Corfu', September '14

Neutrino Mass and Mixing (Theory)

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Plan of the lectures

1 Neutrino mass
Dirac and/or Majorana?
See-saw mechanism
Sterile Neutrinos?

2 Models of neutrino mixing

from Anarchy
to complete order

In the last ~15 years we have learnt a lot about ν's

[important fundamental physics issues involved]

- ν's oscillate (no separate lepton number conservation)
- ν's are massive (at least two of them)
- their masses are very small
- Δm^2_{ij} and mixing angles are measured with fair precision
- Theory: probably ν's are Majorana particles [can explain small masses and large mixings]
- heavy ν_R (see-saw mechanism, O_5) for light ν masses
- an appealing picture: ν's as probes of GUT's, baryogenesis thru leptogenesis....

Yet in spite of impressive progress important experimental open questions remain:
Absolute scale of m^2? Inverse or normal hierarchy?
CP violation? Flavour symmetry? Sterile ν's? DM?..

From the theoretical side, for ν masses and mixings we do not have so far a compelling theoretical picture and many possibilities are still open.

Actually, also for quarks and charged leptons
we do not have a theory of flavour that explains the observed spectrum, mixings and CP violation.

Thus ν's are interesting because they can provide new clues on the flavour problem

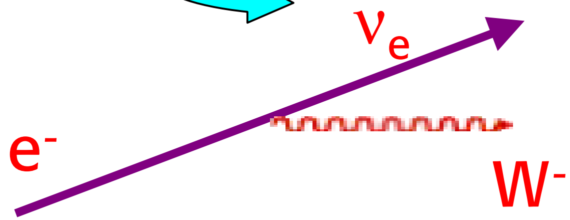
ν Oscillations Imply Different ν Masses

ν_e: same weak isospin doublet as e^-

flavour mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U: mixing matrix



$U = U_{PMNS}$

Pontecorvo
Maki, Nakagawa, Sakata

$\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$

$\nu_\mu = -\sin\theta\,\nu_1 + \cos\theta\,\nu_2$

e.g 2 flav.

Stationary source:

Stodolsky

$\nu_{1,2}$: different mass, different x-dep:

$\nu_a(x) = e^{ip_a x}\,\nu_a$ $\qquad p_a^2 = E^2 - m_a^2$

$$P(\nu_e <-> \nu_\mu) = |< \nu_\mu(L)|\,\nu_e>|^2 = \sin^2(2\theta)\cdot\sin^2(\Delta m^2 L/4E)$$

At a distance L, ν_μ from μ^- decay can produce e^- via charged weak interact's

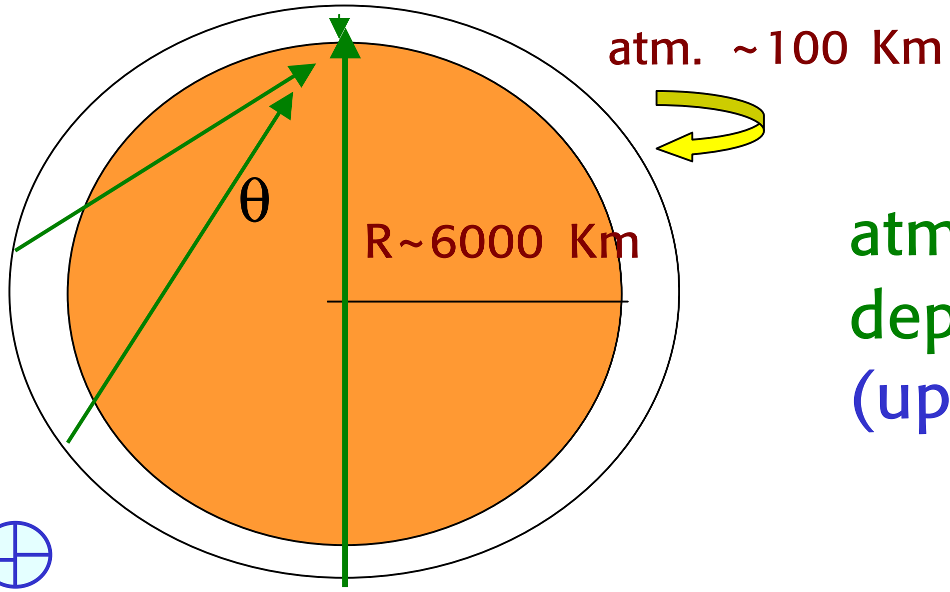
Solar ν's

$\nu_e \to \nu_{\mu,\tau}$

The data imply that the oscill's occur inside the sun thru the MSW effect (resonant oscillation induced by matter)

Mikhaev and Smirnov; Wolfenstein

Atmospheric ν's



$\nu_\mu \to \nu_\tau$

atmospheric ν's traverse different L depending on azimuth θ

(up-down asymm.)

Evolution in vacuum and in matter

$\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$

$\nu_\mu = -\sin\theta\,\nu_1 + \cos\theta\,\nu_2$

$\Delta m^2 = m_2^2 - m_1^2 > 0$

$$i\frac{d}{dt}\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix} = H_{eff}\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix}$$

$$H_{eff} = \frac{\Delta m^2}{4E}\begin{bmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$

In vacuum, for 2 flavours, apart from multiples of the identity

In matter CC int's on electrons introduce a flavour dep.
(coherent forward scattering on electrons)

$$H_{eff} = \frac{\Delta m^2}{4E}\begin{bmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix} + \begin{bmatrix}\sqrt{2}G_F N_e & 0\\ 0 & 0\end{bmatrix}$$

N_e: n. of e per unit V

The mixing angle is changed
A resonance can appear (MSW)

$$\tan 2\theta_m = \frac{\tan 2\theta}{1 - \dfrac{2\sqrt{2}E G_F N_e}{\Delta m^2 \cos 2\theta}}$$

Mikhaev and Smirnov; Wolfenstein

Evidence for solar and atmosph. ν oscillatn's confirmed on earth by K2K, KamLAND, MINOS...

Δm^2 values:

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,

$\Delta m^2_{sol} \sim 7.5\ 10^{-5}\ eV^2$

and mixing angles measur'd:

θ_{12} (solar) large ~ 34°

θ_{23} (atm) large,~ maximal 45°

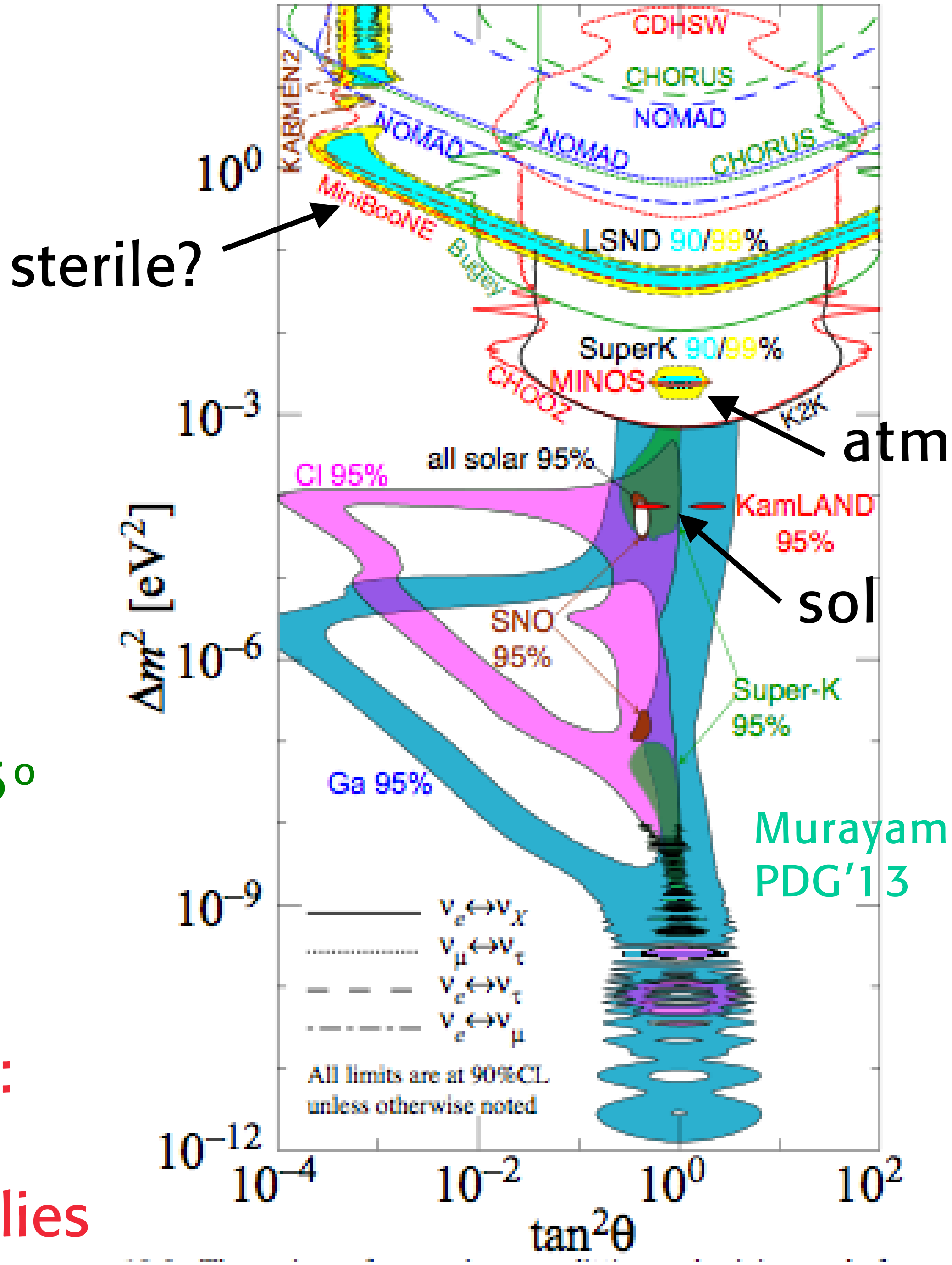
θ_{13} smaller ~ 9°

A 3rd frequency?

Sterile ν's: an open question:

LSND+MiniBooNE

Reactor and Gallium anomalies



Murayama PDG'13

Neutrino Masses

ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} \sim 7.5\ 10^{-5}\ eV^2$

- Direct limits

$m_{"\nu e"} < 2.2$ eV
$m_{"\nu\mu"} < 170$ KeV
$m_{"\nu\tau"} < 18.2$ MeV

End-point tritium β decay (Mainz, Troitsk)
near future: Katrin

sensitivity:
$m_\nu < 0.2$eV (90%CL)

discovery potential:
$m_\nu = 0.3$eV (3σ)
$m_\nu = 0.35$eV (5σ)

- Cosmology $\Omega_\nu h^2 \sim \Sigma_i m_i$ /94eV ($h^2 \sim 1/2$)

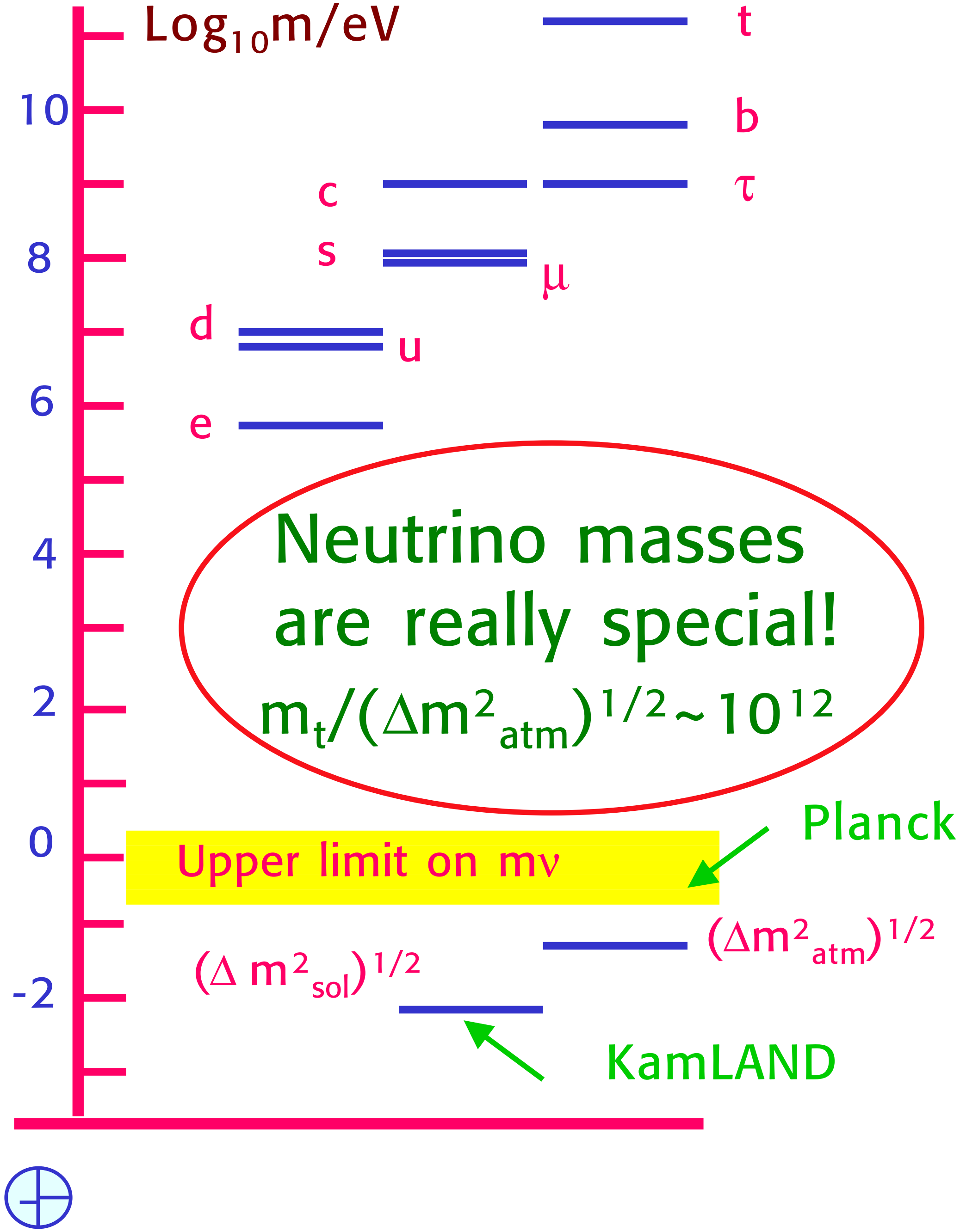
$\Sigma_i m_i < 0.28 - 0.8$ eV 95% Planck +BAO+WMAPPol+HighL

Any ν mass $< 0.09 - 0.27$ eV

depends on cosmology priors

$m_{ee} = |\Sigma\ U_{ei}^2\ m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.2 - 0.7 - ?$ eV (nucl. matrix elements)



Massless ν's?

- no ν_R
- L conserved

But ν_R can well exist and we really have no reason to expect that B and L are exactly conserved

Small ν masses?

- ν_R very heavy
- L not exactly cons.

The SM can be easily extended to include Majorana ν's

How to guarantee a massless neutrino?

1) ν_R does not exist

$\Rightarrow$ No Dirac mass

$$\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L$$

and

2) Lepton Number is conserved

$\Rightarrow$ No Majorana mass

$$\nu^T_R \, \nu_R \text{ or } \nu^T_L \, \nu_L$$

ν mass: completing the SM with ν_R

It is sufficient to introduce 3 RH gauge singlets ν_R
[each completing a 16 of SO(10) for one generation]
and not artificially impose that L is conserved

In the SM, in the absence of ν_R, B and L are "accidental" symmetries [i.e. no renormalizable gauge invariant B and/or L non-conserving vertices can be built from the fields of the theory]

But we know that non perturbative terms (instantons) break B and L (not B-L) and also non renorm. operators:

Weinberg dim-5 operator
$$O_5 = \frac{(Hl)_i^T \lambda_{ij} (Hl)_j}{\Lambda} + \ h.c.$$

With Majorana neutrinos $M\nu^T{}_R\nu_R$ is allowed by SU(2)xU(1) (ν_R is a gauge singlet) and breaks L (and B-L)

Are neutrinos Dirac or Majorana fermions?

Under charge conjugation C: particle <--> antiparticle

For bosons there are many cases of particles that coincide (up to a phase) with their antiparticle:

$$\pi^0, \rho^0, \omega, \gamma, Z^0 .....$$

A fermion that coincides with its antiparticle is called a Majorana fermion

Are there Majorana fermions?
Neutrinos are probably Majorana fermions

The fundamental fermions of the Standard Model:

$$\begin{bmatrix} uuu\nu_e \\ ddde \end{bmatrix} \quad \begin{bmatrix} ccc\nu_\mu \\ sss\mu \end{bmatrix} \quad \begin{bmatrix} ttt\nu_\tau \\ bbb\tau \end{bmatrix}$$

- Of all fundamental fermions only ν's are neutral

  If lepton number L conservation is violated then no conserved charge distinguishes neutrinos from antineutrinos $\longrightarrow$

  Majorana ν's : neutrinos and antineutrinos coincide

  neutrinos are their own antiparticles

- ν's have very small masses

  The two facts are probably related

The field of an electron (massive, charged) has 4 components

In fact there are 4 dof: e^-, e^+, $h = +, -$
(h is the helicity: component of spin along momentum)

$$
\begin{tikzcd}[column sep=huge, row sep=large]
|e^{-}, h = + > \arrow[r, "\text{Lorentz boost}"] \arrow[d, "\text{TCP}"'] & |e^{-}, h = - > \arrow[d, "\text{TCP}"] \\
|e^{+}, h = - > \arrow[r, "\text{Lorentz boost}"] & |e^{+}, h = + >
\end{tikzcd}
$$

For a massless neutrino $|\nu_L> = |\nu, h = --1>$ and
$|\overline{\nu}_R> = |\overline{\nu}, h = +1>$ can be enough because massless particles go at the speed of light (no boost can flip h)

Now we know that (at least two) neutrinos have non vanishing masses, although very small

The evidence for non vanishing masses arises from ν oscillations: Δm^2_{atm} ~2.5 10^{-3} eV^2, Δm^2_{sol} ~7.5 10^{-5} eV^2

Still for a completely neutral neutrino there is the possibility that neutrino and antineutrino coincide (Majorana neutrino)

For a massive Majorana neutrino only two states are enough

Each neutrino mass eigenstate of definite helicity coincides with its own antiparticle

ν's have no electric charge.
Their only charge is lepton number L.

IF L is not conserved (not a good quantum number)
ν and $\bar{\nu}$ are not really different

$$|\nu, h= -1/2 > \xrightarrow{\text{TCP, "Lorentz"}} |\bar{\nu}, h= +1/2>$$

For a massive Majorana neutrino only two states are enough

A Majorana neutrino is identical with its charge conjugated

$$C|\nu> = |\bar{\nu}> = |\nu>$$

Each neutrino mass eigenstate of definite helicity coincides with its own antiparticle

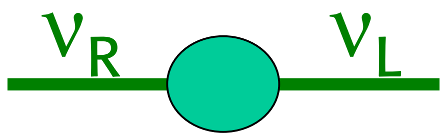
ν masses:

recall: ν_R : ann $|\nu_R>$ creates $|\overline{\nu}_L>$

$\overline{\nu}_L$: ann $|\overline{\nu}_R>$ creates $|\nu_L>$

For massive fermions L,R
refer to chirality, not helicity

Dirac mass: $\overline{\nu}_L \nu_R + \overline{\nu}_R \nu_L$



(needs ν_R)

Lepton number (L)-conserving

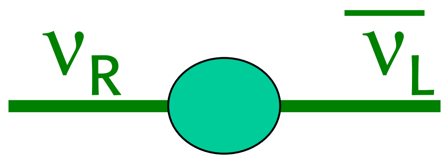
Don't confuse left-chirality and lepton n.

Majorana mass: $\overline{\nu^c}\nu \rightarrow \nu^T{}_R C \nu_R$ or $\nu^T{}_L C \nu_L$

$$\boxed{\psi^c = C\overline{\psi}^T}$$

$C = i\gamma^2\gamma^0$

Violates L, B-L by $|\Delta L| = 2$



short-hand: $\longrightarrow$ $\nu^T{}_R \, \nu_R$ or $\nu^T{}_L \, \nu_L$

Weak isospin I

$\nu_L \Rightarrow I = 1/2,\ I_3 = 1/2$

$\nu_R \Rightarrow I = 0,\ I_3 = 0$

Dirac Mass:

$$\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L \qquad |\Delta I| = 1/2$$

Can be obtained from Higgs doublets: $\bar{\nu}_L \nu_R H$

For Dirac ν's no explanation of small masses

Majorana Mass:

- $\nu^T{}_L \nu_L \qquad |\Delta I| = 1$

Non ren., dim. 5 operator: $\nu^T{}_L\ \nu_L HH$

- $\nu^T{}_R \nu_R \qquad |\Delta I| = 0 \longrightarrow$ Directly compatible with SU(2)xU(1)!

See-Saw Mechanism

Minkowski; Glashow; Yanagida;
Gell-Mann, Ramond , Slansky;
Mohapatra, Senjanovic…..

$M\nu^T{}_R\nu_R$ allowed by SU(2)xU(1)

Large Majorana mass M (as large as the cut-off)

$m_D\bar{\nu}_L\nu_R$

Dirac mass m_D from Higgs doublet(s)

$$\begin{array}{cc} & \begin{array}{cc}\nu_L & \nu_R\end{array} \\ \begin{array}{c}\nu_L \\ \nu_R\end{array} & \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix} \end{array}$$

$M >> m_D$

Eigenvalues

$$|\nu_{light}| = \frac{m_D{}^2}{M} \quad , \quad \nu_{heavy} = M$$

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{GUT}$

$$m_\nu \sim \frac{m^2}{M}$$

$m: \leq m_t \sim v \sim 200$ GeV
M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV
$m \sim v \sim 200$ GeV

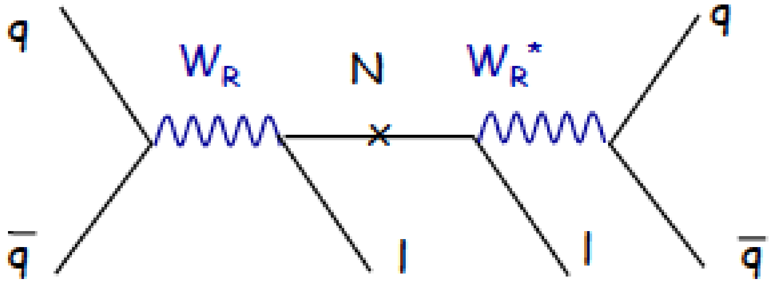
$M \sim 10^{14} - 10^{15}$ GeV

This is so impressive that, in my opinion, models with ν_R at the EW scale or around are strongly disfavoured

Models with ν_R at the EW scale: possible signals at the LHC

Example: Low energy W_R from L-R symmetry

Keung, Senjanovic '83

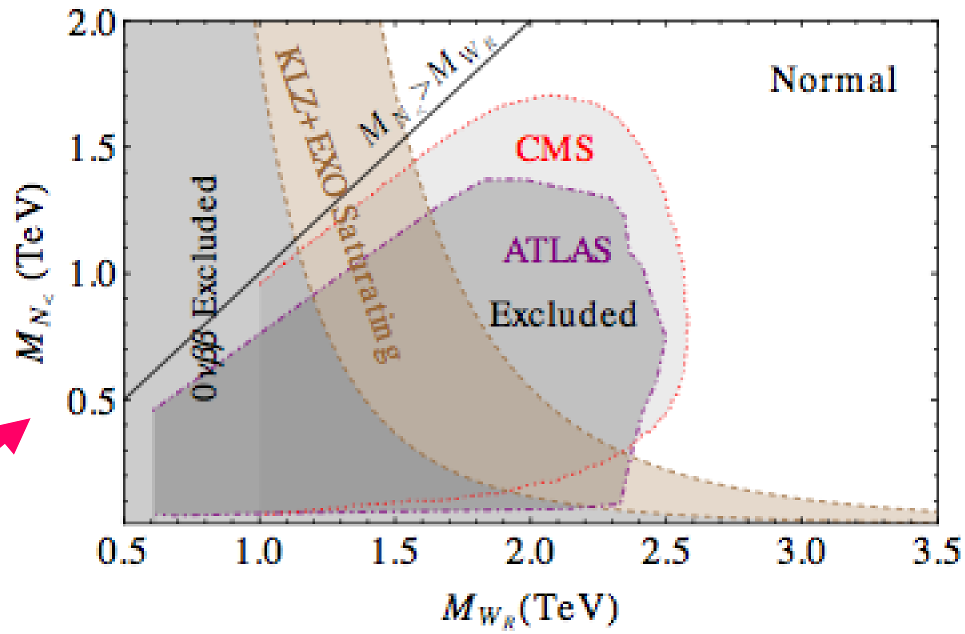


equal sign leptons

[analogous to $0\nu\beta\beta$]

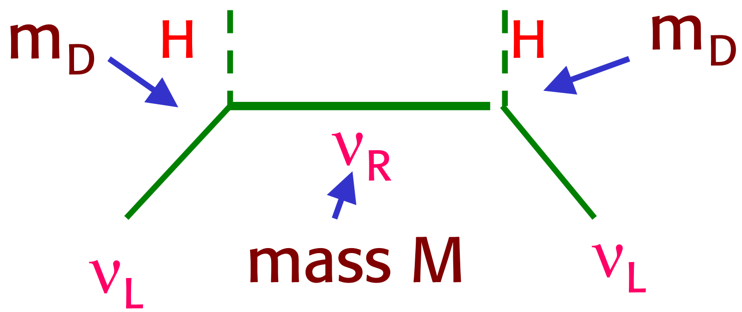
Limits from LHC and $0\nu\beta\beta$

Bhupal Dev et al '13



See-saw diagrams

Type 1

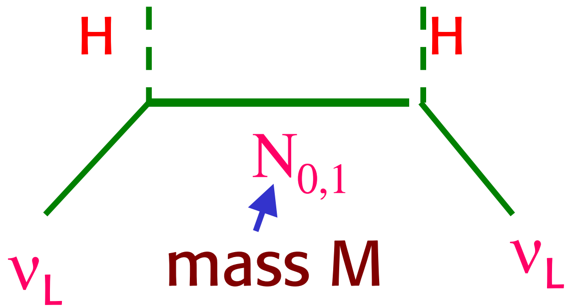


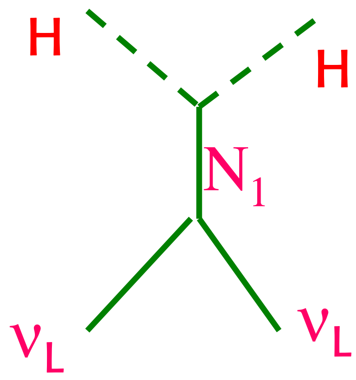
$\nu_L^T m_\nu \nu_L$

$m_\nu = m_D^T M^{-1} m_D$

More in general: non ren. O_5 operator

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell H H$$





e.g from $I_W=1$:Type 2

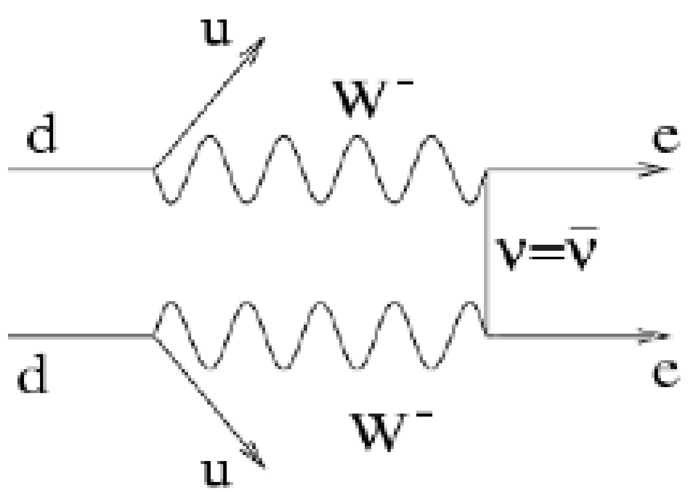
$N_{0,1}$: new particle $I_w=0,1$

Whatever the underlying dynamics O_5 is a general effective description of light Majorana neutrino masses

ν oscillations point to very large values of M

All we know from experiment on ν masses strongly indicates that ν's are Majorana particles and that L is not conserved (but a direct proof still does not exist).

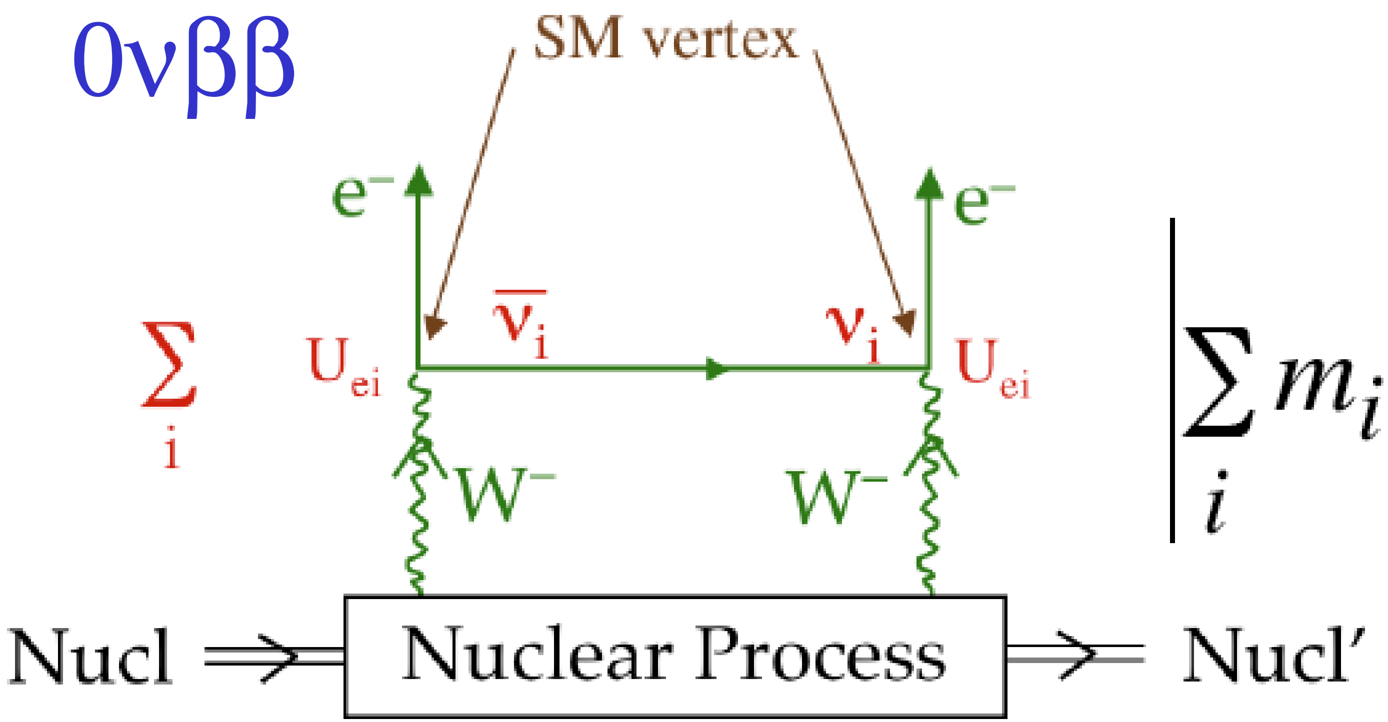
Detection of $0\nu\beta\beta$ (neutrinoless double beta decay) would be a proof of L non conservation ($\Delta L=2$).
Thus a big effort is devoted to improving present limits and possibly to find a signal.



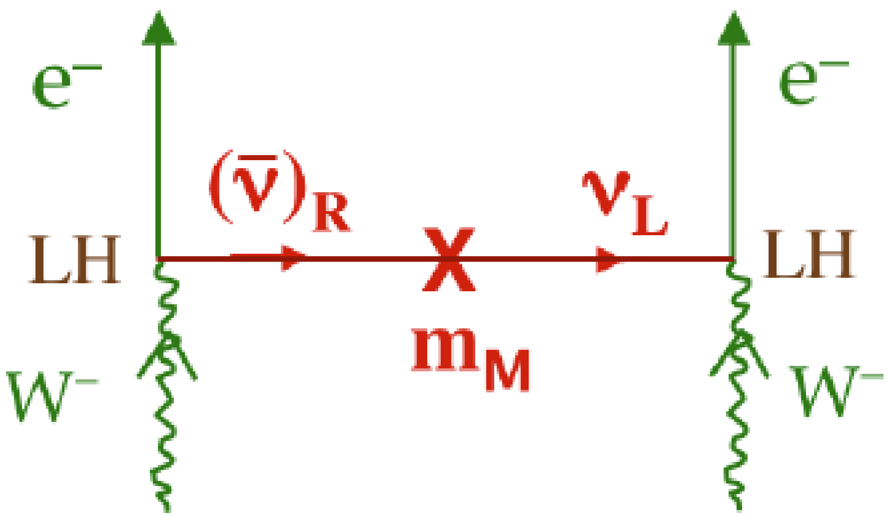
$0\nu\beta\beta$ = dd -> uue^-e^-

Heidelberg-Moscow
IGEX
Cuoricino-Cuore
GERDA
•••••

$0\nu\beta\beta$



$$\left|\sum_i m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

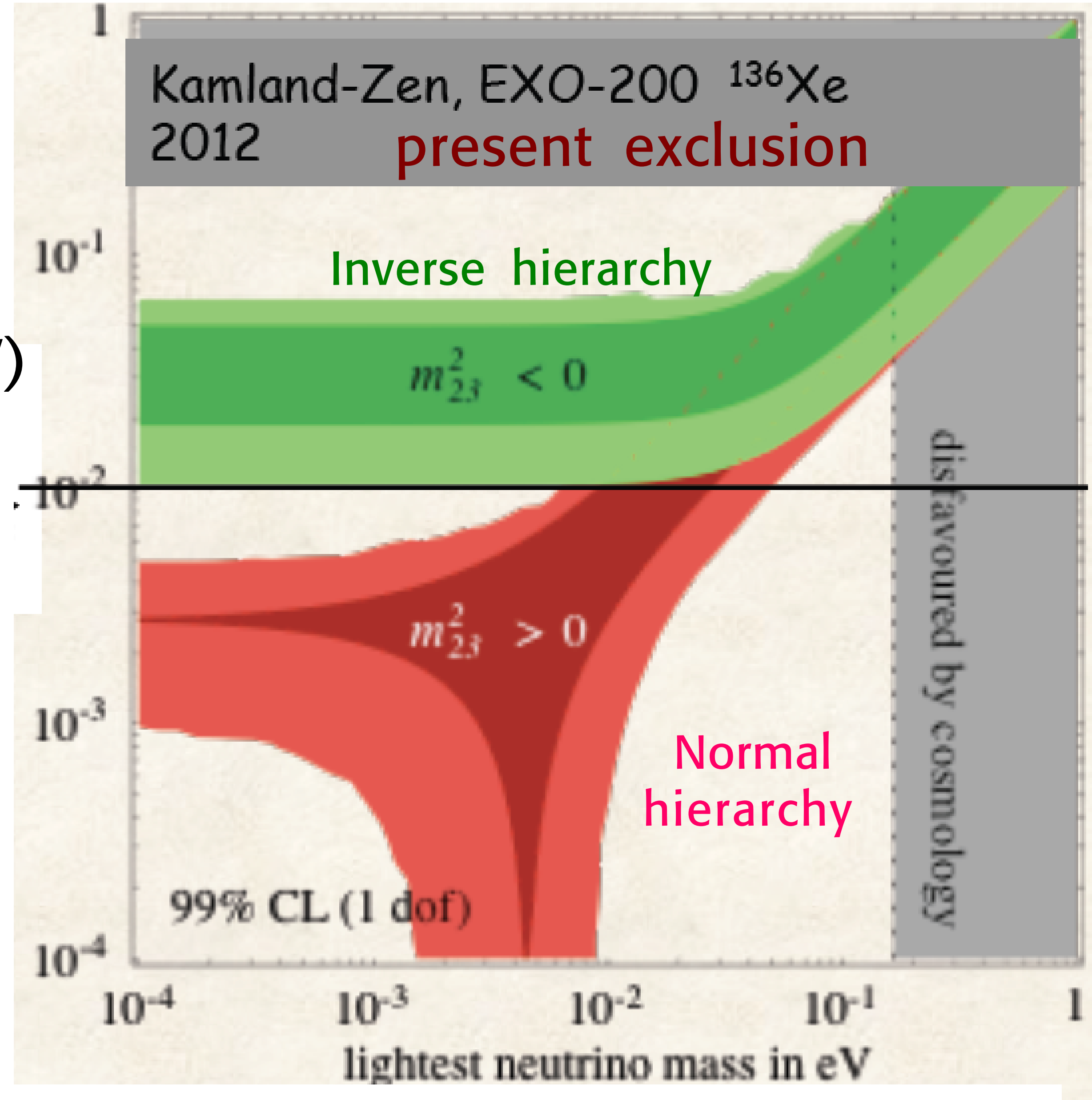


$0\nu\beta\beta$ signal would establish Majorana ν's

Feruglio, Strumia, Vissani '02
Dell'Oro, Marcocci, Vissani '14

$m_{ee} = |\Sigma U_{ej}^2\, m_j\, e^{i\alpha j}|$

next generation
10 meV



$0\nu\beta\beta$ signal
would establish
Majorana ν's,
measure m_{ee} and
indicate hierarchy

Determining the type of spectrum is still an open problem
Better outlook now that θ_{13} has been measured and is large

Present results on neutrinoless DBD

Fiorini

Isotope	Technique	$\tau^{0\nu}_{1/2}$ (y)	$<m_{\beta\beta}>$ eV
^{48}Ca	CaF_2 scint	$>1.4\times10^{22}$	<7-45
^{76}Ge (HM)	Ge diode	$>1.9\times10^{25}$	<(0.3-1.27)
^{76}Ge (IGEX)	Ge diode	$>1.6\times10^{25}$	<(0.33-1.35)
^{76}Ge (Klapdor 2004)	Ge diode	1.2×10^{25}	.38
^{76}Ge (Klapdor 2006)	Ge diode	2.2×10^{25}	.28
^{76}Ge (GERDA I)	Ge diode	$>2.1\times10^{25}$	<(.29-1.1)
^{76}Ge (GERDA+HM+IGEX)	Ge diode	$>3\times10^{25}$	<(.25-.98)
^{82}Se	Foil&track	$>.6\times10^{23}$	<(0.89-2.)
^{96}Zr	Foil&track	$>9.2\times10^{21}$	<(7.2-19.5)
^{100}Mo	Foil&track	$>1.1\times10^{24}$	<(0.31-.79)
^{116}Cd	Scintillator	$>1.7\times10^{23}$	<1.7
^{128}Te	Geochem	$>7.7\times10^{24}$	<(1.1-1.35)
^{130}Te	Bolometer	$>2.8\times10^{24}$	<(0.3-.7)
^{136}Xe	EXO	$>1.6\times10^{25}$	<140-380
^{136}Xe	Kamland Zen	$>1.9\times10^{25}$	**<128-349**
^{136}Xe	EXO+Kamzen		<120-250
^{150}Nd	Foil TPC	$>1.8\times10^{22}$	

here Ettore forgot the dot: 0.140 etc

Baryogenesis

$n_B/n_\gamma \sim 10^{-10}$, $n_B >> n_{Bbar}$

Conditions for baryogenesis: (Sacharov '67)

- B non conservation (obvious)
- C, CP non conserv'n (B-B^{bar} odd under C, CP)
- No thermal equilib'm ($n = \exp[\mu - E/kT]$; $\mu_B = \mu_{Bbar}$, $m_B = m_{Bbar}$ by CPT

If several phases of BG exist at different scales the asymm. created by one out-of-equilib'm phase could be erased in later out-of-equilib'm phases: BG at lowest scale best

Possible epochs and mechanisms for BG:

- At the weak scale in the SM Excluded
- At the weak scale in the MSSM Excluded
  in a more general 2HDM Disfavoured
- Near the GUT scale via Leptogenesis Very attractive

A great extra bonus of see-saw with heavy Majorana ν_R's

BG via Leptogenesis near the GUT scale (after inflation)

Buchmuller,Yanagida,
Plumacher, Ellis, Lola,
Giudice et al, Fujii et al
.....

Only survives if Δ(B-L) is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{11\text{-}12}$ GeV)

L non conserv. & CP violat.'n in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from
ν oscill's is compatible with BG via (thermal) LG

Buchmuller, Di Bari, Plumacher;
Giudice et al; Pilaftsis et al;
Hambye et al

m ~ eV Sterile Neutrinos

A White Paper: K.N. Abazajian et al, ArXiv:1204.5789

Sterile ν's? A number of "hints" with some "tensions"

(they do not make an evidence but pose an important experimental problem that needs clarification)

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ $\nu_\mu \rightarrow \nu_e$ $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

- LSND and MiniBoone (appearance)
- Reactor anomaly ($\bar{\nu}_e$ disappearance)
- Gallium (ν_e disappearance)

These data hint at sterile neutrinos at ~ 1 eV which would represent a major discovery in particle physics

Important information also from

- $\nu_\mu / \nu^{bar}{}_\mu$ disappearence expts (MINOS, CDHSW, CCFR...)
- Neutrino counting from cosmology

Cosmology is fully compatible with N_{eff} ~3 but could accept one sterile neutrino

The bound from nucleosynthesis is the most stringent (assuming thermal properties at decoupling)

- BBN:
  $N_s = 0.22 \pm 0.59$ [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313, astro-ph/0408033]

  $N_s = 0.64^{+0.40}_{-0.35}$ [Izotov, Thuan, ApJL 710 (2010) L67, arXiv:1001.4440]

N_s

- BBN: $N_s < 1.2$ (95% CL) Mangano, Serpico, 1103.1261

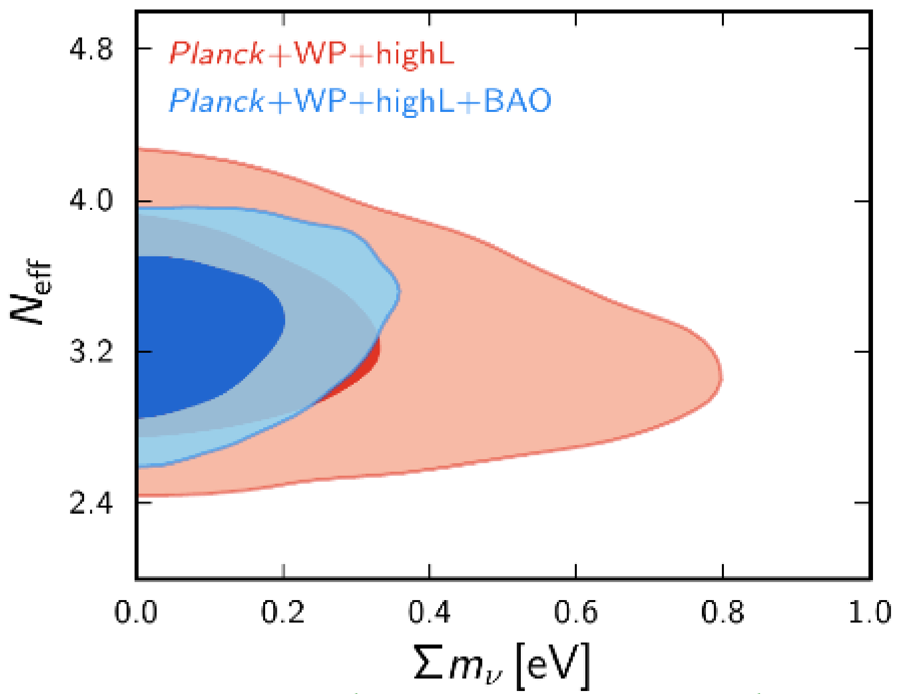
- BBN: $N_s < 1.54$ (95% CL) [M. Pettini, et al, arXiv:0805.0594]

From Cosmology

Simultaneous constraints on $\sum m_\nu$ and $N_{\rm eff}$

Spinelli, Neutrino 2014

[Planck 2013 results.XVI.]



CMB alone not much sensitive to Σm_ν

- assumption: 3 active neutrinos coexisting with extra massless species
- $\sum m_\nu$ and $N_{\rm eff}$ different impact on CMB: **no significant correlation**
- results adding BAO:

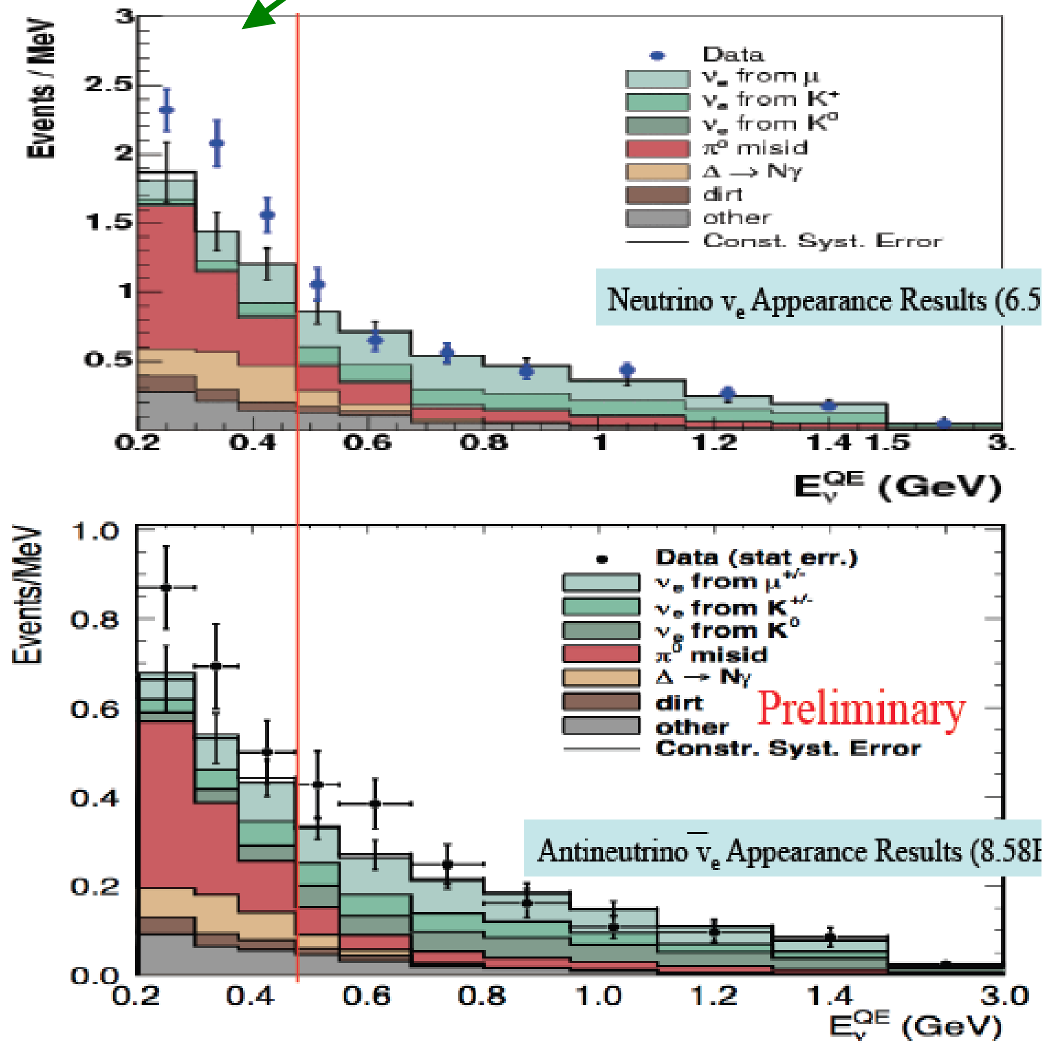
$$\begin{cases} N_{\rm eff} = 3.32 \pm 0.27 \ (68\% CL) \\ \sum m_\nu < 0.28 \text{ eV} \ (95\% CL) \end{cases}$$

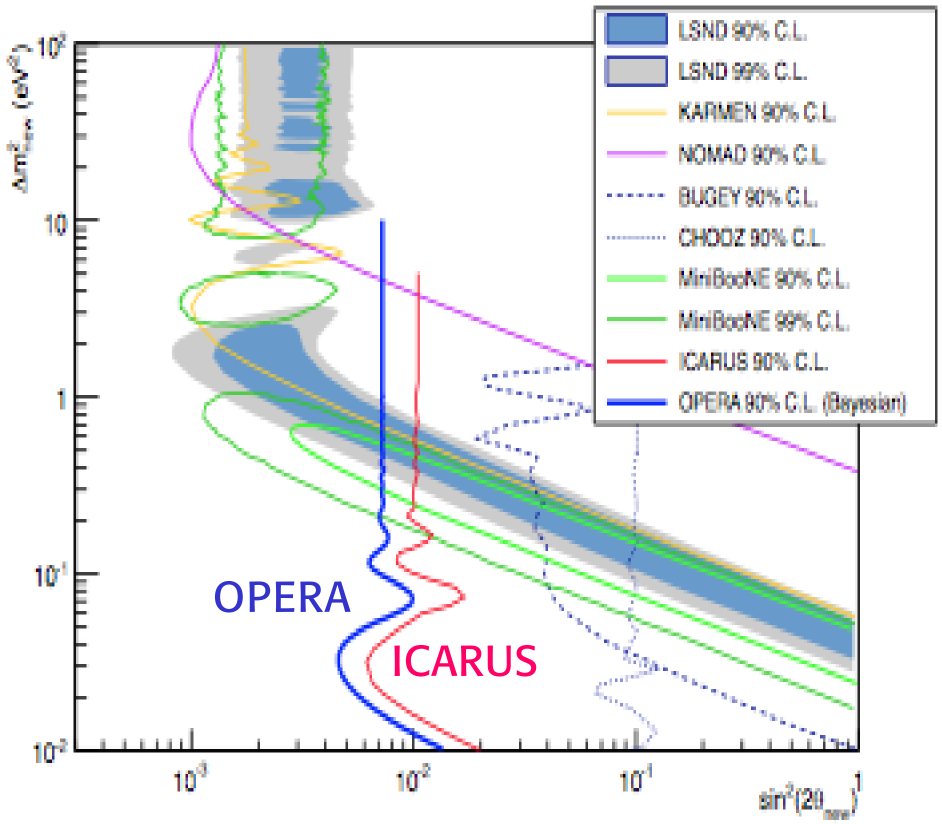
Appearence accelerator experiments

LSND, KARMEN, ICARUS
MiniBoone

Unidentified excess at low energy!!

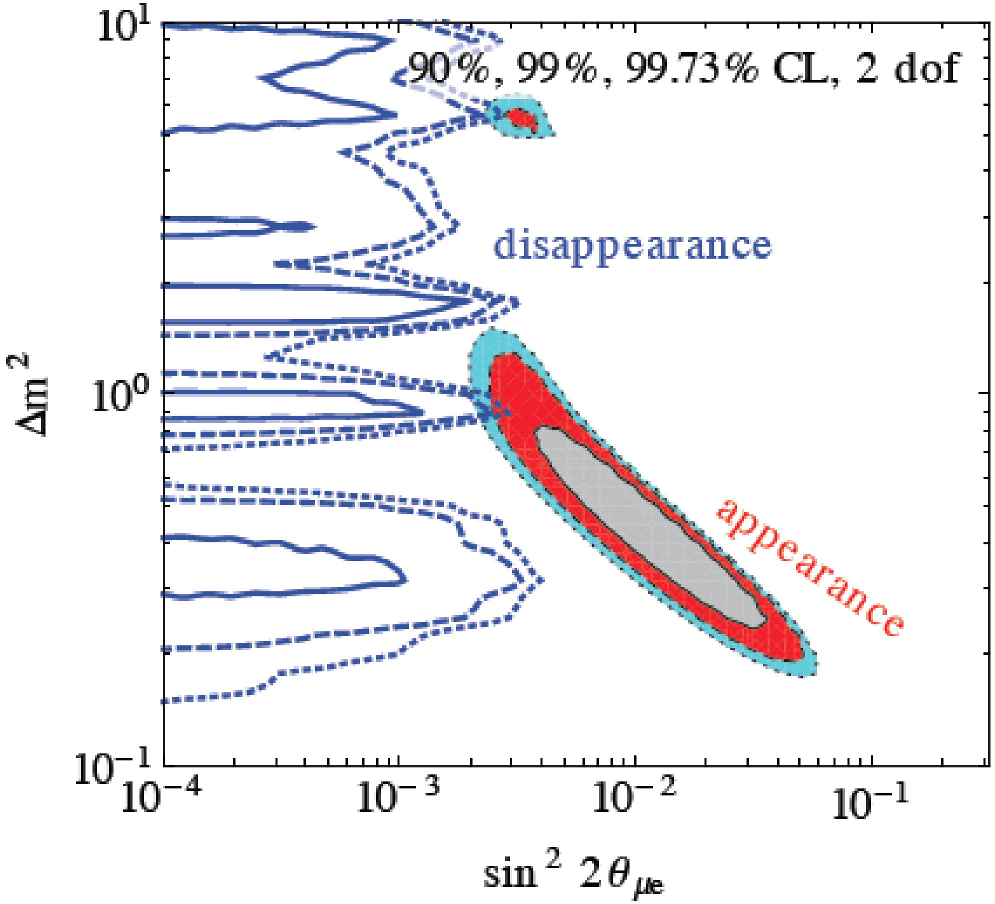
MiniBooNE supports LSND in $\bar{\nu}_\mu$ but not in ν_μ (or CP viol.?)





No signal in ν_μ disappearance experiments (CDHSW, MINOS, CCFR, MiniBooNE-SciBooNE) creates a tension with LSND (if no CP viol.)

Kopp et al '13



For example, in 3+1 models here is the clash between appearance (LSND, MiniBoone.....) and disappearance (MINOS...)

$$\boxed{\sin^2 2\theta_{\mu e} \approx \frac{1}{4}\sin^2 2\theta_{ee} \sin^2 2\theta_{\mu\mu}}$$

app. wants this large

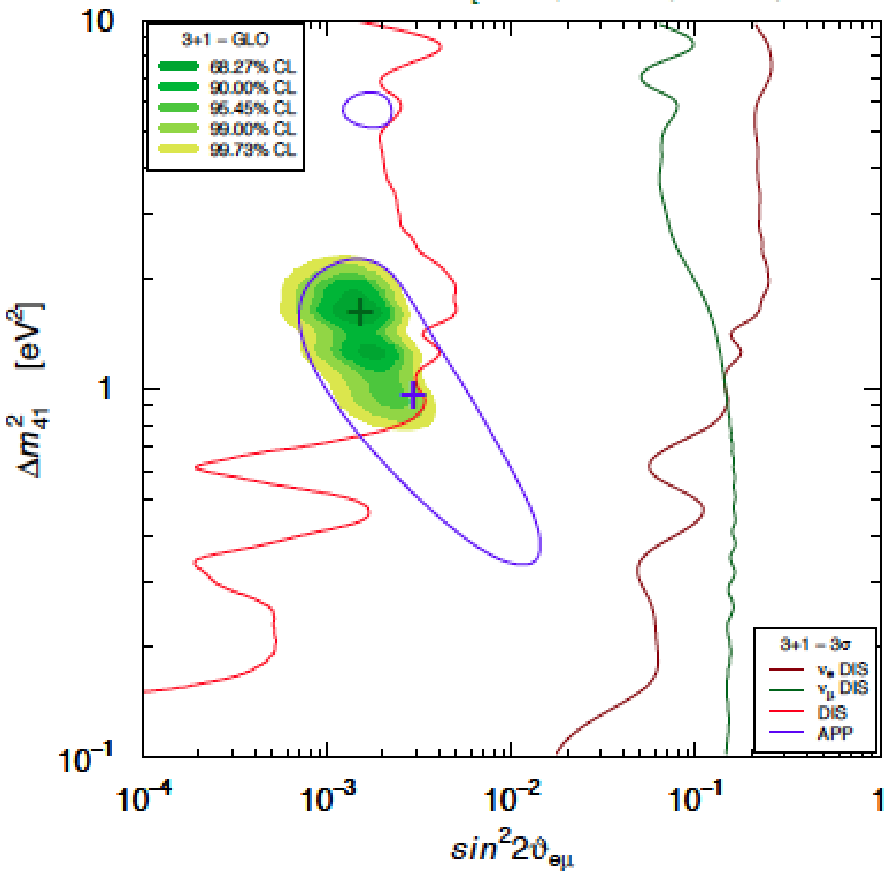
disapp. wants this small

Giunti et al are more positive on the 3+1 fit

The difference comes from the low energy MiniBooNe data (not included here)

3+1 Global Fit

[Giunti, Laveder, Y.F. Li, H.W. Long, arXiv:1308.5288]



MiniBooNE $E > 475\,\mathrm{MeV}$

GoF = 29%   PGoF = 9%

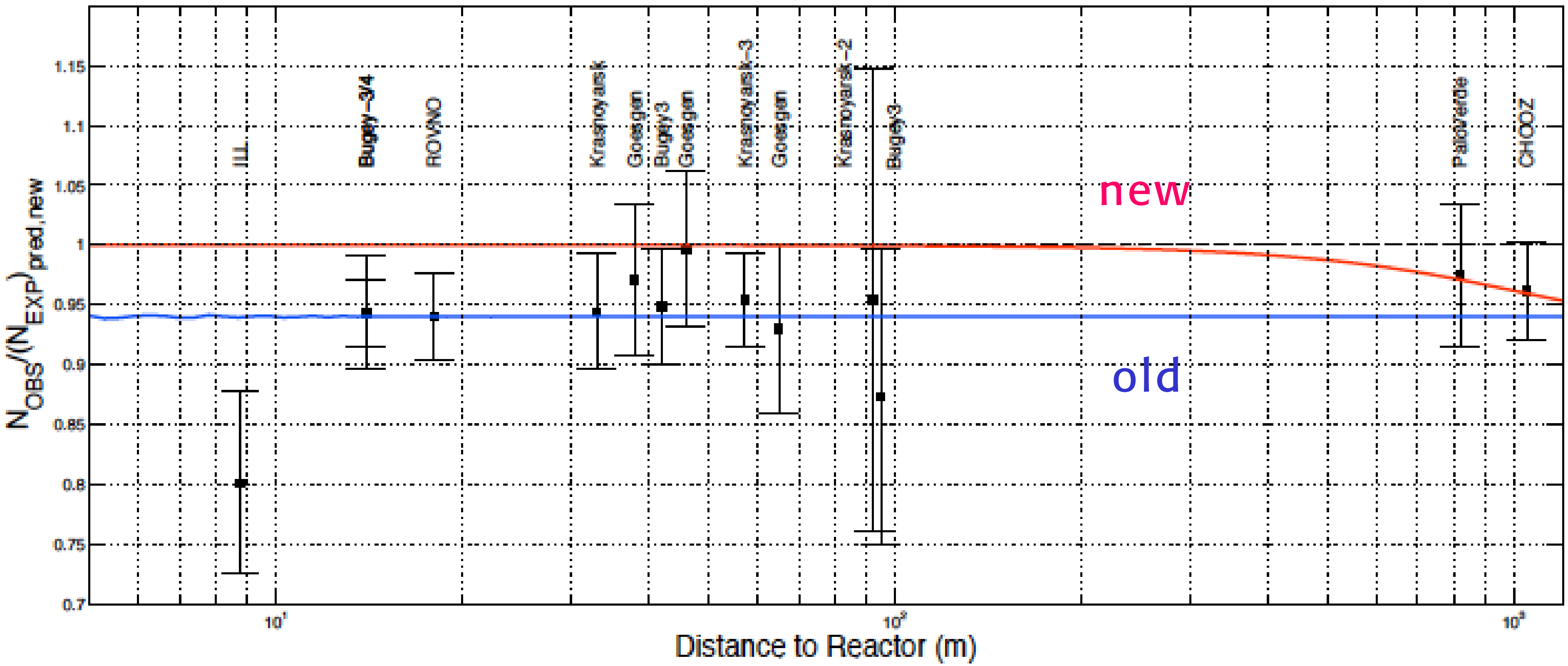
- APP $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$: LSND (Y), MiniBooNE (?), OPERA (N), ICARUS (N), KARMEN (N), NOMAD (N), BNL-E776 (N)
- DIS ν_e & $\bar{\nu}_e$: Reactors (Y), Gallium (Y), ν_eC (N), Solar (N)
- DIS ν_μ & $\bar{\nu}_\mu$: CDHSW (N), MINOS (N), Atmospheric (N), MiniBooNE/SciBooNE (N)

No Osc. excluded at 6.2σ

$\Delta\chi^2/\mathrm{NDF} = 46.2/3$

The reactor anomaly

Lasserre

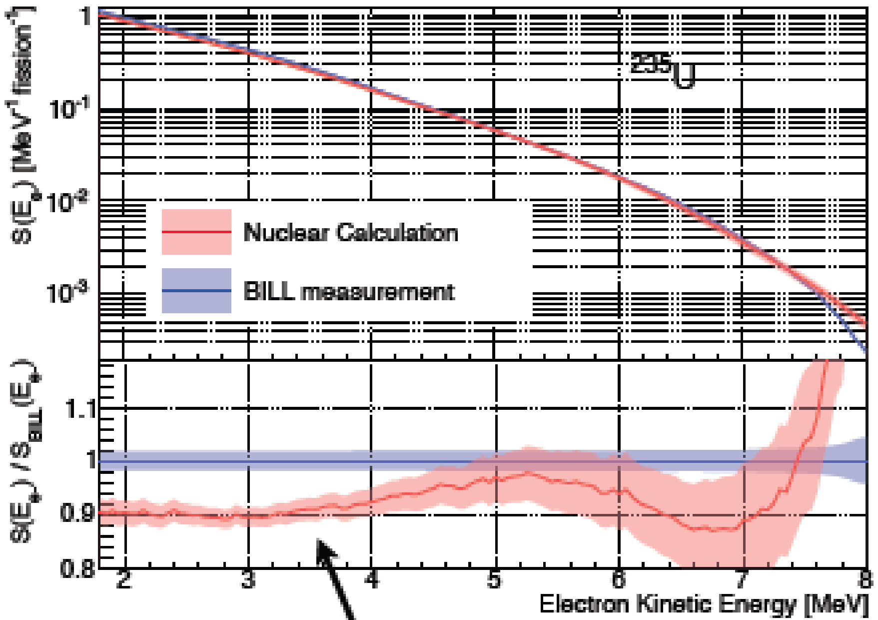


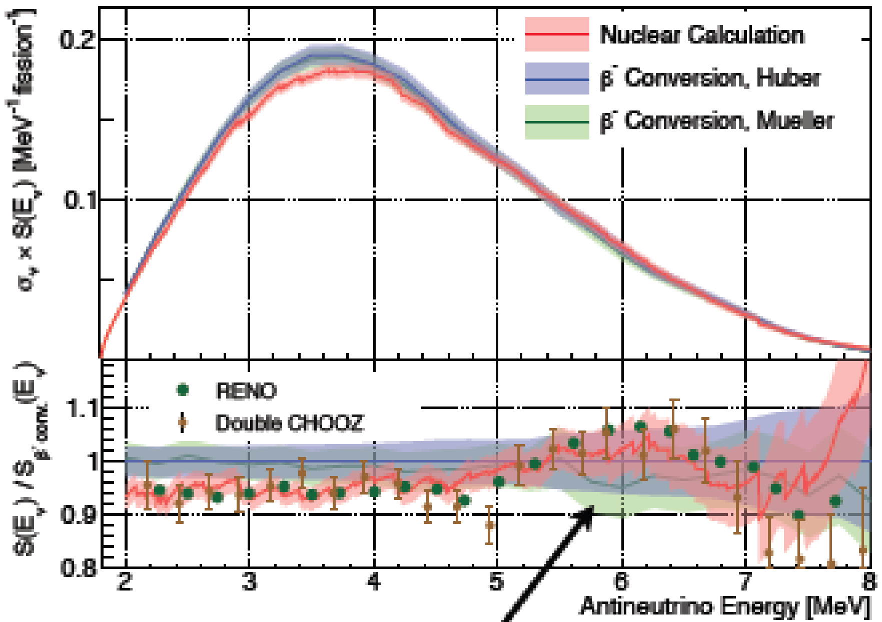
Systematic errors not shown in this figure (estimated in paper)!
Certainly of the same order of the shift (eg Hayes et al '13).
They could well be larger than estimated

new ab-initio calculations

Dwyer, Langford, 1407.1281

suggest th errors of order of the effect





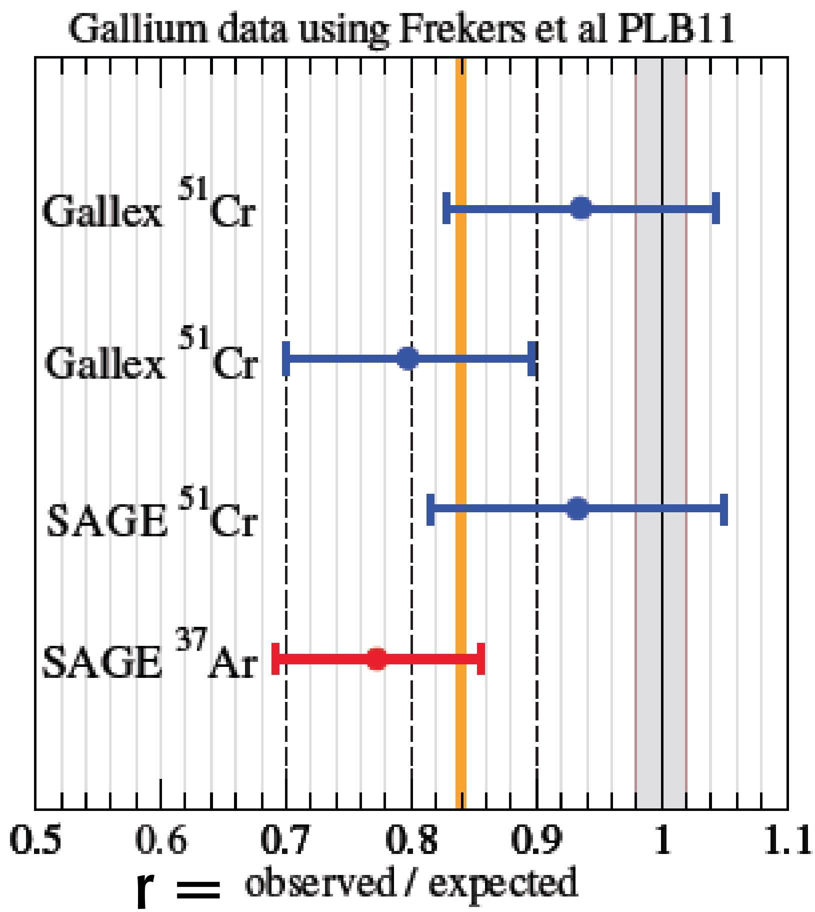
fails to predict beta spectr by ~10%

predicts bump around 6 MeV in agreement with data

The Gallium Anomaly

The neutrino flux from a known radioactive source put inside the Gallex and SAGE detectors shows a deficit wrt expectations

Depends on theoretical crossections and related errors



$$r = 0.84^{+0.054}_{-0.051} \quad (2.9\ \sigma)$$

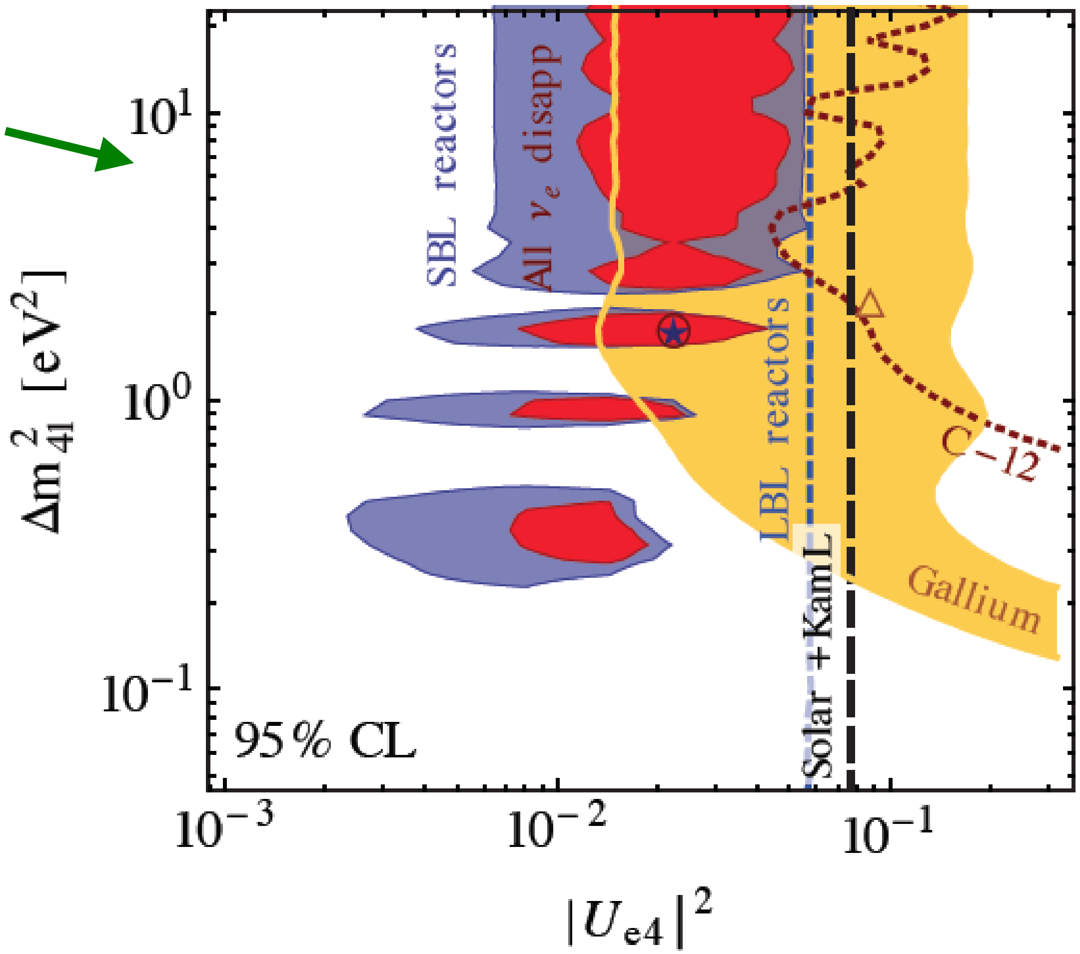
The reactor anomaly (below 100m baseline: SBL) came after a revision of the theoretical flux and of cross-sections

Mueller et al '11; Huber '11

Similarly the Ga anomaly depends on assumed cross section and errors Kaether et al '10; Abdurashitov et al '09

Kopp et al '13

SBL reactors and gallium in 3+1 models

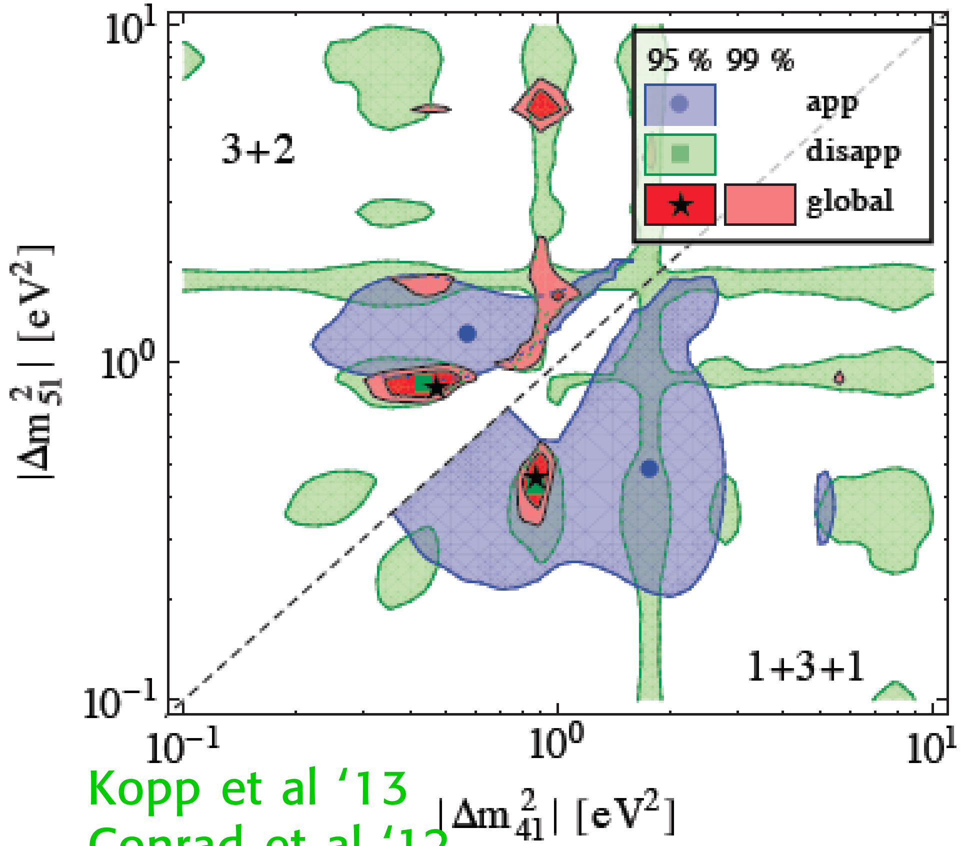


SBL reactors and gallium are not in tension with other measurements

Global fits to all data (1 or 2 sterile neutrinos)

3+1 not very good but acceptable

No great advantage from 3+2 or 1+3+1 (this second is better)



Kopp et al '13
Conrad et al '12

	Δm^2_{41} [eV^2]	$\lvert U_{e4}\rvert$	$\lvert U_{\mu 4}\rvert$	Δm^2_{51} [eV^2]	$\lvert U_{e5}\rvert$	$\lvert U_{\mu 5}\rvert$
3+1	0.93	0.15	0.17			
3+2	0.47	0.13	0.15	0.87	0.14	0.13
1+3+1	−0.87	0.15	0.13	0.47	0.13	0.17

In all fits (3+1or2, 1+3+1) the Δm^2 values are in tension with the cosmology mass bound:

$$\Sigma m_\nu < 0.28 - 0.8 \text{ eV}$$

(partial thermalization?)

The issue of sterile ν's is very important ⟶ experiment

e.g. Icarus at FNAL, Antonello et al, ArXiv:1312.7252

Conclusion

The observed smallness of ν masses is well explained in terms of Majorana masses, inversely proportional to the large energy scale where L non conserving interactions are relevant

The indicated energy scale is close to M_{GUT} where we indeed expect baryon and lepton number non conservation

The observation of $0\nu\beta\beta$ would prove L non conservation and that ν's are Majorana particles.

The observation of $0\nu\beta\beta$ would be as important as that of proton decay

The possible existence of eV sterile neutrinos needs to be clarified by experiment