Summer School and Workshop on the Standard Model and Beyond
Corfu Summer Institute, September 4, 2014

Phys. Rev. D **90**, 033011 (2014)

Radiative charged-lepton mass generation in multi-Higgs doublet models

João Penedo
(IST Physics Dep. and CFTP – Lisboa, Portugal)

in collaboration with Filipe R. Joaquim

DF DEPARTAMENTO DE FÍSICA TÉCNICO LISBOA
CFTP Centro de Física Teórica de Partículas

COMPETE
QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL PORTUGAL 2007.2013
UNIÃO EUROPEIA Fundo Europeu de Desenvolvimento Regional
FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

The electroweak sector of the Standard Model (SM)

EW gauge symmetry group:

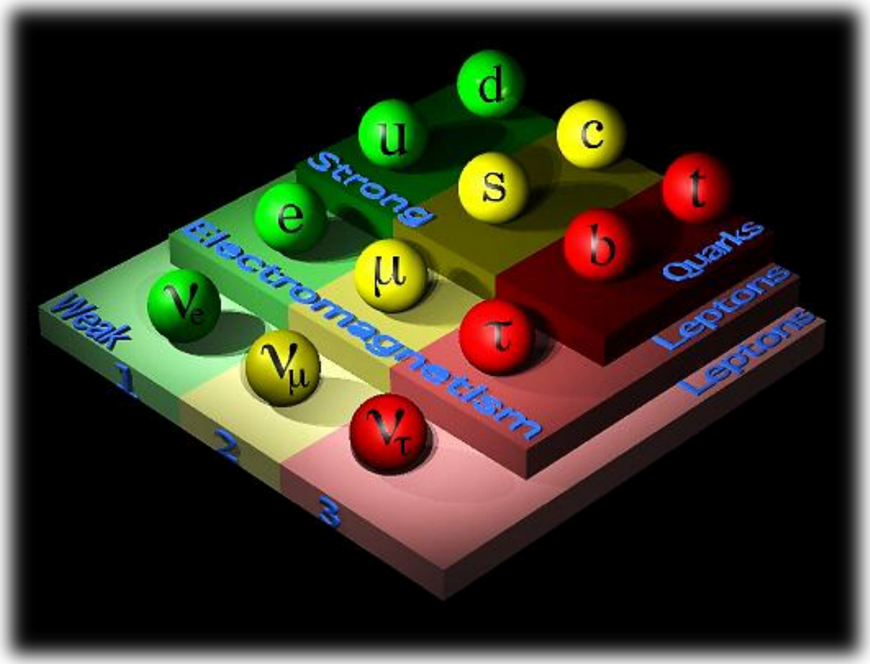
$$\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$$

Ordinary spacetime derivative replaced by a covariant one:

$$D_\mu = \partial_\mu - i\,g\,A^i_\mu\,I^i - i\,g'\,B_\mu\,Y$$

Higgs doublet:

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (2, 1/2)$$



Yukawa Lagrangian:

$$-\mathcal{L}_{\text{Yukawa}} = (\mathbf{Y}^u)_{ij}\bar{q}_{Li}\tilde{\Phi}u_{Rj} + (\mathbf{Y}^d)_{ij}\bar{q}_{Li}\Phi d_{Rj} + (\mathbf{Y}^\ell)_{ij}\bar{\ell}_{Li}\Phi e_{Rj} + \text{H.c.}$$

$$\tilde{\Phi} \equiv i\sigma_2\Phi^*$$

The electroweak sector of the Standard Model (SM)

EW gauge symmetry group:

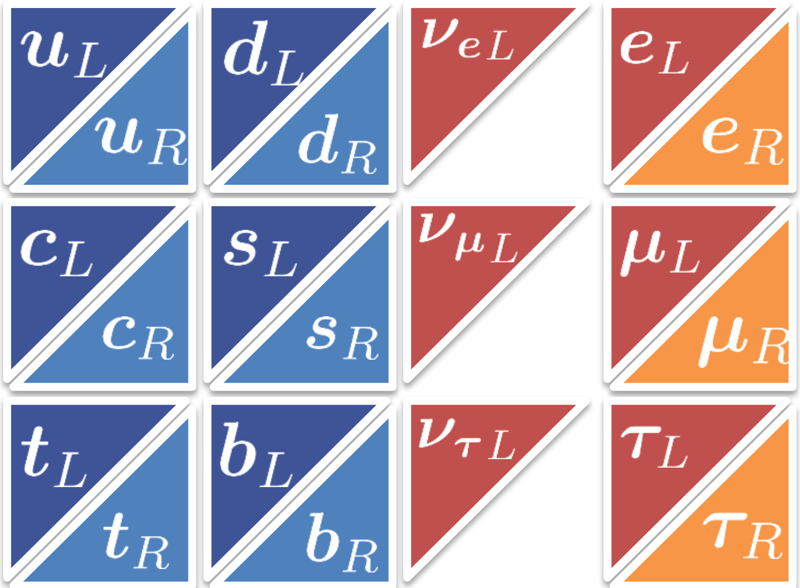
$$\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$$

Ordinary spacetime derivative replaced by a covariant one:

$$D_\mu = \partial_\mu - i\, g\, A_\mu^i\, I^i - i\, g'\, B_\mu\, Y$$

Higgs doublet:

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (2, 1/2)$$



Yukawa Lagrangian:

$$-\mathcal{L}_{\text{Yukawa}} = (\mathbf{Y}^u)_{ij}\bar{q}_{Li}\tilde{\Phi}u_{Rj} + (\mathbf{Y}^d)_{ij}\bar{q}_{Li}\Phi d_{Rj} + (\mathbf{Y}^\ell)_{ij}\bar{\ell}_{Li}\Phi e_{Rj} + \text{H.c.}$$

$$\tilde{\Phi} \equiv i\sigma_2\Phi^*$$

The electroweak sector of the Standard Model (SM)

EW gauge symmetry group:

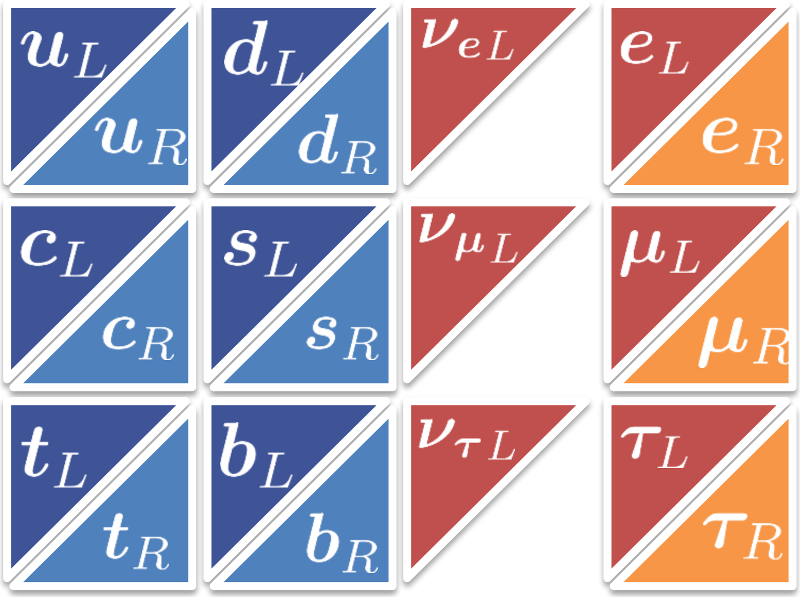
$$\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$$

Ordinary spacetime derivative replaced by a covariant one:

$$D_\mu = \partial_\mu - i\, g\, A^i_\mu\, I^i - i\, g'\, B_\mu\, Y$$

Spontaneous symmetry breaking:

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to \begin{pmatrix} 0 \\ v + \frac{H}{\sqrt{2}} \end{pmatrix}_{v = \langle \phi^0 \rangle}$$



Fermion masses in the SM:

$$\mathbf{M}_{d,\ell} = v\,\mathbf{Y}^{d,\ell} \qquad \mathbf{M}_u = v^*\,\mathbf{Y}^u$$

$$\mathbf{V}^{X\dagger}\mathbf{Y}^X\mathbf{U}^X = \mathrm{diag}(y_1^X, y_2^X, y_3^X)$$

$$y_i^X \geq 0$$

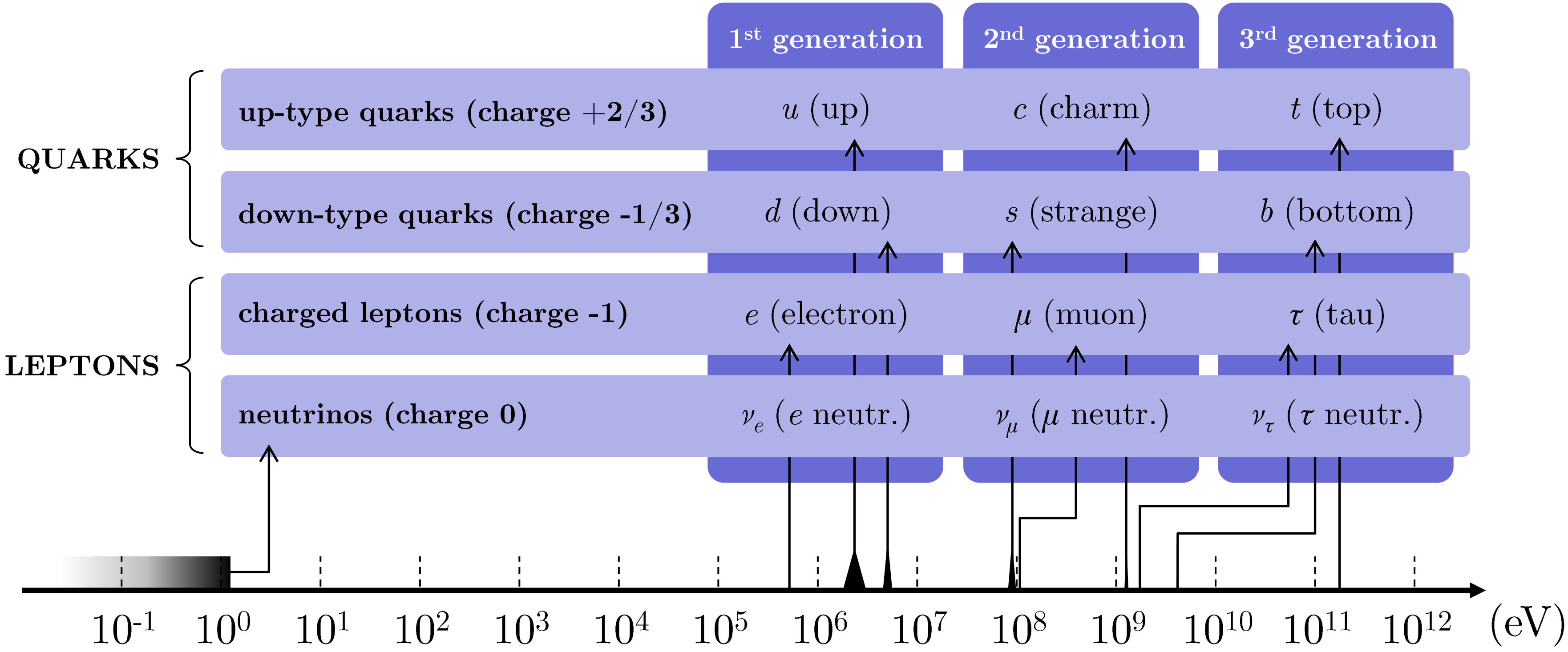
$$X \in \{u, d, \ell\}$$

EW gauge boson masses:

$$m_W = \frac{g}{\sqrt{g^2 + g'^2}}\, m_Z$$

$$m_\gamma = 0$$

The puzzle of fermion masses



Electron

0.511 MeV

Muon

105.7 MeV

Tau

1777 MeV

$\sim 1 : 200$

$\sim 1 : 20$

Mechanisms to explain hierarchies

IDEA: U(1)'s broken by the VEVs $\langle S \rangle$ of scalar fields, giving couplings proportional to powers of small parameters $\epsilon \sim \langle S \rangle / M_S$.

Froggatt and Nielsen, '79

$$\mathbf{Y}_{ij}\,\overline{\psi}_i\,\Phi\,\psi_j \left(\frac{S}{M_S}\right)^{n_{ij}} \;\rightarrow\; \left(\mathbf{Y}_{ij}\,\epsilon^{n_{ij}}\right)\overline{\psi}_i\,\Phi\,\psi_j$$

IDEA: Hierarchies are generated through quantum corrections.

't Hooft, '71; Weinberg, '72; Georgi and Glashow, '72; Mohapatra '74; Barr and Zee '77...
...and many more

Radiative corrections to Yukawa couplings in the SM

$$\left.\frac{d\mathbf{Y}^{\ell}}{dt}\right|_{1\text{ loop}} = \frac{1}{16\pi^2}\,\beta^{(1)}$$

renormalization scale

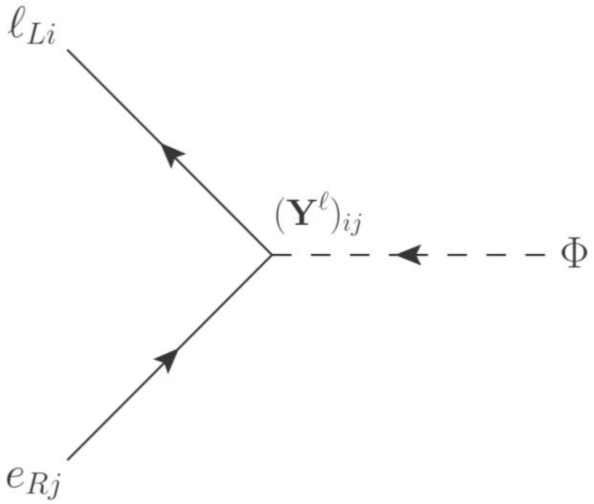
$$t = \log\left(\frac{\mu}{\Lambda}\right)$$

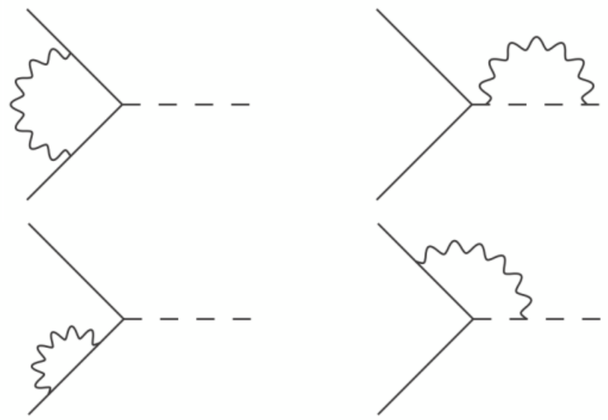
reference scale

Beta function for charged-lepton Yukawas in the SM (at one loop):

$$\beta^{(1)} = (\boxed{\alpha_g} + \alpha_Y)\,\mathbf{Y}^{\ell} + \frac{3}{2}\mathbf{Y}^{\ell}\mathbf{Y}^{\ell\dagger}\mathbf{Y}^{\ell}$$

Cheng, Eichten, and Li, '74





$$\alpha_g = -9g^2/4 - 15g'^2/4$$

Radiative corrections to Yukawa couplings in the SM

$$\left.\frac{d\mathbf{Y}^\ell}{dt}\right|_{1\ \text{loop}} = \frac{1}{16\pi^2}\,\beta^{(1)}$$

renormalization scale

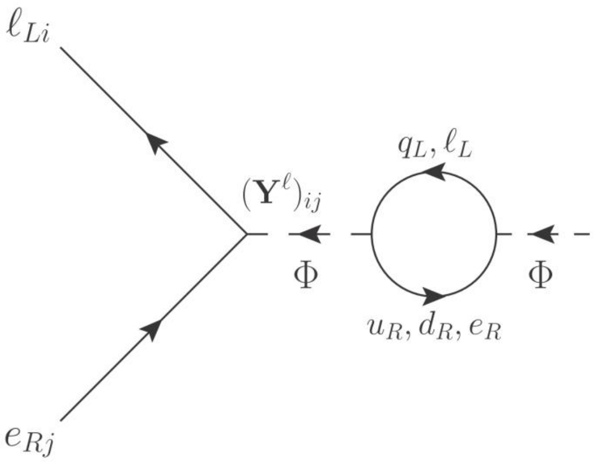
$$t = \log\left(\frac{\mu}{\Lambda}\right)$$

reference scale

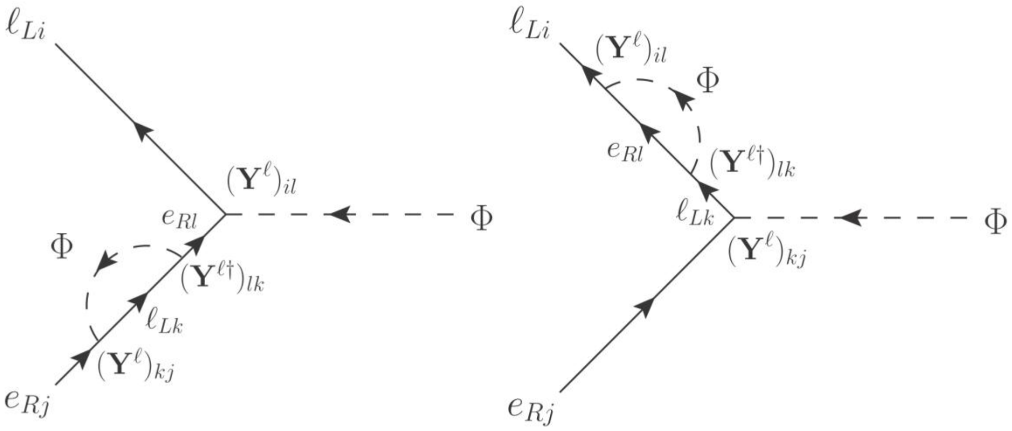
Beta function for charged-lepton Yukawas in the SM (at one loop):

$$\beta^{(1)} = (\alpha_g + \boxed{\alpha_Y})\,\mathbf{Y}^\ell + \boxed{\frac{3}{2}\mathbf{Y}^\ell\mathbf{Y}^{\ell\dagger}\mathbf{Y}^\ell}$$

Cheng, Eichten, and Li, '74



$$\alpha_Y = 3\mathrm{Tr}(\mathbf{Y}^{u\dagger}\mathbf{Y}^u + \mathbf{Y}^d\mathbf{Y}^{d\dagger}) + \mathrm{Tr}(\mathbf{Y}^\ell\mathbf{Y}^{\ell\dagger})$$



Radiative corrections to Yukawa couplings in the SM

$$\left.\frac{d\mathbf{Y}^\ell}{dt}\right|_{1\text{ loop}} = \frac{1}{16\pi^2}\,\beta^{(1)}$$

renormalization scale

$$t = \log\left(\frac{\mu}{\Lambda}\right)$$

reference scale

Beta function for charged-lepton Yukawas in the SM (at one loop): $$\beta^{(1)} = (\alpha_g + \alpha_Y)\mathbf{Y}^\ell + \frac{3}{2}\mathbf{Y}^\ell\mathbf{Y}^{\ell\dagger}\mathbf{Y}^\ell$$

Cheng, Eichten, and Li, ’74

Mass eigenvalues are corrected proportionally to themselves

$$\mathbf{Y}^\ell \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

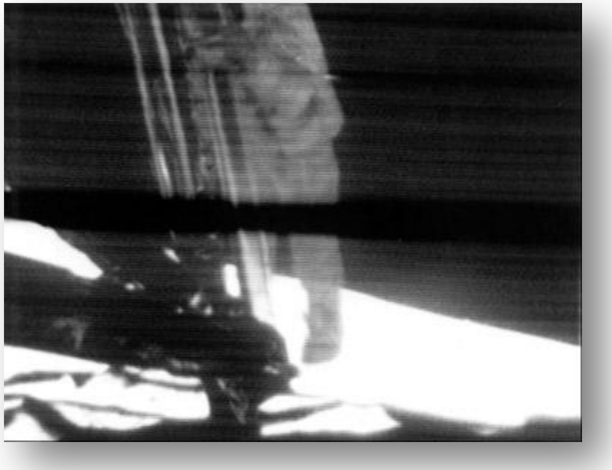
approximate
texture unchanged

The matter-antimatter puzzle

Evidence for a Baryon Asymmetry of the Universe (BAU):

Local
- Earth (accelerators, radioactivity)
- Solar wind
- Planetary probes
- Antinuclei $\left(\overline{\mathrm{D}},\overline{\mathrm{He}}\right)$

Nonlocal
- Cosmic rays (collisions in the ISM)
- γ-ray background



$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.19 \pm 0.15) \times 10^{-10}$$

PDG, '12

$$Y_B \equiv \frac{n_b - n_{\bar{b}}}{s} \simeq \frac{\eta}{7.04}$$

The Sakharov Conditions

- B symmetry violation
- C and CP violation
- Departure from thermal equilibrium

Sakharov, '67

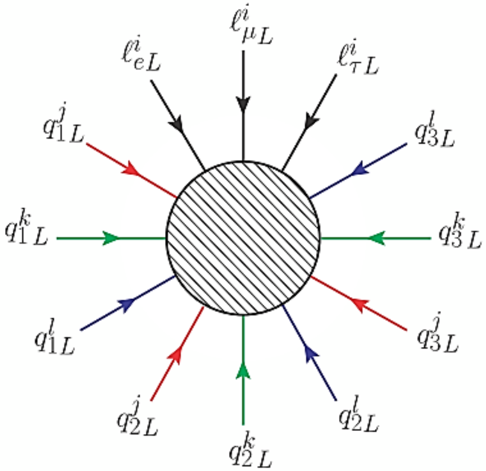
To avoid fine-tuning of initial conditions, the BAU must be dynamically generated

Can we solve the puzzle within the SM?

The Sakharov conditions in the SM

EW Sphalerons:

$$\Delta \mathrm{B} = \Delta \mathrm{L} = 3\,\Delta N_{\mathrm{CS}}$$



Klinkhamer and Manton, ’84; Kuzmin, Rubakov, and Shaposhnikov, ’85

The Sakharov Conditions

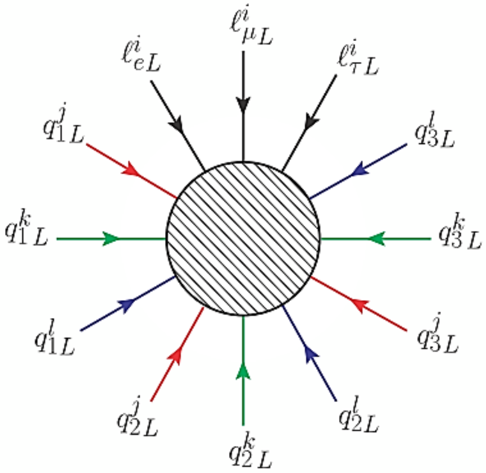
- B symmetry violation
- C and CP violation
- Departure from thermal equilibrium

Sakharov, ’67

The Sakharov conditions in the SM

EW Sphalerons:

$$\Delta\mathrm{B} = \Delta\mathrm{L} = 3\,\Delta N_{\mathrm{CS}}$$



Klinkhamer and Manton, ’84; Kuzmin, Rubakov, and Shaposhnikov, ’85

Not enough CP violation:

$$J_{CP}/T_{EW}^{12} \sim 10^{-19}$$

“The result is many orders of magnitude below what observation requires (...)”

Gavela, Lozano, Orloff, and Pène, ’94

The Sakharov Conditions

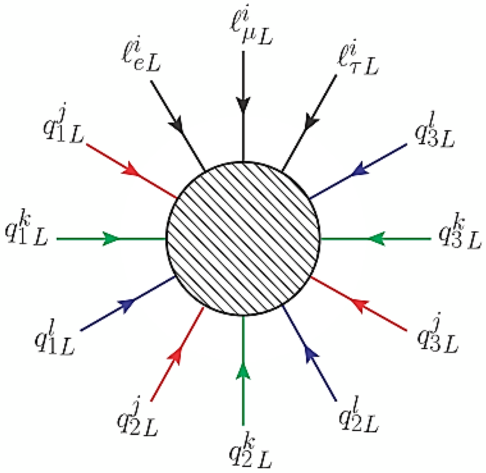
- B symmetry violation
- C and CP violation
- Departure from thermal equilibrium

Sakharov, ’67

The Sakharov conditions in the SM

EW Sphalerons:

$$\Delta \mathrm{B} = \Delta \mathrm{L} = 3\,\Delta N_{\mathrm{CS}}$$



Klinkhamer and Manton, '84; Kuzmin, Rubakov, and Shaposhnikov, '85

Not enough CP violation:

$$J_{CP}/T_{EW}^{12} \sim 10^{-19}$$

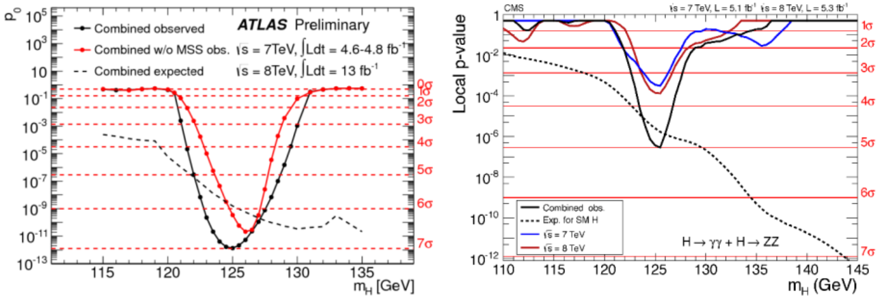
"The result is many orders of magnitude below what observation requires (...)"

Gavela, Lozano, Orloff, and Pène, '94

A strong enough (1st order) phase transition requires

$$m_H \lesssim 80 \text{ GeV}$$

Kajantie, Laine, Rummukainen, and Shaposhnikov, '96



CMS and ATLAS collaborations, '12

The Sakharov Conditions

- B symmetry violation
- C and CP violation
- Departure from thermal equilibrium

Sakharov, '67

Multi-Higgs doublet models (NHDMs)

Extending the SM **scalar sector** by adding Higgs doublets:

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ \phi_a^0 \end{pmatrix} \sim (2, 1/2)$$

$$a = 1, \ldots, N$$

T.D. Lee, ’73

The Sakharov Conditions

- B symmetry violation ✓
- C and CP violation (new sources) ✓
- Departure from thermal equilibrium (extended parameter space: 1st order PT) ✓

Turok and Zadrozny, ‘91; Cline and Lemieux, ’97...

Yukawa Lagrangian:

$$-\mathcal{L}_{\text{Yukawa}} = (\mathbf{Y}_a^u)_{ij} \bar{q}_{Li} \tilde{\Phi}_a u_{Rj} + (\mathbf{Y}_a^d)_{ij} \bar{q}_{Li} \Phi_a d_{Rj} + (\mathbf{Y}_a^\ell)_{ij} \bar{\ell}_{Li} \Phi_a e_{Rj} + \text{H.c.}$$

$$\tilde{\Phi}_a \equiv i\sigma_2 \Phi_a^*$$

Fermion masses:

$$v_a = \langle \phi_a^0 \rangle$$

$$\mathbf{M}_{d,\ell} = \sum_a v_a \mathbf{Y}_a^{d,\ell} \qquad \mathbf{M}_u = \sum_a v_a^* \mathbf{Y}_a^u$$

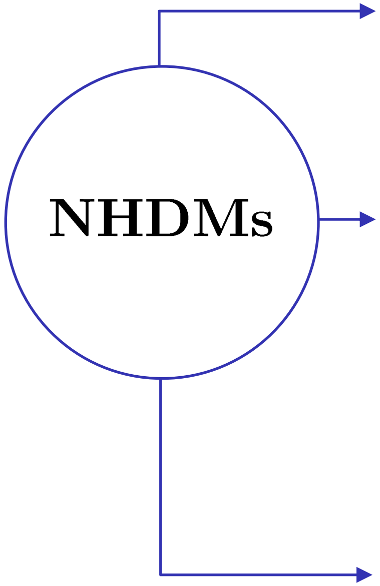
$$\mathbf{V}_a^{X\dagger} \mathbf{Y}_a^X \mathbf{U}_a^X = \text{diag}(y_{a1}^X, y_{a2}^X, y_{a3}^X)$$

$$y_{i,a}^X \geq 0$$

$$X \in \{u, d, \ell\}$$

for a recent review on 2HDMs, see: Branco et al., ’12

Multi-Higgs doublet models (NHDMs)



MOTIVATION

Electroweak baryogenesis ✓

BONUS

ρ parameter still unit at tree level

$$\rho = \frac{\sum_a \left[I_a(1+I_a) - (I_a^3)^2\right] v_a^2}{2\sum_a (I_a^3)^2 v_a^2}$$

B. Lee, '72; Langacker, '81

PROBLEMS

Flavour-changing neutral currents (FCNCs)

constraints from $\mu \to e\gamma$ and mass differences
in meson – anti-meson systems...

see, for instance, (2HDMs):
Mahmoudi and Stål, '10
Crivellin, Kokulu, and Greub, '13

Usual solution: **discrete symmetries**

Unwanted contributions to oblique parameters S, T and U

Haber and O'Neil, '11

Solution to both: **decoupling limit**

Haber and Nir, '90

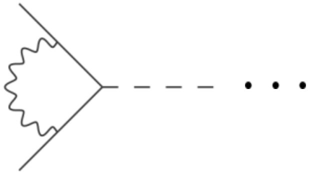
Radiative corrections to Yukawa couplings in NHDMs

Beta functions for charged-lepton Yukawas in the NHDM (at one loop)

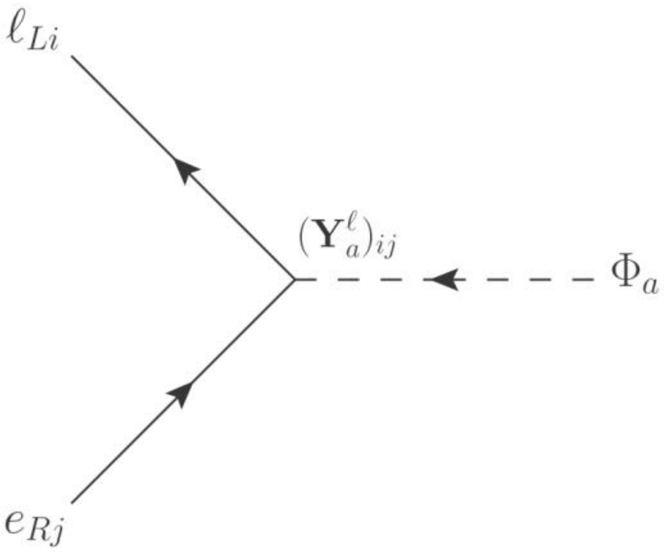
$$\beta_a^{(1)} = \boxed{\alpha_g}\mathbf{Y}_a^\ell + \boxed{\alpha_Y^{ab}}\mathbf{Y}_b^\ell + \mathbf{Y}_a^\ell\mathbf{Y}_b^{\ell\dagger}\mathbf{Y}_b^\ell + \frac{1}{2}\mathbf{Y}_b^\ell\mathbf{Y}_b^{\ell\dagger}\mathbf{Y}_a^\ell$$

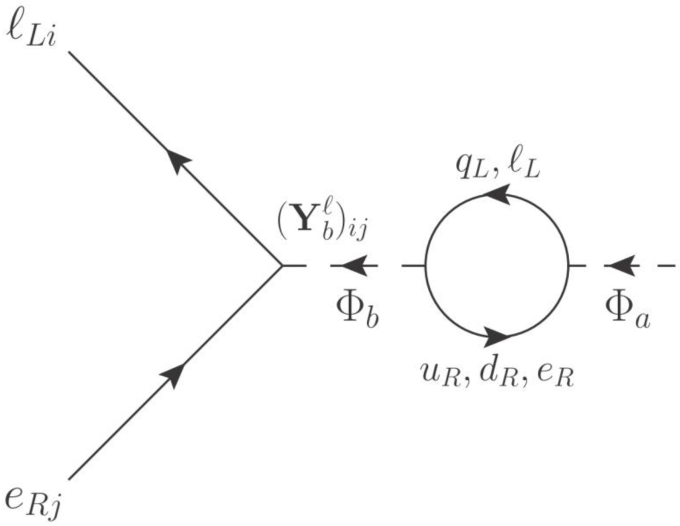
$$\left.\frac{d\mathbf{Y}_a^\ell}{dt}\right|_{(1)} = \frac{1}{16\pi^2}\beta_a^{(1)}$$

Grimus and Lavoura, '05



$$\alpha_g = -9g^2/4 - 15g'^2/4$$





$$\alpha_Y^{ab} = 3\mathrm{Tr}(\mathbf{Y}_a^{u\dagger}\mathbf{Y}_b^u + \mathbf{Y}_a^d\mathbf{Y}_b^{d\dagger}) + \mathrm{Tr}(\mathbf{Y}_a^\ell\mathbf{Y}_b^{\ell\dagger})$$

Corrections independent of the structure of $\mathbf{Y}_a^\ell$

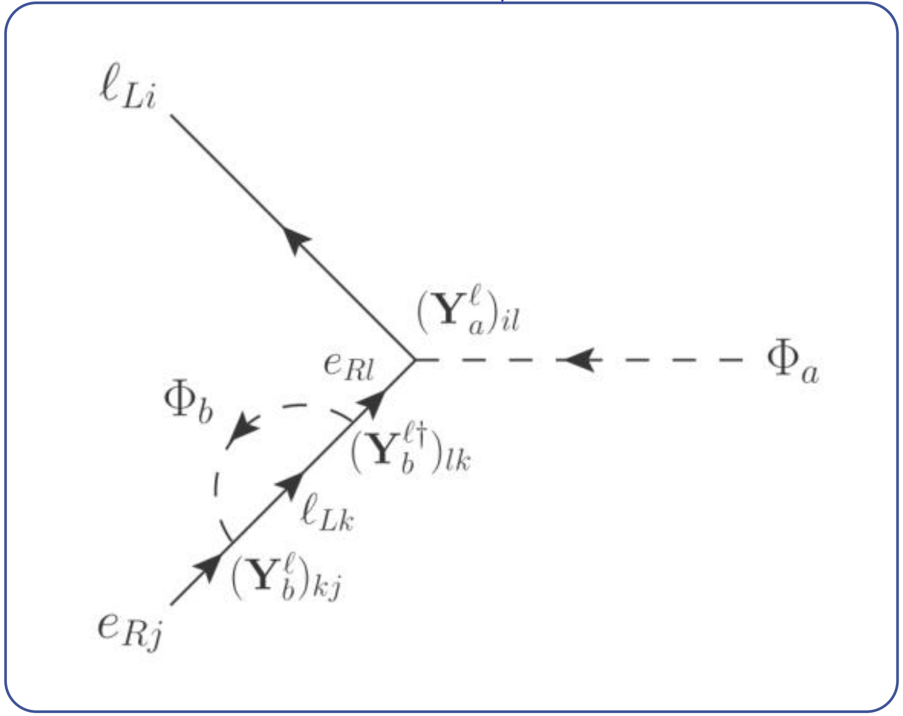
Radiative corrections to Yukawa couplings in NHDMs

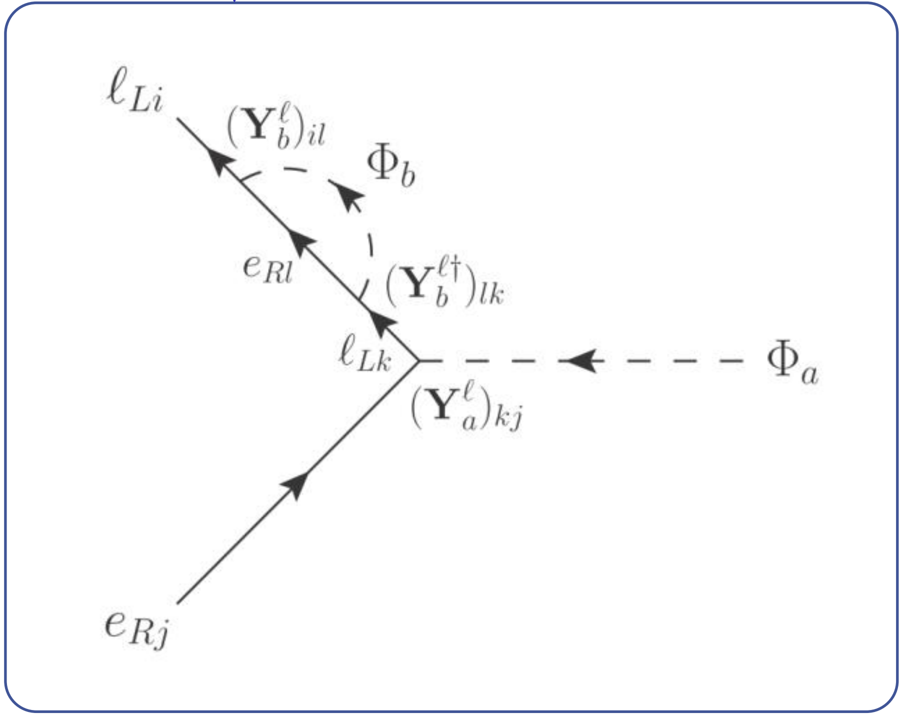
Beta functions for charged-lepton Yukawas in the NHDM (at one loop)

$$\beta_a^{(1)} = \alpha_g \mathbf{Y}_a^\ell + \alpha_Y^{ab}\mathbf{Y}_b^\ell + \boxed{\mathbf{Y}_a^\ell \mathbf{Y}_b^{\ell\dagger}\mathbf{Y}_b^\ell} + \boxed{\frac{1}{2}\mathbf{Y}_b^\ell\mathbf{Y}_b^{\ell\dagger}\mathbf{Y}_a^\ell}$$

$$\left.\frac{d\mathbf{Y}_a^\ell}{dt}\right|_{(1)} = \frac{1}{16\pi^2}\beta_a^{(1)}$$

Grimus and Lavoura, ’05





Radiative corrections to Yukawa couplings in NHDMs

Leading-log approximation:

$$\mathbf{Y}^\ell_a \to \mathbf{Y}^{\ell(1)}_a \simeq \mathbf{Y}^\ell_a + t\,\frac{d\mathbf{Y}^\ell_a}{dt}\bigg|_{1\text{ loop}} = \mathbf{Y}^\ell_a + \frac{\log(\mu/\Lambda)}{16\pi^2}\,\beta^{(1)}_a$$

Structure of the 1-loop beta function:

$$\boxed{\beta^{(1)}_a = \sum_{\text{all }\Phi_k\text{'s}} \mathbf{X}_{a,k}\,\mathbf{Y}^\ell_k}$$

with

$$\mathbf{X}_{a,k} = \left(\delta_{ak}\,\alpha_g + \alpha^{ak}_Y\right)\mathbb{1} + \mathbf{Y}^\ell_a \mathbf{Y}^{\ell\,\dagger}_k + \frac{1}{2}\,\delta_{ak}\,\mathbf{Y}^\ell_a \mathbf{Y}^{\ell\,\dagger}_a$$

Rank (number of nonzero eigenvalues) of corrected Yukawas:

$$r(\mathbf{Y}^{\ell(1)}_a) = ?$$

$$r(\mathbf{AB}) \leq \min\{r(\mathbf{A}), r(\mathbf{B})\}$$
$$r(\mathbf{A}+\mathbf{B}) \leq r(\mathbf{A}) + r(\mathbf{B})$$

Radiative corrections to Yukawa couplings in NHDMs

Leading-log approximation:

$$\mathbf{Y}_a^\ell \to \mathbf{Y}_a^{\ell(1)} \simeq \mathbf{Y}_a^\ell + \frac{\log(\mu/\Lambda)}{16\pi^2} \sum_k \mathbf{X}_{a,k}\, \mathbf{Y}_k^\ell$$

Structure of the 1-loop beta function:

$$\boxed{\beta_a^{(1)} = \sum_{\text{all } \Phi_k\text{'s}} \mathbf{X}_{a,k}\, \mathbf{Y}_k^\ell}$$

with

$$\mathbf{X}_{a,k} = \left(\delta_{ak}\, \alpha_g + \alpha_Y^{ak}\right) \mathbb{1} + \mathbf{Y}_a^\ell \mathbf{Y}_k^{\ell\,\dagger} + \frac{1}{2}\, \delta_{ak}\, \mathbf{Y}_a^\ell \mathbf{Y}_a^{\ell\,\dagger}$$

Rank (number of nonzero eigenvalues) of corrected Yukawas:

$$r(\mathbf{Y}_a^{\ell(1)}) = ?$$

$$r(\mathbf{A}\mathbf{B}) \leq \min\{r(\mathbf{A}), r(\mathbf{B})\}$$
$$r(\mathbf{A}+\mathbf{B}) \leq r(\mathbf{A}) + r(\mathbf{B})$$

Radiative corrections to Yukawa couplings in NHDMs

Leading-log approximation:

$$\mathbf{Y}^{\ell}_{a} \to \mathbf{Y}^{\ell(1)}_{a} \simeq \sum_{k} \left[\delta_{ak} \mathbb{1} + \frac{\log(\mu/\Lambda)}{16\pi^2} \mathbf{X}_{a,k} \right] \mathbf{Y}^{\ell}_{k}$$

Structure of the 1-loop beta function:

$$\boxed{\beta^{(1)}_{a} = \sum_{\text{all } \Phi_k\text{'s}} \mathbf{X}_{a,k}\, \mathbf{Y}^{\ell}_{k}}$$

with
$$\mathbf{X}_{a,k} = \left(\delta_{ak}\, \alpha_g + \alpha^{ak}_{Y}\right) \mathbb{1} + \mathbf{Y}^{\ell}_{a} \mathbf{Y}^{\ell\,\dagger}_{k} + \frac{1}{2}\, \delta_{ak}\, \mathbf{Y}^{\ell}_{a} \mathbf{Y}^{\ell\,\dagger}_{a}$$

Rank (number of nonzero eigenvalues) of corrected Yukawas:

$$r(\mathbf{Y}^{\ell(1)}_{a}) = ?$$

$$r(\mathbf{AB}) \leq \min\{r(\mathbf{A}), r(\mathbf{B})\}$$
$$r(\mathbf{A}+\mathbf{B}) \leq r(\mathbf{A}) + r(\mathbf{B})$$

Radiative corrections to Yukawa couplings in NHDMs

Leading-log approximation:

$$\mathbf{Y}^\ell_a \to \mathbf{Y}^{\ell(1)}_a \simeq \sum_k \Big[\underbrace{\delta_{ak}\mathbb{1} + \frac{\log(\mu/\Lambda)}{16\pi^2}\mathbf{X}_{a,k}}_{\equiv \mathbf{X}'_{a,k}} \Big] \mathbf{Y}^\ell_k$$

Structure of the 1-loop beta function:

$$\boxed{\beta^{(1)}_a = \sum_{\text{all } \Phi_k\text{'s}} \mathbf{X}_{a,k}\, \mathbf{Y}^\ell_k}$$

with

$$\mathbf{X}_{a,k} = \big(\delta_{ak}\,\alpha_g + \boxed{\alpha^{ak}_Y}\big)\,\mathbb{1} + \mathbf{Y}^\ell_a \mathbf{Y}^{\ell\,\dagger}_k + \frac{1}{2}\,\delta_{ak}\,\mathbf{Y}^\ell_a \mathbf{Y}^{\ell\,\dagger}_a$$

term which generally confers maximum rank to $\mathbf{X}_{a,k}$ and $\mathbf{X}'_{a,k}$

Rank (number of nonzero eigenvalues) of corrected Yukawas:

$$r(\mathbf{Y}^{\ell(1)}_a) = r\left(\sum_k \mathbf{X}'_{a,k}\, \mathbf{Y}^\ell_k \right) \leq \min\left\{ 3, \sum_k r(\mathbf{Y}^\ell_k) \right\}$$

$$r(\mathbf{AB}) \leq \min\{r(\mathbf{A}), r(\mathbf{B})\}$$
$$r(\mathbf{A}+\mathbf{B}) \leq r(\mathbf{A}) + r(\mathbf{B})$$

Barring tailored cancellations among Yukawa structures (e.g. trivial case $\mathbf{Y}^\ell_2 \propto \mathbf{Y}^\ell_1$), equality generally holds

Radiative corrections to Yukawa couplings in NHDMs

For all diagonal Yukawas, corrections can typically be of order

$$\delta y_{ai} \sim \frac{3 y_{bi}}{16\pi^2} \log\left(\frac{\Lambda}{m_H}\right) \;,\; i = 1,2,3 \;,\; b \neq a$$

$$\mathbf{Y}_a^u \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

$$\alpha_Y^{ab} \simeq 3\mathrm{Tr}(\mathbf{Y}_a^{u\dagger} \mathbf{Y}_b^u)$$

Working ansatz: only one nonzero vev (*aka* Higgs basis)

$$(v_1, v_2, \ldots, v_N) = (0, 0, \ldots, v) \qquad v_N = v \simeq 174\ \mathrm{GeV}$$

Charged-lepton masses:

$$\mathbf{M}_\ell = v \mathbf{Y}_N^\ell$$

$$\mathbf{V}_N^{\ell\dagger} \mathbf{Y}_N^\ell \mathbf{U}_N^\ell = \mathrm{diag}(y_e, y_\mu, y_\tau)$$

Working ansatz: rank-1 Yukawas (motivated e.g. by hierarchies)

$$\mathbf{Y}_a^\ell \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} ; \begin{pmatrix} \times & 0 & 0 \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Expected result (max. rank):

$$r(\mathbf{Y}_a^{\ell(1)}) = n_m = 3$$

$$r(\mathbf{Y}_a^{\ell(1)}) \leq \min\left\{3, \textstyle\sum_k r(\mathbf{Y}_k^\ell)\right\}$$

Extreme examples of mass generation

2HDM Cannot do it with all rank-1 Yukawas

3HDM A look at the most trivial of examples:

$$(v_1, v_2, v_3) = (0, 0, v)$$

$$\mathbf{Y}_1^{\ell} = \mathrm{diag}(\epsilon_1, 0, 0)$$
$$\mathbf{Y}_2^{\ell} = \mathrm{diag}(0, \epsilon_2, 0)$$
$$\mathbf{Y}_3^{\ell} = \mathrm{diag}(0, 0, \epsilon_3)$$

$$\Rightarrow$$

(only the tau mass at tree-level)

$$m_{e_i}^{(1)} \simeq \frac{v}{16\pi^2} \alpha_Y^{3i} \epsilon_i \log\left(\frac{\Lambda}{m_H}\right)$$

$$i = 1, 2$$

$$\left.\begin{aligned} \Lambda = \Lambda_{\mathrm{GUT}} \sim 10^{16}\ \mathrm{GeV} \\ m_H \sim 1\ \mathrm{TeV} \\ \alpha_Y^{3i} \sim \mathcal{O}(1) \end{aligned}\right\} (\epsilon_1, \epsilon_2, \epsilon_3) \simeq (1.6 \times 10^{-5}, 3.2 \times 10^{-3}, 0.01)$$

no much use would be given to loop suppression in explaining hierarchies. one would have resort, e.g., to a FN mechanism...

If all Yukawa couplings are of the same strength, the magnitude of the corrections could still be as high as 70% of the original value

Extreme examples of mass and mixing generation

3HDM A look at a nontrivial Yukawa structure:

$$(v_1, v_2, v_3) = (0, 0, v)$$

$$\mathbf{Y}_1^\ell = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ -\epsilon_1\,\epsilon & 0 & 0 \\ -\epsilon_1\,\epsilon & 0 & 0 \end{pmatrix} \quad \mathbf{Y}_2^\ell = \begin{pmatrix} 0 & \epsilon_2\,\epsilon & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathbf{Y}_3^\ell = \begin{pmatrix} 0 & 0 & \epsilon_3\,\epsilon \\ 0 & 0 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

$$\mathbf{Y}_3^\ell \to \mathbf{Y}_3^{\ell(1)} = \mathbf{Y}_3^\ell + \delta\mathbf{Y}_3^\ell$$

$$\mathbf{M}_\ell^{(1)} = v\mathbf{Y}_3^{\ell(1)}$$

$$\delta\mathbf{Y}_3^\ell \simeq \frac{\log(m_H/\Lambda)}{16\pi^2} \begin{pmatrix} \alpha_Y^{31}\epsilon_1 & \alpha_Y^{32}\epsilon_2\epsilon & \frac{1}{2}\epsilon_3\epsilon(\epsilon_2^2\epsilon^2 + 3\epsilon_3^2) \\ -\alpha_Y^{31}\epsilon_1\epsilon & \alpha_Y^{32}\epsilon_2 & \frac{1}{2}\epsilon_2^2\epsilon_3\epsilon^2 \\ -\alpha_Y^{31}\epsilon_1\epsilon & 0 & \frac{3}{2}\epsilon_3^3 \end{pmatrix}$$

Extreme examples of mass and mixing generation

3HDM A look at a nontrivial Yukawa structure:

$$(v_1, v_2, v_3) = (0, 0, v)$$

$$\mathbf{Y}_1^\ell = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ -\epsilon_1\,\epsilon & 0 & 0 \\ -\epsilon_1\,\epsilon & 0 & 0 \end{pmatrix} \quad \mathbf{Y}_2^\ell = \begin{pmatrix} 0 & \epsilon_2\,\epsilon & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathbf{Y}_3^\ell = \begin{pmatrix} 0 & 0 & \epsilon_3\,\epsilon \\ 0 & 0 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

$$\mathbf{Y}_3^\ell \to \mathbf{Y}_3^{\ell(1)} = \mathbf{Y}_3^\ell + \delta\mathbf{Y}_3^\ell$$

$$\mathbf{M}_\ell^{(1)} = v\mathbf{Y}_3^{\ell(1)}$$

$$\delta\mathbf{Y}_3^\ell \simeq \frac{\log(m_H/\Lambda)}{16\pi^2}\left(\quad \cdots \quad \right) \Rightarrow$$

(only the tau mass at tree-level;
enough freedom to fit others)

$$m_e^{(1)} \simeq \frac{\alpha_Y^{31}\epsilon_1}{16\pi^2}\sqrt{1+2\epsilon^2}\, v \log\left(\frac{\Lambda}{m_H}\right)$$

$$m_\mu^{(1)} \simeq \frac{\alpha_Y^{32}\epsilon_2}{16\pi^2}\sqrt{1+\epsilon^2}\, v \log\left(\frac{\Lambda}{m_H}\right)$$

Extreme examples of mass and mixing generation

3HDM A look at a nontrivial Yukawa structure: (cont.)

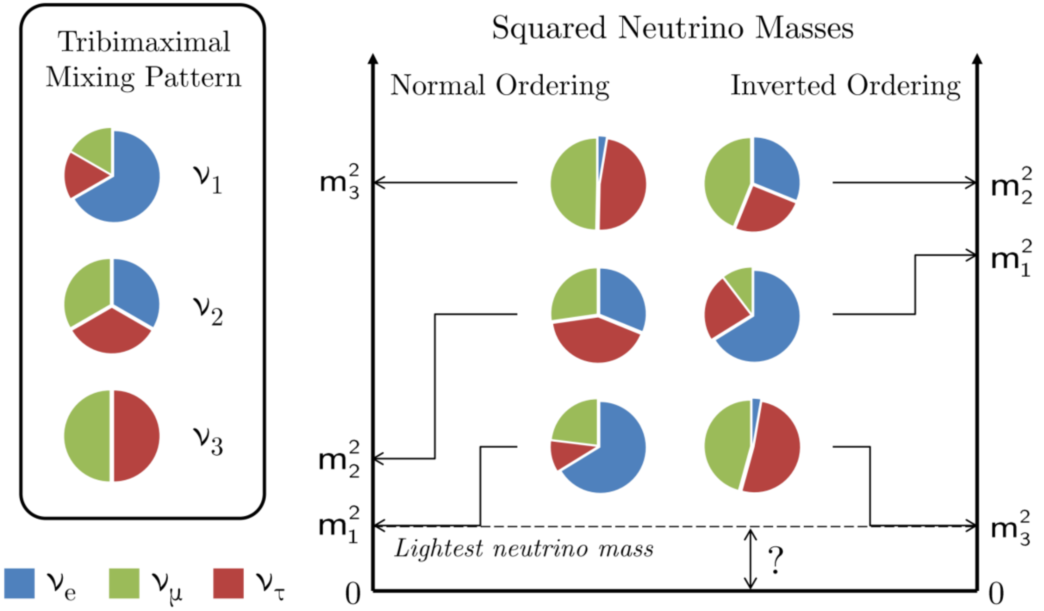
$$\mathbf{V}_L \simeq \begin{pmatrix} -1+\epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1-\dfrac{\epsilon^2}{2} & 0 \\ \epsilon & -\epsilon^2 & 1-\dfrac{\epsilon^2}{2} \end{pmatrix} \xleftarrow[\mathbf{Y}_3^\ell \to \mathbf{Y}_3^{\ell(1)}]{} \mathbf{Y}_1^\ell, \mathbf{Y}_2^\ell, \mathbf{Y}_3^\ell = \ldots$$

A tree-level tribimaximal pattern would be corrected:

$$\mathbf{U}_{\text{TBM}} \to \mathbf{V}_L^\dagger \mathbf{U}_{\text{TBM}}$$

$$\mathbf{U}_{\text{TBM}} = \begin{pmatrix} -\sqrt{\dfrac{2}{3}} & \dfrac{1}{\sqrt{3}} & 0 \\ \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{3}} & -\dfrac{1}{\sqrt{2}} \end{pmatrix}$$

Harrison, Perkins, and Scott, ’02



Extreme examples of mass and mixing generation

3HDM A look at a nontrivial Yukawa structure: (cont.)

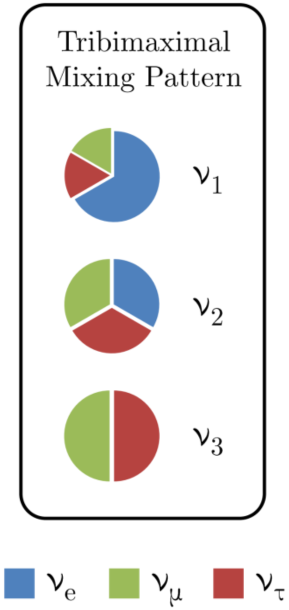
$$\mathbf{V}_L \simeq \begin{pmatrix} -1+\epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1-\dfrac{\epsilon^2}{2} & 0 \\ \epsilon & -\epsilon^2 & 1-\dfrac{\epsilon^2}{2} \end{pmatrix} \xleftarrow[\mathbf{Y}_3^\ell \to \mathbf{Y}_3^{\ell(1)}]{} \mathbf{Y}_1^\ell, \mathbf{Y}_2^\ell, \mathbf{Y}_3^\ell = \ldots$$

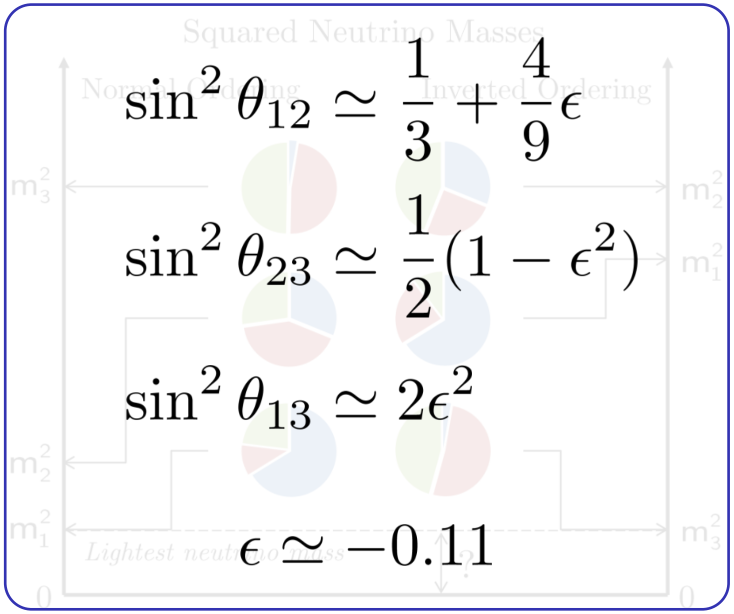
A tree-level tribimaximal pattern would be corrected:

$$\mathbf{U}_{\text{TBM}} \to \mathbf{V}_L^\dagger \mathbf{U}_{\text{TBM}}$$

$$\mathbf{U}_{\text{TBM}} = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Harrison, Perkins, and Scott, ’02





$$\sin^2\theta_{12} \simeq \frac{1}{3} + \frac{4}{9}\epsilon$$

$$\sin^2\theta_{23} \simeq \frac{1}{2}(1-\epsilon^2)$$

$$\sin^2\theta_{13} \simeq 2\epsilon^2$$

$$\epsilon \simeq -0.11$$

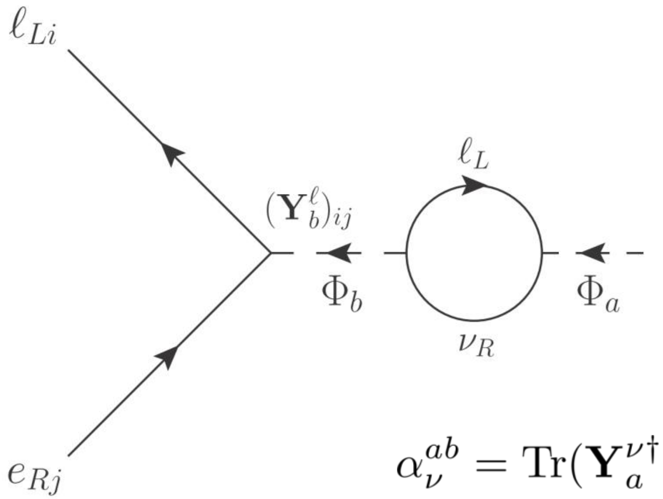
Extending the NHDM with right-handed neutrinos (RHNs)

$$\mathcal{L}_{\text{Yukawa}} \to \mathcal{L}_{\text{Yukawa}} - \left[(\mathbf{Y}^{\nu}_{a})_{ij} \bar{\ell}_{Li} \tilde{\Phi}_a \nu_{Rj} + \text{H.c.} \right]$$

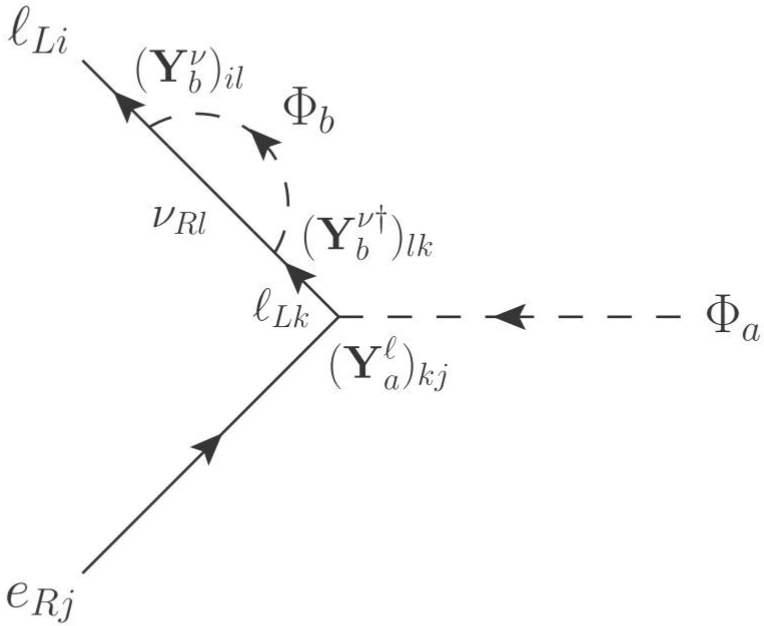
ν_{Rj} with Majorana masses $M_j \gg v$

Beta functions for charged-lepton Yukawas in the NHDM+RHNs (at one loop), $\mu \in [M_i, \Lambda > M_i]$

$$\beta'^{(1)}_a = \beta^{(1)}_a + \boxed{\alpha^{ab}_{\nu}} \mathbf{Y}^{\ell}_b + \boxed{\frac{1}{2} \mathbf{Y}^{\nu}_b \mathbf{Y}^{\nu\dagger}_b \mathbf{Y}^{\ell}_a} - 2 \mathbf{Y}^{\nu}_b \mathbf{Y}^{\nu\dagger}_a \mathbf{Y}^{\ell}_b$$



$$\alpha^{ab}_{\nu} = \text{Tr}(\mathbf{Y}^{\nu\dagger}_a \mathbf{Y}^{\nu}_b)$$



Once again, we need more than one Higgs doublet so that eigenvalues are not corrected proportionally to themselves.

Extending the NHDM with right-handed neutrinos (RHNs)

$$\mathcal{L}_{\text{Yukawa}} \to \mathcal{L}_{\text{Yukawa}} - \left[(\mathbf{Y}^\nu_a)_{ij} \bar{\ell}_{Li} \tilde{\Phi}_a \nu_{Rj} + \text{H.c.} \right]$$

ν_{Rj} with Majorana masses $M_j \gg v$

Beta functions for charged-lepton Yukawas in the NHDM+RHNs (at one loop), $\mu \in [M_i, \Lambda > M_i]$

$$\beta'^{(1)}_a = \beta^{(1)}_a + \alpha^{ab}_\nu \mathbf{Y}^\ell_b + \frac{1}{2} \mathbf{Y}^\nu_b \mathbf{Y}^{\nu\dagger}_b \mathbf{Y}^\ell_a - \boxed{2\mathbf{Y}^\nu_b \mathbf{Y}^{\nu\dagger}_a \mathbf{Y}^\ell_b}$$

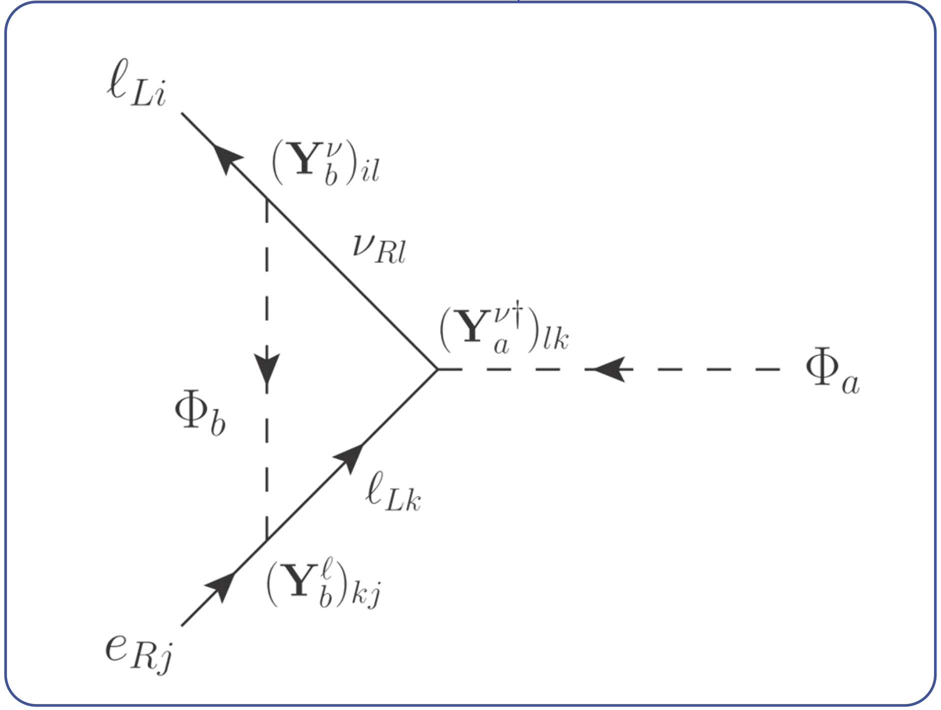
In the ‘trivial’ 3HDM: $i = 1, 2$

$$m^{(1)}_{e_i} \simeq \frac{v}{16\pi^2} \alpha^{21}_\nu \epsilon_i \log\left(\frac{\Lambda}{M_i}\right)$$

$$\mathbf{Y}^\nu_a \sim \mathcal{O}(1) \begin{cases} M_i \sim 10^{14} \\ \alpha^{21}_\nu \sim \mathcal{O}(1) \end{cases}$$

$$(\epsilon_1, \epsilon_2) \simeq (10^{-4}, 0.02)$$

If all Yukawa couplings are of the same strength, the magnitude of the corrections does not exceed ~ 10%



Final Remarks

The SM offers no explanation of mass hierarchies and is insufficient to generate the observed baryon asymmetry

NHDMs arise as a natural extension to the SM capable of solving the BAU puzzle

Charged-lepton masses (for instance) can be radiatively induced in NHDMs through the renormalization group running of Yukawa couplings

Adding right-handed neutrinos provides additional sources for this effect

Thank you / Σας ευχαριστώ

Backup slide

What happens when we impose symmetries?

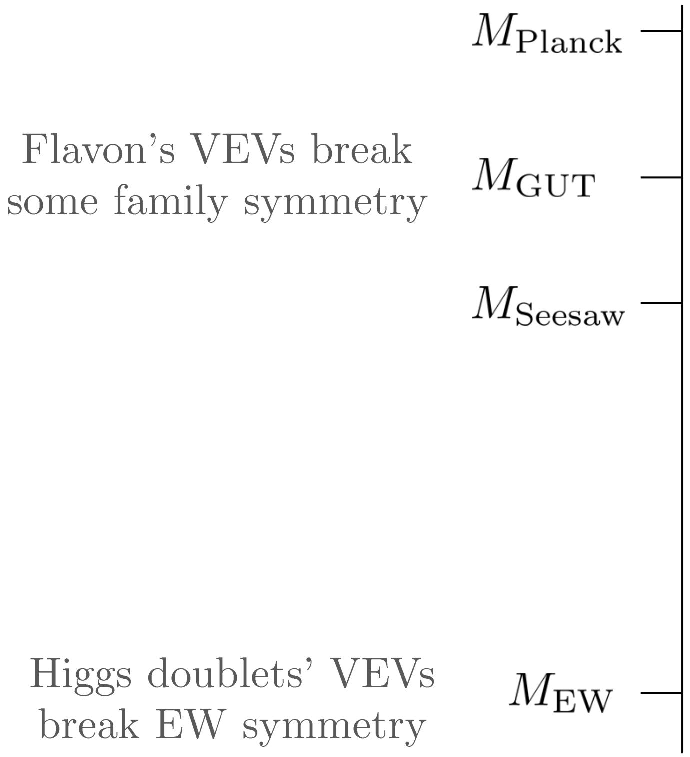
Topic to explore, for instance, in the context of flavour models

Model	u_R	d_R	e_R
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Lepton-specific	Φ_2	Φ_2	Φ_1
Flipped	Φ_2	Φ_1	Φ_2

2HDMs leading to natural flavour conservation

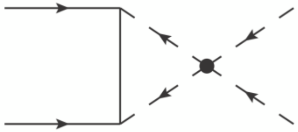
Branco et al., ’12

$$\Rightarrow \alpha_Y^{ab}(a \neq b) = 0$$



Radiatively generate neutrino masses in NHDMs?

Possible with just one RHN! (Ibarra and Simonetto, ’11)



Loop effects not controlled by the RGE running?

Explored recently for symmetric 2HDMs (Kanemura, Kikuchi, and Yagyu, ’14)