2

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Neutrino Mass and Mixing (Theory)

Guido Altarelli

Universita' di Roma Tre
CERN

Models of Neutrino Mixing

Models of ν mixing

$$m_\ell \to \bar{R} m_\ell L$$

$$m_\ell' = V_\ell^\dagger m_\ell U_\ell$$

$$m_\ell^{\dagger\prime} m_\ell' = U_\ell^\dagger m_\ell^\dagger m_\ell U_\ell$$

An interplay of different matrices:

$$U_{PMNS} = U_\ell^\dagger U_\nu$$

neutrino diagonalisat'n

charged lepton diagonalisat'n

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell HH \to \nu_L^T m_\nu \nu_L$$

See-saw

$$m_\nu = m_D^T M^{-1} m_D$$

neutrino Majorana mass

neutrino Dirac mass

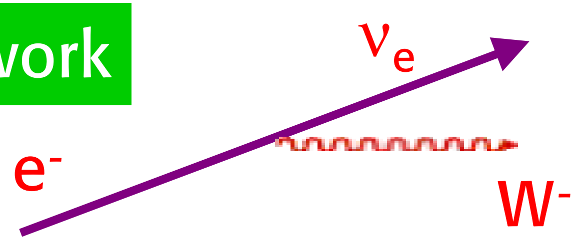
For example, the large ν mixing vs the small q mixing can be due to the Majorana nature of ν's

$$m_\nu' = U_\nu^T m_\nu U_\nu$$

3-ν Models: still the main framework

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour mass



$U = U_{PMNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

s = solar: large

$|s_{13}|$ (not so) small

$$\sim \begin{bmatrix} c_{13}\, c_{12} & c_{13}\, s_{12} & s_{13}e^{-i\delta} \\ \dots & \dots & c_{13}\, s_{23} \\ \dots & \dots & c_{13}\, c_{23} \end{bmatrix}$$

atm.: ~ max

(some signs are conventional)

In general: $U = U^+_e U_\nu$

$$m_\nu \sim U^* \begin{bmatrix} e^{i\alpha_1} m_1 & 0 & 0 \\ 0 & e^{i\alpha_2} m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U^+$$

$\nu_L^T m_\nu \nu_L$

In general 9 parameters:
3 masses, 3 angles,
3 phases

The extra phases appear because the Majorana mass is $\nu_L^T m_\nu \nu_L$ and not $\bar{\nu} m_\nu \nu$

Note:

- m_ν is symmetric
- phases can be included in m_i

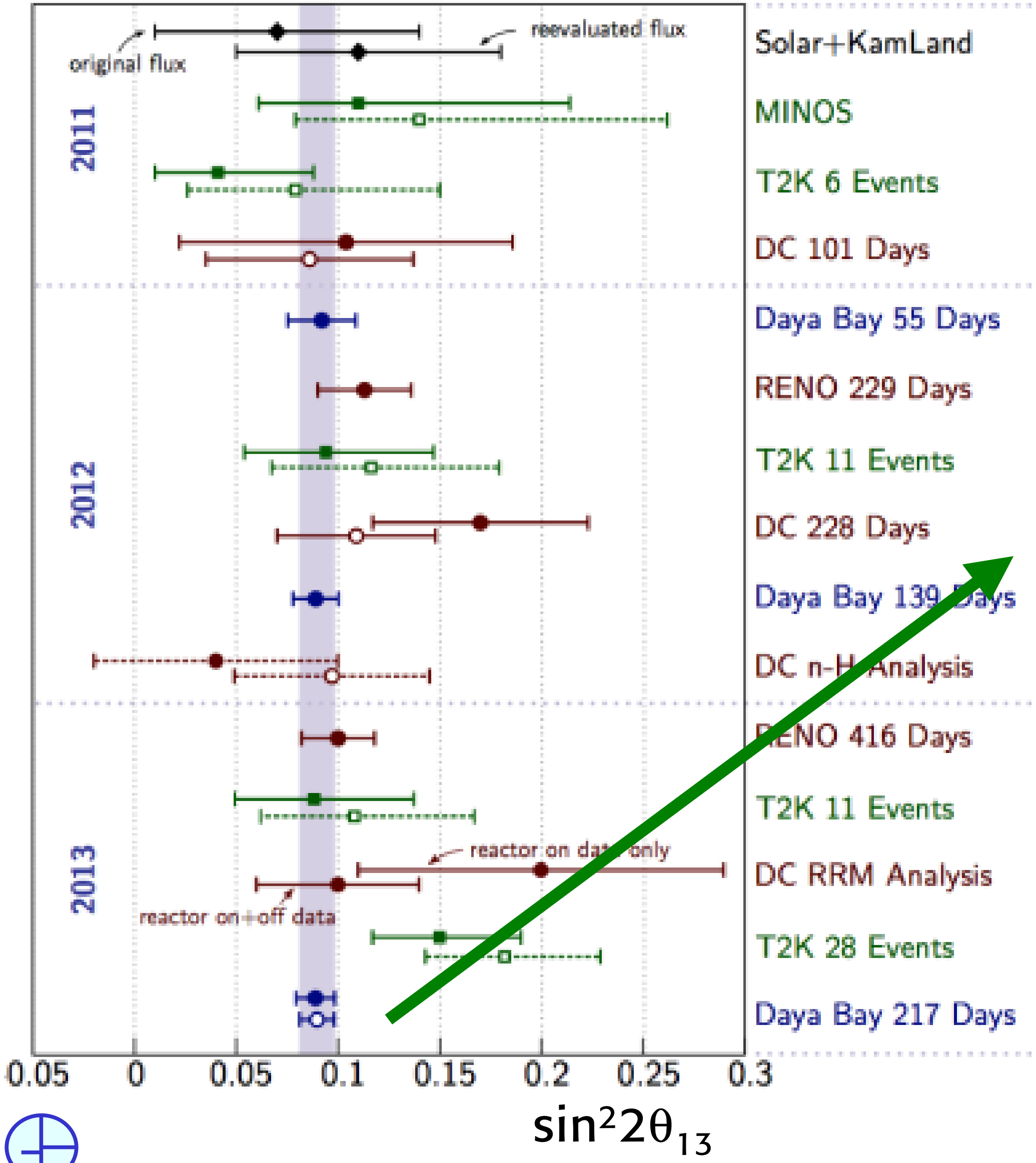
Relation between masses and frequencies:

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \quad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} > \text{ or } < 0$

here by m^2
we mean $|m^2|$

Now we have a good measurement of θ_{13}!!



~10 σ from zero

Daya Bay

$$\sin^2 2\theta_{13} = 0.090^{+0.008}_{-0.009}$$

A large impact on model building and on designing new experiments!
(hierarchy, δ_{CP}...)

Empirically $\sin^2\theta_{13} \simeq \frac{1}{2}\sin^2\theta_C$

or $\theta_{13} \approx \theta_C/\sqrt{2}$

Parameter	Best fit	1σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91 – 3.25
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.37 – 2.49
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.38	2.32 – 2.44
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34	2.15 – 2.54
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40	2.18 – 2.59
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.37	4.14 – 4.70
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.55	4.24 – 5.94
δ/π (NH)	1.39	1.12 – 1.77
δ/π (IH)	1.31	0.98 – 1.60

Capozzi, Fogli et al '14

(free reactor fluxes)

θ_{23} non maximal?

(free reactor fluxes)

a start on $\cos\delta$?

		3σ
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.270 \to 0.344$
$\theta_{12}/^\circ$	$33.48^{+0.77}_{-0.74}$	$31.30 \to 35.90$
$\sin^2\theta_{23}$	$[0.451^{+0.001}_{-0.001}] \oplus 0.577^{+0.027}_{-0.035}$	$0.385 \to 0.644$
$\theta_{23}/^\circ$	$[42.2^{+0.1}_{-0.1}] \oplus 49.4^{+1.6}_{-2.0}$	$38.4 \to 53.3$
$\sin^2\theta_{13}$	$0.0219^{+0.0010}_{-0.0011}$	$0.0188 \to 0.0251$
$\theta_{13}/^\circ$	$8.52^{+0.20}_{-0.21}$	$7.87 \to 9.11$
$\delta_{CP}/^\circ$	251^{+67}_{-59}	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$
$\frac{\Delta m^2_{31}}{10^{-3}\ \text{eV}^2}$ (N)	$[+2.458^{+0.002}_{-0.002}]$	$+2.325 \to +2.599$
$\frac{\Delta m^2_{32}}{10^{-3}\ \text{eV}^2}$ (I)	$-2.448^{+0.047}_{-0.047}$	$-2.590 \to -2.307$

Gonzalez-Garcia et al '14

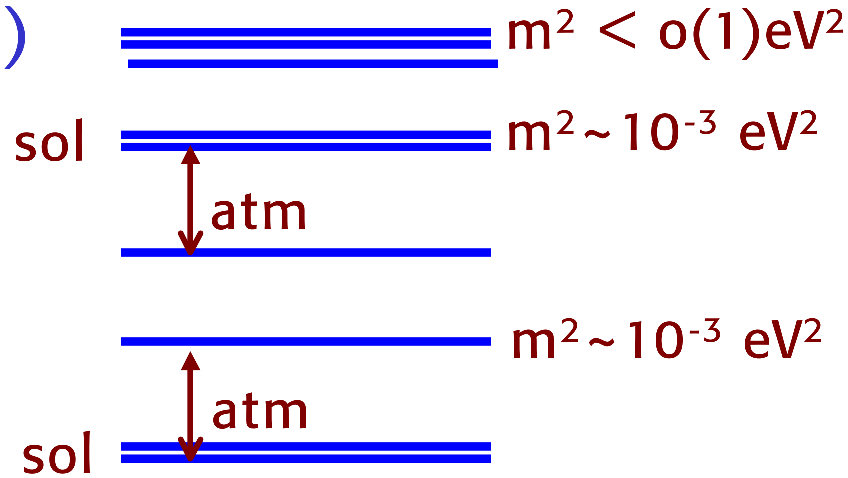
By now all mixing angles are fairly well known!

Not so δ_{CP}, θ_{23} octant, NH vs IH

The current experimental situation on ν masses and mixings has much improved but is still incomplete

- what is the absolute scale of ν masses?
- phase of CP viol., deviation of θ_{23} from $\pi/4$ and its octant
- pattern of spectrum (sign of Δm^2_{atm})

- Degenerate ($m^2 >> \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



- no detection of $0\nu\beta\beta$ (i.e. no proof that ν's are Majorana)
  see-saw?
- are 3 light ν's OK? (are there sterile neutrinos?)

Different classes of models are still possible

$0\nu\beta\beta$ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$$|m_{ee}|=c_{13}^2\,[m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$$

Degenerate: $\sim|m||c_{12}^2+e^{i\alpha}s_{12}^2|\sim|m|(0.3-1)$

$|m_{ee}|\sim |m|\ (0.3-1)\leq 0.23\text{-}1$ eV

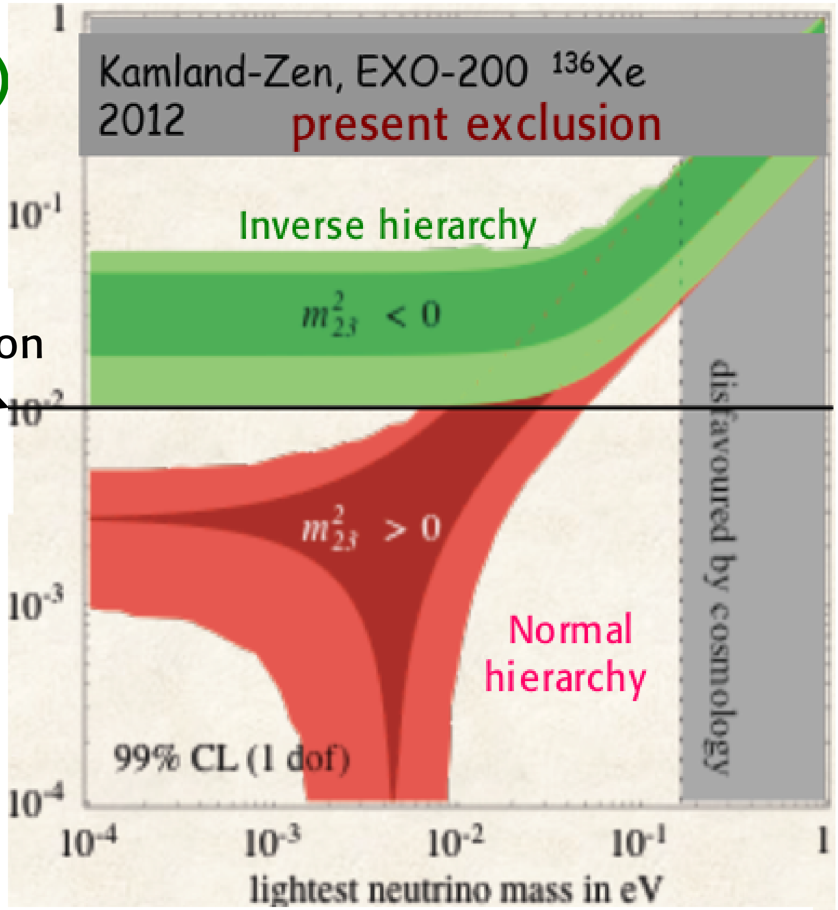
IH: $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim (1.6\text{-}5)\ 10^{-2}$ eV

NH: $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim$ (few) 10^{-3} eV

next generation
10 meV



Present exp. limit: $m_{ee} < 0.12\text{-}0.25$ eV

General remarks for model building

- Finally not too much hierarchy is found in ν masses:

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$

Only a few years ago r could be as small as 10^{-8}!

Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '10

For a normal hierarchy spectrum:

$$\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$$

Comparable to $\lambda_C = \sin\theta_C$:

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

Now we also know that $\theta_{13} \approx \theta_C/\sqrt{2}$

Suggests the same "hierarchy" parameters for q, l, ν (small powers of λ_C)

I now discuss some current ideas on model building

We go from less to more structure

Models with little symmetry are more qualitative.
Some examples:

Anarchy
Semianarchy
Lopsided models
$U(1)_{FN}$
••••••

With more symmetry models are more predictive.
Better data have narrowed the range for each mixing angle
and precise special patterns are suggested
that can be reproduced by specified symmetries :

TriBimaximal (TB), BiMaximal (BM),.......
Discrete non abelian flavour groups A4, S4,.....
Continuous flavour groups

No order for neutrinos -> Anarchy

In the neutrino sector no symmetry, no dynamics is assumed; only chance

Hall, Murayama, Weiner'00

Anarchy and its variants can be embedded in a simple GUT context based on

$SU(5)xU(1)_{flavour}$

Froggatt Nielsen '79

Offers a simple description of hierarchies for quarks and leptons, but only orders of magnitude are predicted (large number of undetermined o(1) parameters)

θ_{13} near the previous bound and θ_{23} non maximal both go in the direction of Anarchy (a great success for Anarchy!)

Anarchy: no order for neutrino mixing

In the neutrino sector no symmetry, no dynamics is needed; only chance

Hall, Murayama, Weiner '00.....
de Gouvea, Murayama '12

$\theta_{12}, \theta_{13}, \theta_{23}$ are just 3 random angles, the value of $r = \Delta m^2_{sun} / \Delta m^2_{atm} \sim 1/30$ is also determined by chance

See-Saw: $m_\nu \sim m^T M^{-1} m$ produces some hierarchy (r small) from random m, M. But θ_{13} and r are still too small

In models based on $SU(5)xU(1)_{FN}$ one gets more success by charge assignments that mitigate anarchy(with the same n. of parameters)

GA, Feruglio, Masina '02,'06
Buchmuller et al, '11
GA, Feruglio, Masina, Merlo '12
Bergstrom, Meloni, Merlo '14

Hierarchy for masses and mixings via horizontal $U(1)_{FN}$ charges.

Froggatt, Nielsen '79

Principle: A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by U(1) if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H: U(1) charges of $\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge $q_\theta = -1$.

If vev $\theta = w$, and $w/M = \lambda$ we get for a generic interaction:

$$\bar{R}_1 m_{12} L_2 H \, (\theta/M)^{\overbrace{q1+q2+qH}^{\Delta_{charge}}} \qquad m_{12} \to m_{12}\, \varepsilon^{q1+q2+qH}$$

Hierarchy: More Δ_{charge} -> more suppression ($\varepsilon = \theta/M$ small)

One can have more flavons ($\varepsilon, \varepsilon'$, ...)
with different charges (>0 or <0) etc -> many versions

Anarchy can be realised in SU(5) by putting all the flavour structure in T ~ 10 and not in F^{bar} ~ 5^{bar}

$m_u \sim 10 . 10$ — strong hierarchy $m_u : m_c : m_t$

$m_d \sim 5^{bar} . 10 \sim m_e^T$ — milder hierarchy $m_d : m_s : m_b$ or $m_e : m_\mu : m_\tau$

Experiment supports that d, e hierarchy is roughly the square root of u hierarchy

$m_\nu \sim \nu_L^T m_\nu \nu_L \sim 5^T . 5$ or for see saw $(5.1)^T$ (1.1) (1.5)

For example, for the simplest flavour group, $U(1)_F$

		1st fam.	2nd	3rd
anarchy	T :	(3,	2,	0)
	F^{bar}:	(0,	0,	0)
	1 :	(0,	0,	0)

A milder ansatz - μ–τ anarchy: no structure only in 23

Consider a matrix like $m_\nu \sim L^T L \sim \begin{bmatrix} \varepsilon^4 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & 1 & 1 \\ \varepsilon^2 & 1 & 1 \end{bmatrix}$ Note: $\theta_{13} \sim \varepsilon^2$, $\theta_{23} \sim 1$

$q(5^{bar}) \sim (2, 0, 0)$

with coeff.s of o(1) and det23~o(1)

["semianarchy", while $\varepsilon \sim 1$ corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{bmatrix} \varepsilon^4 & \varepsilon^2 & 0 \\ \varepsilon^2 & \eta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Normally two masses are of o(1) or r ~1 and $\theta_{12} \sim \varepsilon^2$

But if, accidentally, $\eta \sim \varepsilon^2$, then r is small and θ_{12} is large.

The advantage over anarchy is that θ_{13} is naturally small and a single accident is needed to get both θ_{12} large and r small

Ramond et al,
Buchmuller et al, '11

With see-saw one can do better

G.A., Feruglio, Masina'02
GA, Feruglio, Masina, Merlo '12

1st fam. 2nd 3rd

$q(10)$: (5, 3, 0)
$q(\bar{5})$: (2, 0, 0)
$q(1)$: (1,-1, 0)

Needed: not all charges positive

$q(H) = 0,\ q(\bar{H}) = 0$
$q(\theta) = -1,\ q(\theta') = +1$

In first approx., with $<\theta>/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

with $\lambda \sim \lambda'$

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{bmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{bmatrix}$$

see-saw $\quad m_\nu \sim m_{\nu D}{}^T M_{RR}{}^{-1} m_{\nu D}$

$$m_\nu \sim v_u{}^2/M \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix},$$

$\det_{23} \sim \lambda^2$

lopsided m_D
and M_{33} non zero
guarantees
det 23 suppressed

$r \sim \lambda^4,\ \theta_{13} \sim \lambda^2,\ \theta_{12},\ \theta_{23} \sim 1$

In this model all small parameters are naturally explained in terms of suitable suppression factors fixed by the charges

But too many free parameters!!

Called $AA_{\mu\tau}$ in the following

SU(5)xU(1)

1st fam. 2nd 3rd

Ψ_{10}: (5, 3, 0)
Ψ_5: (2, 0, 0)
Ψ_1: (1,-1, 0)

Equal 2,3 ch. for lopsided

Recall: $m_u \sim 10\ 10$
$m_d = m_e^T \sim \bar{5}\ 10$
$m_{\nu D} \sim \bar{5}\ 1$; $M_{RR} \sim 1\ 1$

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchy (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semianarchy $\mu\tau$-Anarchy $(A_{\mu\tau})$	(3,2,0)	(1,0,0)	(2,1,0)	(0,0)
$\mu\tau$-Apparent Anarchy $(AA_{\mu\tau})$	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchy (H)	(5,3,0)	(2,1,0)	(2,1,0)	(0,0)

No structure for leptons

No automatic det23 = 0

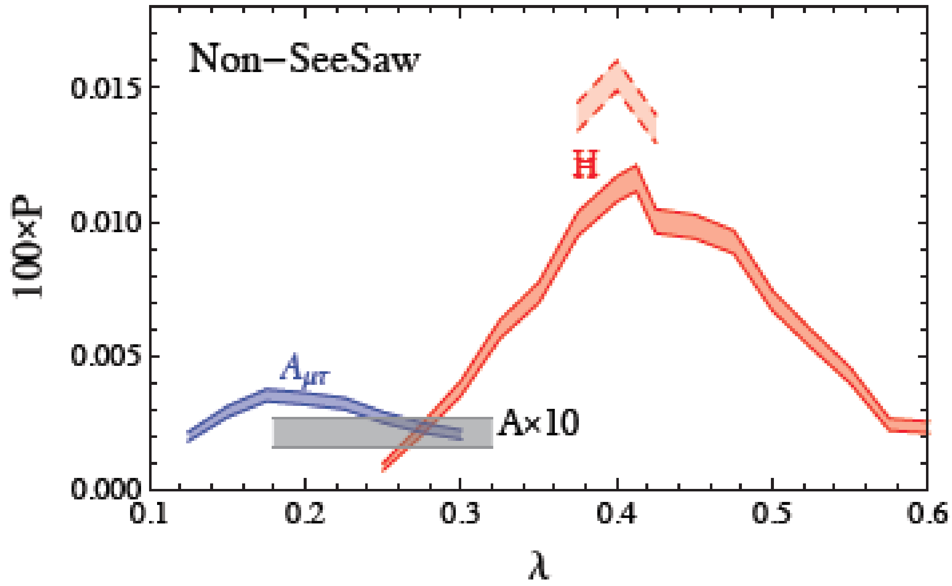
Automatic det23 = 0

all charges non negative

charges of both signs

here r, θ_{23} are suppressed

With suitable charge assignments many relevant patterns can be obtained

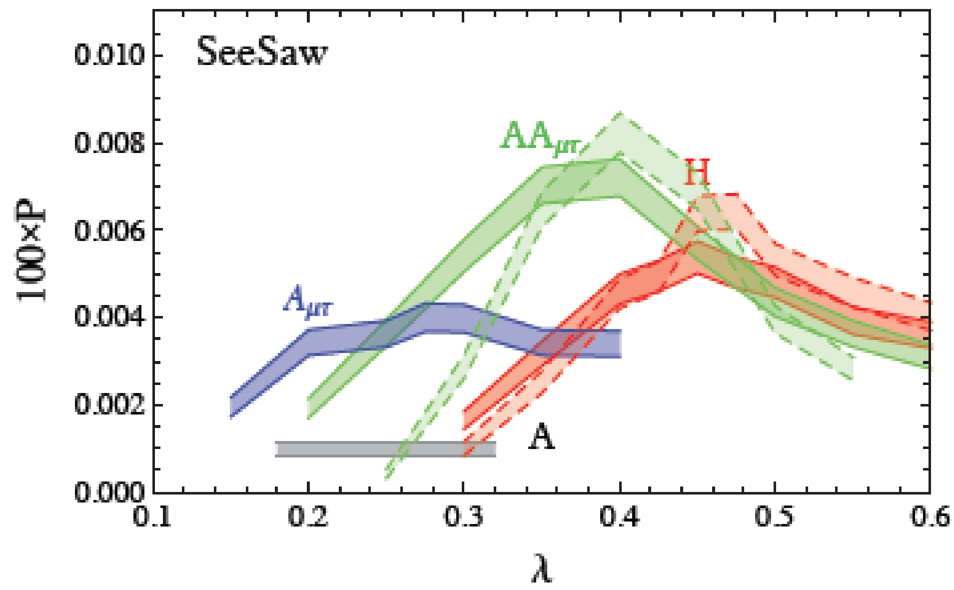


Anarchy (A): both r and θ_{13} small by accident

$\mu\tau$-anarchy ($A_{\mu\tau}$): only r small by accident

H, $AA_{\mu\tau}$: no accidents

GA, Feruglio, Masina '02,'06
GA, Feruglio, Masina, Merlo '12



Optimal values of $\lambda \sim o(\lambda_c)$

A: λ ~ 0.2

$A_{\mu\tau}$: λ ~ 0.2 (non SS), 0.3 (SS)

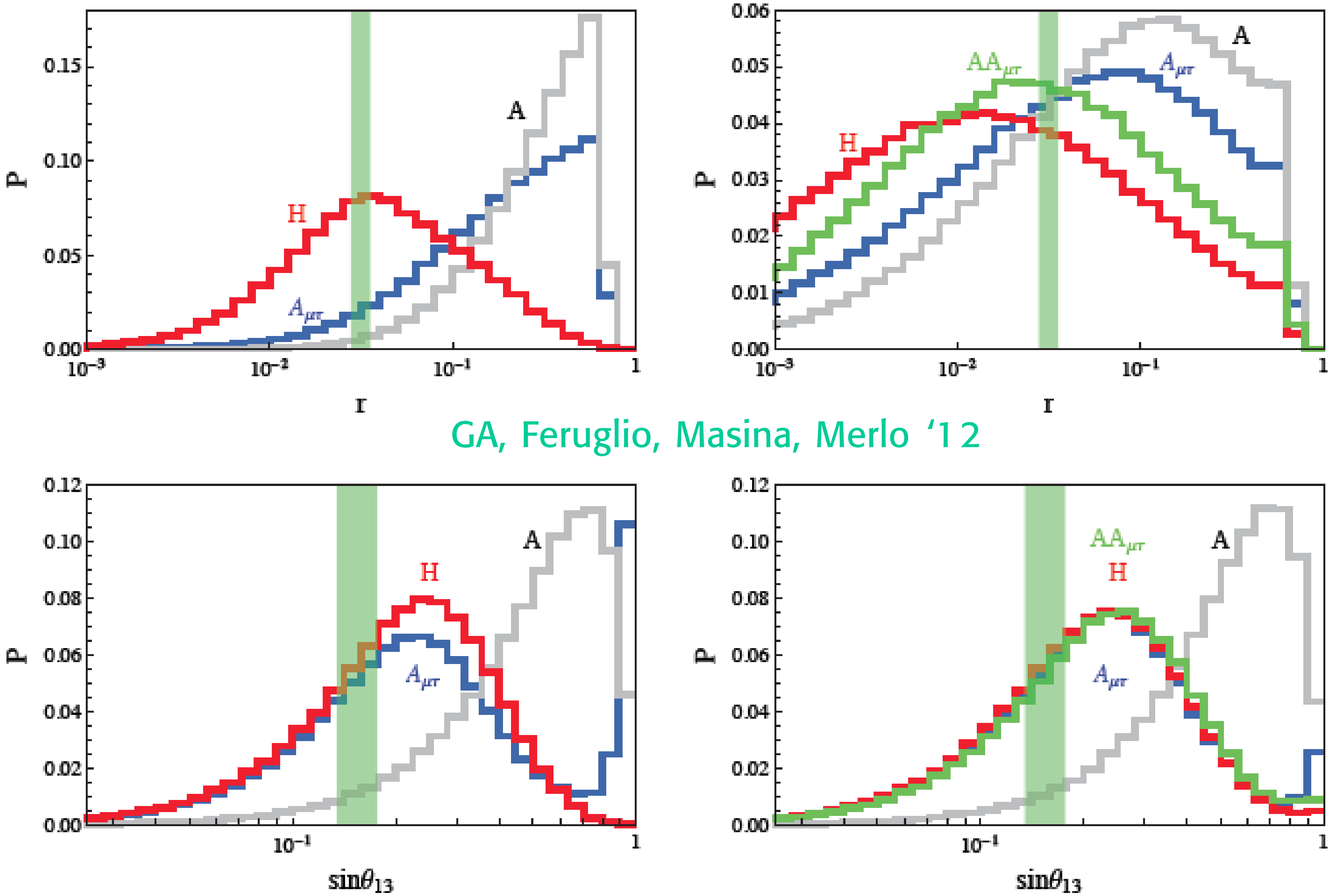
$AA_{\mu\tau}$: λ ~ 0.35

⊕H: λ ~ 0.4 (non SS), 0.45 (SS)

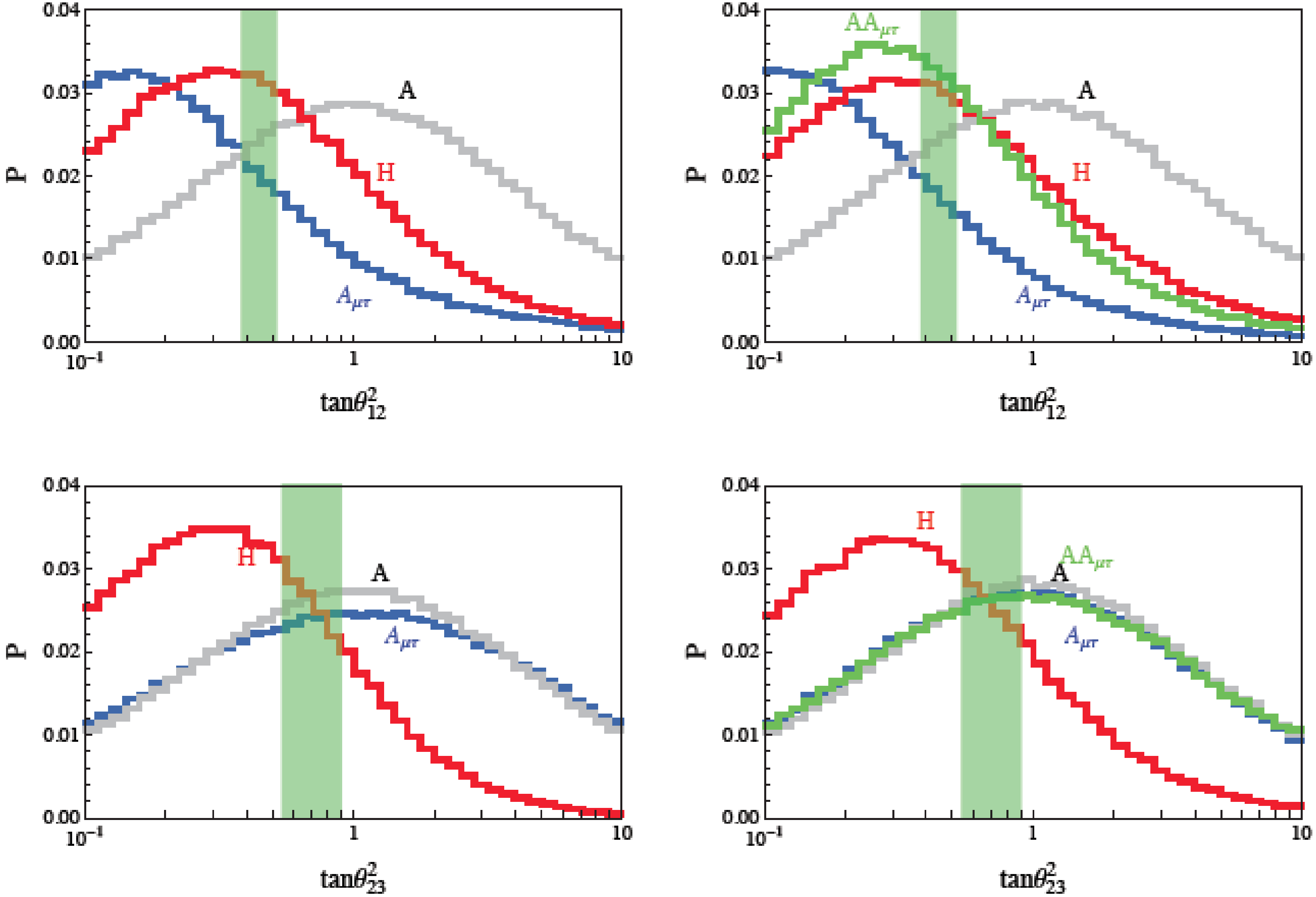
no see-saw

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell HH \to \nu_L{}^T m_\nu \nu_L$$

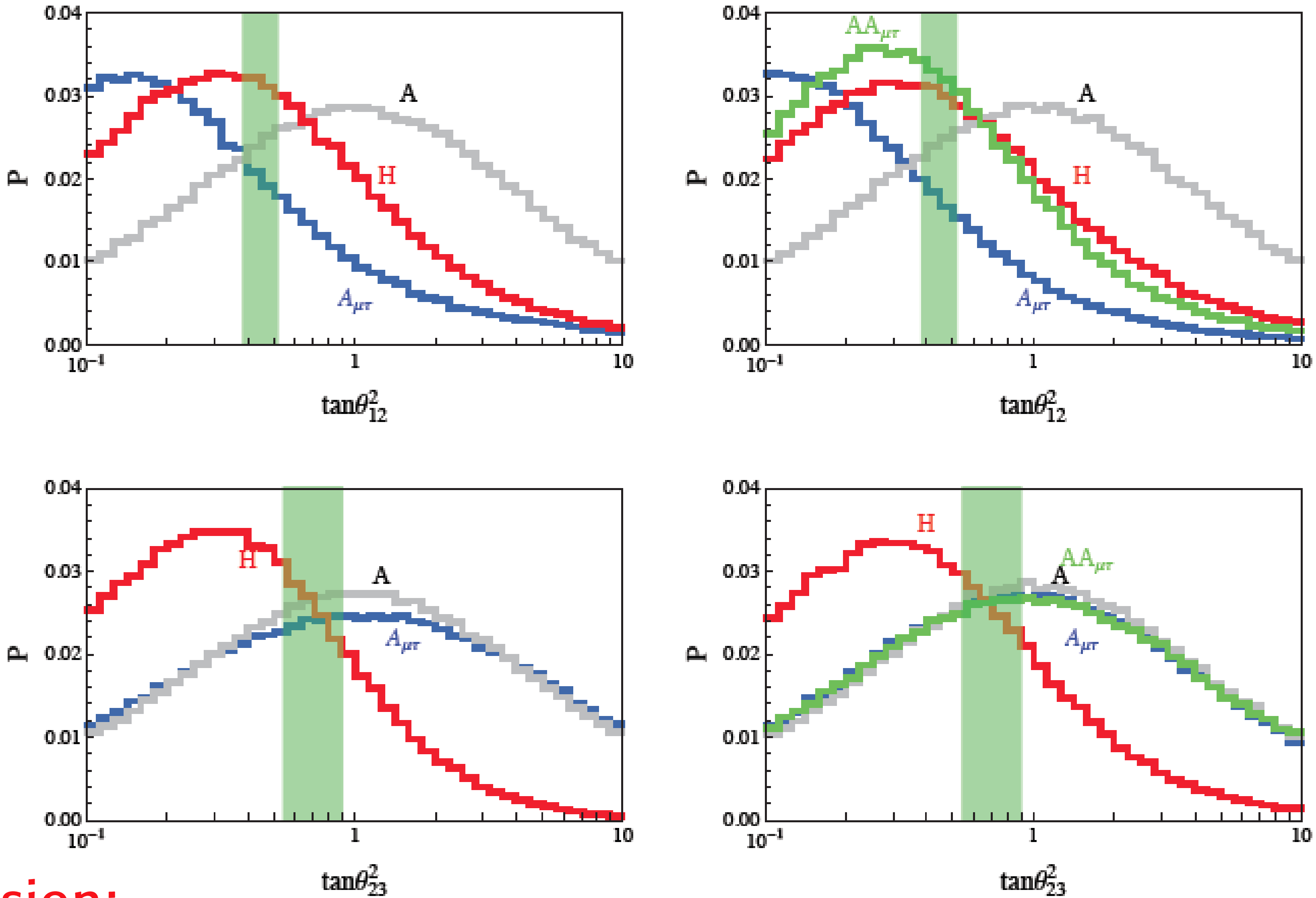
see-saw



GA, Feruglio, Masina, Merlo '12



0.04
0.03
0.02
0.01
0.00
P
10^{-1}
1
10
$\tan\theta_{12}^2$
A
H
$A_{\mu\tau}$
$AA_{\mu\tau}$
$\tan\theta_{23}^2$



Conclusion: If we embed anarchy in GUT's and explain mass hierarchies in terms of FN charges, then more effective variants of anarchy can be built, where chance is somewhat mitigated

From Anarchy and $U(1)_{FN}$ to more symmetry

Larger than U(1) continuous symmetries:

e.g $U(3)_l x U(3)_e$ ----> $U(2)_l x U(2)_e$

Blankenburg, Isidori, Jones-Perez '12
Alonso, Gavela, isidori, Maiani'13

At the other extreme from Anarchy

models with a maximum of order:
based on non abelian discrete flavour groups

(reviews: G.A., Feruglio, Rev.Mod.Phys. 82 (2010) 2701; Kobayashi et al'10;
Grimus, Ludl'11; G.A., Feruglio, Merlo'12 ;
Morisi, Valle'12; King, Luhn'13)

A number of "coincidences" could be hints pointing to the underlying dynamics

TB Mixing

TB mixing is close to the data; less now,
but still: θ_{12}, θ_{23} agree at $< \sim 2\sigma$
and θ_{13} is the smallest angle

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

At 1σ: Capozzi et al '14

$\sin^2\theta_{12} = 1/3$: 0.291- 0.325

$\sin^2\theta_{23} = 1/2$: 0.41 - 0.47

$\sin\theta_{13} = 0$: 0.15 - 0.16

A coincidence or a hint?

Called:
Tri-Bimaximal mixing

Harrison, Perkins, Scott '02

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

θ_{13} largish and θ_{23} non maximal move away from exact TB
(still remains a good first approximation)

LQC: Lepton Quark Complementarity

$\theta_{12} + \theta_C = (46.5 \pm 0.8)^o \sim \pi/4$ ← Gonzalez-Garcia et al '14

Suggests Bimaximal Mixing (BM) corrected by diagonalisation of charged leptons

(in GUT charged leptons may know θ_C)

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

BM: Group S4 GA, Feruglio, Merlo '09....

A coincidence or a hint?

Golden Ratio

GR: Golden Ratio - Group A5

Feruglio, Paris '11; G. J. Jing et al '11

Cooper et al '12, de Madeiros Verzielas et al '13....

$$\sin^2\theta_{12} = \frac{1}{\sqrt{5}\phi} = \frac{2}{5+\sqrt{5}} \approx 0.276$$

$$U_{GR} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

A coincidence or a hint?

Cannot all be true hints, perhaps none

TB or BM or GR mixing naturally lead to discrete flavour groups

For example:

TB Mixing: $U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

In fact this is a particular rotation matrix with specified fixed angles

TB: Group A4, S4, T'..... A vast literature (Ma, Rajasekaran '01.....)

Some recent works: A4 Ferreira et al '13; Morisi et al '13; Gonzalez-Felipe et al '13 Holthausen et al '12; Ben Tov et al '12; King et al '12 ...

S4 Bazzocchi et al '12; Hagedorn et al '12; Zhao '11.....

T' Chen et al '13; Meroni et al '12; Merlo et al '11.....

In A4 models at LO TB mixing is exact

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

The only fine-tuning needed is to account for $r^{1/2} \sim 0.2$
[In most A4, S4 models $r^{1/2} \sim 1$ would be expected as $l, \nu^c \sim 3$]

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle receives generically corrections of the same order $\delta\theta_{ij} \sim o(VEV/\Lambda)$

As the maximum allowed corrections to θ_{12} (and also to θ_{23}) are $o(\lambda_C^2)$, we need $VEV/\Lambda \sim o(\lambda_C^2)$ and we expect:

$$\boxed{\theta_{13} \sim o(\lambda_C^2) \sim 0.05}$$

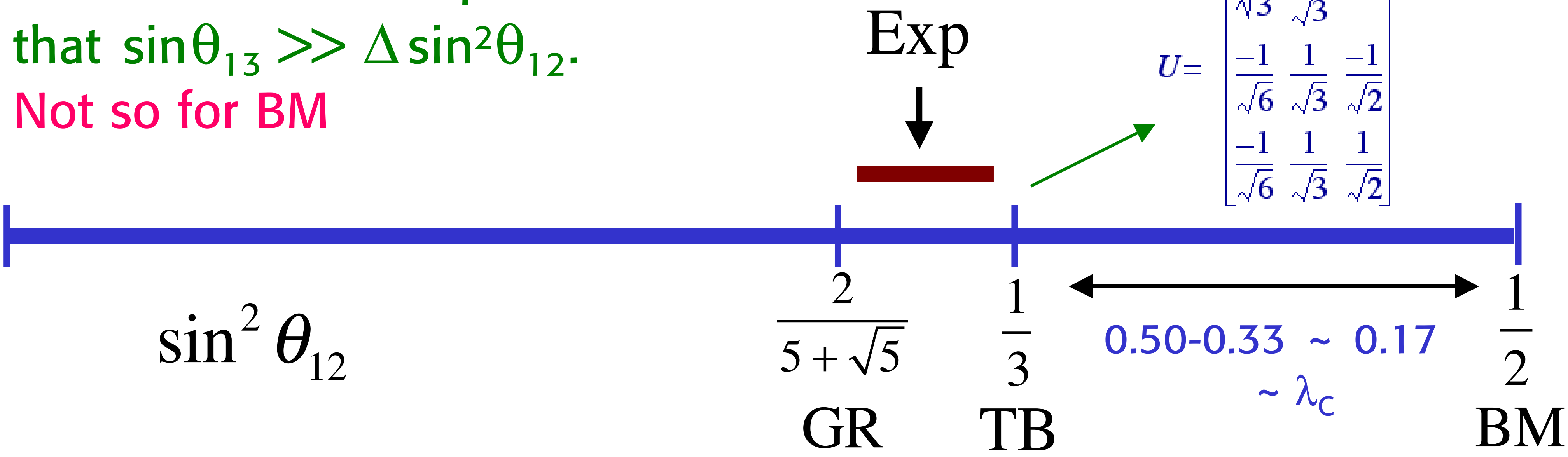
Thus $\theta_{13} \sim 0.15$ disfavours TBM models

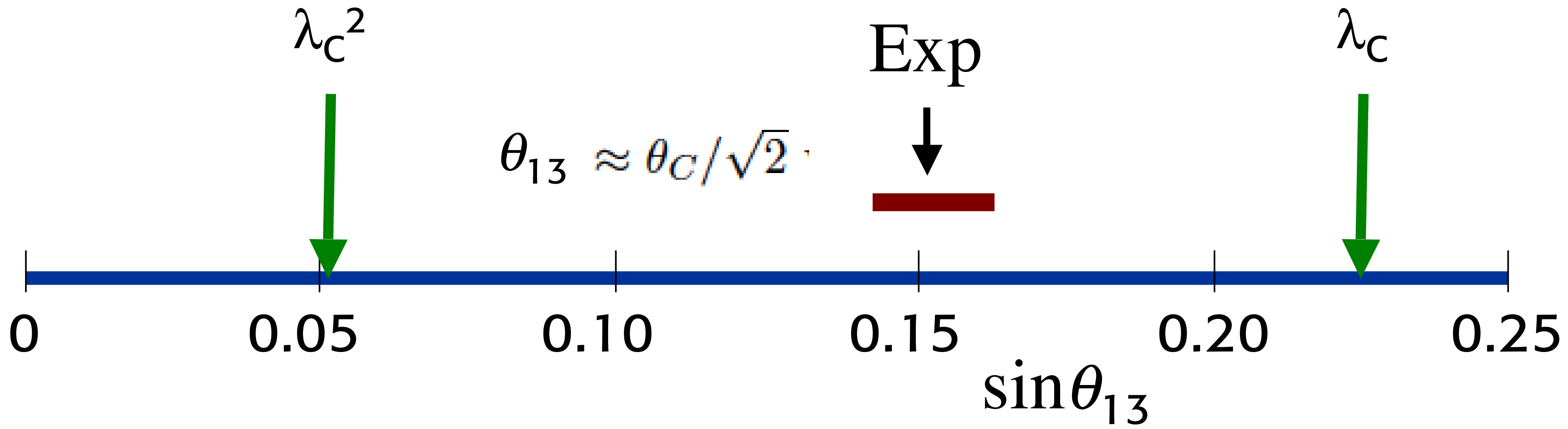
Of course the generic prediction can be altered in ad hoc versions
e.g. Lin '09 has a A4 model where $\theta_{13} \sim o(\lambda_C)$ or by allowing fine tuning

For TB and GR the problem is that $\sin\theta_{13} \gg \Delta\sin^2\theta_{12}$.

Not so for BM

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$





With θ_{13} largish TB models need some additional ingredient

Some selected versions are still perfectly viable

GA, Feruglio, Merlo, Stamou '12

e.g. Lin '09 discussed a natural A4 model where $\theta_{13} \sim o(\lambda_C)$, $\theta_{12} \sim o(\lambda_C^2)$
[ch. lepton and ν sectors are kept separate also at NLO]

Yin Lin: ArXiv:0905.3534

Alternatively:

Symmetry requirements have been relaxed

Hernandez, Smirnov '12

eg

$G_\nu = Z_2$

He, Zee '07 and '11; Grimus, Lavoura '08; Grimus, Lavoura, Singraber '09; Albright, Rodejohann '09; Antusch, King, Luhn, Spinrath '11; King, Luhn'11 Hall, Ross'13...

Larger groups have been studied

eg $\Delta(600)$ (!!)

de A. Toorop, Feruglio, Hagedorn'11; Lam '12 - '13; de Madeiros Verzielas, Ross '12; Holthauser, Lim, Lindner '12; Neder, King, Stuart '13....

CP violation has been included in the symmetry breaking pattern

$$G_\nu = Z_2 \times CP$$

Feruglio, Hagedorn, Ziegler'12 - '13; Ding, King, Luhn, Stuart '13; Girardi, Meroni, Petcov, Spinrath'13; Chen et al '14......

For Bimaximal Mixing θ_{13} is not a problem

Inspired by the "complementarity" relation:

$$\theta_{12} + \theta_C = (46.5 \pm 0.8)^o \sim \pi/4$$

Raidal'04;
Minakata, Smirnov '04

one is led to consider models that give $\theta_{12} = \pi/4$ before corrections from the diagonalization of charged leptons

- θ_{13} large is not problematic in this case!

GA, Feruglio, Masina '04
Frampton et al
King
Antusch et al........

$$U_{PMNS} = U_\ell^\dagger U_\nu \qquad \lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

- In GUT's a possible connection between ch. leptons and θ_C

Normally one obtains $\theta_{12} + o(\theta_C) = \pi/4$ "weak compl."
rather than $\theta_{12} + \theta_C = \pi/4$

TB, BM, GR models all belong to the class of models with $\theta_{13}= 0$ and θ_{23} maximal and θ_{12} generic

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

The most general mass matrix for $\theta_{13}= 0$ and θ_{23} maximal is given by

(after ch. lepton diagonalization!!!):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

μ–τ symmetric

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

Inspired models based on μ–τ symmetry

Grimus, Lavoura..., Ma,.... Mohapatra, Nasri, Hai-Bo Yu ....

By adding $\sin^2\theta_{12} \sim 1/2$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Bimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$$m_1 = x + \sqrt{2}y$$

$$m_2 = x - \sqrt{2}y$$

$$m_3 = 2z - x$$

BM corresponds to $\tan^2\theta_{12}=1$
while exp.: $\tan^2\theta_{12}= 0.45 \pm 0.04$
so a large correction is needed

The 3 remaining parameters are the mass eigenvalues

A model, based on S4, has been constructed where BM mixing holds in 1st approximation and is then corrected by terms $o(\lambda_C)$ from the diagonalisation of charged leptons

GA, Feruglio, Merlo '09
GA, Feruglio, Merlo, Stamou '12
Bazzocchi, Merlo '12

$$\theta_{12} = \theta_{23} = \pi/4, \ \theta_{13} = 0$$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Why and how discrete groups, in particular S4, work?

BM mixing corresponds to m
in the basis where
charged leptons are diagonal

$$m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

Crucial point 1:
m is the most general matrix invariant under
SmS = m and $A_{23}mA_{23}$= m with: $S^2 = A_{23}{}^2 = 1$

$$S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3
symmetry

Invariance under S can be made automatic in S4 while
invariance under A_{23} happens if the flavon content is suitable

Crucial point 2:

Charged lepton masses:
a generic diagonal matrix
is defined by invariance under T
(or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda}\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

a possible T is

$$T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix}$$

An essential observation is that

$T^4 = 1$

S, T are contained in S4

$$T^4 = S^2 = (ST)^3 = (TS)^3 = 1$$

S4: permutations of 4 objects. 24 transformations

Irr. representations: 3, 3′, 2, 1, 1′

S4: Group of permutations of 4 objects (24 transformations)

Irreducible representations: 1, 1′, 2, 3, 3′

$$S^2 = T^4 = (ST)^3 = (TS)^3 = 1$$

1 $\qquad T = 1 \qquad S = 1$

2 $\qquad T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad S = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

3 $\qquad T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix} \qquad S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

1 <-> 1′ and 3<-> 3′ by changing S, T <-> -S, -T

Crucial point 3: S4 must be broken: the alignment

The model is invariant under the flavour group S4

But there are flavons $\phi_l, \chi_l, \phi_\nu, \xi_\nu$ with VEV's that break S4:

ϕ_l, χ_l break S4 down to G_T, the subgroup generated by 1, T in the charged lepton sector

ϕ_ν, ξ_ν break S4 down to G_S, the subgroup generated by 1, S in the neutrino sector

$\phi \sim 3, \chi \sim 3', \xi \sim 1$

Exact BM broken by higher dimension operators

In a good model this aligment along subgroups of S4 occurs in a natural way

In this model BM mixing is exact at LO

For the special flavon content chosen, at NLO θ_{12} and θ_{13} are corrected only from the charged lepton sector by terms of $o(\lambda_C)$ (large correction!) while θ_{23} gets smaller corrections at NNLO (great!)
[for a generic flavon content also $\delta\theta_{23} \sim o(\lambda_C)$]

Original prediction of the model before the data:

θ_{13} is relatively large, of $o(\lambda_C)$.

This appears now to be the case!!

Matrix for ch lepton diagonaliz'n

GA, Feruglio, Merlo, Stamou '12
Bazzocchi, Merlo '12

$$V_e = \begin{pmatrix} 1 & c^e_{12}\,\theta & c^e_{13}\,\theta \\ -c^{e*}_{12}\,\theta & 1 & c^e_{23}\,\theta \\ -c^{e*}_{13}\,\theta & -c^{e*}_{23}\,\theta & 1 \end{pmatrix}$$

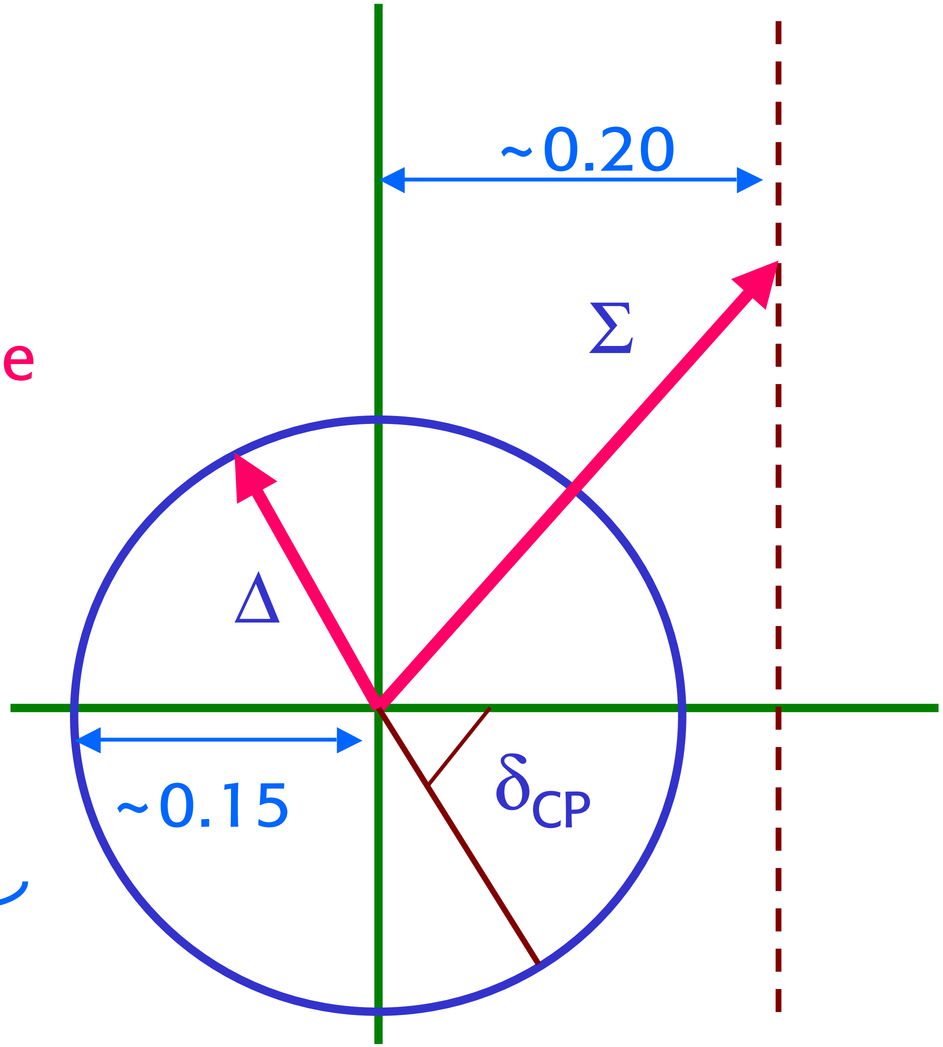
c^e_{ij} complex with o(1) abs value

$$\delta_{CP} = \pi + \arg\left(c^e_{12} - c^e_{13}\right)$$

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\,|c^e_{12} - c^e_{13}|\,\xi \leftarrow |\Delta|$$

$$\sin^2\theta_{12} = \frac{1}{2} - \underbrace{\frac{1}{\sqrt{2}}\mathcal{R}e(c^e_{12} + c^e_{13})}_{\mathrm{Re}\Sigma}\,\xi$$

$$(\sin\theta^2_{23})_{BM} = \frac{1}{2} + \mathcal{R}e(c^e_{23})\,\xi$$



$$\delta_{CP} = \pi + \arg\left(c^e_{12} - c^e_{13}\right)$$

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\underbrace{|c^e_{12} - c^e_{13}|}_{}\,\xi \leftarrow |\Delta|$$

$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}\mathcal{R}e\underbrace{(c^e_{12} + c^e_{13})}_{\mathrm{Re}\Sigma}\,\xi$$

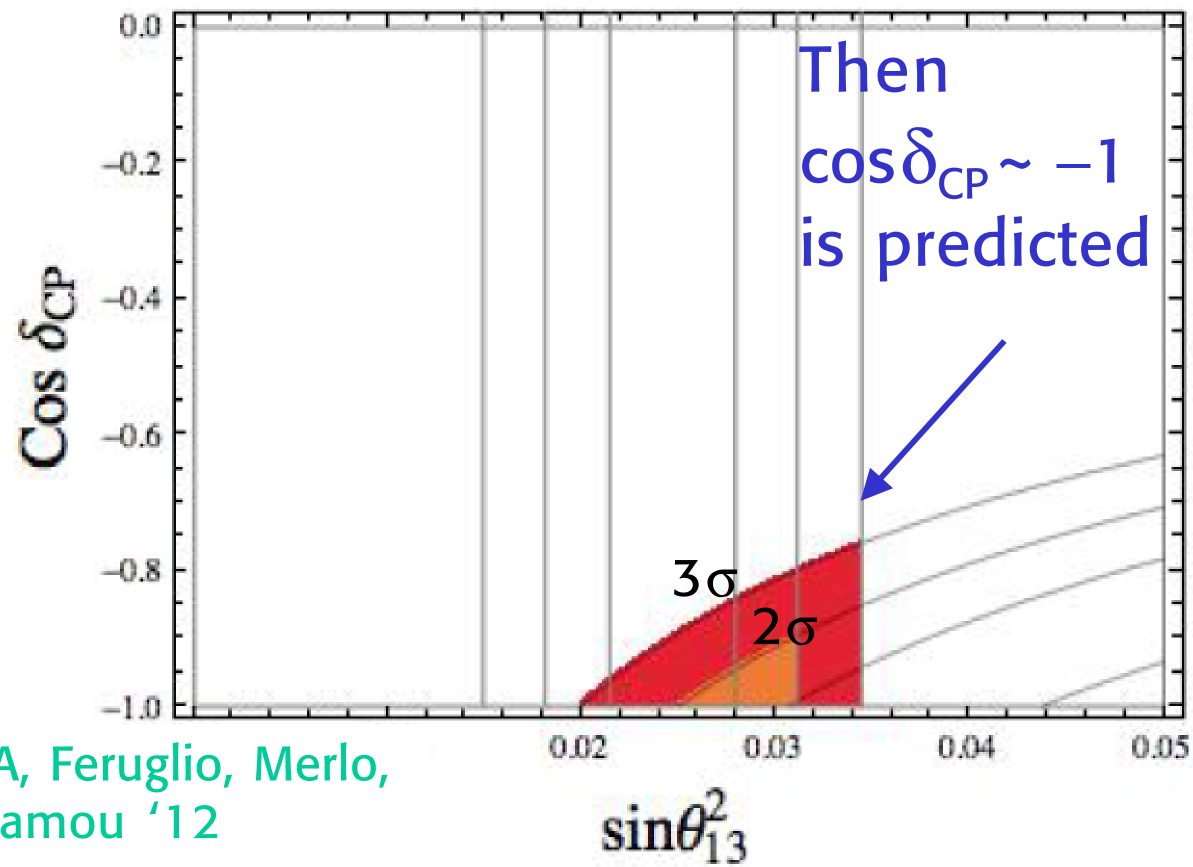
For dominance of a single c^e, e.g. $c^e_{13}=0$ we have $\Delta=\Sigma$ and a sum rule

$$\sin^2\theta_{12} = \frac{1}{2} + \sin\theta_{13}\cos\delta_{CP}$$

equivalent to

$\theta_{12} \sim \pi/4 + \sin\theta_{13}\cos\delta_{CP}$

Masina '05



GA, Feruglio, Merlo, Stamou '12

If the same dominance of ch. lepton corr.s is assumed for TB mixing

$\theta_{12} \sim \theta^{TB}_{12} + \sin\theta_{13}\cos\delta_{CP}$

then $\cos\delta_{CP} \sim 0$

GUT extension to obtain $\theta_{13} \sim o(\lambda_C)$

SU(5) in extra dimensions

GA, Feruglio, Hagedorn '08 (TB)
Meloni '11
GA, Meloni '14 (in preparation)

Field	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$	φ_ν	ξ_ν	φ_ℓ	χ_ℓ	θ	θ'	φ_ν^0	ξ_ν^0	ψ_ℓ^0	χ_ℓ^0
SU(5)	$\bar{5}$	10	10	10	5	$\bar{5}$	1	1	1	1	1	1	1	1	1	1
S_4	3_1	1	1	1	1	1	3_1	1	3_1	3_2	1	1	3_1	1	2	3_2
Z_3	ω	ω	1	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	1	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	0	0	0	2	2	2	2
$U(1)_{FN}$	0	2	1	0	0	0	0	0	0	0	-1	-1	0	0	0	0
	br	bu	bu	br	bu	bu	br	br	br	br	br	br	br	br	br	br

bulk brane

The order of all quark and lepton mixing angles are reproduced in terms of powers of $\lambda \sim o(\lambda_C)$

$V_{us} = \lambda_C \sim \lambda$ $\qquad$ $V_{ub} \sim \lambda^3$ $\qquad$ $V_{cb} \sim \lambda^2$

$\theta_{13} \sim \lambda$ $\qquad$ $\theta_{12} \sim 1/2 - o(\lambda)$ $\qquad$ $\theta_{23} \sim 1/2 +/- o(\lambda^2)$

$\oplus$ [but the 1st gen. masses m_e, m_u and also r must be fine tuned]

GUT's, quarks and discrete symmetries

While it is possible to build GUT models with TB or BM mixing for neutrinos, it is true that no support for discrete flavour groups is found from quarks

In fact a common strategy for building such a GUT model is to arrange that, at the LO, for quark mass matrices, only the 33 matrix element is non vanishing, due to an additional flavour symmetry (e.g. $U(1)_{FN}$) or dynamical effect (e.g. geometrical factors in extra dim. models)

All other matrix elements come from higher orders and show a suitable suppression pattern

Thus the small mass ratios and mixings for quarks are generated as corrections and are largely independent from the discrete symmetry of the lepton sector

Conclusion

Neutrino physics deals with fundamental issues,
is being vigorously studied and our knowledge has much increased in the last 15 years

But many crucial problems remain open: Dirac/Majorana, $|m^2_i|$, hierarchy (normal or inverse), CP viol., sterile ν's, ...

Data on mixing angles are much better now but models of neutrino mixing still span a wide range from anarchy to discrete flavour groups

In the near future it will not be easy to decide from the data which ideas are right

The main problem of discrete flavour groups is not so much that θ_{13} is large but that there is no hint from quarks for them

So far no real illumination came from leptons to be combined with the quark sector for a more complete theory of flavour