Majorana Neutrinos and Quasidegeneracy at the origin of Large Leptonic Mixing

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Invited talk at the Workshop on SM and Beyond, Corfu 2014

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Talk based on two works :

GCB, M. N. Rebelo, J. I. Silva-Marcos and D. Wegman, arXiv:1405.5120 [hep-ph]

GCB, M. N. Rebelo and J. I. Silva-Marcos, Phys. Rev. Lett. 82 (1999) 683 [hep-ph/9810328].

Work Partially supported by:

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

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European Union

QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL
PORTUGAL 2007.2013

COMPETE
PROGRAMA OPERACIONAL FACTORES DE COMPETITIVIDADE

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The Limit of Exact Degeneracy

Without loss of generality, one can choose to work in a Weak Basis where the charged lepton mass matrix is diagonal, real.

Assume three left-handed neutrinos and consider a Majorana mass term with the form:

$$L_{mass} = -(\nu_{L\alpha})^T C^{-1} (M_0)_{\alpha\beta} \nu_{L\beta} + h.c.$$

where $\nu_{L\alpha}$ stand for the left-handed weak eigenstates and M_0 is a 3×3 symmetric complex mass matrix.

In general, M is diagonalised by a unitary matrix U through:

$$U_0^T M_0 U_0 = \mathrm{diag}\,(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$$

it follows that in the limit of exact neutrino mass degeneracy, M_0 can be written:

$$M_0 = \mu\, S_0$$

where μ is the common neutrino mass and $S_0 = U_0^* U_0^\dagger$. In the limit of exact degeneracy, a novel feature arises, namely M_0 is proportional to the symmetric unitary matrix S_0.

In the case of neutrinos with different CP parities the most general matrix S_0 can be parametrized in the form:

$$S_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & s\phi & -c\phi \end{bmatrix} \begin{bmatrix} c_\theta & s_\theta & 0 \\ s\theta & -c\theta & 0 \\ 0 & 0 & e^{i\alpha} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & s\phi & -c\phi \end{bmatrix}, \text{ or}$$

$$S_0 = O_{23}(\phi)\, O_{12}(\theta) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\alpha} \end{bmatrix} O_{23}(\phi)$$

Using the fact that:

$$\boxed{S_0 = U_0^* U_0^\dagger}$$

One concludes that the leptonic mixing matrix U_0 is given by:

$$U_0 = O_{23}(\phi)\, O_{12}\left(\frac{\theta}{2}\right) \begin{bmatrix} 1 & & \\ & i & \\ & & e^{-i\alpha/2} \end{bmatrix}$$

An important point :

U_0 always has one zero entry which in the above parametrization appears in the (1, 3) position.

This may be a hint that the limit of exact degeneracy is a good starting point to perform a small perturbation around it, leading to the lifting of the degeneracy and the generation of a non-zero U_{e3}

Since S_0 is a unitary matrix one can consider S_0 unitarity triangles which are analogous to the ones encountered in the U^{PMNS}, but with a <u>different physical meaning</u>!

$$M_0 = \mu S_0$$

Under a **Weak basis transformation** corresponding to a rephasing of both ν_L and the charged lepton fields, S_0 transforms as:

$$S_0 = L\, S_0\, L$$

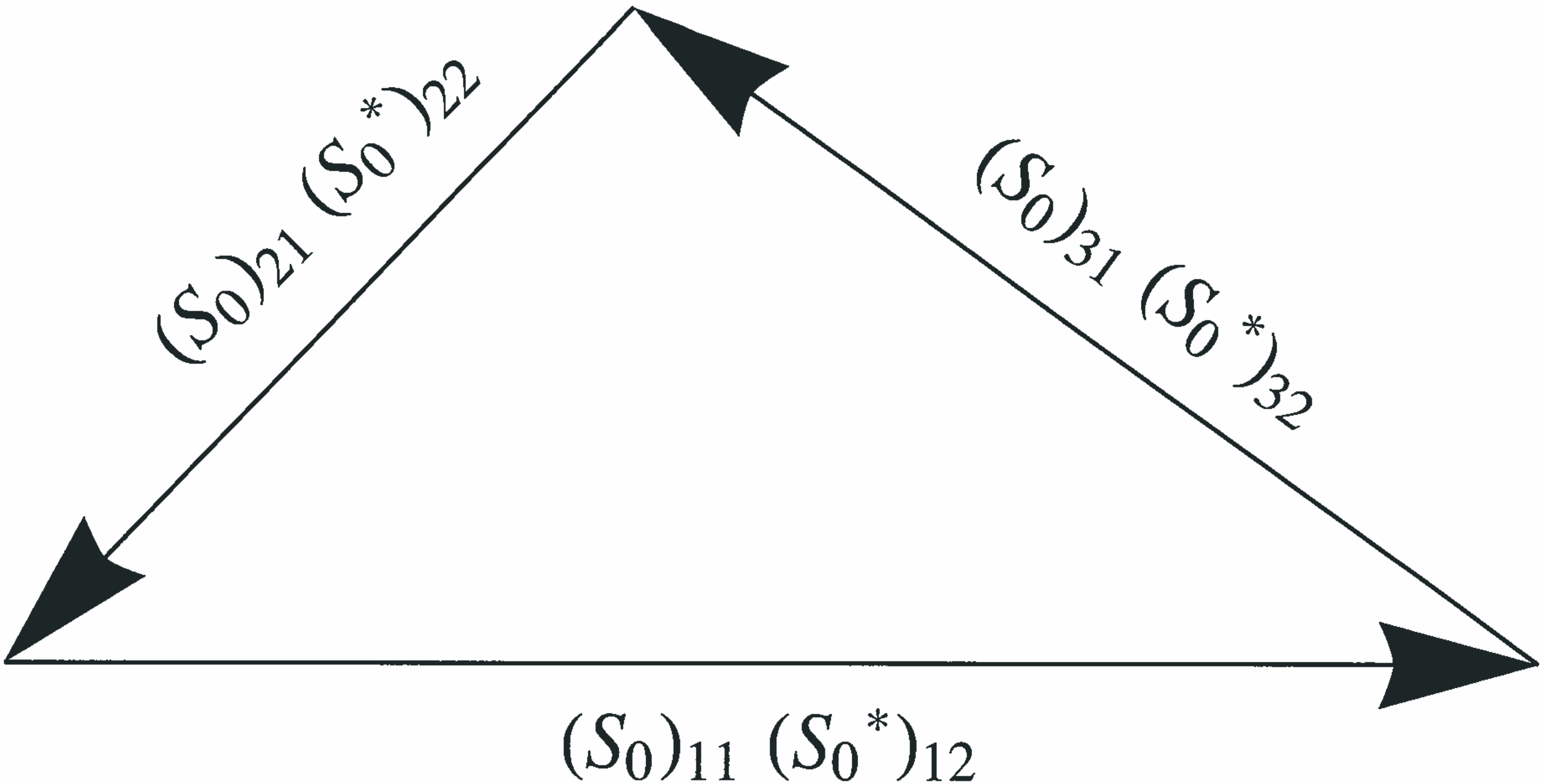
where $L = \mathrm{diag}\,(e^{i\varphi_1}\ e^{i\varphi_2}\ e^{i\varphi_3})$. As a result the individual phases of S_0 have no physical meaning

On the physical meaning of

S_0 triangles

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

$$\boxed{S_{12}\, S_{12}^{*} + S_{21}\, S_{22}^{*} + S_{31}\, S_{32}^{*} = 0}$$



Under rephasing, the triangle rotates.
However, its area remains invariant.
Question: What is the physical meaning of this area?

In order to answer this question, one has to recall the conditions for CP invariance when one has n generations of left-handed Majorana neutrinos. Under CP the lepton fields transform as:

$$(CP)\, l_L\, (CP)^\dagger = U_L\, \gamma^0 C\, \bar{l}_L^{\,T}$$

$$(CP)\, \nu_L\, (CP)^\dagger = U_L\, \gamma^0 C\, \bar{\nu}_L^{\,T}$$

$$(CP)\, l_R\, (CP)^\dagger = U_R\, \gamma^0 C\, \bar{l}_R^{\,T}$$

where U_L, U_R are unitary matrices acting in flavour space.

In order to have CP invariance in the leptonic sector, the mass matrices m_ν, m_ℓ have to satisfy the conditions:

$$U_L^T (m_\nu) U_L = - m_\nu^*$$

$$U_L^\dagger (m_\ell) U_R = m_\ell^*$$

$$\Downarrow$$

$$U_L^\dagger \, h_\ell \, U_L = h_\ell^*$$

From the previous Eqs., one can derive that a necessary condition for CP invariance, for an arbitrary number of generations is:

$$\mathrm{tr}\left[m_\nu m_\nu^\dagger, h_\ell\right]^3 = 0$$

For 3 generations one has:

$$\mathrm{tr}\left[m_\nu m_\nu^\dagger, h_\ell\right]^3 \propto i\,(m_\mu^2 - m_e^2)(m_\tau^2 - m_\mu^2)(m_\tau^2 - m_e^2)\times$$

$$\times\,(m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2)\left[\mathrm{Im}\, Q_\ell\right]$$

Dirac type CP violation

Q_ℓ invariant quartet of U_{PMNS}

Can one have a WB invariant sensitive to Majorana-type CP Violation?

The best procedure to find such an invariant is to consider the case of two Majorana neutrinos, where one does know that there is no Dirac-type CP violation, but there is Majorana type CP-Violation

The simplest invariant of this type is:

$$I_{\text{Majorana}} \equiv \text{Im}\, \text{tr}\left(h_\ell\, m^* m\, m^* h^* m\right)$$

For two generations:

$$\boxed{I_{\text{Maj}} = \frac{1}{2}\, m_1 m_2 \left(m_2^2 - m_1^2\right)\left(M_\mu^2 - M_e^2\right)^2 \times \sin^2(2\theta)\, \sin 2\gamma}$$

Invariants are very clever!

The invariant $I_{Maj.}$ vanishes in the limit of degenerate neutrinos even for 3 generations.

Question: Can one have a CP-odd WB invariant which does not vanish even in the limit of 3 exactly mass degenerate Majorana neutrinos?

Answer: Yes!

$$I_{deg} \equiv \mathrm{tr}\left[(m\,h\,m^*),\, h^*\right]^3$$

One can write $I_{deg.}$ in terms of physical quantities :

Area of S_0 triangle !!

$$I_{deg} = 6i\,\Delta m\,\mathrm{Im}\left[(S_0)_{11}(S_0)_{22}(S_0)^*_{12}(S_0)^*_{21}\right]$$

$$= \frac{3i}{2}\,\Delta m \cos\theta \sin^2\theta \sin^2(2\phi)\sin\alpha$$

$$\Delta m = \mu^6 (m_\tau^2 - m_\mu^2)^2 (m_\tau^2 - m_e^2)^2 (m_\mu^2 - m_e^2)^2$$

μ — degenerate neutrino mass

Experimental Data

Table 1: Neutrino oscillation parameter summary. For Δm^2_{31}, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, and δ the upper (lower) row corresponds to normal (inverted) neutrino mass hierarchy.

Parameter	Best fit	1σ range
Δm^2_{21} $[10^{-5} eV^2]$	7.62	7.43 – 7.81
Δm^2_{31} $[10^{-3} eV^2]$	2.55	2.46 – 2.61
Δm^2_{31} $[10^{-3} eV^2]$	2.43	2.37 – 2.50
$\sin^2\theta_{12}$	0.320	0.303 – 0.336
$\sin^2\theta_{23}$	0.613 (0.427)	0.400 –0. 461 and 0.573 – 0.635
$\sin^2\theta_{23}$	0.600	0.569 – 0.626
$\sin^2\theta_{13}$	0.0246	0.0218 –0.0275
$\sin^2\theta_{13}$	0.0250	0.0223 – 0.0276
δ	0.80 π	0 –2 π
δ	-0.03 π	0 –2 π

Lifting the Degeneracy

- For definiteness and without loss of generality, we work in the weak Basis where the charged lepton mass matrix is diagonal and real.
  - Several textures for leptonic mixing have been proposed. In most schemes, the pattern of leptonic mixing is predicted, but the spectrum of masses is not constrained by the simetries.

- Therefore, it is consistent to consider these schemes, together with the hypothesis of quasi-degeneracy of Majorana neutrinos.

- Until recently, one of the most favoured Ansätze, from the experimental point of view, was the tribimaximal Ansatz.

- The discovery of $\theta_{13} \neq 0$ motivated a series of studies of how to generate $\theta_{13} \neq 0$ through a small perturbation of the tribimaximal Ansatz.

The distinctive feature of our analysis:
we start from a non-trivial limit of 3 exactly degenerate Majorana neutrinos, where the mixing can be written:

$$U_0(\theta,\varphi)K \quad ; \quad K = \text{diag.}(1, i, e^{-i\alpha/2})$$

Lifting the degeneracy corresponds to adding a small perturbation to S_0

$$M = \mu\left(S_0 + \varepsilon^2 Q_0\right)$$

$$\varepsilon^2 \equiv \frac{\Delta m^2_{31}}{2\mu^2}$$

- We assume that the physics responsible to the lifting of the degeneracy, does not introduce new sources of CP violation, beyond the phase α, already present in the degeneracy limit:

$$U_{PMNS} = U_0 \, O$$

where O is an orthogonal matrix parametrized by small angles:

$$O = O_{12}(\phi_1)\, O_{13}(\phi_3)\, O_{23}(\phi_2)$$

The matrix Q_0 is determined by:

$$\varepsilon^2 Q_0 = U_0^* \, O \left(\frac{1}{\mu} D_\nu - \mathbb{1} \right) O^T U_0^\dagger$$

$$D_\nu = \text{diag.}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$$

Let us consider:

$$U_0 = U_{TBM} \cdot K \quad ; \quad K = \text{diag.}\,(1, i\, e^{-i\alpha/2})$$

$$U_{TBM} = \begin{bmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \end{bmatrix} \rightarrow$$

corresponds to $\phi = 45°$ and $\cos\theta/2 = 2/\sqrt{6}$, $\theta/2 = 35.26°$

In this scenario, one may find a particularly simple solution: one can reach agreement with experiment, by choosing a matrix O with only one parameter different from 0, namely ϕ_2. In this case

$$\boxed{\sin^2(\theta_{13}) = \frac{\sin^2(\phi_2)}{3}}$$

$$\boxed{\sin^2\theta_{12} = \frac{1-\sin^2\phi_2}{3-\sin^2\phi_2} = \frac{1/3-|U_{13}|^2}{1-|U_{13}|^2}}$$

$$\boxed{\sin^2(\theta_{23}) = \frac{1}{2} - \frac{\sqrt{6}\,\sin\alpha/2\,\sin\phi_2\cos\phi_2}{3-\sin^2\phi_2}}$$

With the Standard Parametrization one has:

$$I_{CP} \equiv \mathrm{Im}\, Q_{leptonic} = \frac{1}{8}\Big|\sin(2\theta_{12})\sin(2\theta_{13}) \cdot$$

$$\cdot \sin(2\theta_{23})\cos\theta_{13} \cdot \sin\delta\Big|$$

In our framework

$$I_{CP} = \left| \frac{\cos\alpha/2\ \sin\phi/2\ \cos\phi/2}{3\sqrt{6}} \right|$$

and I_{CP} is predicted to be of order 10^{-2}.

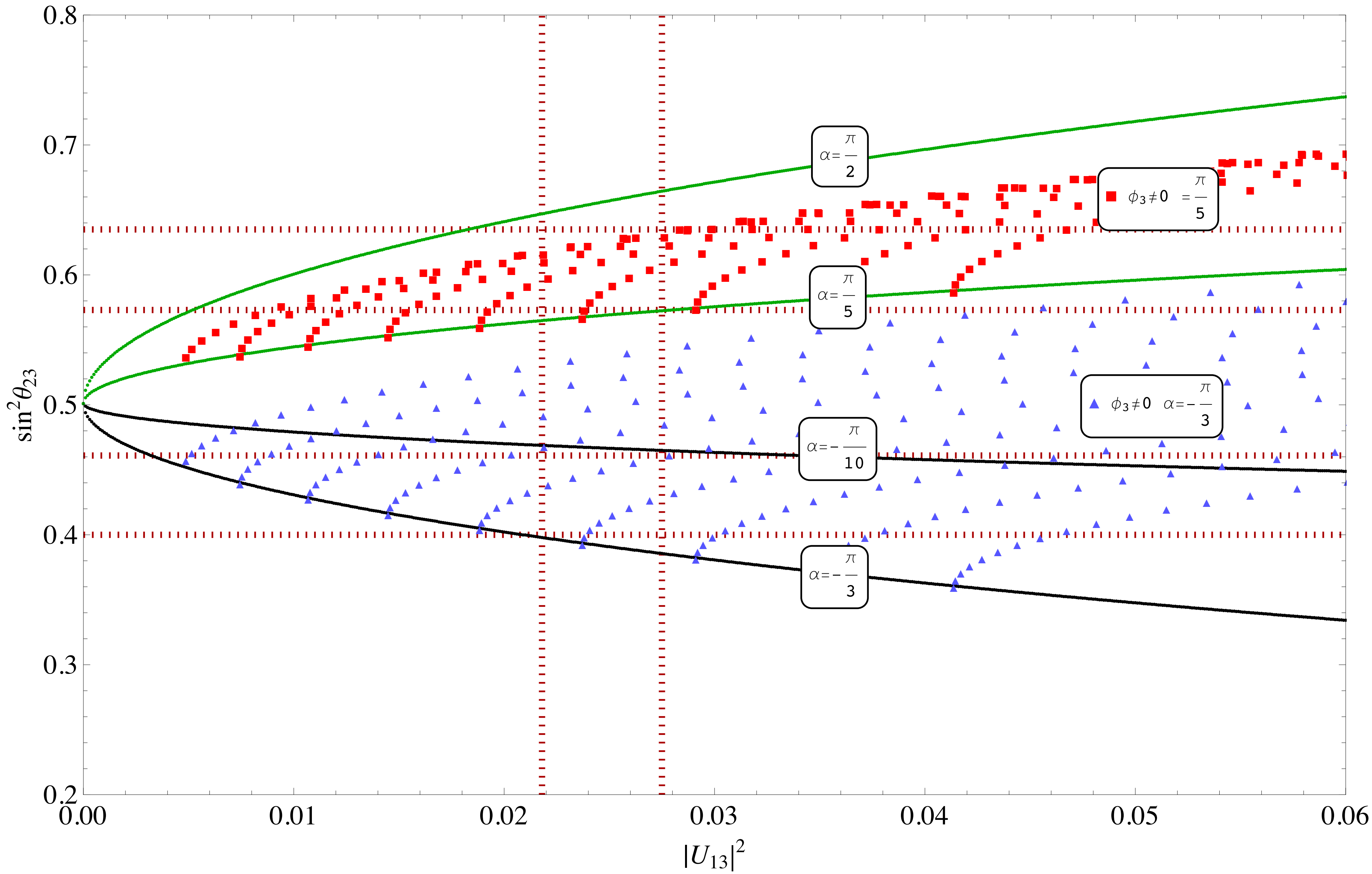


Figure 2: $\sin^2\theta_{23}$ versus $|U_{13}|^2$ obtained by perturbing tribimaximal mixing with $\phi_3 = 0$. Each curve corresponds to a fixed α and to $\phi_1 = 0$, therefore ϕ_2 is the only variable. The points drifting away from each curve were obtained by varying also ϕ_3.

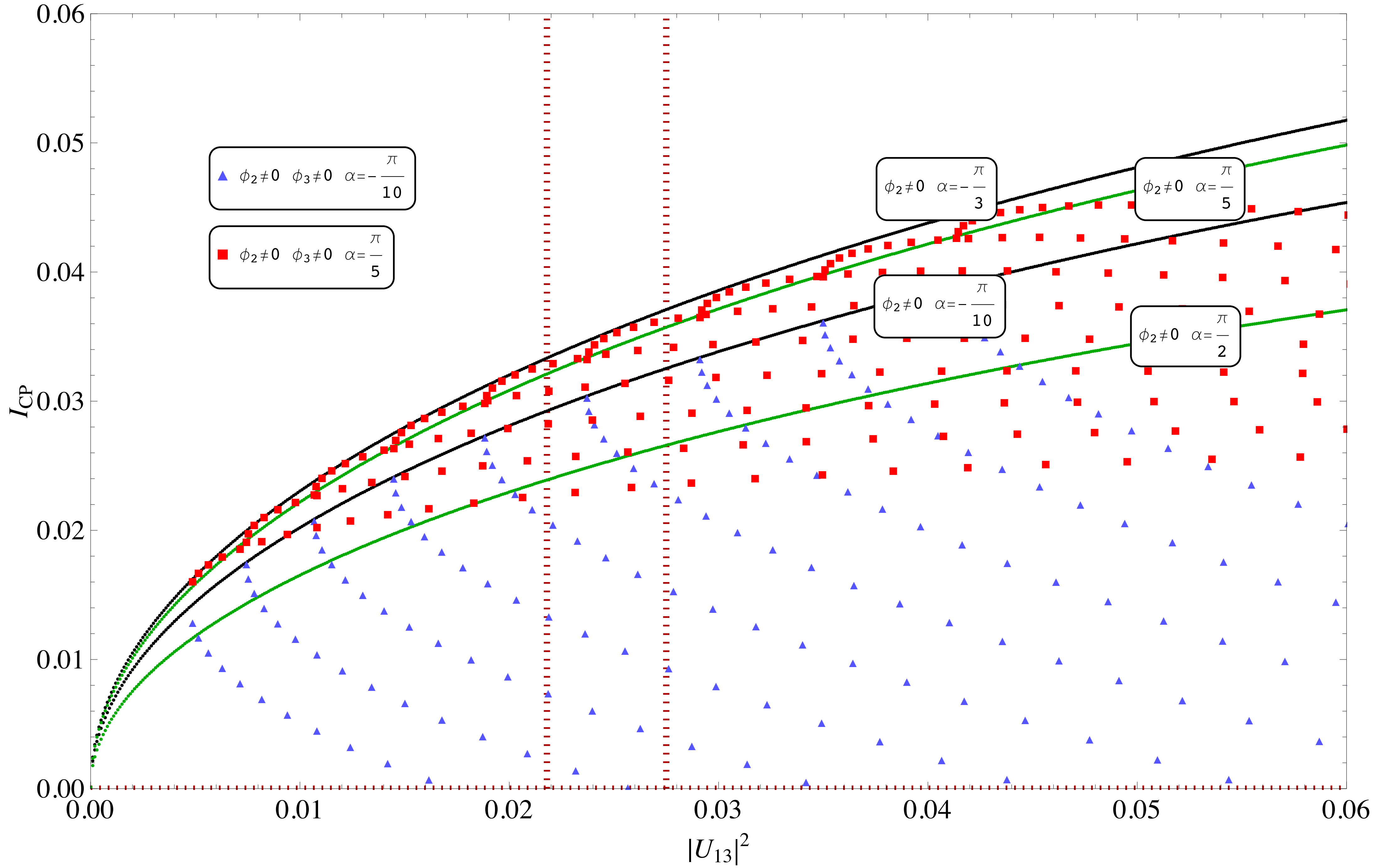


Figure 3: I_{CP} versus $|U_{13}|^2$ obtained by perturbing tribimaximal mixing with $\phi_3 = 0$. Each curve corresponds to a fixed α and to $\phi_1 = 0$, therefore ϕ_2 is the only variable. The points drifting away from each curve were obtained by varying also ϕ_3.

Conclusions

- We present a novel proposal for the understanding of the observed pattern of leptonic mixing which relies on the assumption that neutrinos are Majorana particles,

  with quasi-degenerate masses.
- In our scenario, there is only one CP violating phase α which is already present in the exactly degenerate limit
- Upon the lifting of the degeneracy this phase α generates both Dirac and Majorana-type CP violation.