Component Lagrangian description of arbitrary superhelicity

Massless, irreducible representations of the $4D$, $\mathcal{N} = 1$ Super-Poincaré group

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Quantum Symmetries and Strings

Corfu, 19 September 2014

based on: *JHEP 1406 (2014) 098*
arXiv:1310.7386
arXiv:1310.7385

Outline

Motivation

- "*The underlying structure is of considerable mathematical interest*" (P. Dirac)
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- Compactify $10D$ to $4D$
  (study constraints of $\mathcal{SUSY}$ in $4D$ for a choice independent answer)

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$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) =& A(x) + \theta^\alpha \psi_\alpha(x) + \bar{\theta}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}}(x) + \theta^2 F(x) + \bar{\theta}^2 G(x) \\ &+ \theta^\alpha \bar{\theta}^{\dot{\alpha}} V_{\alpha\dot{\alpha}} + \theta^2 \bar{\theta}^{\dot{\alpha}} \bar{\lambda}_{\dot{\alpha}}(x) + \bar{\theta}^2 \theta^\alpha \rho_\alpha(x) + \theta^2 \bar{\theta}^2 D(x)\end{aligned}$$

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- Component fields are the coefficients in the expansion.

$$\Phi \sim \theta^n \bar{\theta}^m \Phi_{(n,m)} \to \Phi_{(n,m)} \sim \partial^n_\alpha \bar{\partial}^m_{\dot{\alpha}} \Phi|_{\theta=0,\ \bar{\theta}=0}$$

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$$\Phi^{(0,0)}_{\alpha(n)\dot{\alpha}(m)}=\Phi_{\alpha(n)\dot{\alpha}(m)}|$$
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- $\mathcal{SUSY}$ transformation law for the components

$$\delta_S\Phi_{(n,m)}=-\left(\epsilon^\alpha\mathrm{D}_\alpha+\bar{\epsilon}^{\dot{\alpha}}\bar{\mathrm{D}}_{\dot{\alpha}}\right)f(\mathrm{D}^n,\bar{\mathrm{D}}^m)\Phi|$$

Superspace & Component Action integrals

- The superspace action integral has the form

$$S = \int d^8 z \mathcal{L} = \int d^4 x d^2\theta d^2\bar{\theta} \mathcal{L}$$

- Conventions and useful properties

$$\left[\mathrm{D}_\alpha, \bar{\mathrm{D}}_{\dot{\alpha}}\right] = i\partial_{\alpha\dot{\alpha}},\ \mathrm{D}^2 = \frac{1}{2}\mathrm{D}^\alpha \mathrm{D}_\alpha,\ \mathrm{D}_\alpha \mathrm{D}_\beta = -C_{\alpha\beta}\mathrm{D}^2,\ (\mathrm{D}_\alpha)^\dagger = -\bar{\mathrm{D}}_{\dot{\alpha}}$$
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$$\int d\theta = 0 \quad , \int d\theta\theta = 1 \quad , \int d\theta_\alpha \theta^\beta = \delta_\alpha{}^\beta \quad , \int d\theta_\alpha f(\theta) = \partial_\alpha f(\theta) = \mathrm{D}_\alpha f(\theta)|_{\theta=0,\ \bar{\theta}=0}$$

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$$S = \int d^4 x d^2 \theta d^2 \bar{\theta} \mathcal{L} = \int d^4 x \begin{cases} \mathrm{D}^2 \bar{\mathrm{D}}^2 \mathcal{L}|_{\theta=0,\ \bar{\theta}=0} \\ \bar{\mathrm{D}}^2 \mathrm{D}^2 \mathcal{L}|_{\theta=0,\ \bar{\theta}=0} \end{cases}$$

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Irreducible Representations

Representation theory forces on us the following:

- $m \neq 0, \;\; Y = s$

$$\begin{aligned} \Psi_{\alpha(s)\dot\alpha(s-1)} : \;\; & \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot\alpha(s-1)} = 0, \\ & \mathrm{D}^{\alpha_s} \Psi_{\alpha(s)\dot\alpha(s-1)} = 0, \\ & \partial^{\gamma\dot\gamma} \Psi_{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-2)} = 0, \\ i\partial_{\alpha_s}{}^{\dot\alpha_s} \bar{\Psi}_{\alpha(s-1)\dot\alpha(s)} + m\Psi_{\alpha(s)\dot\alpha(s-1)} = 0 \end{aligned}$$

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- $m \neq 0, \;\; Y = s + \frac{1}{2}$

$$\begin{aligned} H_{\alpha(s)\dot\alpha(s)} : \; & \mathrm{D}^{\alpha_s} H_{\alpha(s)\dot\alpha(s)} = 0 \\ & H_{\alpha(s)\dot\alpha(s)} = \bar H_{\alpha(s)\dot\alpha(s)} \\ & \Box H_{\alpha(s)\dot\alpha(s)} = m^2 H_{\alpha(s)\dot\alpha(s)} \end{aligned}$$

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Superspace Approach: Building Blocks

Superspace action integral $\xrightarrow{\text{equations of motion}}$ $\bar{\mathrm{D}}_{\dot{\gamma}} F_{\alpha(2s)} = 0,\ \mathrm{D}^{\alpha_{2s}} F_{\alpha(2s)} = 0,\ F_{\alpha(2s)}$

Kuzenko, Postnikov & Sibiryakov 1993

Kuzenko, Sibiryakov 1993

Gates, Koutrolikos 2011,2014

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$$S = \int d^8z \Big\{ -\frac{1}{2} c\Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c.$$
$$+ c\Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}_{\alpha_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)}$$
$$- cV^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c.$$
$$+ \frac{1}{2} cV^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}^{\gamma} \bar{\mathrm{D}}^2 \mathrm{D}_{\gamma} V_{\alpha(s-1)\dot{\alpha}(s-1)} \Big\}$$

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- Invariant under

$$\delta_G \Psi_{\alpha(s)\dot{\alpha}(s-1)} = -\mathrm{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \left[\frac{1}{(s-1)!}\right] \bar{\mathrm{D}}_{(\dot{\alpha}_{s-1}} \Lambda_{\alpha(s)\dot{\alpha}(s-2))}$$

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- Bianchi identities

$$\mathrm{D}^2 T_{\alpha(s)\dot\alpha(s-1)} + \frac{1}{s!} \mathrm{D}_{(\alpha_s} G_{\alpha(s-1))\dot\alpha(s-1)} = 0,\quad G_{\alpha(s-1)\dot\alpha(s-1)} = \frac{\delta S}{\delta V^{\alpha(s-1)\dot\alpha(s-1)}}$$

$$\bar{\mathrm{D}}^{\dot\alpha_{s-1}} T_{a(s)\dot\alpha(s-1)} = 0 \quad , \quad T_{\alpha(s)\dot\alpha(s-1)} = \frac{\delta S}{\delta \Psi^{\alpha(s)\dot\alpha(s-1)}}$$

Existence of $F_{\alpha(2s)}$

and on-shell description of arbitrary integer superhelicity

- The following identity, can be proven

$$
\begin{aligned}
\bar{\mathrm{D}}^{\dot{\alpha}_{2s}}\bar{F}_{\dot{\alpha}(2s)} =& -\frac{i}{(2s-1)!c}\partial^{\alpha_s}{}_{(\dot{\alpha}_{2s-1}}\cdots\partial^{\alpha_1}{}_{\dot{\alpha}_s}T_{\alpha(s)\dot{\alpha}(s-1))}\\
&+\frac{B}{(2s-1)!}\bar{\mathrm{D}}^2\partial^{\alpha_{s-1}}{}_{(\dot{\alpha}_{2s-1}}\cdots\partial^{\alpha_1}{}_{(\dot{\alpha}_{s+1}}\bar{T}_{\alpha(s-1)\dot{\alpha}(s))}\\
&+\frac{1+2cB}{(2s-1)!2c}\bar{\mathrm{D}}_{(\dot{\alpha}_{2s-1}}\partial^{\alpha_{s-1}}{}_{\dot{\alpha}_{2s-2}}\cdots\partial^{\alpha_1}{}_{\dot{\alpha}_s}G_{\alpha(s-1)\dot{\alpha}(s-1))}\\
&+\frac{1}{(2s-1)!2c}\bar{\mathrm{D}}_{(\dot{\alpha}_{2s-1}}\mathrm{D}^{\alpha_s}\partial^{\alpha_{s-1}}{}_{\dot{\alpha}_{2s-2}}\cdots\partial^{\alpha_1}{}_{\dot{\alpha}_s}T_{\alpha(s)\dot{\alpha}(s-1)}
\end{aligned}
$$

where

$$
\bar{F}_{\dot{\alpha}(2s)} = \frac{1}{(2s)!}\mathrm{D}^2\bar{\mathrm{D}}_{(\dot{\alpha}_{2s}}\partial^{\alpha_{s-1}}{}_{\dot{\alpha}_{2s-1}}\cdots\partial^{\alpha_1}{}_{\dot{\alpha}_{s+1}}\bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s))}
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&+\frac{B}{(2s-1)!}\bar{\mathrm{D}}^2\partial^{\alpha_{s-1}}{}_{(\dot{\alpha}_{2s-1}}\cdots\partial^{\alpha_1}{}_{(\dot{\alpha}_{s+1}}\bar{T}_{\alpha(s-1)\dot{\alpha}(s))} \\
&+\frac{1+2cB}{(2s-1)!2c}\bar{\mathrm{D}}_{(\dot{\alpha}_{2s-1}}\partial^{\alpha_{s-1}}{}_{\dot{\alpha}_{2s-2}}\cdots\partial^{\alpha_1}{}_{\dot{\alpha}_s}G_{\alpha(s-1)\dot{\alpha}(s-1))} \\
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\end{aligned}
$$

where

$$
\bar{F}_{\dot{\alpha}(2s)} = \frac{1}{(2s)!}\mathrm{D}^2\bar{\mathrm{D}}_{(\dot{\alpha}_{2s}}\partial^{\alpha_{s-1}}{}_{\dot{\alpha}_{2s-1}}\cdots\partial^{\alpha_1}{}_{\dot{\alpha}_{s+1}}\bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s))}
$$

- On-shell ($T_{\alpha(s)\dot{\alpha}(s-1)} = 0, \; G_{\alpha(s-1)\dot{\alpha}(s-1)} = 0$) we get the required constraint

$$
\mathrm{D}^{\alpha_{2s}}F_{\alpha(2s)} = 0, \; \bar{\mathrm{D}}_{\dot{\gamma}}F_{\alpha(2s)} = 0
$$

Outline

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Example: The vector multiplet, $Y = \frac{1}{2}$

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$$
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- Use the definitions of the components:

$$\begin{aligned}\bar{\mathrm{D}}^2\mathrm{D}^2H| &= H^{(2,2)} + \frac{1}{4}\Box H^{(0,0)} - \frac{i}{2}\partial^{\beta\dot{\beta}}H^{(1,1)}_{\beta\dot{\beta}} \\ \mathrm{D}^{\gamma}\bar{\mathrm{D}}^2\mathrm{D}_{\gamma}H| &= 2H^{(2,2)} - \frac{1}{2}\Box H^{(0,0)} \\ \bar{\mathrm{D}}_{\dot{\rho}}\mathrm{D}_{\rho}H| &= H^{(1,1)}_{\rho\dot{\rho}} + \frac{i}{2}\partial_{\rho\dot{\rho}}H^{(0,0)}\end{aligned}$$

Example: The vector multiplet, $Y = \frac{1}{2}$ (Cont'd.)

- We get for the bosonic action:

$$\mathcal{L}_B^c = 2H^{(2,2)}H^{(2,2)} - H^{(2,2)}\Box H^{(0,0)} + H^{(1,1)\alpha\dot{\alpha}}\Box H^{(1,1)}_{\alpha\dot{\alpha}} - \frac{1}{2}H^{(1,1)\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\partial^{\beta\dot{\beta}}H^{(1,1)}_{\beta\dot{\beta}} + \frac{1}{8}H^{(0,0)}\Box\Box H^{(0,0)}$$

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- Similar for the fermionic action

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- The components are defined as the coefficients in the Taylor expansion of the superfields participating in the superspace lagrangian. The lagrangian is quadratic to the superfields therefore we get this huge number of terms.
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Observations:

There are three different *types* of components that appear in a superfield

- Type $\mathcal{D}$: The Dynamical components $(H^{(1,1)}_{\alpha\dot{\alpha}})$
  dynamics, helicity content, construct gauge invariant objects (field strengths)
- Type $\mathcal{A}_l$: The low dimensionality Auxiliary components $(H^{(0,0)})$
  closure of *susy* algebra within a superfield, algebraic terms in their gauge transformations, lagrangian and *susy* transformations do not depend on them
- Set $\mathcal{A}_h$: The high dimensionality Auxiliary components $(H^{(2,2)})$
  off-shell *susy* invariance of the action, appear in an algebraic way in the lagrangian, can be redefined in order to 1) eliminate type $\mathcal{A}_l$ components and 2) bring the action in a *diagonal* form

Diagonal form of the lagrangian

Diagonal form requirement:

- The *diagonal* form of the component lagrangian is:
  $\mathcal{L}^{c} = \mathcal{L}_{\lambda=Y} + \mathcal{L}_{\lambda=Y+\frac{1}{2}} + \sum$ quadratic monomials of $\mathcal{A}_h$ components
- Acceptable quadratic monomials are: A^2 or AB but not $A^2 + AB$.
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Consequences of the diagonal form requirement:

- Makes obvious that on-shell the theory describes helicities Y and $Y + 1/2$
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Suggestion: It might be smart to define the $\mathcal{A}_h$ components using objects that are already gauge invariant and automatically vanish on-shell. Are there any ?

Enter superfields $T_{\alpha(s)\dot{\alpha}(s-1)}$, $G_{\alpha(s-1)\dot{\alpha}(s-1)}$

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- The auxiliary components (mass dimensions: $\frac{3}{2}$, 2, $\frac{5}{2}$) of the theory are:
  - For bosons: $\bar{\mathrm{D}}_{(\dot{\alpha}_s} T_{\alpha(s)\dot{\alpha}(s-1))}|$, $D^{\alpha_s} T_{\alpha(s)\dot{\alpha}(s-1)}|$, $G_{\alpha(s-1)\dot{\alpha}(s-1)}|$, $\mathrm{D}_{(\alpha(s+1)} T_{\alpha(s))\dot{\alpha}(s-1)}|$
  - For fermions: $T_{\alpha(s)\dot{\alpha}(s-1)}|$, $\mathrm{D}^2 T_{\alpha(s)\dot{\alpha}(s-1)}|$

Extract the component lagrangian

- Express the superspace action in terms of the equations of motion

$$S = \int d^8z \Big\{ \ \frac{1}{2} \Psi^{\alpha(s)\dot{\alpha}(s-1)} T_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\ + \frac{1}{2} V^{\alpha(s-1)\dot{\alpha}(s-1)} G_{\alpha(s-1)\dot{\alpha}(s-1)} \Big\}$$

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- The fermionic piece is:

$$\begin{aligned}\mathcal{L}_F =& \frac{1}{2}\mathrm{D}^2\bar{\mathrm{D}}^2\Psi^{\alpha(s)\dot\alpha(s-1)}| T_{\alpha(s)\dot\alpha(s-1)}| \\ &+\frac{1}{2}\left(\bar{\mathrm{D}}^2\Psi^{\alpha(s)\dot\alpha(s-1)} - \frac{1}{s!}\bar{\mathrm{D}}^2\mathrm{D}^{(\alpha_s} V^{\alpha(s-1))\dot\alpha(s-1)}\right)|\mathrm{D}^2 T_{\alpha(s)\dot\alpha(s-1)}| \\ &-\frac{1}{2}\frac{1}{(s+1)!s!}\mathrm{D}^{(\alpha_{s+1}}\bar{\mathrm{D}}^{(\dot\alpha_s}\Psi^{\alpha(s))\dot\alpha(s-1))}|\frac{1}{(s+1)!s!}\mathrm{D}_{(\alpha_{s+1}}\bar{\mathrm{D}}_{(\dot\alpha_s} T_{\alpha(s))\dot\alpha(s-1))}| \\ &+\frac{1}{2}\frac{s}{s+1}\frac{1}{s!}\mathrm{D}_\gamma\bar{\mathrm{D}}^{(\dot\alpha_s}\Psi^{\gamma\alpha(s-1)\dot\alpha(s-1))}|\frac{1}{s!}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}_{(\dot\alpha_s} T_{\alpha(s)\dot\alpha(s-1))}| \\ &-\frac{s-1}{2s}\bar{\mathrm{D}}^2\mathrm{D}_\gamma V^{\gamma\alpha(s-2)\dot\alpha(s-1)}|\mathrm{D}^{\alpha_{s-1}} G_{\alpha(s-1)\dot\alpha(s-1)}| \\ &+c.c.\end{aligned}$$

Extract the component lagrangian

- Express the superspace action in terms of the equations of motion

$$S = \int d^8z \Big\{ \frac{1}{2}\Psi^{\alpha(s)\dot{\alpha}(s-1)} T_{\alpha(s)\dot{\alpha}(s-1)} + c.c. + \frac{1}{2} V^{\alpha(s-1)\dot{\alpha}(s-1)} G_{\alpha(s-1)\dot{\alpha}(s-1)} \Big\}$$

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\mathcal{L}_F = & \frac{1}{2}\mathrm{D}^2\bar{\mathrm{D}}^2\Psi^{\alpha(s)\dot{\alpha}(s-1)}| \, T_{\alpha(s)\dot{\alpha}(s-1)}| \\
& + \frac{1}{2}\left(\bar{\mathrm{D}}^2\Psi^{\alpha(s)\dot{\alpha}(s-1)} - \frac{1}{s!}\bar{\mathrm{D}}^2\mathrm{D}^{(\alpha_s} V^{\alpha(s-1))\dot{\alpha}(s-1)}\right) |\mathrm{D}^2 T_{\alpha(s)\dot{\alpha}(s-1)}| \\
& - \frac{1}{2}\frac{1}{(s+1)!s!}\mathrm{D}^{(\alpha_{s+1}}\bar{\mathrm{D}}^{(\dot{\alpha}_s}\Psi^{\alpha(s))\dot{\alpha}(s-1))}| \frac{1}{(s+1)!s!}\mathrm{D}_{(\alpha_{s+1}}\bar{\mathrm{D}}_{(\dot{\alpha}_s} T_{\alpha(s))\dot{\alpha}(s-1))}| \\
& + \frac{1}{2}\frac{s}{s+1}\frac{1}{s!}\mathrm{D}_\gamma\bar{\mathrm{D}}^{(\dot{\alpha}_s}\Psi^{\gamma\alpha(s-1)\dot{\alpha}(s-1))}| \frac{1}{s!}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}_{(\dot{\alpha}_s} T_{\alpha(s)\dot{\alpha}(s-1))}| \\
& - \frac{s-1}{2s}\bar{\mathrm{D}}^2\mathrm{D}_\gamma V^{\gamma\alpha(s-2)\dot{\alpha}(s-1)}|\mathrm{D}^{\alpha_{s-1}} G_{\alpha(s-1)\dot{\alpha}(s-1)}| \\
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Identities for T and G

$$\frac{1}{(s+1)!s!}\mathrm{D}_{(\alpha_{s+1}}\bar{\mathrm{D}}_{(\dot\alpha_s}T_{\alpha(s))\dot\alpha(s-1))}=-\frac{ic}{(s+1)!}\partial_{(\alpha_{s+1}}{}^{\dot\alpha_{s+1}}\left[\frac{1}{(s+1)!s!}\bar{\mathrm{D}}_{(\dot\alpha_{s+1}}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1)))\dot\alpha(s))}\right]$$
$$+\frac{ic}{(s+1)!s!}\frac{s}{s+1}\partial_{(\alpha_{s+1}(\dot\alpha_s}\left[\frac{1}{s!}\bar{\mathrm{D}}^{\dot\gamma}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1)))\dot\gamma\dot\alpha(s-1))}\right]$$

$$\frac{1}{s!}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}_{(\dot\alpha_s}T_{\alpha(s)\dot\alpha(s-1))}=\frac{i}{s!}\frac{s+1}{s}\partial^{\alpha_s}{}_{(\dot\alpha_s}T_{\alpha(s)\dot\alpha(s-1))}+\frac{s+1}{s}\bar{\mathrm{D}}^2\bar{T}_{\alpha(s-1)\dot\alpha(s)}$$
$$-ic\partial^{\alpha_s\dot\alpha_{s+1}}\left[\frac{1}{(s+1)!s!}\bar{\mathrm{D}}_{(\dot\alpha_{s+1}}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1))\dot\alpha(s))}\right]$$
$$-\frac{ic}{s!}\frac{2s+1}{s(s+1)}\partial^{\alpha_s}{}_{(\dot\alpha_s}\left[\frac{1}{s!}\bar{\mathrm{D}}^{\dot\gamma}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1))\dot\gamma\dot\alpha(s-1))}\right]$$
$$-\frac{ic}{s!s!}\frac{s^2-1}{s}\partial_{(\alpha_{s-1}(\dot\alpha_s}\left[\bar{\mathrm{D}}^2\mathrm{D}^{\gamma}V_{\gamma\alpha(s-2))\dot\alpha(s-1))}\right]$$

$$\mathrm{D}^{\alpha_{s-1}}G_{\alpha(s-1)\dot\alpha(s-1)}=i\partial^{\alpha_{s-1}\dot\alpha_s}\bar{T}_{\alpha(s-1)\dot\alpha(s)}-ic\partial^{\alpha_{s-1}\dot\alpha_s}\left[\frac{1}{s!}\mathrm{D}^{\gamma}\bar{\mathrm{D}}_{(\dot\alpha_s}\Psi_{\gamma\alpha(s-1)\dot\alpha(s-1))}\right]$$
$$-ic\frac{s-1}{s!}\partial^{\alpha_{s-1}}{}_{(\dot\alpha_{s-1}}\left[\mathrm{D}^2\bar{\mathrm{D}}^{\dot\gamma}V_{\alpha(s-1)\dot\gamma\dot\alpha(s-2))}\right]$$

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$$\frac{1}{(s+1)!s!}\mathrm{D}_{(\alpha_{s+1}}\bar{\mathrm{D}}_{(\dot\alpha_s}T_{\alpha(s))\dot\alpha(s-1))} = -\frac{ic}{(s+1)!}\partial_{(\alpha_{s+1}}{}^{\dot\alpha_{s+1}}\left[\frac{1}{(s+1)!s!}\bar{\mathrm{D}}_{(\dot\alpha_{s+1}}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1)))\dot\alpha(s))}\right] + \frac{ic}{(s+1)!s!}\frac{s}{s+1}\partial_{(\alpha_{s+1}(\dot\alpha_s}\left[\frac{1}{s!}\bar{\mathrm{D}}^{\dot\gamma}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1)))\dot\gamma\dot\alpha(s-1))}\right]$$

$$\frac{1}{s!}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}_{(\dot\alpha_s}T_{\alpha(s)\dot\alpha(s-1))} = \frac{i}{s!}\frac{s+1}{s}\partial^{\alpha_s}{}_{(\dot\alpha_s}T_{\alpha(s)\dot\alpha(s-1))} + \frac{s+1}{s}\bar{\mathrm{D}}^2\bar{T}_{\alpha(s-1)\dot\alpha(s)} - ic\partial^{\alpha_s\dot\alpha_{s+1}}\left[\frac{1}{(s+1)!s!}\bar{\mathrm{D}}_{(\dot\alpha_{s+1}}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1))\dot\alpha(s))}\right] - \frac{ic}{s!}\frac{2s+1}{s(s+1)}\partial^{\alpha_s}{}_{(\dot\alpha_s}\left[\frac{1}{s!}\bar{\mathrm{D}}^{\dot\gamma}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1))\dot\gamma\dot\alpha(s-1))}\right] - \frac{ic}{s!s!}\frac{s^2-1}{s}\partial_{(\alpha_{s-1}(\dot\alpha_s}\left[\bar{\mathrm{D}}^2\mathrm{D}^{\gamma}V_{\gamma\alpha(s-2))\dot\alpha(s-1))}\right]$$

$$\mathrm{D}^{\alpha_{s-1}}G_{\alpha(s-1)\dot\alpha(s-1)} = i\partial^{\alpha_{s-1}\dot\alpha_s}\bar{T}_{\alpha(s-1)\dot\alpha(s)} - ic\partial^{\alpha_{s-1}\dot\alpha_s}\left[\frac{1}{s!}\mathrm{D}^{\gamma}\bar{\mathrm{D}}_{(\dot\alpha_s}\Psi_{\gamma\alpha(s-1)\dot\alpha(s-1))}\right] - ic\frac{s-1}{s!}\partial^{\alpha_{s-1}}{}_{(\dot\alpha_{s-1}}\left[\mathrm{D}^2\bar{\mathrm{D}}^{\dot\gamma}V_{\alpha(s-1)\dot\gamma\dot\alpha(s-2))}\right]$$

Definition of dynamical components

- Specific combinations appear again and again. Let's define:

$$\frac{1}{s!(s+1)!}\mathrm{D}_{(\alpha_{s+1}}\bar{\mathrm{D}}_{(\dot{\alpha}_s}\Psi_{\alpha(s))\dot{\alpha}(s-1))}| \equiv N_1\psi_{\alpha(s+1)\dot{\alpha}(s)}$$

$$\frac{1}{s!}\bar{\mathrm{D}}^{\dot{\alpha}_s}\mathrm{D}_{(\alpha_s}\bar{\Psi}_{\alpha(s-1))\dot{\alpha}(s)}| \equiv N_2\psi_{\alpha(s)\dot{\alpha}(s-1)}$$

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$$\mathrm{D}^2\bar{\mathrm{D}}^{\dot{\alpha}_{s-1}}V_{\alpha(s-1)\dot{\alpha}(s-1)}| \equiv N_3\psi_{\alpha(s-1)\dot{\alpha}(s-2)}$$

- Put everything together:

$$\begin{aligned}\mathcal{L}_F =& -\frac{1}{2c}T^{\alpha(s)\dot{\alpha}(s-1)}|\left(2\mathrm{D}^2T_{\alpha(s)\dot{\alpha}(s-1)}+\frac{i}{s!}\partial_{(\alpha_s}{}^{\dot{\alpha}_s}\bar{T}_{\alpha(s-1))\dot{\alpha}(s)}\right)|+c.c.\\
&-ic|N_1|^2\ \bar{\psi}^{\alpha(s)\dot{\alpha}(s+1)}\partial^{\alpha_{s+1}}{}_{\dot{\alpha}_{s+1}}\psi_{\alpha(s+1)\dot{\alpha}(s)}\\
&-ic\frac{s}{s+1}N_1N_2\ \psi^{\alpha(s+1)\dot{\alpha}(s)}\partial_{\alpha_{s+1}\dot{\alpha}_s}\psi_{\alpha(s)\dot{\alpha}(s-1)}+c.c.\\
&+ic\frac{2s+1}{(s+1)^2}|N_2|^2\ \bar{\psi}^{\alpha(s-1)\dot{\alpha}(s)}\partial^{\alpha_s}{}_{\dot{\alpha}_s}\psi_{\alpha(s)\dot{\alpha}(s-1)}\\
&+ic\frac{s-1}{s}N_2N_3\ \psi^{\alpha(s)\dot{\alpha}(s-1)}\partial_{\alpha_s\dot{\alpha}_{s-1}}\psi_{\alpha(s-1)\dot{\alpha}(s-2)}+c.c.\\
&+ic\left(\frac{s-1}{s}\right)^2|N_3|^2\ \bar{\psi}^{\alpha(s-2)\dot{\alpha}(s-1)}\partial^{\alpha_{s-1}}{}_{\dot{\alpha}_{s-1}}\psi_{\alpha(s-1)\dot{\alpha}(s-2)}\end{aligned}$$

- To get an exact match with the theory of helicity $\lambda = s + 1/2$

$$c = -1 \ , \ N_2 = 1 \ , \ N_1 = 1 \ , \ N_3 = -\frac{s}{s-1}$$

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- The final form of the fermionic component lagrangian is:

$$\begin{aligned}\mathcal{L}_F = & \ \rho^{\alpha(s)\dot{\alpha}(s-1)}\beta_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\ & + i\ \bar{\psi}^{\alpha(s)\dot{\alpha}(s+1)}\partial^{\alpha_{s+1}}{}_{\dot{\alpha}_{s+1}}\psi_{\alpha(s+1)\dot{\alpha}(s)} \\ & + i\left[\frac{s}{s+1}\right]\psi^{\alpha(s+1)\dot{\alpha}(s)}\partial_{\alpha_{s+1}\dot{\alpha}_s}\psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\ & - i\left[\frac{2s+1}{(s+1)^2}\right]\bar{\psi}^{\alpha(s-1)\dot{\alpha}(s)}\partial^{\alpha_s}{}_{\dot{\alpha}_s}\psi_{\alpha(s)\dot{\alpha}(s-1)} \\ & + i\ \psi^{\alpha(s)\dot{\alpha}(s-1)}\partial_{\alpha_s\dot{\alpha}_{s-1}}\psi_{\alpha(s-1)\dot{\alpha}(s-2)} + c.c. \\ & - i\ \bar{\psi}^{\alpha(s-2)\dot{\alpha}(s-1)}\partial^{\alpha_{s-1}}{}_{\dot{\alpha}_{s-1}}\psi_{\alpha(s-1)\dot{\alpha}(s-2)}\end{aligned}$$

$$\begin{aligned}\rho_{\alpha(s)\dot{\alpha}(s-1)} & \equiv T_{\alpha(s)\dot{\alpha}(s-1)}| \\ \beta_{\alpha(s)\dot{\alpha}(s-1)} & \equiv \mathrm{D}^2 T_{\alpha(s)\dot{\alpha}(s-1)}| \\ & \quad + \frac{i}{2s!}\partial_{(\alpha_s}{}^{\dot{\alpha}_s}\bar{T}_{\alpha(s-1))\dot{\alpha}(s)}| \\ \psi_{a(s+1)\dot{\alpha}(s)} & \equiv \frac{1}{s!(s+1)!}\mathrm{D}_{(\alpha_{s+1}}\bar{\mathrm{D}}_{(\dot{\alpha}_s}\Psi_{\alpha(s))\dot{\alpha}(s-1))}| \\ \psi_{\alpha(s)\dot{\alpha}(s-1)} & \equiv \frac{1}{s!}\bar{\mathrm{D}}^{\dot{\alpha}_s}\mathrm{D}_{(\alpha_s}\bar{\Psi}_{\alpha(s-1))\dot{\alpha}(s)}| \\ \psi_{\alpha(s-1)\dot{\alpha}(s-2)} & \equiv -\frac{s-1}{s}\mathrm{D}^2\bar{\mathrm{D}}^{\dot{\alpha}_{s-1}}V_{\alpha(s-1)\dot{\alpha}(s-1)}|\end{aligned}$$

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$$\begin{aligned}
\mathcal{L}_F = & \ \rho^{\alpha(s)\dot\alpha(s-1)}\beta_{\alpha(s)\dot\alpha(s-1)} + c.c. \\
& + i\,\bar\psi^{\alpha(s)\dot\alpha(s+1)}\partial^{\alpha_{s+1}}{}_{\dot\alpha_{s+1}}\psi_{\alpha(s+1)\dot\alpha(s)} \\
& + i\left[\frac{s}{s+1}\right]\psi^{\alpha(s+1)\dot\alpha(s)}\partial_{\alpha_{s+1}\dot\alpha_s}\psi_{\alpha(s)\dot\alpha(s-1)} + c.c. \\
& - i\left[\frac{2s+1}{(s+1)^2}\right]\bar\psi^{\alpha(s-1)\dot\alpha(s)}\partial^{\alpha_s}{}_{\dot\alpha_s}\psi_{\alpha(s)\dot\alpha(s-1)} \\
& + i\,\psi^{\alpha(s)\dot\alpha(s-1)}\partial_{\alpha_s\dot\alpha_{s-1}}\psi_{\alpha(s-1)\dot\alpha(s-2)} + c.c. \\
& - i\,\bar\psi^{\alpha(s-2)\dot\alpha(s-1)}\partial^{\alpha_{s-1}}{}_{\dot\alpha_{s-1}}\psi_{\alpha(s-1)\dot\alpha(s-2)}
\end{aligned}$$

$$\begin{aligned}
\rho_{\alpha(s)\dot\alpha(s-1)} &\equiv T_{\alpha(s)\dot\alpha(s-1)}| \\
\beta_{\alpha(s)\dot\alpha(s-1)} &\equiv \mathrm{D}^2 T_{\alpha(s)\dot\alpha(s-1)}| \\
&\quad + \frac{i}{2s!}\partial_{(\alpha_s}{}^{\dot\alpha_s}\bar T_{\alpha(s-1))\dot\alpha(s)}| \\
\psi_{a(s+1)\dot\alpha(s)} &\equiv \frac{1}{s!(s+1)!}\mathrm{D}_{(\alpha_{s+1}}\bar{\mathrm{D}}_{(\dot\alpha_s}\Psi_{\alpha(s))\dot\alpha(s-1))}| \\
\psi_{\alpha(s)\dot\alpha(s-1)} &\equiv \frac{1}{s!}\bar{\mathrm{D}}^{\dot\alpha_s}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1))\dot\alpha(s)}| \\
\psi_{\alpha(s-1)\dot\alpha(s-2)} &\equiv -\frac{s-1}{s}\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha_{s-1}}V_{\alpha(s-1)\dot\alpha(s-1)}|
\end{aligned}$$

- and the gauge transformations of the fields are

$$\delta_G\psi_{\alpha(s+1)\dot\alpha(s)} = \frac{1}{s!(s+1)!}\partial_{(\alpha_{s+1}(\dot\alpha_s}\xi_{\alpha(s))\dot\alpha(s-1))} \ , \quad \delta_G\rho_{\alpha(s)\dot\alpha(s-1)} = 0,$$

$$\delta_G\psi_{\alpha(s)\dot\alpha(s-1)} = -\frac{1}{s!}\partial_{(\alpha_s}{}^{\dot\alpha_s}\bar\xi_{\alpha(s-1))\dot\alpha(s)} \ , \qquad \delta_G\beta_{\alpha(s)\dot\alpha(s-1)} = 0,$$

$$\delta_G\psi_{\alpha(s-1)\dot\alpha(s-2)} = \frac{s-1}{s}\partial^{\alpha_s\dot\alpha_{s-1}}\xi_{\alpha(s)\dot\alpha(s-1)} \ , \qquad \text{with } \xi_{\alpha(s)\dot\alpha(s-1)} = -i\mathrm{D}^2 L_{\alpha(s)\dot\alpha(s-1)}|$$

Definitions for the bosonic components

- The bosonic components are:

$$U_{\alpha(s+1)\dot{\alpha}(s-1)} \equiv \frac{1}{(s+1)!}\mathrm{D}_{(\alpha(s+1)}T_{\alpha(s))\dot{\alpha}(s-1)}|$$

$$u_{\alpha(s)\dot{\alpha}(s)} \equiv \frac{1}{2s!}\left\{\bar{\mathrm{D}}_{(\dot{\alpha}_s}T_{\alpha(s)\dot{\alpha}(s-1))} - \mathrm{D}_{(\alpha_s}\bar{T}_{\alpha(s-1))\dot{\alpha}(s)}\right\}|$$

$$v_{\alpha(s)\dot{\alpha}(s)} \equiv -\frac{i}{2s!}\left\{\bar{\mathrm{D}}_{(\dot{\alpha}_s}T_{\alpha(s)\dot{\alpha}(s-1))} + \mathrm{D}_{(\alpha_s}\bar{T}_{\alpha(s-1))\dot{\alpha}(s)}\right\}|$$

$$A_{\alpha(s-1)\dot{\alpha}(s-1)} \equiv G_{\alpha(s-1)\dot{\alpha}(s-1)}| - \frac{s}{2s+1}\left(\mathrm{D}^{\alpha_s}T_{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}^{\dot{\alpha}_s}\bar{T}_{\alpha(s)\dot{\alpha}(s-1)}\right)|$$

$$S_{\alpha(s-1)\dot{\alpha}(s-1)} \equiv \frac{1}{2}\left\{\mathrm{D}^{\alpha_s}T_{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}^{\dot{\alpha}_s}\bar{T}_{\alpha(s)\dot{\alpha}(s-1)}\right\}|$$

$$P_{\alpha(s-1)\dot{\alpha}(s-1)} \equiv -\frac{i}{2}\left\{\mathrm{D}^{\alpha_s}T_{\alpha(s)\dot{\alpha}(s-1)} - \bar{\mathrm{D}}^{\dot{\alpha}_s}\bar{T}_{\alpha(s)\dot{\alpha}(s-1)}\right\}|$$

$$h_{\alpha(s)\dot{\alpha}(s)} \equiv \frac{1}{\sqrt{2}}\left\{\frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{\Psi}_{\alpha(s-1))\dot{\alpha}(s)} - \frac{1}{s!}\bar{\mathrm{D}}_{(\dot{\alpha}_s}\Psi_{\alpha(s)\dot{\alpha}(s-1))} - \frac{1}{2s!s!}\left[\mathrm{D}_{(\alpha_s},\bar{\mathrm{D}}_{(\dot{\alpha}_s}\right]V_{\alpha(s-1))\dot{\alpha}(s-1))}\right\}|$$

$$h_{\alpha(s-2)\dot{\alpha}(s-2)} \equiv -\frac{1}{2\sqrt{2}}\frac{s-1}{s^2}\left[\mathrm{D}^{\alpha_{s-1}},\bar{\mathrm{D}}^{\dot{\alpha}_{s-1}}\right]V_{\alpha(s-1)\dot{\alpha}(s-1)}|$$

Bosonic component lagrangian

- The bosonic lagrangian is:

$$
\begin{aligned}
\mathcal{L}_B = & u^{\alpha(s)\dot\alpha(s)} u_{\alpha(s)\dot\alpha(s)} + v^{\alpha(s)\dot\alpha(s)} v_{\alpha(s)\dot\alpha(s)} - \left[\frac{2s+1}{4s}\right] A^{\alpha(s-1)\dot\alpha(s-1)} A_{\alpha(s-1)\dot\alpha(s-1)} \\
& - \left[\frac{s^2}{s+1}\right] P^{\alpha(s-1)\dot\alpha(s-1)} P_{\alpha(s-1)\dot\alpha(s-1)} - \left[\frac{s^2}{(2s+1)(s+1)}\right] S^{\alpha(s-1)\dot\alpha(s-1)} S_{\alpha(s-1)\dot\alpha(s-1)} \\
& - \frac{1}{2} U^{\alpha(s+1)\dot\alpha(s-1)} U_{\alpha(s+1)\dot\alpha(s-1)} + c.c. \\
& + h^{\alpha(s)\dot\alpha(s)} \Box h_{\alpha(s)\dot\alpha(s)} - \frac{s}{2} h^{\alpha(s)\dot\alpha(s)} \partial_{\alpha_s\dot\alpha_s} \partial^{\gamma\dot\gamma} h_{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-1)} \\
& + s(s-1)\ h^{\alpha(s)\dot\alpha(s)} \partial_{\alpha_s\dot\alpha_s} \partial_{\alpha_{s-1}\dot\alpha_{s-1}} h_{\alpha(s-2)\dot\alpha(s-2)} \\
& - s(2s-1)\ h^{\alpha(s-2)\dot\alpha(s-2)} \Box h_{\alpha(s-2)\dot\alpha(s-2)} \\
& - \left[\frac{s(s-2)^2}{2}\right] h^{\alpha(s-2)\dot\alpha(s-2)} \partial_{\alpha_{s-2}\dot\alpha_{s-2}} \partial^{\gamma\dot\gamma} h_{\gamma\alpha(s-3)\dot\gamma\dot\alpha(s-3)}
\end{aligned}
$$

Bosonic component lagrangian

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\begin{aligned}
\mathcal{L}_B =& u^{\alpha(s)\dot{\alpha}(s)} u_{\alpha(s)\dot{\alpha}(s)} + v^{\alpha(s)\dot{\alpha}(s)} v_{\alpha(s)\dot{\alpha}(s)} - \left[\frac{2s+1}{4s}\right] A^{\alpha(s-1)\dot{\alpha}(s-1)} A_{\alpha(s-1)\dot{\alpha}(s-1)} \\
& - \left[\frac{s^2}{s+1}\right] P^{\alpha(s-1)\dot{\alpha}(s-1)} P_{\alpha(s-1)\dot{\alpha}(s-1)} - \left[\frac{s^2}{(2s+1)(s+1)}\right] S^{\alpha(s-1)\dot{\alpha}(s-1)} S_{\alpha(s-1)\dot{\alpha}(s-1)} \\
& - \frac{1}{2}\, U^{\alpha(s+1)\dot{\alpha}(s-1)} U_{\alpha(s+1)\dot{\alpha}(s-1)} + c.c. \\
& + h^{\alpha(s)\dot{\alpha}(s)} \Box h_{\alpha(s)\dot{\alpha}(s)} - \frac{s}{2}\, h^{\alpha(s)\dot{\alpha}(s)} \partial_{\alpha_s\dot{\alpha}_s} \partial^{\gamma\dot{\gamma}} h_{\gamma\alpha(s-1)\dot{\gamma}\dot{\alpha}(s-1)} \\
& + s(s-1)\; h^{\alpha(s)\dot{\alpha}(s)} \partial_{\alpha_s\dot{\alpha}_s} \partial_{\alpha_{s-1}\dot{\alpha}_{s-1}} h_{\alpha(s-2)\dot{\alpha}(s-2)} \\
& - s(2s-1)\; h^{\alpha(s-2)\dot{\alpha}(s-2)} \Box h_{\alpha(s-2)\dot{\alpha}(s-2)} \\
& - \left[\frac{s(s-2)^2}{2}\right] h^{\alpha(s-2)\dot{\alpha}(s-2)} \partial_{\alpha_{s-2}\dot{\alpha}_{s-2}} \partial^{\gamma\dot{\gamma}} h_{\gamma\alpha(s-3)\dot{\gamma}\dot{\alpha}(s-3)}
\end{aligned}
$$

- their gauge transformations are:

$$
\delta_G U_{\alpha(s+1)\dot{\alpha}(s-1)} = 0, \quad \delta_G u_{\alpha(s)\dot{\alpha}(s)} = 0, \quad \delta_G V_{\alpha(s)\dot{\alpha}(s)} = 0,
$$

$$
\delta_G A_{\alpha(s-1)\dot{\alpha}(s-1)} = 0, \quad \delta_G S_{\alpha(s-1)\dot{\alpha}(s-1)} = 0, \quad \delta_G P_{\alpha(s-1)\dot{\alpha}(s-1)} = 0,
$$

$$
\delta_G h_{\alpha(s)\dot{\alpha}(s)} = \frac{1}{s!s!} \partial_{(\alpha_s(\dot{\alpha}_s} \zeta_{\alpha(s-1))\dot{\alpha}(s-1))}, \quad \delta_G h_{\alpha(s-2)\dot{\alpha}(s-2)} = \frac{s-1}{s^2} \partial^{\alpha_{s-1}\dot{\alpha}_{s-1}} \zeta_{\alpha(s-1)\dot{\alpha}(s-1)}
$$

where $\zeta_{\alpha(s-1)\dot{\alpha}(s-1)} = \frac{i}{2\sqrt{2}} \left(\mathrm{D}^{\alpha_s} L_{\alpha(s)\dot{\alpha}(s-1)} - \bar{\mathrm{D}}^{\dot{\alpha}_s} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)} \right) |$

Off-shell d.o.f: Bosons

The bosonic degrees of freedom involved in the theory are:

fields	*d.o.f*	*redundancy*	*net*
$h_{\alpha(s)\dot{\alpha}(s)}$	$(s+1)^2$	s^2	s^2+2
$h_{\alpha(s-2)\dot{\alpha}(s-2)}$	$(s-1)^2$		
$u_{\alpha(s)\dot{\alpha}(s)}$	$(s+1)^2$	0	$(s+1)^2$
$v_{\alpha(s)\dot{\alpha}(s)}$	$(s+1)^2$	0	$(s+1)^2$
$A_{\alpha(s-1)\dot{\alpha}(s-1)}$	s^2	0	s^2
$U_{\alpha(s+1)\dot{\alpha}(s-1)}$	$2(s+2)s$	0	$2(s+2)s$
$S_{\alpha(s-1)\dot{\alpha}(s-1)}$	s^2	0	s^2
$P_{\alpha(s-1)\dot{\alpha}(s-1)}$	s^2	0	s^2
		Total	$8s^2+8s+4$

Off-shell d.o.f: Fermions

The Fermionic degrees of freedom are:

fields	*d.o.f*	*redundancy*	*net*
$\psi_{\alpha(s+1)\dot{\alpha}(s)}$	$2(s+2)(s+1)$	$2(s+1)s$	$4s^2+4s+4$
$\psi_{\alpha(s)\dot{\alpha}(s-1)}$	$2(s+1)s$		
$\psi_{\alpha(s-1)\dot{\alpha}(s-2)}$	$2s(s-1)$		
$\rho_{\alpha(s)\dot{\alpha}(s-1)}$	$2(s+1)s$	0	$2(s+1)s$
$\beta_{\alpha(s)\dot{\alpha}(s-1)}$	$2(s+1)s$	0	$2(s+1)s$
		Total	$8s^2+8s+4$

Supersymmetric transformations for the components

- $\mathcal{SUSY}$ transformations for the components

$$\delta_S \Phi_{(n,m)} = -\left(\epsilon^{\alpha} \mathrm{D}_{\alpha} + \bar{\epsilon}^{\dot{\alpha}} \bar{\mathrm{D}}_{\dot{\alpha}}\right) f(\mathrm{D}^{n}, \bar{\mathrm{D}}^{m}) \Phi|$$

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- *Note*: The dynamical components ($\in \mathcal{D}$) are treated as equivalence classes

$$\mathcal{D} \sim \mathcal{D} + \partial\left(\zeta\right) \quad \rightarrow \quad \delta_S\{\mathcal{D}\} \sim \delta_S\{\mathcal{D}\} + \partial\left(\delta_S\zeta\right)$$

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$$\mathcal{D} \sim \mathcal{D} + \partial(\zeta) \quad \to \quad \delta_S\{\mathcal{D}\} \sim \delta_S\{\mathcal{D}\} + \partial(\delta_S\zeta)$$
- For the fermions we get:

$$\delta_S \psi_{\alpha(s+1)\dot{\alpha}(s)} = -\frac{1}{s!}\bar{\epsilon}_{(\dot{\alpha}_s} U_{\alpha(s+1)\dot{\alpha}(s-1))} - \frac{1}{(s+1)!}\epsilon_{(\alpha_{s+1}}\left[u_{\alpha(s))\dot{\alpha}(s)} - iv_{\alpha(s))\dot{\alpha}(s)}\right] + \frac{i\sqrt{2}}{(s+1)!}\bar{\epsilon}^{\dot{\beta}}\partial_{(\alpha_{s+1}\dot{\beta}}h_{\alpha(s))\dot{\alpha}(s)}$$

$$\delta_S \psi_{\alpha(s)\dot{\alpha}(s-1)} = \bar{\epsilon}^{\dot{\alpha}_s}\left[u_{\alpha(s)\dot{\alpha}(s)} + iv_{\alpha(s)\dot{\alpha}(s)}\right] - \frac{s}{(2s+1)s!}\epsilon_{(\alpha_s}S_{\alpha(s-1))\dot{\alpha}(s-1)} - \frac{is}{s!}\epsilon_{(\alpha_s}P_{\alpha(s-1))\dot{\alpha}_(s-1)} + \frac{s+1}{(2s)s!}\epsilon_{(\alpha_s}A_{\alpha(s-1))\dot{\alpha}(s-1)} + i\frac{s-1}{\sqrt{2}}\epsilon^{\beta}\partial_{\beta}{}^{\dot{\alpha}_s}h_{\alpha(s)\dot{\alpha}(s)} + i\frac{(s+1)s(s-1)}{\sqrt{2}s!s!}\epsilon_{(\alpha_s}\partial_{\alpha_{s-1}(\dot{\alpha}_{s-1}}h_{\alpha(s-2))\dot{\alpha}(s-2))}$$

$$\delta_S \psi_{\alpha(s-1)\dot{\alpha}(s-2)} = \frac{(s-1)(2s+1)}{2s^2}\bar{\epsilon}^{\dot{\alpha}_{s-1}}A_{\alpha(s-1)\dot{\alpha}(s-1)} + \frac{i(s-1)^2}{\sqrt{2}s(s-1)!^2}\bar{\epsilon}^{\dot{\alpha}_{s-1}}\partial_{(\alpha_{s-1}(\dot{\alpha}_{s-1}}h_{\alpha(s-2))\dot{\alpha}(s-2)} - \frac{i\sqrt{2}(s-1)^2}{s(s-1)!^2}\partial_{(\alpha_{s-1}}{}^{\dot{\alpha}_{s-1}}\bar{\epsilon}_{(\dot{\alpha}_{s-1}}h_{\alpha(s-2))\dot{\alpha}(s-2))}$$

δ_S for fermions (Cont'd.)

and for the auxiliary components

$$
\begin{aligned}
\delta_S\beta_{\alpha(s)\dot\alpha(s-1)} =& -i\bar\epsilon^{\dot\beta}\partial^{\alpha_{s+1}}{}_{\dot\beta}U_{\alpha(s+1)\dot\alpha(s-1)} - \frac{i}{2s!}\bar\epsilon^{\dot\alpha_{s+1}}\partial_{(\alpha_s}{}^{\dot\alpha_s}\bar U_{\alpha(s-1))\dot\alpha(s+1)} \\
&+\frac{i}{2s!}\epsilon^{\beta}\partial_{(\alpha_s}{}^{\dot\alpha_s}\left[u_{\beta\alpha(s-1))\dot\alpha(s)} - iv_{\beta\alpha(s-1))\dot\alpha(s)}\right] + \frac{i}{2}\frac{1}{s!s!}\bar\epsilon^{\dot\alpha_s}\partial_{(\alpha_s(\dot\alpha_s}A_{\alpha(s-1))\dot\alpha(s-1))} \\
&+\frac{i}{2}\left[\frac{2s^2-1}{(s+1)(2s+1)}\right]\frac{1}{s!s!}\bar\epsilon^{\dot\alpha_s}\partial_{(\alpha_s(\dot\alpha_s}S_{\alpha(s-1))\dot\alpha(s-1))} \\
&+\frac{1}{2}\left[\frac{2s^2-2s-1}{s+1}\right]\frac{1}{s!s!}\bar\epsilon^{\dot\alpha_s}\partial_{(\alpha_s(\dot\alpha_s}P_{\alpha(s-1))\dot\alpha(s-1))} \\
&-\frac{i}{2}\left[\frac{(s-1)^2}{s(s+1)}\right]\frac{1}{s!(s-1)!}\bar\epsilon_{(\dot\alpha_{s-1}}\partial_{(\alpha_s}{}^{\dot\gamma}S_{\alpha(s-1))\dot\gamma\dot\alpha(s-2))} \\
&+\frac{1}{2}\left[\frac{(s-1)(3s+1)}{s(s+1)}\right]\frac{1}{s!(s-1)!}\bar\epsilon_{(\dot\alpha_{s-1}}\partial_{(\alpha_s}{}^{\dot\gamma}P_{\alpha(s-1))\dot\gamma\dot\alpha(s-2))} \\
&-\sqrt{2}\bar\epsilon^{\dot\alpha_s}\Box h_{\alpha(s)\dot\alpha(s)} + \frac{s}{\sqrt{2}}\frac{1}{s!s!}\bar\epsilon^{\dot\alpha_s}\partial_{(\alpha_s(\dot\alpha_s}\partial^{\gamma\dot\gamma}h_{\gamma\alpha(s-1))\dot\gamma\dot\alpha(s-1))} \\
&-\frac{s(s-1)}{\sqrt{2}}\frac{1}{s!s!}\bar\epsilon^{\dot\alpha_s}\partial_{(\alpha_s(\dot\alpha_s}\partial_{\alpha_{s-1}\dot\alpha_{s-1}}h_{\alpha(s-2))\dot\alpha(s-2))}
\end{aligned}
$$

$$
\begin{aligned}
\delta_S\rho_{\alpha(s)\dot\alpha(s-1)} =& -\epsilon^{\alpha_{s+1}}U_{\alpha(s+1)\dot\alpha(s-1)} + \frac{s}{(s+1)!}\epsilon_{(\alpha_s}\left[S_{\alpha(s-1))\dot\alpha(s-1)} + iP_{\alpha(s-1))\dot\alpha(s-1)}\right] \\
&-\bar\epsilon^{\dot\alpha_s}\left[u_{\alpha(s)\dot\alpha(s)} + iv_{\alpha(s)\dot\alpha(s)}\right]
\end{aligned}
$$

δ_s for bosons

for the dynamical components

$$\delta_S h_{\alpha(s)\dot\alpha(s)} = \frac{1}{\sqrt{2}}\bar\epsilon^{\dot\alpha_{s+1}}\bar\psi_{\alpha(s)\dot\alpha(s+1)} - \frac{1}{\sqrt{2}(s+1)}\frac{1}{s!}\bar\epsilon_{(\dot\alpha_s}\psi_{\alpha(s)\dot\alpha(s-1))} + c.c.$$
$$+\frac{1}{\sqrt{2}s!}\epsilon_{(\alpha_s}\bar\rho_{\alpha(s-1))\dot\alpha(s)} + c.c.$$

$$\delta_S h_{\alpha(s-2)\dot\alpha(s-2)} = -\frac{1}{\sqrt{2}s}\epsilon^{\alpha_{s-1}}\psi_{\alpha(s-1)\dot\alpha(s-2)} + c.c.$$

for the auxiliary components

$$\delta_S U_{\alpha(s+1)\dot\alpha(s-1)} = \frac{1}{(s+1)!}\epsilon_{(\alpha_{s+1}}\beta_{\alpha(s))\dot\alpha(s-1)} - \frac{i}{2}\frac{1}{(s+1)!}\epsilon_{(\alpha_{s+1}}\partial_{\alpha_s}{}^{\dot\alpha_s}\bar\rho_{\alpha(s-1))\dot\alpha(s)}$$
$$-\frac{i}{(s+1)!}\bar\epsilon^{\dot\beta}\partial_{(\alpha_{s+1}\dot\beta}\rho_{\alpha(s))\dot\alpha(s-1)} - \frac{i}{(s+1)!}\bar\epsilon^{\dot\alpha_s}\partial_{(\alpha_{s+1}}{}^{\dot\alpha_{s+1}}\bar\psi_{\alpha(s))\dot\alpha(s+1)}$$
$$-i\frac{s}{s+1}\frac{1}{(s+1)!s!}\bar\epsilon^{\dot\alpha_s}\partial_{(\alpha_{s+1}(\dot\alpha_s}\psi_{\alpha(s))\dot\alpha(s-1))}$$

δ_s for bosons (Cont'd.)

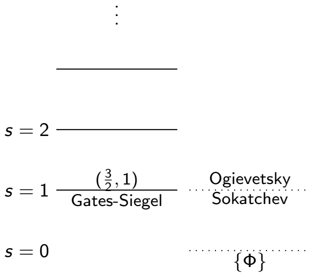
$$\delta_S A_{\alpha(s-1)\dot\alpha(s-1)} = i\frac{(s-1)(s+1)}{s(2s+1)(s-1)!}\bar\epsilon_{(\dot\alpha_{s-1}}\partial^{\alpha_s\dot\gamma}\rho_{\alpha(s)\dot\gamma\dot\alpha(s-2))} - \frac{i}{(2s+1)s!}\bar\epsilon^{\dot\alpha_s}\partial^{\alpha_s}{}_{(\dot\alpha_s}\rho_{\alpha(s)\dot\alpha(s-1)} + c.c.$$
$$+ i\frac{s}{2s+1}\bar\epsilon^{\dot\alpha_s}\partial^{\alpha_s\dot\alpha_{s+1}}\bar\psi_{\alpha(s)\dot\alpha(s+1)} - \frac{i}{(s+1)s!}\bar\epsilon^{\dot\alpha_s}\partial^{\alpha_s}{}_{(\dot\alpha_s}\psi_{\alpha(s)\dot\alpha(s-1))} + c.c.$$
$$+ i\frac{s-1}{s!}\epsilon_{(\alpha_{s-1}}\partial^{\gamma\dot\alpha_s}\bar\psi_{\gamma\alpha(s-2))\dot\alpha(s)} + \frac{i(s+1)}{(2s+1)(s-1)!s!}\bar\epsilon^{\dot\alpha_s}\partial_{(\alpha_{s-1}(\dot\alpha_s}\bar\psi_{\alpha(s-2))\dot\alpha(s-1))} + c.c.$$
$$- i\frac{s-1}{s!(s-1)!}\epsilon_{(\alpha_{s-1}}\partial^{\gamma}{}_{(\dot\alpha_{s-1}}\psi_{\gamma\alpha(s-2))\dot\alpha(s-2))} + c.c.$$

$$\delta_S\left(u_{\alpha(s)\dot\alpha(s)} + iv_{\alpha(s)\dot\alpha(s)}\right) = \frac{i}{(s+1)!}\epsilon^{\alpha_{s+1}}\partial_{(\alpha_{s+1}}{}^{\dot\alpha_{s+1}}\bar\psi_{\alpha(s))\dot\alpha(s+1)} - i\frac{s}{(s+1)s!}\epsilon_{(\alpha_s}\partial^{\gamma\dot\alpha_{s+1}}\bar\psi_{\gamma\alpha(s-1))\dot\alpha(s+1)}$$
$$+ i\frac{s}{(s+1)(s+1)!s!}\epsilon^{\alpha_{s+1}}\partial_{(\alpha_{s+1}(\dot\alpha_s}\psi_{\alpha(s))\dot\alpha(s-1))}$$
$$+ i\frac{2s+1}{(s+1)^2 s!^2}\epsilon_{(\alpha_s}\partial^{\gamma}{}_{(\dot\alpha_s}\psi_{\gamma\alpha(s-1))\dot\alpha(s-1))} + \frac{i}{s!s!}\epsilon_{(\alpha_s}\partial_{\alpha_{s-1}(\dot\alpha_s}\bar\psi_{\alpha(s-2))\dot\alpha(s-1))}$$
$$+ \frac{1}{s!}\epsilon_{(\alpha_s}\bar\beta_{\alpha(s-1))\dot\alpha(s)} + \frac{i}{2s!^2}\epsilon_{(\alpha_s}\partial^{\gamma}{}_{(\dot\alpha_s}\rho_{\gamma\alpha(s-1))\dot\alpha(s-1))}$$

$$\delta_S\left(S_{\alpha(s-1)\dot\alpha(s-1)} + iP_{\alpha(s-1)\dot\alpha(s-1)}\right) = \epsilon^{\alpha_s}\beta_{\alpha(s)\dot\alpha(s-1)} + \frac{s+1}{s}\bar\epsilon^{\dot\alpha_s}\bar\beta_{\alpha(s-1)\dot\alpha(s)} - \frac{i}{2\ s!}\epsilon^{\alpha_s}\partial_{(\alpha_s}{}^{\dot\alpha_s}\bar\rho_{\alpha(s-1)\dot\alpha(s)}$$
$$- i\frac{s-1}{(2s)s!}\bar\epsilon^{\dot\alpha_s}\partial^{\alpha_s}{}_{(\dot\alpha_s}\rho_{\alpha(s)\dot\alpha(s-1))} + i\frac{s-1}{s!}\bar\epsilon_{(\dot\alpha_{s-1}}\partial^{\alpha_s\dot\gamma}\rho_{\alpha(s)\dot\gamma\dot\alpha(s-2))} - i\bar\epsilon^{\dot\alpha_s}\partial^{\alpha_s\dot\alpha_{s+1}}\bar\psi_{\alpha(s)\dot\alpha(s+1)}$$
$$+ i\frac{2s+1}{s(s+1)s!}\bar\epsilon^{\dot\alpha_s}\partial^{\alpha_s}{}_{(\dot\alpha_s}\psi_{\alpha(s)\dot\alpha(s-1)} + i\frac{s+1}{s\ (s-1)!s!}\bar\epsilon^{\dot\alpha_s}\partial_{(\alpha_{s-1}(\dot\alpha_s}\bar\psi_{\alpha(s-2))\dot\alpha(s-1))}$$

Outline

Map of Arbitrary Superhelicities



Integer Superhelicity $Y = s$

$$\{\Psi_{\alpha(s)\dot{\alpha}(s-1)} \ , \ V_{\alpha(s-1)\dot{\alpha}(s-1)}\}$$

$$8s^2 + 8s + 4$$

$h_{\alpha(s)\dot{\alpha}(s)}$	$\psi_{\alpha(s+1)\dot{\alpha}(s)}$
$h_{\alpha(s-2)\dot{\alpha}(s-2)}$	$\psi_{\alpha(s)\dot{\alpha}(s-1)}$
$A_{\alpha(s-1)\dot{\alpha}(s-1)}$	$\psi_{\alpha(s-1)\dot{\alpha}(s-2)}$
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$P_{\alpha(s-1)\dot{\alpha}(s-1)}$	
$U_{\alpha(s+1)\dot{\alpha}(s-1)}$	

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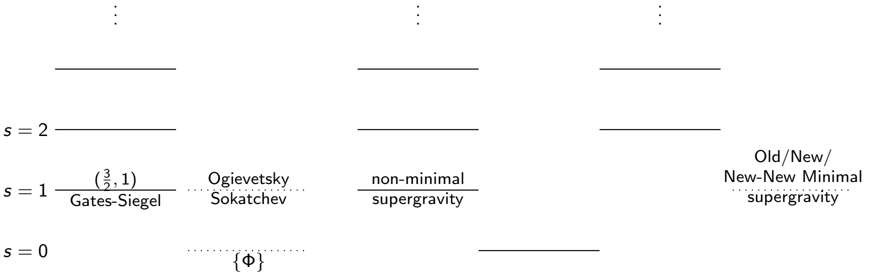

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$u_{\alpha(s)\dot\alpha(s)}$	$\beta_{\alpha(s)\dot\alpha(s-1)}$
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$S_{\alpha(s-1)\dot\alpha(s-1)}$	
$P_{\alpha(s-1)\dot\alpha(s-1)}$	
$U_{\alpha(s+1)\dot\alpha(s-1)}$	

Half-Integer Superhelicity $Y=s+1/2$

$\{H_{\alpha(s)\dot\alpha(s)}\ ,\ \chi_{\alpha(s)\dot\alpha(s-1)}\}$

$8s^2+8s+4$ (I)

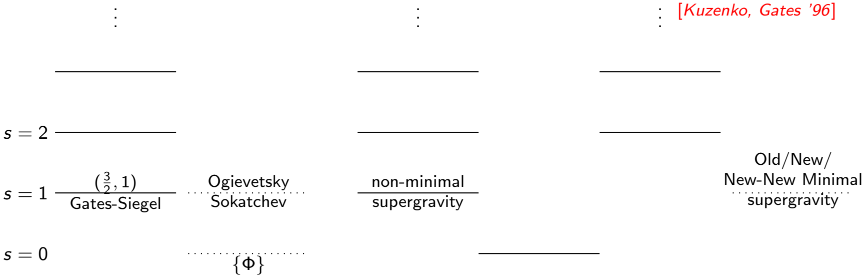
$h_{\alpha(s+1)\dot\alpha(s+1)}$	$\psi_{\alpha(s+1)\dot\alpha(s)}$
$h_{\alpha(s-1)\dot\alpha(s-1)}$	$\psi_{\alpha(s)\dot\alpha(s-1)}$
$A_{\alpha(s)\dot\alpha(s)}$	$\psi_{\alpha(s-1)\dot\alpha(s-2)}$
$u_{\alpha(s)\dot\alpha(s)}$	$\beta_{\alpha(s)\dot\alpha(s-1)}$
$v_{\alpha(s)\dot\alpha(s)}$	$\rho_{\alpha(s)\dot\alpha(s-1)}$
$S_{\alpha(s-1)\dot\alpha(s-1)}$	
$P_{\alpha(s-1)\dot\alpha(s-1)}$	
$U_{\alpha(s)\dot\alpha(s-2)}$	

$\{H_{\alpha(s)\dot\alpha(s)}\ ,\ \chi_{\alpha(s-1)\dot\alpha(s-2)}\}$

$8s^2+4$ (II)

$h_{\alpha(s+1)\dot\alpha(s+1)}$	$\psi_{\alpha(s+1)\dot\alpha(s)}$
$h_{\alpha(s-1)\dot\alpha(s-1)}$	$\psi_{\alpha(s)\dot\alpha(s-1)}$
$A_{\alpha(s)\dot\alpha(s)}$	$\psi_{\alpha(s-1)\dot\alpha(s-2)}$
$u_{\alpha(s-1)\dot\alpha(s-1)}$	$\beta_{\alpha(s-1)\dot\alpha(s-2)}$
$v_{\alpha(s-1)\dot\alpha(s-1)}$	$\rho_{\alpha(s-1)\dot\alpha(s-2)}$
$S_{\alpha(s-2)\dot\alpha(s-2)}$	
$P_{\alpha(s-2)\dot\alpha(s-2)}$	
$U_{\alpha(s)\dot\alpha(s-2)}$	

Map of Arbitrary Superhelicities, Hints of $\mathcal{N}=2$



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$\{\Psi_{\alpha(s)\dot\alpha(s-1)}\ ,\ V_{\alpha(s-1)\dot\alpha(s-1)}\}$ — $8s^2+8s+4$

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$u_{\alpha(s)\dot\alpha(s)}$	$\beta_{\alpha(s)\dot\alpha(s-1)}$
$v_{\alpha(s)\dot\alpha(s)}$	$\rho_{\alpha(s)\dot\alpha(s-1)}$
$S_{\alpha(s-1)\dot\alpha(s-1)}$	
$P_{\alpha(s-1)\dot\alpha(s-1)}$	
$U_{\alpha(s+1)\dot\alpha(s-1)}$	

$\{H_{\alpha(s)\dot\alpha(s)}\ ,\ \chi_{\alpha(s)\dot\alpha(s-1)}\}$ — $8s^2+8s+4$ (I)

$h_{\alpha(s+1)\dot\alpha(s+1)}$	$\psi_{\alpha(s+1)\dot\alpha(s)}$
$h_{\alpha(s-1)\dot\alpha(s-1)}$	$\psi_{\alpha(s)\dot\alpha(s-1)}$
$A_{\alpha(s)\dot\alpha(s)}$	$\psi_{\alpha(s-1)\dot\alpha(s-2)}$
$u_{\alpha(s)\dot\alpha(s)}$	$\beta_{\alpha(s)\dot\alpha(s-1)}$
$v_{\alpha(s)\dot\alpha(s)}$	$\rho_{\alpha(s)\dot\alpha(s-1)}$
$S_{\alpha(s-1)\dot\alpha(s-1)}$	
$P_{\alpha(s-1)\dot\alpha(s-1)}$	
$U_{\alpha(s)\dot\alpha(s-2)}$	

$\{H_{\alpha(s)\dot\alpha(s)}\ ,\ \chi_{\alpha(s-1)\dot\alpha(s-2)}\}$ — $8s^2+4$ (II)

$h_{\alpha(s+1)\dot\alpha(s+1)}$	$\psi_{\alpha(s+1)\dot\alpha(s)}$
$h_{\alpha(s-1)\dot\alpha(s-1)}$	$\psi_{\alpha(s)\dot\alpha(s-1)}$
$A_{\alpha(s)\dot\alpha(s)}$	$\psi_{\alpha(s-1)\dot\alpha(s-2)}$
$u_{\alpha(s-1)\dot\alpha(s-1)}$	$\beta_{\alpha(s-1)\dot\alpha(s-2)}$
$v_{\alpha(s-1)\dot\alpha(s-1)}$	$\rho_{\alpha(s-1)\dot\alpha(s-2)}$
$S_{\alpha(s-2)\dot\alpha(s-2)}$	
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- Applying this approach, we get the off-shell lagrangian description of the arbitrary superhelicity, irreducible representations of the $4D$, $N = 1$ Super Poincaré group {These are the basic building blocks one should use in order to go beyond free theory}

Summary

- Projecting from Superspace to components typically is a cumbersome process
- I demonstrated an efficient method to organize the calculation, that gives us a better understanding of where the various components come from.
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- Having the proper definition of components, allows us to find their *susy* transformations
- A naive counting of the off-shell d.o.f provides hints how to construct the $\mathcal{N} = 2$ representations

Now, that's how you project

Thank you !

Superspace Approach: Building Blocks

Superspace action integral $\xrightarrow{\text{equations of motion}}$ $\bar{\mathrm{D}}_{\dot{\gamma}} F_{\alpha(2s)} = 0, \; \mathrm{D}^{\alpha_{2s}} F_{\alpha(2s)} = 0, \; F_{\alpha(2s)}$

Kuzenko, Postnikov & Sibiryakov 1993

Kuzenko, Sibiryakov 1993

Gates, Koutrolikos 2011,2014

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Problem: 1)looks like $\Psi_{\alpha(s)\dot{\alpha}(s-1)}$ (A_m) is the fundamental object but $F_{\alpha(2s)}$ (F_{mn}) carries the physical d.o.f

2)d.o.f of $F_{\alpha(2s)}$ (F_{mn}) $\neq$ d.o.f of $\Psi_{\alpha(s)\dot{\alpha}(s-1)}$ (A_m)

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- $A_m\sim A_m+R_m$
- $\partial_m R_n-\partial_n R_m=0$
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Construction of the Superspace Action

- $[\Psi] = 1/2$ $\quad (\Psi \sim \theta\bar{\theta}\psi_{(1,1)}$, $\psi_{(1,1)}$: propagating fermion)

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S = \int d^8z\ & a_1 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \mathrm{D}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\
& + a_2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\
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- It must be invariant under:

$$\delta_G \Psi_{\alpha(s)\dot{\alpha}(s-1)} = \frac{1}{s!} \mathrm{D}_{(\alpha_s} K_{\alpha(s-1))\dot{\alpha}(s-1)} + \frac{1}{(s-1)!} \bar{\mathrm{D}}_{(\dot{\alpha}_{s-1}} \Lambda_{\alpha(s)\dot{\alpha}(s-2))}$$

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- Its deformation under gauge transformation is:

$$\delta_G S = \int d^8z \Bigg[\Bigg\{ -2a_1 \mathrm{D}_{\alpha_s} \Psi^{\alpha(s)\dot{\alpha}(s-1)} + a_4 \bar{\mathrm{D}}_{\dot{\alpha}_s} \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \Bigg\} \mathrm{D}^\beta \bar{\mathrm{D}}_{\dot{\alpha}_{s-1}} \Lambda_{\beta\alpha(s-1)\dot{\alpha}(s-2)}$$
$$+ \Bigg\{ -a_3 \left[\frac{s-1}{s} \right] \bar{\mathrm{D}}_{\dot{\alpha}_s} \mathrm{D}_{\alpha_{s-1}} \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)}$$
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Gauge Invariance: $\delta_G S = 0$

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- Choose coefficients for gauge invariance? No choice of free parameters
- Add compensators? No choice for compensator with appropriate dimensions and index structure
- Constraint the gauge parameters, choose coefficients and add compensator
  $\mathrm{D}^{\beta} K_{\beta\alpha(s-2)\dot\alpha(s-1)} = 0 \rightarrow K_{\alpha(s-1)\dot\alpha(s-1)} = \mathrm{D}^{\alpha_s} L_{\alpha(s)\dot\alpha(s-1)}$,
  $a_1 = a_4 = 0,\ 2a_2 = -a_3$,
  $V_{\alpha(s-1)\dot\alpha(s-1)},\ \delta_G V_{\alpha(s-1)\dot\alpha(s-1)} = \mathrm{D}^{\alpha_s} L_{\alpha(s)\dot\alpha(s-1)} + \bar{\mathrm{D}}^{\dot\alpha_s} \bar{L}_{\alpha(s-1)\alpha(s)}$

Gauge invariant action & Bianchi Identities

- Update the action, with the presence of $V_{\alpha(s-1)\dot{\alpha}(s-1)}$

$$
\begin{aligned}
S = \int d^8z \Big\{ & -\frac{1}{2} a_3 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\
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& - a_3 V^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\
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& + b_2 V^{\alpha(s-1)\dot{\alpha}(s-1)} \left\{ \mathrm{D}^2, \bar{\mathrm{D}}^2 \right\} V_{\alpha(s-1)\dot{\alpha}(s-1)} \\
& + b_3 V^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}_{\alpha_{s-1}} \bar{\mathrm{D}}^2 \mathrm{D}^{\gamma} V_{\gamma\alpha(s-2)\dot{\alpha}(s-1)} + c.c. \\
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$$+ b_2 V^{\alpha(s-1)\dot{\alpha}(s-1)} \left\{ \mathrm{D}^2, \bar{\mathrm{D}}^2 \right\} V_{\alpha(s-1)\dot{\alpha}(s-1)}$$
$$+ b_3 V^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}_{\alpha_{s-1}} \bar{\mathrm{D}}^2 \mathrm{D}^{\gamma} V_{\gamma\alpha(s-2)\dot{\alpha}(s-1)} + c.c.$$
$$\left. + b_4 \; V^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}_{\alpha_{s-1}} \bar{\mathrm{D}}_{\dot{\alpha}_{s-1}} \mathrm{D}^{\gamma} \bar{\mathrm{D}}^{\dot{\gamma}} V_{\gamma\alpha(s-2)\dot{\gamma}\dot{\alpha}(s-2)} + c.c. \right\}$$

- Invariant under

$$\delta_G \Psi_{\alpha(s)\dot{\alpha}(s-1)} = -\mathrm{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \left[\frac{1}{(s-1)!} \right] \bar{\mathrm{D}}_{(\dot{\alpha}_{s-1}} \Lambda_{\alpha(s)\dot{\alpha}(s-2))}$$
$$\delta_G V_{\alpha(s-1)\dot{\alpha}(s-1)} = \mathrm{D}^{\alpha_s} L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}^{\dot{\alpha}_s} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)}$$

Gauge invariant action & Bianchi Identities

- Update the action, with the presence of $V_{\alpha(s-1)\dot{\alpha}(s-1)}$

$$S = \int d^8z \left\{ -\frac{1}{2} a_3 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \right.$$
$$+ a_3 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{\mathrm{D}}^{\dot{\alpha}_s} \mathrm{D}_{\alpha_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)}$$
$$- a_3 V^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}^{\alpha_s} \bar{\mathrm{D}}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c.$$
$$+ b_1 V^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}^{\gamma} \bar{\mathrm{D}}^2 \mathrm{D}_{\gamma} V_{\alpha(s-1)\dot{\alpha}(s-1)}$$
$$+ b_2 V^{\alpha(s-1)\dot{\alpha}(s-1)} \left\{ \mathrm{D}^2, \bar{\mathrm{D}}^2 \right\} V_{\alpha(s-1)\dot{\alpha}(s-1)}$$
$$+ b_3 V^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}_{\alpha_{s-1}} \bar{\mathrm{D}}^2 \mathrm{D}^{\gamma} V_{\gamma\alpha(s-2)\dot{\alpha}(s-1)} + c.c.$$
$$\left. + b_4 \ V^{\alpha(s-1)\dot{\alpha}(s-1)} \mathrm{D}_{\alpha_{s-1}} \bar{\mathrm{D}}_{\dot{\alpha}_{s-1}} \mathrm{D}^{\gamma} \bar{\mathrm{D}}^{\dot{\gamma}} V_{\gamma\alpha(s-2)\dot{\gamma}\dot{\alpha}(s-2)} + c.c. \right\}$$

- Invariant under

$$\delta_G \Psi_{\alpha(s)\dot{\alpha}(s-1)} = -\mathrm{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \left[\frac{1}{(s-1)!} \right] \bar{\mathrm{D}}_{(\dot{\alpha}_{s-1}} \Lambda_{\alpha(s)\dot{\alpha}(s-2))}$$
$$\delta_G V_{\alpha(s-1)\dot{\alpha}(s-1)} = \mathrm{D}^{\alpha_s} L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}^{\dot{\alpha}_s} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)}$$

- Bianchi identities

$$\mathrm{D}^2 T_{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{s!} \mathrm{D}_{(\alpha_s} G_{\alpha(s-1))\dot{\alpha}(s-1)} = 0, \quad G_{\alpha(s-1)\dot{\alpha}(s-1)} = \frac{\delta S}{\delta V^{\alpha(s-1)\dot{\alpha}(s-1)}}$$
$$\bar{\mathrm{D}}^{\dot{\alpha}_{s-1}} T_{a(s)\dot{\alpha}(s-1)} = 0 \quad , \quad T_{\alpha(s)\dot{\alpha}(s-1)} = \frac{\delta S}{\delta \Psi^{\alpha(s)\dot{\alpha}(s-1)}}$$

The Superspace action integral

- Bianchi identities fix all free coefficients

$$b_1 = \frac{1}{2}a_3 \ , \ \ b_3 = 0 \ , \ \ b_2 = 0 \ , \ \ b_4 = 0$$

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$$\begin{aligned} S = \int d^8z \Big\{ & -\frac{1}{2}c\Psi^{\alpha(s)\dot{\alpha}(s-1)}\bar{\mathrm{D}}^2\Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\ & + c\Psi^{\alpha(s)\dot{\alpha}(s-1)}\bar{\mathrm{D}}^{\dot{\alpha}_s}\mathrm{D}_{\alpha_s}\bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \\ & - cV^{\alpha(s-1)\dot{\alpha}(s-1)}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^2\Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\ & + \frac{1}{2}cV^{\alpha(s-1)\dot{\alpha}(s-1)}\mathrm{D}^{\gamma}\bar{\mathrm{D}}^2\mathrm{D}_{\gamma}V_{\alpha(s-1)\dot{\alpha}(s-1)} \Big\} \end{aligned}$$

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- The equations of motion are

$$\begin{aligned} T_{\alpha(s)\dot\alpha(s-1)} &= -c\bar{\mathrm{D}}^2\Psi_{\alpha(s)\dot\alpha(s-1)} + \frac{c}{s!}\bar{\mathrm{D}}^{\dot\alpha_s}\mathrm{D}_{(\alpha_s}\bar\Psi_{\alpha(s-1))\dot\alpha(s)} \\ & + \frac{c}{s!}\bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_s}V_{\alpha(s-1))\dot\alpha(s-1)} \\ G_{\alpha(s-1)\dot\alpha(s-1)} &= -c\left(\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^2\Psi_{\alpha(s)\dot\alpha(s-1)} + \bar{\mathrm{D}}^{\dot\alpha_s}\mathrm{D}^2\bar\Psi_{\alpha(s-1)\alpha(s)}\right) \\ & + c\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma V_{\alpha(s-1)\dot\alpha(s-1)} \end{aligned}$$