Anisotropic fluid on the brane from 5-dimensional tidal effects

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Introduction

Gravity – a truly higher dimensional theory at short ranges

Brane-world scenario – matter fields are confined to the 4-dimensional spacetime (the brane) whereas gravity propagates in the full spacetime (the bulk) – R. Maartens, PRD 62, 084023 (2000); M. Sasaki, T. Shiromizu and K. Maeda, PRD 62, 024008 (2000); C. Germani and R. Maartens, PRD 64, 124010 (2001).

– covariant equations describing both the 5-dimensional gravity in the bulk and the 4-dimensional gravity on the brane – T. Shiromizu, K. Maeda and M. Sasaki, PRD 62, 024012 (2000).

They become the standard Einstein equations in the low energy limit if the nonlocal term E_{ab} is negligible.

Germani and Maartens – the vacum exterior of a state is not a Schwarzschild spacetime in the RS braneworld model but has stresses induced by 5-dimensional gravitational effects. – non-Schwarzschild exterior matches the star interior on the brane.

Expanding bubble spacetime.

Witten bubble – a vacuum 5-dimensional spacetime – describes the decay of a standard KK vacuum to a zero energy bubble configuration via a tunneling process (E. Witten, NPB 195, 481 (1982)).

– an analytical continuation of the 5-dimensional Schw. metric.

$$ds^2 = \frac{dr^2}{1-\frac{R^2}{r^2}} + r^2\left(d\chi^2 + \sin^2\chi\, d\Omega_2^2\right) + \left(1-\frac{R^2}{r^2}\right)dy^2 \quad (2.1)$$

with $r \geq R$, y – periodic, of range $2\pi R$,

The Lorenzian evolution: $\chi \to igt + \frac{\pi}{2}$

$$ds^2 = -g^2 r^2 dt^2 + \frac{dr^2}{1-\frac{R^2}{r^2}} + r^2\cosh^2 gt\, d\Omega^2 + \left(1-\frac{R^2}{r^2}\right)dy^2 \quad (2.2)$$

(2.2) results from a semiclassical decay process of $M^4 \times S^1$ which is unstable against a process of barrier penetration (D.R. Brill and M.D. Matlin, PRD 39, 3151 (1989); G. Horowitz and T. Wiseman, arXiv: 1107.5563 (gr-qc)

S. Copley and T. Jacobson, PRD 49, 6261 (1994):

- Riemann tensor - nonzero components:

$$R^{rt}{}_{rt} = R^{r\theta}{}_{r\theta} = R^{r\varphi}{}_{r\varphi} = R^{yt}{}_{yt} =$$
$$= -R^{\varphi t}{}_{\varphi t} = -R^{t\theta}{}_{t\theta} = -\frac{R^2}{r^4}\,,\quad R^{ry}{}_{ry} = \frac{3R^2}{r^4}$$

- Kretschmann scalar: $R^{abcd}R_{abcd} = \frac{72R^4}{r^8}$
- Weyl tensor: $C^a{}_{bcd} = R^a{}_{bcd}$.
  - time independent and finite.

Take a congruence of static observers

$$u^a = \left(\frac{1}{gr}, 0, 0, 0, 0\right), \quad u^b u_b = -1.$$
$$(t, r, \theta, \varphi, y).$$

Kinematical quantities:

- expansion scalar

$$\theta \equiv \nabla_b u^b = \frac{2}{r}\tanh gt$$

- acceleration 4-vector

$$a^b \equiv u^a \nabla_a u^b = \left(0, \frac{1}{r}\left(1 - \frac{R^2}{r^2}\right), 0, 0, 0\right)$$

$$\sqrt{a_b a^b} = \frac{1}{r}\sqrt{1 - \frac{R^2}{r^2}}$$

- shear tensor

$$\sigma_{ab} = \frac{1}{2}\left(q^c{}_b \nabla_c u_a + q^c{}_a \nabla_c u_b\right) - \frac{1}{4}\theta q_{ab} +$$
$$+ \frac{1}{2}\left(a_b u_a + a_a u_b\right), \quad \text{with}$$

$$\sigma^a{}_b = \left(0, -\frac{1}{2r}\tanh gt, \frac{1}{2r}\tanh gt, \frac{1}{2r}\tanh gt, -\frac{1}{2r}\tanh gt\right)$$

where $h_{ab} = g_{ab} + u_a u_b$

θ and σ_{ab} are not time-symmetric.

$\theta \to \pm 2/r$ when $t \to \pm\infty$. With

$\dot\theta \equiv u^a \nabla_a \theta = 2/r^2\cosh^2 gt$, $\sigma^{ab}\sigma_{ab} = (1/r^2)\tanh^2 gt$, $\nabla_b a^b = 2/r^2$,

$\omega_{ab} = 0$, $R_{ab} = 0$,

the Raychaudhuri equation

$$\dot\theta - \nabla_b a^b + \sigma^{ab}\sigma_{ab} - \omega^{ab}\omega_{ab} + \frac{\theta^2}{4} = -R_{ab}u^a u^b$$

is fulfilled for the chosen congruence of worldlines.

- Apparent horizon is obtained from

$$g^{ab}\nabla_a R \nabla_b R = 0,$$

with $R(r,t) = r\cosh gt$ is the areal radius. Hence

$$r_{AH}(t) = R\cosh gt \longrightarrow$$

a radial null geodesic

With $r = \rho + b^2/\rho$, the metric (2.2) becomes

$$ds^2 = \left(1 + \frac{b^2}{\rho^2}\right)^2 \left(-g^2\rho^2 dt^2 + d\rho^2 + \rho^2\cosh^2 gt\, d\Omega^2\right)$$

$$(y = \text{const.},\ R = 2b) \qquad (2.3)$$

which is conformally flat.

We have $\rho = \sqrt{\bar{r}^2 - \bar{t}^2}$, $\bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2}$, and $(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ – Minkowski coordinates.

Therefore, $\bar{r}_{AH}(\bar{t}) = \bar{t} + R/2$, i.e. the Minkowski light cone. Hence

$$ds^2 = \left(1 + \frac{b^2}{\bar{x}_a \bar{x}^a}\right)^2 \eta_{cd}\, d\bar{x}^c d\bar{x}^d. \qquad (2.4)$$

(2.4) – more suitable for the spacetime viewed by an inertial observer than the Minkowsi space. The difference – only close to the AH (H.C. J. Korean Phys. Soc. 57, 419 (2010)) (near $r = R$ or $t = 0$). – WH throat (H.C., Internal Report IC (93/23, ICTP)

D. Ida, T. Shiromizu and H. Ochiai (PRD 65, 023504 (2002) reached a similar result, to whom (2.3) looks like an Einstein–Rosen bridge

3. Brane-world stress tensor

- Take a general bulk spacetime with 5 dimensions.
- the induced metric on the brane:

$$h_{ab} = g_{ab} - n_a n_b$$

From the Gauss–Codazzi equations one obtains:

$$G_{ab} = \left({}^5R_{ab} - \frac{1}{2} g_{ab}\, {}^5R\right) h^c{}_c h^d{}_d + {}^5R_{cd} n^c n^d h_{ab} \tag{3.1}$$
$$+ K \cdot K_{ab} - K^c{}_a K_{bc} - \frac{1}{2}\left(K^2 - K^{cd} K_{cd}\right) h_{ab} - E_{ab}$$

(Shiromizu et al.), K_{ab} – the extrinsic curvature of the hypersurface, $K \equiv K^a{}_a$ is its trace, $K_{ab} = h^c{}_a h^d{}_b \nabla_c n_d$, and

$$E_{ab} = {}^5R^c{}_{def}\, n_c n^e h^d{}_a h^f{}_b \tag{3.2}$$

is rooted from nonlocal effects of the free gravitational field in the bulk, transmitted through the projection of the Weyl tensor.

brane: the hypersurface $y = 0$.

$n^a = \left(0, 0, 0, 0, 1/\sqrt{1 - R^2/r^2}\right)$, so that we have ${}^5R_{ab} = 0$, $K_{ab} = 0$

$$G_{ab} = -E_{ab}$$

From (3.2) one obtains

$$E^t_t = E^\theta_\theta = E^\varphi_\varphi = -\frac{R^2}{r^4}, \quad E^r_r = \frac{3R^2}{r^4}$$

whence

$$8\pi G_4 T_{ab} = \left(\frac{R^2}{r^4}, -\frac{3R^2}{r^4}, \frac{R^2}{r^4}, \frac{R^2}{r^4}\right) \tag{3.3}$$

$$(G_4 = G_5/R)$$

The brane geometry (the dimensionally reduced Witten bubble)

$$ds^2 = -g^2 r^2 dt^2 + \frac{dr^2}{1 - \frac{R^2}{r^2}} + r^2 \cosh^2 gt\, d\Omega^2$$

The energy density of the "dark fluid"

$$\rho = -T^t_{\ t} = -\frac{R^2}{8\pi G_4 r^4} < 0,$$

because ρ is rooted from the non-local gravitational DOF from the bulk into the brane ("tidal effects)

If we take $R \approx l_p$, $\rightarrow \rho \approx -\hbar c/8\pi r^4$.

$\rightarrow \rho$ has a purely quantum origin (similar with Casimir energy density).

The tidal acceleration orthogonal to the brane is

$$A^y = -E_{ab} v^a v^b = -\frac{R^2}{r^4} < 0$$

where $v^a = (1/g_{tt}, 0, 0, 0)$ is the 4-velocity of observers comoving with the fluid. A^y is toward the brane $\rightarrow$ localization of gravity close to the brane is enforced by a negative A^y. and negative energy density on the brane.

Anisotropic fluid on the brane

$T^a_{\ b}$ from (3,3) gives

$$\rho = \frac{1}{3} p_r = -p_\theta = -p_\varphi$$

Define an average pressure

$$p = \frac{p_r + p_\theta + p_\varphi}{3}$$

which gives $p=\rho/3$, as for a null fluid.

The general expression

$$T_{ab}=\rho v_a v_b+p f_{ab}+\pi_{ab}+q_a v_b+q_b v_a\,, \qquad (4.1)$$

where $f_{ab}=g_{ab}-n_a n_b+v_a v_b=h_{ab}+v_a v_b$ is the metric felt by comoving observers on the brane, q_a – the heat flux and π_{ab} – the anisotropic stress tensor.

We have $q_a=0$ and

$$\frac{8\pi G_4}{3}\rho=8\pi G_4 p=-\frac{R^2}{3r^4}\;;\;\pi^r{}_r=-2\pi^\theta{}_\theta=$$
$$=-2\pi^\varphi{}_\varphi=-\frac{R^2}{3\pi G_4 r^4}\,, \qquad (4.2)$$

where p is the average pressure, $\pi_{ab}v^b=0$, $\pi^a{}_a=0$.

Take $\pi^a{}_b$ of the form

$$\pi^a{}_b=2\eta\,\sigma^a{}_b+\zeta\,\Theta\,f^a{}_b \qquad (4.3)$$

where $\sigma^a{}_b$ and Θ are defined on the brane. η is the shear viscosity and ζ – the bulk viscosity coefficient.

The shear viscosity tensor is

$$\sigma^r{}_r=-2\sigma^\theta{}_\theta=-2\sigma^\varphi{}_\varphi=-\frac{2}{3r}\tanh gt \qquad (4.4)$$

with $\sigma^{ab}\sigma_{ab}=(2/3r^2)\tanh gt$.

One finds that

$$\eta(r,t) = \frac{2R^2}{r^3 \tanh gt} > \zeta = 0$$

η is divergent at $t=0$ and tends to $\pm 2R^2/r^3$ when $t \to \pm\infty$, at constant r. For example,

$$\eta(t=\infty) = \eta_\infty = (c^3/G_4)\, 2R^2/r^3.$$

Taking $R \simeq l_P$, $r \simeq 1$ cm, one has $\eta_\infty \simeq 10^{-27}$ g/cm.s, but $r \simeq 10^{-8}$ cm gives $\eta_\infty \simeq 10^{-5}$ g/cm.s, much less then water viscosity at 20°C, which is $\approx 10^{-2}$ g/cm.s.

One finds that, with

$\Theta = (2/r)\tanh gt$, $\dot\Theta = 2/r^2\cosh^2 gt$,

$\nabla_b a^b = (2/r^2) - (R^2/r^4)$, $\sigma^{ab}\sigma_{ab} =$

$= (2/3r^2)\tanh^2 gt$, $R_{ab}v^a v^b = -R^2/r^4$,

the Raychaudhuri equation

$$\dot\Theta - \nabla_a a^a + \sigma^{ab}\sigma_{ab} - \omega^{ab}\omega_{ab} + \frac{\Theta^2}{3} = -R_{ab}v^a v^b,$$

is obeyed.

Conclusions

- the decay of the standard KK vacuum to a zero energy bubble configuration via a tunneling process $\to$ the expanding Witten bubble solution.

- a Ricci-flat solution of the KK equations.
- the Lorentzian evolution of the bubble is obtained through the analytical continuation $\chi \to igt + \pi/2$.

Riemann (Weyl) tensors, Kretschmann scalar - static and finite.
- metric - time dependent, $\to$

$$r_{AH}(t) = R \cosh gt$$

- in Minkowski coordinates:

$$\bar{r}_{AH}(\bar{t}) = \bar{t} + R/2,$$

i.e., the light cone.
- gravitational from the bulk carries informations on the brane (nonlocal tidal effects) through the tensor E_{ab}, which is rooted from the 5-dimensional Weyl tensor.

$E_{ab} = -T_{ab}$, on the brane.
- $\rho \simeq -\hbar c/8\pi r^4$, if $R \simeq l_P$.
- the fluid on the brain - anisotropic.

$T^a{}_b \neq 0$, with $\rho = 0$, $\eta(r,t)$,

$\eta_\infty = 2R^2/r^3$.