masses Squared For The Scalars Reduce To Those Displayed For The model With no-Scale Symmetry.

Approximating the Inflationary Dynamics

- The Slow-Roll Parameters Are Determined **Performing Expansions** About $x_\phi \simeq 0$

$$\widehat{\epsilon} = \frac{(2 + k_{S\Phi} c_{\mathcal{R}} x_\phi^4)^2}{3 f_\Phi^2} \quad \text{AND} \quad \widehat{\eta} = \frac{4 + 4k_{S\Phi} c_{\mathcal{R}}^2 x_\phi^6 + 2c_{\mathcal{R}} x_\phi^2 \left(k_{S\Phi} x_\phi^2 - 1\right)}{3 f_\Phi^2}.$$

- The Number of e-foldings Is Calculated As in the Case Of no-Scale Symmetry I.e.

$$\widehat{N}_\star \simeq \frac{3c_{\mathcal{R}}}{4} \frac{\phi_\star^2 - \phi_{\rm f}^2}{m_{\rm P}^2} \Rightarrow x_\star \equiv \frac{\phi_\star}{m_{\rm P}} \simeq 2\sqrt{\widehat{N}_\star / 3c_{\mathcal{R}}}.$$

- $\phi_\star$ Decreases as $c_{\mathcal{R}}$ or λ Increases. Indeed, $\phi_\star \leq m_{\rm P} \quad \Rightarrow \quad c_{\mathcal{R}} \geq 4\widehat{N}_\star/3.$
- The Power Spectrum Normalization Implies A **Mild Dependence of λ on $k_{S\Phi}$**

$$A_{\rm s}^{1/2} = \frac{\lambda f_\Phi^2(x_\star)}{4\sqrt{2}\pi c_{\mathcal{R}}^2 x_\star^2 (2 + k_{S\Phi} c_{\mathcal{R}} x_\star^4)} \Rightarrow \lambda \simeq 2\pi\sqrt{2A_{\rm s}} c_{\mathcal{R}} \left(\frac{3}{\widehat{N}_\star} + 4k_{S\Phi} \frac{2\widehat{N}_\star}{3c_{\mathcal{R}}}\right).$$

- A Clear Efficient **Dependence of $n_{\rm s}$ on $k_{S\Phi}$** arises.

$$n_{\rm s} \simeq 1 - \frac{2}{\widehat{N}_\star} + \left(\frac{4}{9}\right)^{1/2} \frac{\widehat{N}_\star}{c_{\mathcal{R}}} \frac{32k_{S\Phi} + 27/\widehat{N}_\star^3}{12}.$$

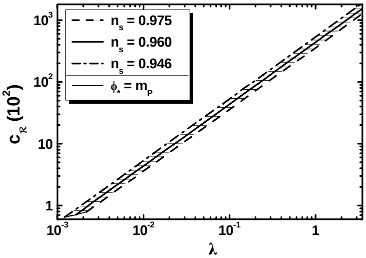
- r Remains Pretty Close To The One Obtained In the Case Of no-Scale Symmetry since $k_{S\Phi} \neq 0$ is Accompanied By **Large Denominators** Where $c_{\mathcal{R}} \gg 1$ is involved

$$r \simeq \frac{12}{\widehat{N}_\star^2} + 32\frac{2k_{S\Phi}}{3c_{\mathcal{R}}} + 64\frac{k_{S\Phi}^2 \widehat{N}_\star^2}{27c_{\mathcal{R}}^2}.$$

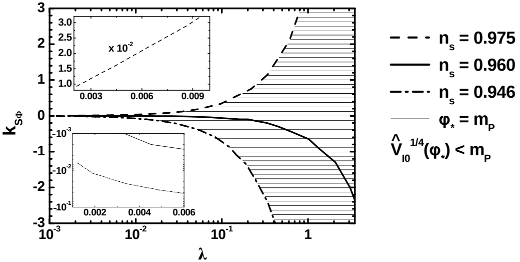
Testing Against Observations

Imposing the Observational Constraints for $k_S = 1, k_\Phi = 0.5$ and $T_{\rm rh} = 10^9$ GeV we Obtain the Following Regions Allowed By the PLANCK Data:

- In the $\lambda - c_{\mathcal{R}}$ Plane



- In the $\lambda - k_{S\Phi}$ Plane



We Observe the Following:

- $c_{\mathcal{R}}$ Remains Proportional to λ and Increases as $n_{\rm s}$ Decreases.
- For $\lambda < 0.05 \Leftrightarrow \phi_\star > 0.01 m_{\rm P}$, a Tuning Of The Order 0.01 Is Required in the $k_{S\Phi}$-values
- For $\lambda > 0.05 \Leftrightarrow \phi_\star < 0.01 m_{\rm P}$, Less Tuning As Regards The $k_{S\Phi}$-Values Is Required.

• The Resulting r Does not Deviate A Lot From That Obtained in the Previous Case, $r \simeq 0.004$. Therefore **No Consistency With the Bicep2 Data** Can Be Reported And **We Are Obliged to Totally Lift** The Assumptions Of No-Scale SUGRA.

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If We **Totaly** Lift The Assumptions Of No-Scale SUGRA, W and K Take The Form (With $k_S, k_\Phi, k_{S\Phi} \sim 1$):

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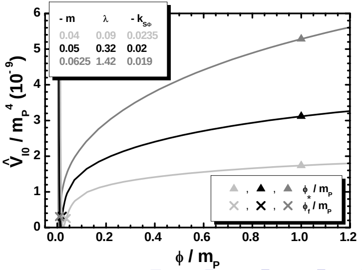

Stability of the Inflationary Trajectory

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Fields	Eingestates	Masses Squared
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2 real scalars	$\widehat{s},\ \widehat{\bar{s}}$	$\widehat{m}_s^2 = \lambda^2 m_{\rm P}^2 \left(2 - 6m - c_{\mathcal{R}} x_\phi^2 + 6k_S(1+m) f_2^2\right)/6(1+m)c_{\mathcal{R}}^{3(1+m)} x_\phi^{2(1+3m)}$
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Stability of the Inflationary Trajectory

The Mass Spectrum Along The Inflationary Trajectory

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Approximating the Inflationary Dynamics

- The Slow-Roll Parameters Are Rather Complicate, E.g., $\widehat{\epsilon} = (2 + 3m - 3mc_{\mathcal{R}} x_\phi^2 + (1+3m)k_{S\Phi} c_{\mathcal{R}} x_\phi^4)^2/3(1+m) f_{S\Phi}^2 f_\Phi^2$
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$$A_{\rm s}^{1/2} \simeq \frac{\lambda c_{\mathcal{R}}^{-3m/2} x_\star^{2-3m}}{4\sqrt{2}\pi(2 - 3c_{\mathcal{R}} m x_\star^2 + k_{S\Phi} c_{\mathcal{R}} x_\star^4)} \;\Rightarrow\; \lambda \simeq \frac{4\pi\sqrt{2A_{\rm s}}(k_{S\Phi} + 9c_{\mathcal{R}} m^2 e_m(1 + 2e_m)}{3^{1+3m/2}(|m| e_m)^{1+3m/2}}.$$

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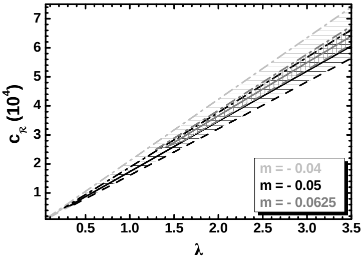
- For $m < 0$, r Increases Efficiently since **$c_{\mathcal{R}}$ Appears both in the Numerator and Denominator**, I.e.,

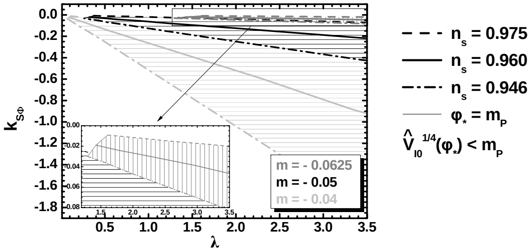
$$r \simeq 16\frac{\left(k_{S\Phi} + 3k_{S\Phi} m + 9c_{\mathcal{R}} m^2 e_m(1 + 2e_m)\right)^2}{3(1+m)(1+3me_m)^2(k_{S\Phi} + 3c_{\mathcal{R}} m e_m)^2}.$$

Testing Against Observations

Imposing the Observational Constraints for $k_S = 0.1$, $k_\Phi = 0.5$ and $T_{\rm rh} = 10^9$ GeV we Obtain the Following Regions Allowed By the PLANCK Data:

- In the $\lambda - c_{\mathcal{R}}$ Plane
- In the $\lambda - k_{S\Phi}$ Plane



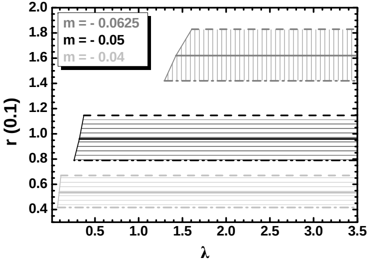


We Observe the Following:

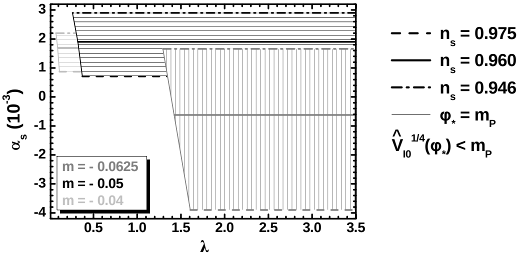
- $c_{\mathcal{R}}$ **Remains Almost Proportional to** λ But a Stronger Dependence on $k_{S\Phi}$ Arises.
- As $|m|$ Increases, The Required λ and $c_{\mathcal{R}}$ values Increase.
- As $|m|$ Increases, the **Tuning of $k_{S\Phi}$ Increases** Since $k_{S\Phi}$ Approaches zero.
- As $|m|$ Increases Above 0, r and $n_{\rm s}$ Increase Above 0.004 and 0.964.
  The **Decrease of $k_{S\Phi}$ below 0 is imperative** in order to Achieve Acceptable $n_{\rm s}$ Values.

Independently of λ, The Resulting r's are Consistent With The Planck and Bicep2 Data Whereas the α_s's Remain Within the Validity of ΛCDM Model

- In the $\lambda - r$ Plane



- In the $\lambda - \alpha_s$ Plane



INDEPENDENTLY OF λ, THE RESULTING r'S ARE CONSISTENT WITH THE PLANCK AND BICEP2 DATA WHEREAS THE α_s'S REMAIN WITHIN THE VALIDITY OF ΛCDM MODEL

- IN THE $\lambda - r$ PLANE
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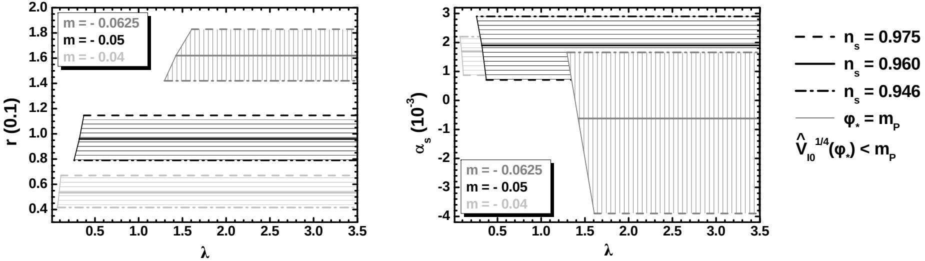

CONCLUSIONS

- FOR **ALL** THE ANALYZED MODELS, WE CONCLUDE THAT:
  - IGI CAN BE REALIZED IN SUGRA ADOPTING A **LOGARITHMIC KÄHLER POTENTIAL** WHICH INCLUDES AN HOLOMORPHIC FUNCTION;
  - DESPITE THE PRESENCE OF A LARGE ENOUGH COEFFICIENT IN THIS FUNCTION, **THE MODELS ARE VALID UP TO m_P**.
  - INFLATION CAN BE ATTAINED EVEN FOR **SUB-PLANCKIAN VALUES OF THE ORIGINAL INFLATON**.

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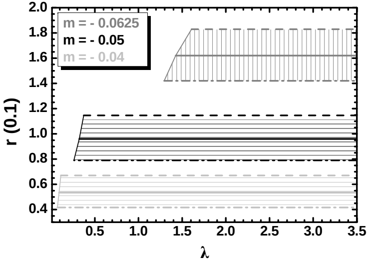

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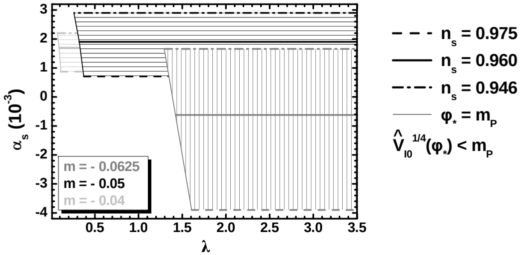

Conclusions

- For **All** the Analyzed Models, We Conclude That:
  - IGI can Be Realized in SUGRA Adopting a **Logarithmic Kähler Potential** Which Includes an Holomorphic Function;
  - Despite the Presence of a Large Enough Coefficient In this Function, **The Models Are Valid Up to m_P**.
  - Inflation Can be Attained Even for **Sub-Planckian Values of the Original Inflaton**.
- As Regards IGI **With** no-Scale-Type Kähler Manifold, **No Mixing Terms** Between the Inflaton and The Accompanying Field Are Allowed in the Kähler Potential And The **Mass** of Inflaton At the Vacuum Is Predicted to Be $\widehat{m} = 3 \cdot 10^{13}$ GeV.

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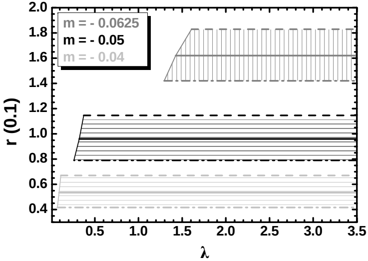

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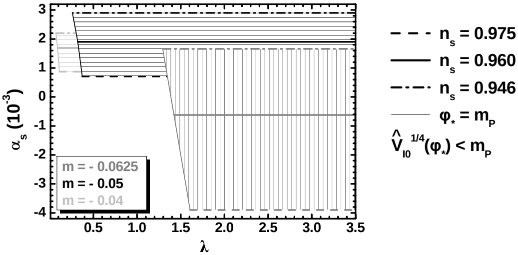

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- As Regards IGI **Without** no-Scale-type Kähler Manifold, a Mild Adjustment of $k_{S\Phi}$ and m Assists us to Precisely Reconcile The Models With The **Current Planck and Bicep2 Data** For A little Larger $\widehat{m}$ Values.