Second Order Fermions and the Standard Model

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For different views suggest different kinds of modifications which might be made and hence are not equivalent in the hypotheses one generates from them in one's attempt to understand what is not yet understood.

Richard Feynman, Nobel Lecture

Outline

I. From first to second order fermions

II. The Standard Model
- A. Quark Sector
- B. Higgs Sector

III. Future Prospects and Conclusion
- A. Anomalies
- B. Unitarity
- C. Conclusions

I. From first to second order fermions

Dirac fermions

We start with the Dirac Lagrangian coupled to Electrodynamics in $3+1$ dimensions with the metric $\eta_{\mu\nu} = (-,+,+,+)$. We have:

$$\mathcal{L}_D = -i\bar{\Psi}\not{D}\Psi - m\bar{\Psi}\Psi, \quad \not{D}\Psi = (\not{\partial} + ie\not{A})\Psi \tag{1}$$

with $\not{D} = \gamma^\mu D_\mu$.
The Lagrangian has the usual $U(1)$ gauge symmetry:

$$\Psi \ \mapsto \ e^{-ie\alpha(x)}\Psi, \quad A_\mu \ \mapsto \ A_\mu + \partial_\mu\alpha \tag{2}$$

Two-component fermions

Using the isomorphism

$$SO(1,3) \sim SL(2,\mathbb{C}) \tag{3}$$

And the soldering form (also referred to as "tetrad" or "vielbein"):

$$\theta_\mu^{AA'} \ : \ x^\mu \mapsto x^{AA'} \equiv \theta_\mu^{AA'} x^\mu \tag{4}$$

One identifies Minkowski spacetime with the set of hermitian (2×2) matrices.

Two-component fermions

We can further identify:

$$\Psi = \begin{pmatrix} \chi \\ \xi^\dagger \end{pmatrix}, \quad \gamma^\mu = \begin{pmatrix} 0 & \sqrt{2}\theta^\mu \\ \sqrt{2}\theta^\mu & 0 \end{pmatrix} \tag{5}$$

where we omit spinor indices.

- In order to go into a two-component fermions formalism $\rightarrow$ rewrite the Dirac Lagrangian in terms of these new quantities.

Integrating one chirality out

The Dirac Lagrangian is then:

$$\mathcal{L}_D = -i\sqrt{2}\chi^\dagger_{A'} D^{A'A}\chi_A - i\sqrt{2}\xi^\dagger_{A'} D^{A'A}\xi_A - m(\chi^A\xi_A + \chi^\dagger_{A'}\xi^{\dagger A'}) \tag{6}$$

Equations of motion

From the field equations for the primed spinors we get:

$$\xi^{\dagger A'} = -\frac{i\sqrt{2}}{m} D^{A'A}\chi_A, \qquad \chi^{\dagger A'} = -\frac{i\sqrt{2}}{m} D^{A'A}\xi_A. \tag{7}$$

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- Quadratic Lagrangian for the primed spinors
- Carry out the Berezin path-integral by effectively reinserting the EOM in the Lagrangian

Second Order QED

The Lagrangian for QED_2 is given by:

$$\mathcal{L}_{\text{chiral}} = -2D_{A'}{}^{A}\chi_A D^{A'B}\xi_B - m^2\chi^A\xi_A. \tag{8}$$

Together with the equations of motion:

$$2D_{A'}{}^{A}D^{A'B}\xi_B + m^2\xi^A = 0, \quad 2D_{A'}{}^{A}D^{A'B}\chi_B + m^2\chi^A = 0 \tag{9}$$

The unprimed spinor fields satisfy the generalised Klein-Gordon equation with non-zero bundle curvature.

Second Order QED

- The first order equations of motion are now seen as non-trivial reality conditions:

$$m\xi^{\dagger\, A'} = -i\sqrt{2}D^{A'A}\chi_A, \qquad m\chi^{\dagger\, A'} = -i\sqrt{2}D^{A'A}\xi_A \quad (10)$$

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- The fields are now normalised to have canonical mass dimension 1 (as seen from the Lagrangian).

Feynman rules

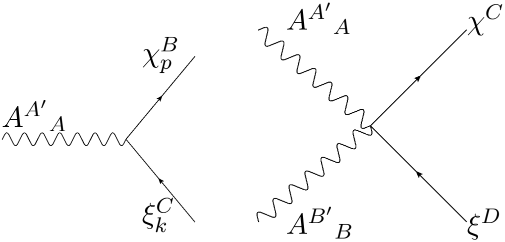
The propagator becomes very simple:

$$\langle 0|T\{\chi_A(p)\xi_B(-p)\}|0\rangle \equiv D(p)_{AB} = \frac{-i}{p^2+m^2}\epsilon_{AB} \tag{11}$$

Feynman rules

We have two interaction vertices with Feynman rules (incoming momenta):

$$2ie\left[{k_C}^{A'}\epsilon_{BA} + {p_B}^{A'}\epsilon_{CA}\right], \qquad -2ie^2\epsilon^{A'B'}\epsilon_{AB}\epsilon_{CD} \tag{12}$$



Feynman rules

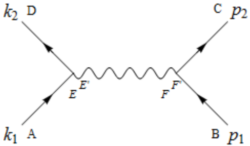
Computing Feynman amplitudes simply amounts to contracting spinors indices with the simple rules:

$$\lambda^A = \epsilon^{AB}\lambda_B, \quad \lambda_A = \lambda^B \epsilon_{BA} \tag{13}$$

No gamma matrices algebra to worry about!

$e^-\mu^- \to e^-\mu^-$ scattering

Simplest QED process: electron-muon scattering at tree level

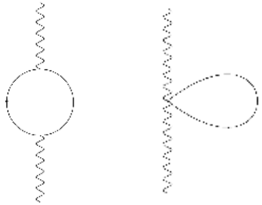


Amputated amplitude $\mathcal{M}_{ABCD}$ for an incoming electron with momentum k_1 scattered off an incoming muon with momentum p_1. We have:

$$\mathcal{M}_{ABCD} = -\frac{4ie^2}{q^2}\Big[(k_1\cdot p_1)_{AB}\,\epsilon_{CD} - (k_2\cdot p_1)_{DB}\,\epsilon_{AC} - (k_1\cdot p_2)_{AC}\,\epsilon_{BD} + (k_2\cdot p_2)_{CD}\,\epsilon_{AB}\Big] \quad (14)$$

One-loop charge renormalisation

Although there are half the number of fermions in our theory, fermions loops are equivalent in both formalisms:



$$i\Pi^{1-\text{loop}}(k)^{A'}{}_{A}{}^{B'}{}_{B}$$
$$= (-1)4e^2 \int \frac{d^4p}{(2\pi)^4} \frac{\left[p^{A'}{}_{B}(p+k)^{B'}{}_{A} + (p+k)^{A'}{}_{B}p^{B'}{}_{A} - \frac{1}{2}\left((p+k)^2+p^2\right)\epsilon^{A'B'}\epsilon_{AB}\right]}{\left[p^2+m^2\right]\left[(p+k)^2+m^2\right]}$$
$$+ (-1)4e^2 \int \frac{d^4p}{(2\pi)^4} \frac{\epsilon^{A'B'}\epsilon_{AB}}{\left[p^2+m^2\right]} \tag{15}$$

Which exhibits the contributions from both diagrams.

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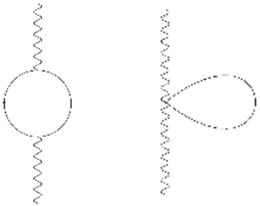

Rewritten:

$$i\Pi^{1-\text{loop}}(k)^{A'}{}_{A}{}^{B'}{}_{B} = (-1)4e^2 \int \frac{d^D p}{(2\pi)^D} \frac{\left[p^{A'}{}_{B}(p+k)^{B'}{}_{A} + (p+k)^{A'}{}_{B}\, p^{B'}{}_{A} + m^2 \epsilon^{A'B'} \epsilon_{AB}\right]}{\left[p^2+m^2\right]\left[(p+k)^2+m^2\right]} \tag{16}$$

We have the usual one-loop integral to compute.

Yang-Mills

- Feynman rules obtained for the second-order fermions are similar to Yang-Mills':

$$igf^{abc}\left[\eta^{\mu\nu}(k_1-k_2)^\rho+\text{perms}\right], \qquad -\frac{i}{4}g^2\left[f^{abe}f^{cde}(\eta^{\mu\rho}\eta^{\nu\sigma}-\eta^{\mu\sigma}\eta^{\nu\rho})+\text{perms}\right] \tag{17}$$

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- Main difference comes from the groups structures (both spacetime and internal)
- Opens new directions for unification (?)

II. The Standard Model

A. Quark Sector

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Quark sector

Using index-free notations, the Lagrangian for the quark sector of the Standard Model reads:

$$\begin{aligned}\mathcal{L}_{quark} = & -i\sqrt{2}Q^{\dagger i}DQ_i - i\sqrt{2}\bar{u}^{\dagger i}D\bar{u}_i - i\sqrt{2}\bar{d}^{\dagger i}D\bar{d}_i \\ & + Y_u^{ij}\phi^T\varepsilon Q_i\bar{u}_j - Y_d^{ij}\phi^\dagger Q_i\bar{d}_j \\ & - (Y_u^\dagger)^{ij}\bar{u}_i^\dagger Q_j^\dagger\varepsilon\phi^* - (Y_d^\dagger)^{ij}\bar{d}_i^\dagger Q_j^\dagger\phi.\end{aligned} \tag{18}$$

As before $D^{AA'} \equiv \theta^{\mu AA'}D_\mu$ and the quantities Y^{ij} are arbitrary complex 3×3 Yukawa mass matrices.

Equations of motion

The equations of motion for the unprimed spinors are:

$$\begin{aligned} Q_i^\dagger: \quad i\sqrt{2}DQ^i &= -(\epsilon\phi^*)\,\bar{u}_j^\dagger (Y_u^\dagger)^{ji} - \phi\,\bar{d}_j^\dagger (Y_d^\dagger)^{ji} \\ \bar{u}_i^\dagger: \quad i\sqrt{2}D\bar{u}^i &= -(Y_u^\dagger)^{ij} Q_j^\dagger\,(\epsilon\phi^*) \\ \bar{d}_i^\dagger: \quad i\sqrt{2}D\bar{d}^i &= -(Y_d^\dagger)^{ij} Q_j^\dagger\,\phi \end{aligned} \tag{19}$$

They relate, as before, primed to unprimed spinors.

$SU(2)$ structure

Some structure is making itself explicit in the equations of motion $\rightarrow$ We combine the components of the Higgs field into the following 2×2 matrix:

$$\rho\Phi^\dagger := (\epsilon\phi^*, \phi) \equiv \begin{pmatrix} (\phi^0)^* & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix} \tag{20}$$

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Upon field redefinitions and combination of the singlets into a row:

$$\bar{Q}_i := \left(\bar{u}_i \ , \ \bar{d}_i\right) \tag{21}$$

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The equations of motion become:

$Q_i^\dagger$:	$i\sqrt{2} D Q_i = \quad -\rho \ \Phi^\dagger \left(\bar{Q}^\dagger \Lambda \right)_i$
$\bar{Q}_i^\dagger$:	$i\sqrt{2} D \bar{Q}_i = \quad -\rho \ Q_i^\dagger \Phi^\dagger$

(22)

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Second-order Lagrangian

We now substitute the primed spinors obtained from the above field equations into the Lagrangian and obtain the following second-order Lagrangian:

$$\mathcal{L}_{quarks} = -\frac{2}{\rho} D\bar{Q}^i DQ_i - \rho \left(\Lambda\bar{Q}\right)^i Q_i, \tag{25}$$

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- Much simpler Lagrangian: half the terms from the quark sector
- However, non-polynomical in the Higgs scalar field $\rho \rightarrow$ absorb the Higgs field and write:

$$\mathcal{L}^{(2)}_{quarks} = -2\mathcal{D}\bar{Q}^i \mathcal{D}Q_i - \rho^2 \left(\Lambda\bar{Q}\right)^i Q_i \tag{26}$$

Physical Bosons and their masses

The Higgs sector Lagrangian can be rewritten in terms of physical quantities as follows:

$$\begin{aligned}\mathcal{L}_{Higgs} &= -|D_\mu \phi|^2 - V\left(|\phi|^2\right) \\ &= -\left(\partial_\mu \rho\right)^2 - \frac{(g_2 \rho)^2}{2}\left(W^+ W^- + \frac{1}{2\cos^2(\theta_W)} Z_\mu Z^\mu\right) - V(\rho^2)\end{aligned} \tag{27}$$

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- To extract the mass terms for the W, Z bosons no symmetry breaking was needed
- The Higgs sector was merely reformulated in terms of the physical $\mathrm{SU}(2)$-invariant degrees of freedom of the theory.

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- However, it turns out that the measure in the path integral formulation of the theory is not. The measure transforms as

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where the function $\mathcal{A}(x)$ is the anomaly:

$$\mathcal{A}(x) = -\frac{1}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \tag{29}$$

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- Leads to the anomalous conservation of the axial current:

$$\partial_\mu\langle j_5^\mu\rangle = \mathcal{A}(x) \tag{30}$$

Second-order formalism

- The second order Lagrangian is explicitly *not* invariant under the (local or global) chiral transformations. We have, at the classical level:

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- However, the massless Lagrangian vanishes on the surface of the reality conditions. It is only in this sense that the massless theory is invariant under the chiral transformations.

First-order result recovered

- At the quantum level:

$$\partial^\mu j^5_\mu \equiv \partial^A{}_{A'} \langle j^{5\,A'}{}_A \rangle = \mathcal{A}^{chiral} - 2i\langle \mathcal{L} \rangle \equiv \mathcal{A}, \tag{32}$$

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  where we have introduced the notation $\mathcal{A}$ for the full anomaly.
- The anomaly can be calculated non-pertubatively or in perturbation theory.
- One finds as usual:

$$D^A{}_{A'} \langle j^{5\,A'}{}_A \rangle = -\frac{1}{(4\pi)^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \tag{33}$$

Non-hermitity

The Lagrangian

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- How can the theory be unitary?
- We have to impose reality conditions!
- In our case, we have a non-trivial real-structure that involves a derivative operator:

$$\dagger \;\mapsto\; \frac{i}{m}\mathcal{D}, \quad \left(\frac{i}{m}\mathcal{D}\right)^2 = I_V$$

Reality conditions

- Imposed linearly (without any coupling to the gauge fields) on the free theory, this leads to a positive definite Hamiltonian $H_0 \sim a^\dagger a$.

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- Imposed linearly (without any coupling to the gauge fields) on the free theory, this leads to a positive definite Hamiltonian $H_0 \sim a^\dagger a$.
- It is checked perturbativaly, that unitarity holds. Moreover, the new vertex is essential to guarantee full non-linear reality.
- Equivalence results between the two theories can be established, leading to proofs of unitarity.

Conclusions

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- The perturbative proof of unitarity for this non-hermitian theory is being developed.
- Implications on supersymmetric field theories?

Thank you!