Beyond the SM

Corfu, September, 2014

Graham Ross

I. Motivation

The Standard Model:

Local Gauge Symmetry: $SU(3)\times SU(2)\times U(1)$

$$L_{SM} = L_{YM} + L_{WD} + L_{Yu} + L_{H}$$

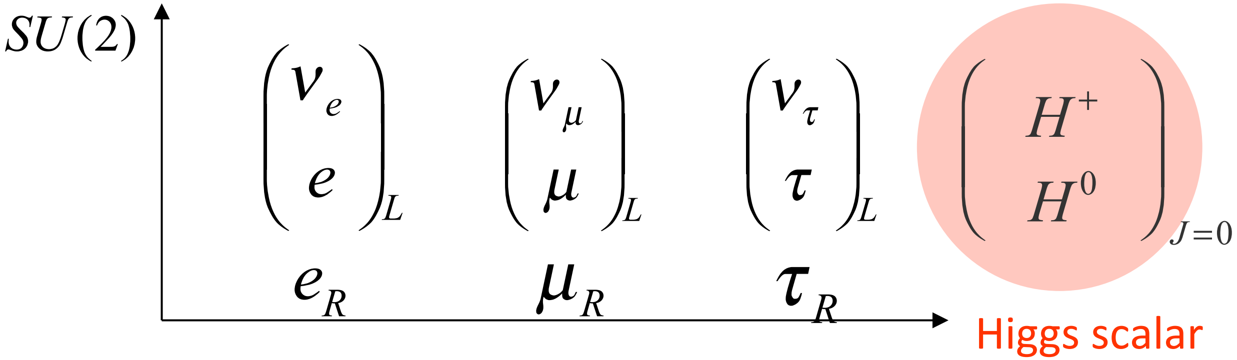
$$\boxed{\begin{aligned} L_{YM} &= L_{QCD} + L_{I_W} + L_{Y} \\ &= -\frac{1}{4g_3^2}\sum_{A=1}^{8}\overset{\dagger}{G}{}^{A}_{\mu\nu}G^{\mu\nu A} - \frac{1}{4g_2^2}\sum_{a=1}^{3}F^{a}_{\mu\nu}F^{\mu\nu a} - \frac{1}{4g_1^2}B_{\mu\nu}B^{\mu\nu}\end{aligned}}$$

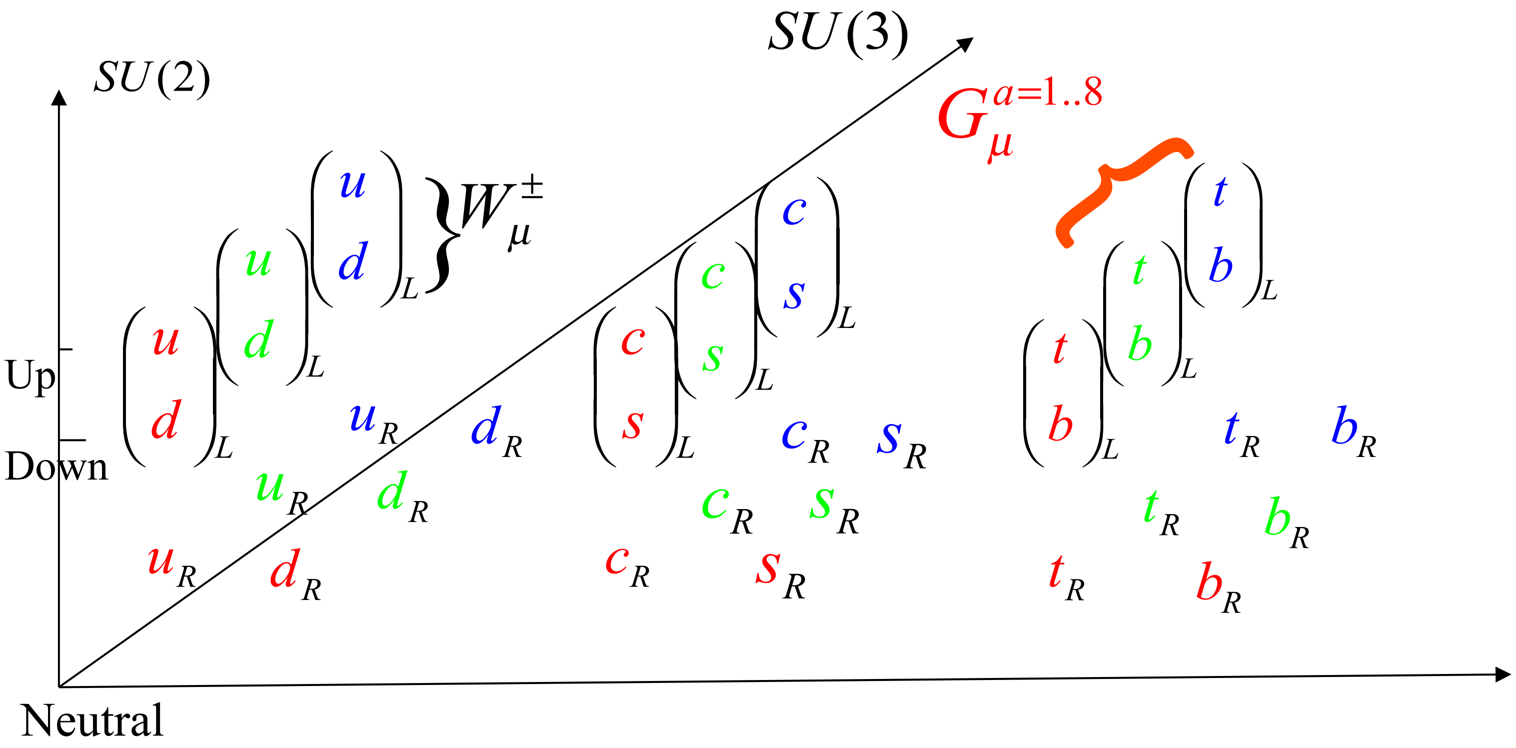
†Adjoint representation: Dimension $SU(n):\ n^2-1$

Multiplet structure –'chiral'

$SU(3)\otimes SU(2)\otimes U(1)$

$$L^K_{fermion} = \overline{\psi}\gamma_\mu D^\mu\psi$$





Spontaneously broken

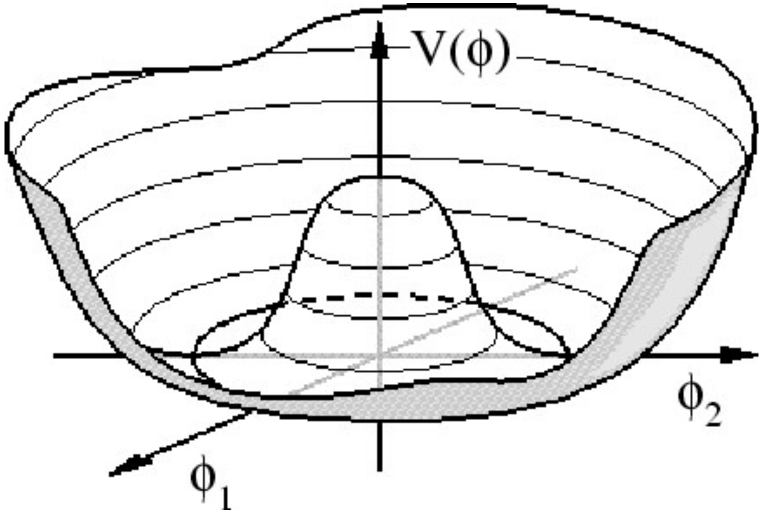
$$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}_{J=0}$$

Higgs scalar

$$\boxed{L_H = \left(D_\mu H\right)^\dagger \left(D^\mu H\right) - V(H)}$$

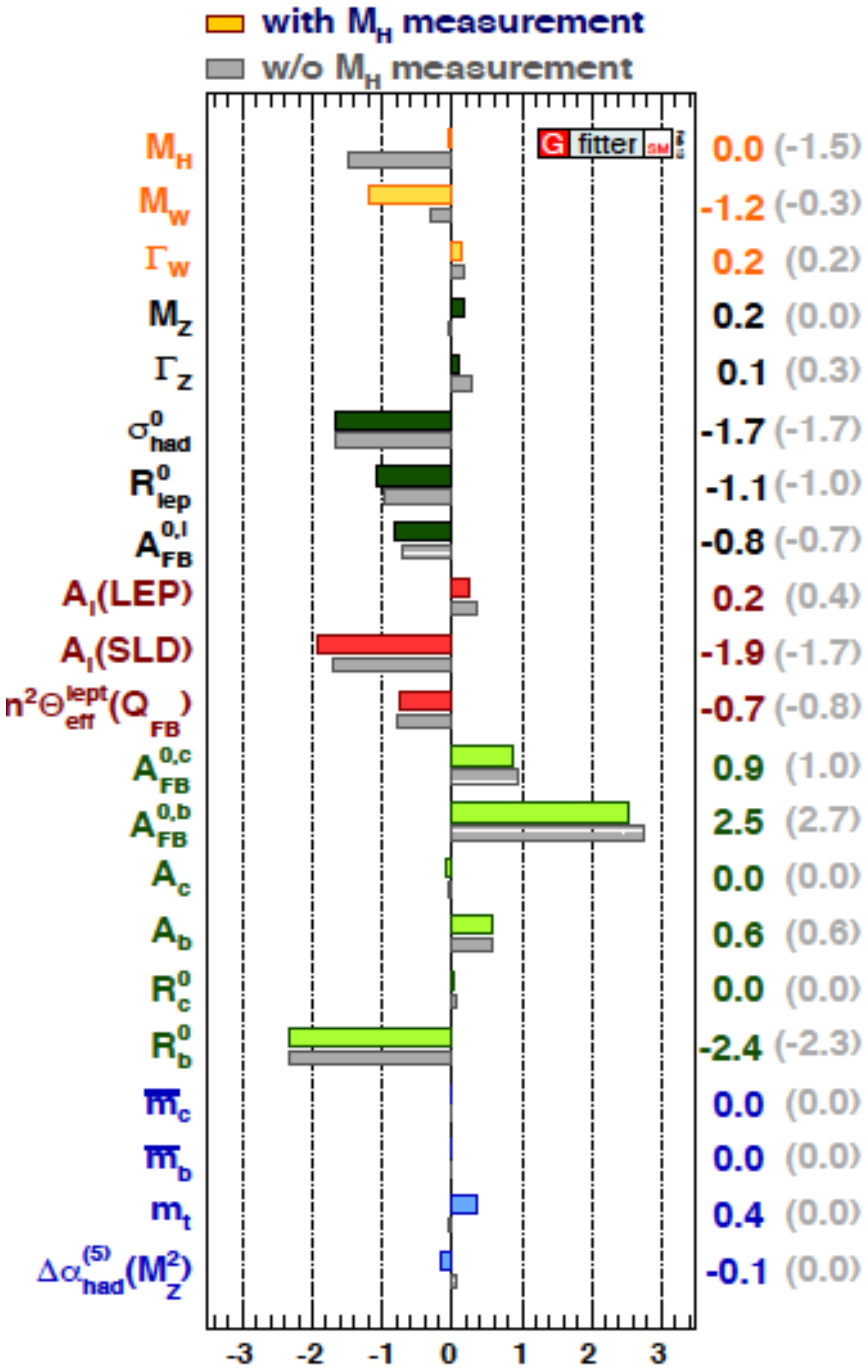
$$V = -m^2 |H|^2 + \lambda |H|^4$$

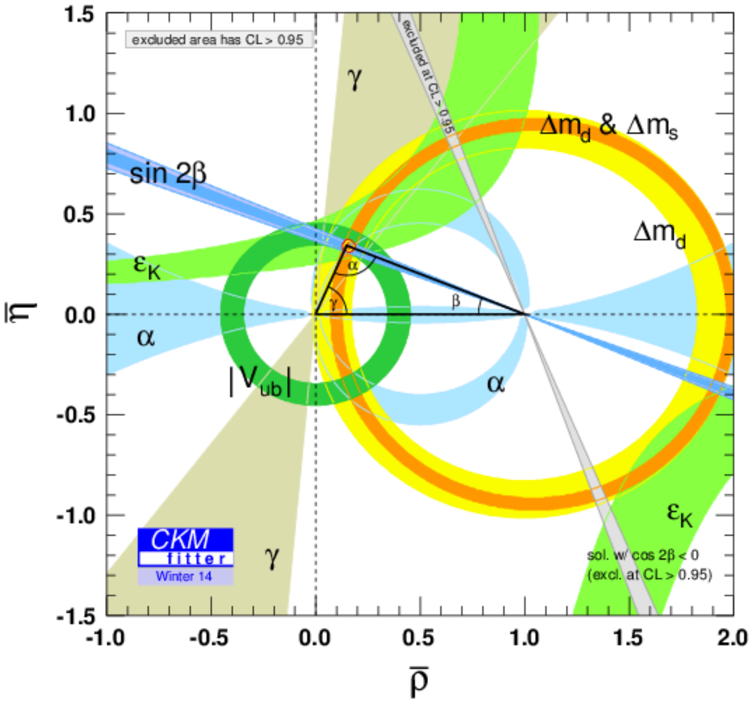
$$H = e^{i\theta.\sigma} \begin{pmatrix} 0 \\ \dfrac{v+h}{\sqrt{2}} \end{pmatrix}$$

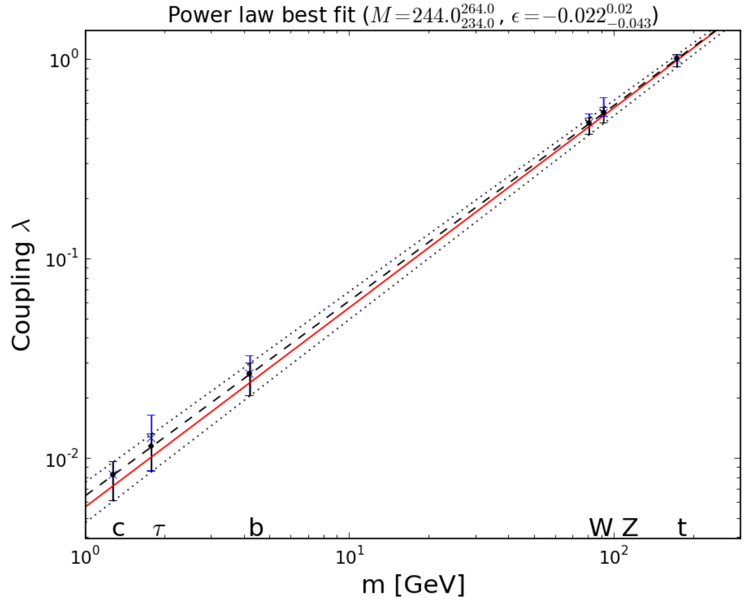


$SU(2)$: adjoint rep 3 dimensional

$\Rightarrow$ 3 Goldstone modes, θ_i (in absence of gauge interactions)


with M_H measurement
w/o M_H measurement
G fitter SM
M_H 0.0 (-1.5)
M_W -1.2 (-0.3)
Γ_W 0.2 (0.2)
M_Z 0.2 (0.0)
Γ_Z 0.1 (0.3)
σ^0_{had} -1.7 (-1.7)
R^0_{lep} -1.1 (-1.0)
$A^{0,l}_{FB}$ -0.8 (-0.7)
A_l(LEP) 0.2 (0.4)
A_l(SLD) -1.9 (-1.7)
$\sin^2\Theta^{lept}_{eff}(Q_{FB})$ -0.7 (-0.8)
$A^{0,c}_{FB}$ 0.9 (1.0)
$A^{0,b}_{FB}$ 2.5 (2.7)
A_c 0.0 (0.0)
A_b 0.6 (0.6)
R^0_c 0.0 (0.0)
R^0_b -2.4 (-2.3)
$\overline{m}_c$ 0.0 (0.0)
$\overline{m}_b$ 0.0 (0.0)
m_t 0.4 (0.0)
$\Delta\alpha^{(5)}_{had}(M^2_Z)$ -0.1 (0.0)
-3 -2 -1 0 1 2 3


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
$|V_{ub}|$
α
β
γ
ε_K
CKM fitter Winter 14
γ
sol. w/ cos 2β < 0 (excl. at CL > 0.95)
$\bar{\eta}$
$\bar{\rho}$
1.5 1.0 0.5 0.0 -0.5 -1.0 -1.5
-1.0 -0.5 0.0 0.5 1.0 1.5 2.0


Power law best fit ($M=244.0^{264.0}_{234.0}$, $\epsilon=-0.022^{0.02}_{-0.043}$)
Coupling λ
m [GeV]
10^0 10^{-1} 10^{-2}
10^0 10^1 10^2
c τ b W Z t

The Standard Model – unanswered questions

- Complicated choice of multiplets
- Fractional and integral charges?
- Neutrino masses?
- Many parameters 16 (25)
- Only partial unification $A_\mu^\gamma = \sin\theta_W W_\mu^3 + \cos\theta_W\, B_\mu$
- The hierarchy problem
- Strong CP problem
- Dark matter, baryogenesis, inflation…..

II. Grand Unification

$$SU(3)\otimes SU(2)\otimes U(1)\overset{?}{\subset}\mathbf{G}$$

$$(3,2)+2.(3,1)+(1,2)+(1,1)\overset{?}{\subset}\mathbf{R}$$

$\mathbf{G} \geq$ Rank 4 (# diagonal generators)

II. Grand Unification

$$SU(3) \otimes SU(2) \otimes U(1) \overset{?}{\subset} \mathbf{G}$$

$$(3,2) + 2.(3,1) + (1,2) + (1,1) \overset{?}{\subset} \mathbf{R}$$

$\mathbf{G} \geq \text{Rank } 4$ (# diagonal generators)

$SU(5)$... unique viable rank 4 possibility

Georgi Glashow

$SU(5)$: Group of 5×5 complex unitary matrices with determinant 1

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$50 - 25 - 1 = 24$ independent matrices - adjoint representation

$U = \exp\left(-i\sum_{i=1}^{24}\beta^i L^i\right),\qquad U^\dagger U = 1 \quad \Rightarrow \quad L^i$ Hermitian generators

$SU(5)$: Group of 5×5 complex unitary matrices with determinant 1

$50-25-1=24$ independent matrices - adjoint representation

$$U=\exp\left(-i\sum_{i=1}^{24}\beta^i L^i\right),\qquad U^\dagger U=1\quad\Rightarrow\quad L^i\ \text{Hermitian generators}$$

$$SU(3)\times SU(2)\times U(1)\subset SU(5)$$

$$L^{a=1..8}=\begin{bmatrix} \multicolumn{3}{c}{\lambda^a} & 0 & 0\\ & & & 0 & 0\\ & & & 0 & 0\\ 0&0&0&0&0\\ 0&0&0&0&0\end{bmatrix}\qquad SU(3)$$

$$\left.\begin{aligned} L^{a=9,10}&=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&\multicolumn{2}{c}{\sigma_{1,2}}\\0&0&0& & \end{bmatrix}\\ L^{11}&=\text{Diagonal }(0,0,0,1,-1)\end{aligned}\right\}SU(2)$$

$$L^{12}=\text{Diagonal }(-2,-2,-2,3,3)\qquad U(1)$$

Fermions

Convenient to use Weyl notation for fermions

Weyl spinors

$(\frac{1}{2},0) \quad (0,\frac{1}{2})$

$\psi_L \qquad \psi_{\mathrm{R}}$

$SO(3,1) \sim SU(2) \times SU(2)$

2-component spinors of SU(2)

Rotations and Boosts

$\psi_{L(R)} \to S_{L(R)}\,\psi_{L(R)}$

$S_{L(R)} = e^{i\frac{\alpha}{2}.\sigma}$: Rotations

$S_{L(R)} = e^{\mp\frac{\nu}{2}.\sigma}$: Boosts

†

Weyl basis

$$\gamma_0 = \begin{bmatrix} 0 & -I \\ -I & 0 \end{bmatrix}, \quad \gamma_i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & o \end{bmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}$$

† $\psi_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)\psi_{Dirac}$

Fermions

Convenient to use Weyl notation for fermions

Weyl spinors

$$(\tfrac{1}{2},0) \quad (0,\tfrac{1}{2})$$

$$\psi_L \qquad \psi_{\mathrm{R}}$$

$SO(3,1) \sim SU(2)\times SU(2)$

2-component spinors of SU(2)

Rotations and Boosts

$$\psi_{L(R)} \rightarrow S_{L(R)}\,\psi_{L(R)}$$

†

$$S_{L(R)} = e^{i\frac{\alpha}{2}.\sigma} \quad : \ \text{Rotations}$$

$$S_{L(R)} = e^{\mp\frac{\nu}{2}.\sigma} \quad : \ \text{Boosts}$$

Can construct LH spinors out of RH antispinors and vice-versa

$$\boxed{\psi_L^c \equiv \sigma_2 \psi_R^* \sim (\frac{1}{2},0)}$$

$SU(5)$: Group of 5×5 complex unitary matrices with determinant 1

$50-25-1=24$ independent matrices - adjoint representation

$$U=\exp\left(-i\sum_{i=1}^{24}\beta^i L^i\right), \qquad U^\dagger U=1 \quad \Rightarrow \quad L^i$$ Hermitian generators

Fermions:

$$L^K_{fermion} = \psi^\dagger_R \sigma_\mu D^\mu \psi_R$$

Fundamental representation $\psi_{5R} \equiv \begin{bmatrix} n^1 \\ n^2 \\ n^3 \\ n^4 \\ n^5 \end{bmatrix}_R$

$SU(5)$: Group of 5×5 complex unitary matrices with determinant 1

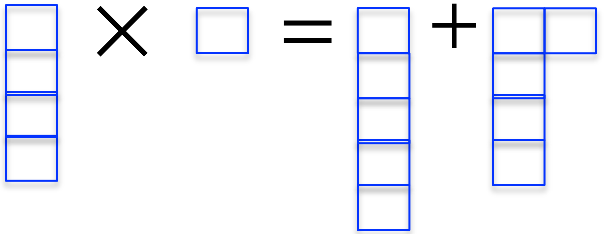
$50-25-1=24$ independent matrices - adjoint representation

$$U=\exp\left(-i\sum_{i=1}^{24}\beta^i L^i\right),\qquad U^\dagger U=1 \quad\Rightarrow\quad L^i$$ Hermitian generators

In terms of fundamental representation, $\left(\psi_5\right)^a = n^a$

$$\boxed{L^K_{fermion} = \psi^\dagger_R \sigma_\mu D^\mu \psi_R}$$

$\bar{5}\times 5 = 1+24$



Young Tableau

$$T^\beta_\alpha = \varepsilon_{\alpha abcd} n^a n^b n^c n^d \times n^\beta$$

$24=\frac{5\times 4\times 3\times 2\times 6}{1\times 2\times 3\times 5}$

$SU(5)$: Group of 5×5 complex unitary matrices with determinant 1

$50-25-1=24$ independent matrices - adjoint representation

$$U=\exp\left(-i\sum_{i=1}^{24}\beta^i L^i\right),\qquad U^\dagger U=1\quad\Rightarrow\quad L^i$$ Hermitian generators

Covariant derivative:

Gauge bosons V_μ^a

$(3,1)+(1,2)$

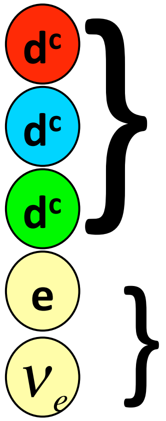
Define $$\frac{1}{\sqrt{2}}V_\mu\equiv\frac{1}{2}\sum_{a=1}^{24}V_\mu^a L^a,\qquad \left(D_\mu\psi_5\right)^i=\left[\delta^i_j\partial_\mu-\frac{ig}{2}\sum_{a=1}^{24}V_\mu^a\left(L^a\right)^i_j\right]\psi_5^j$$

$$V_\mu=\begin{bmatrix} G_1^1-\frac{2B}{\sqrt{30}} & G_2^1 & G_2^1 & \overline{X}_1 & \overline{Y}_1 \\ G_1^2 & G_2^2-\frac{2B}{\sqrt{30}} & G_3^2 & \overline{X}_2 & \overline{Y}_2 \\ G_1^3 & G_2^3 & G_3^3-\frac{2B}{\sqrt{30}} & \overline{X}_3 & \overline{Y}_3 \\ X_1 & X_2 & X_3 & \frac{W_\mu^3}{\sqrt{2}}+\frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W_\mu^3}{\sqrt{2}}+\frac{3B}{\sqrt{30}} \end{bmatrix},$$

Grand Unification

$$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$

$$\left(\bar{5}\right)_{La} :$$



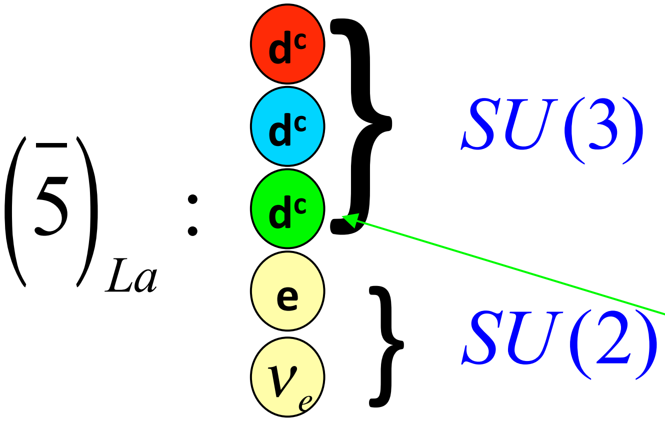
$SU(3)$

$SU(2)$

$$(\bar{3},1)+(1,2)$$

Grand Unification

$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$



$$3Q_{d^c} + Q_{e^-} = 0$$

$$\boxed{Q_{d^c} = 1/3}$$

Grand Unification

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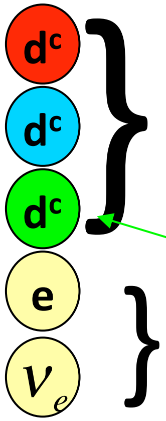


$SU(3)$

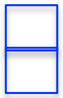
$SU(2)$

$$3Q_{d^c} + Q_{e^-} = 0$$

$$\boxed{Q_{d^c} = 1/3}$$

Remaining 10 states?

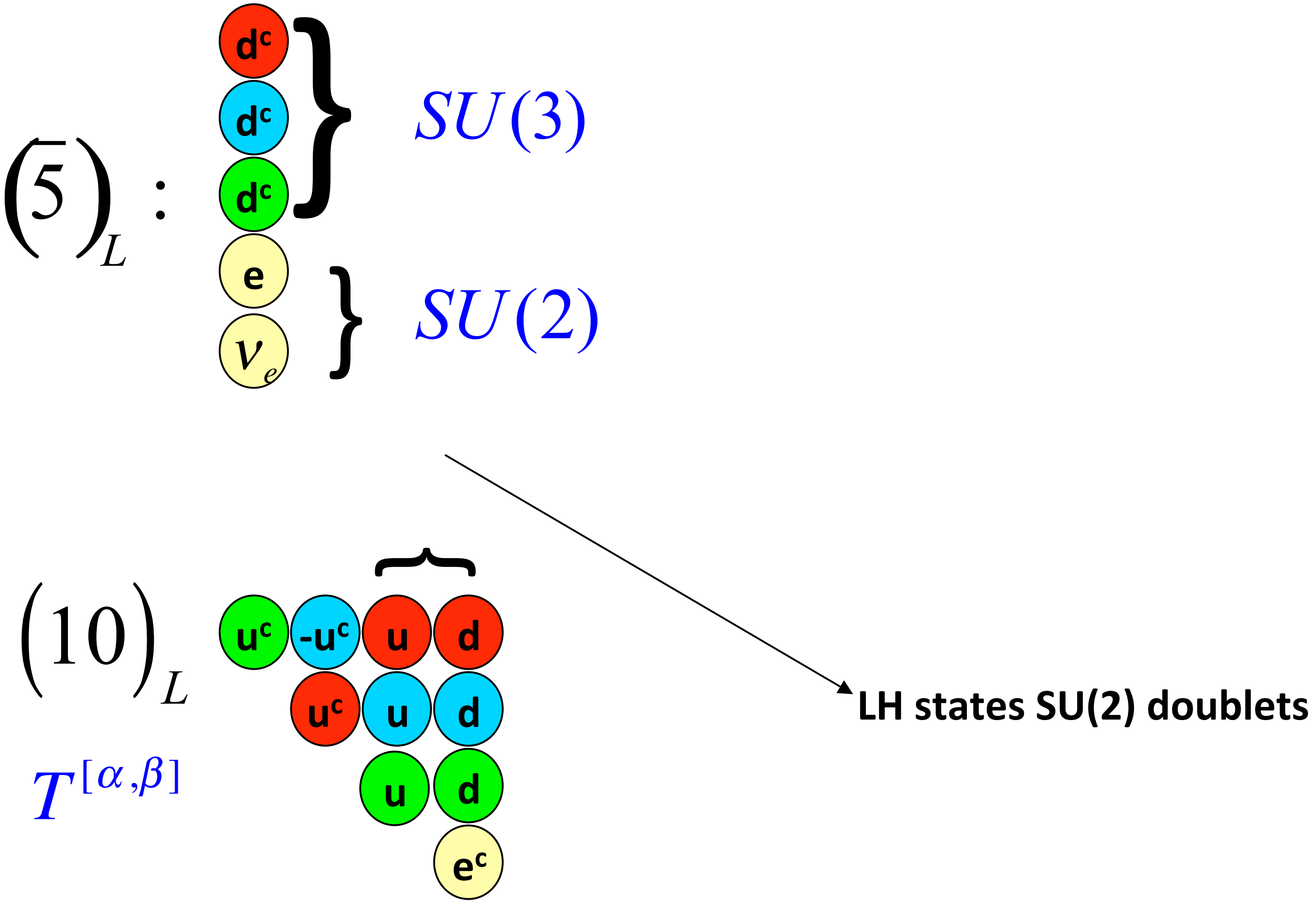
$$T^{[\alpha,\beta]}$$



$$\frac{n(n-1)}{1\times 2} = 10$$

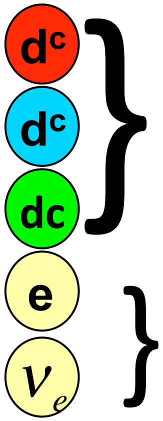
Grand Unification

$$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$



Gauge Couplings

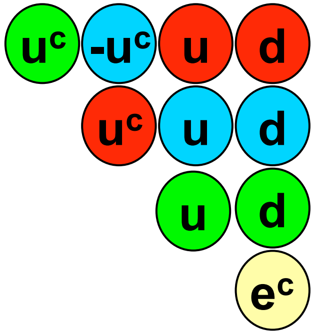
$(\bar{5})_L$:



$SU(3)$

$SU(2)$

$(10)_L$:



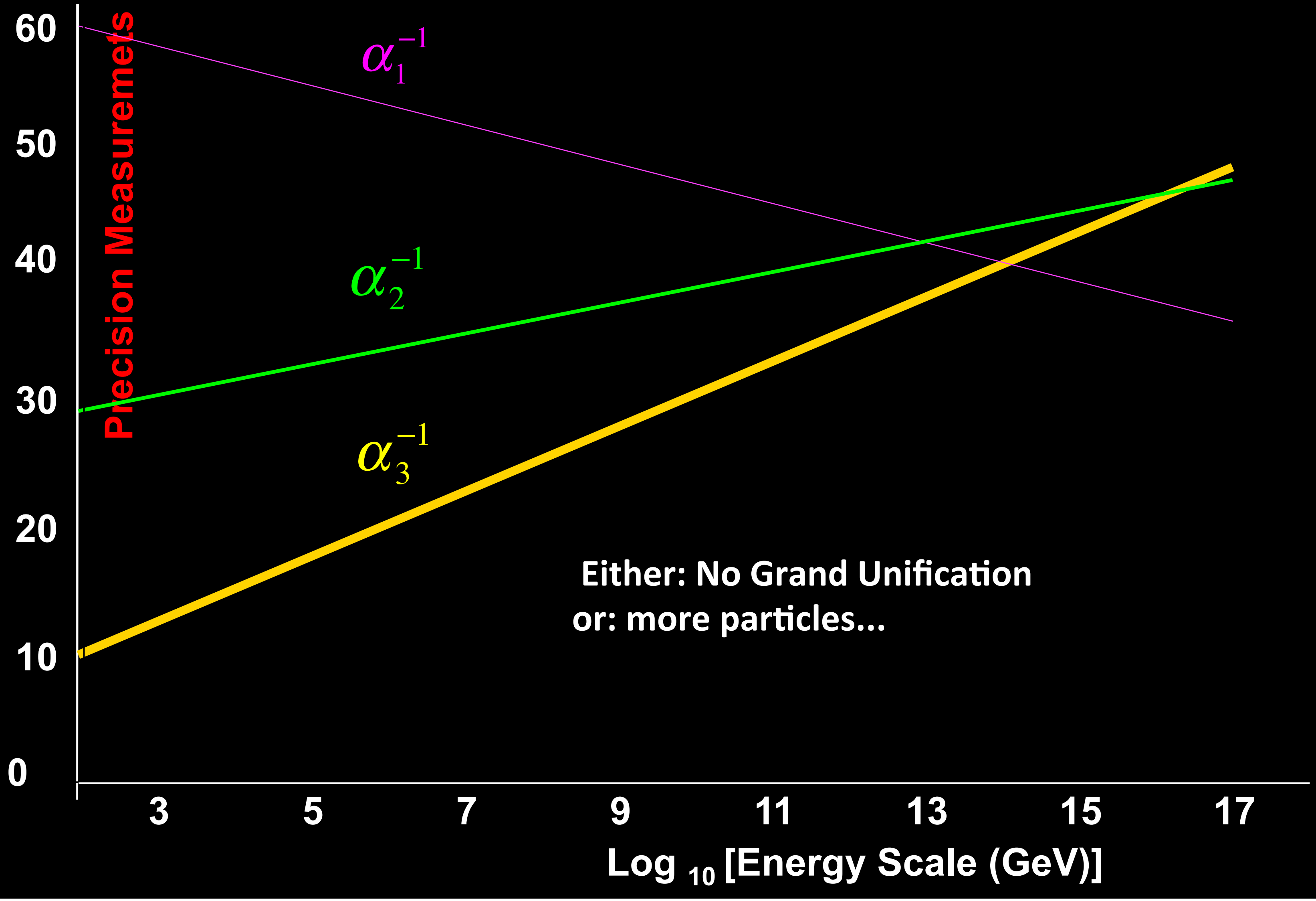
$$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$

M_X

$$g_5 \supset g_3 \qquad g_2 \qquad g_1$$

$$\boxed{g_1(M_X) = g_2(M_X) = g_3(M_X) = g_5}$$

$$M_X \sim 10^{14}\, GeV$$

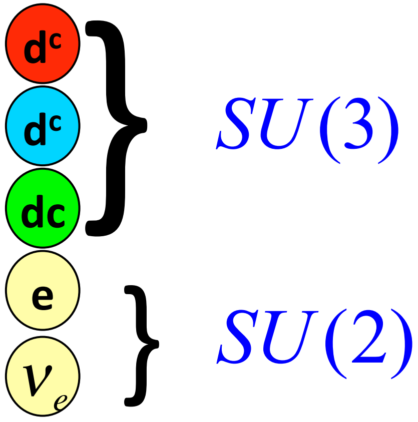


Either: No Grand Unification
or: more particles...

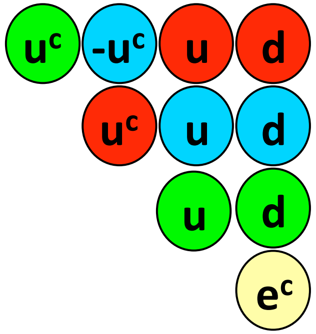
Grand Unification

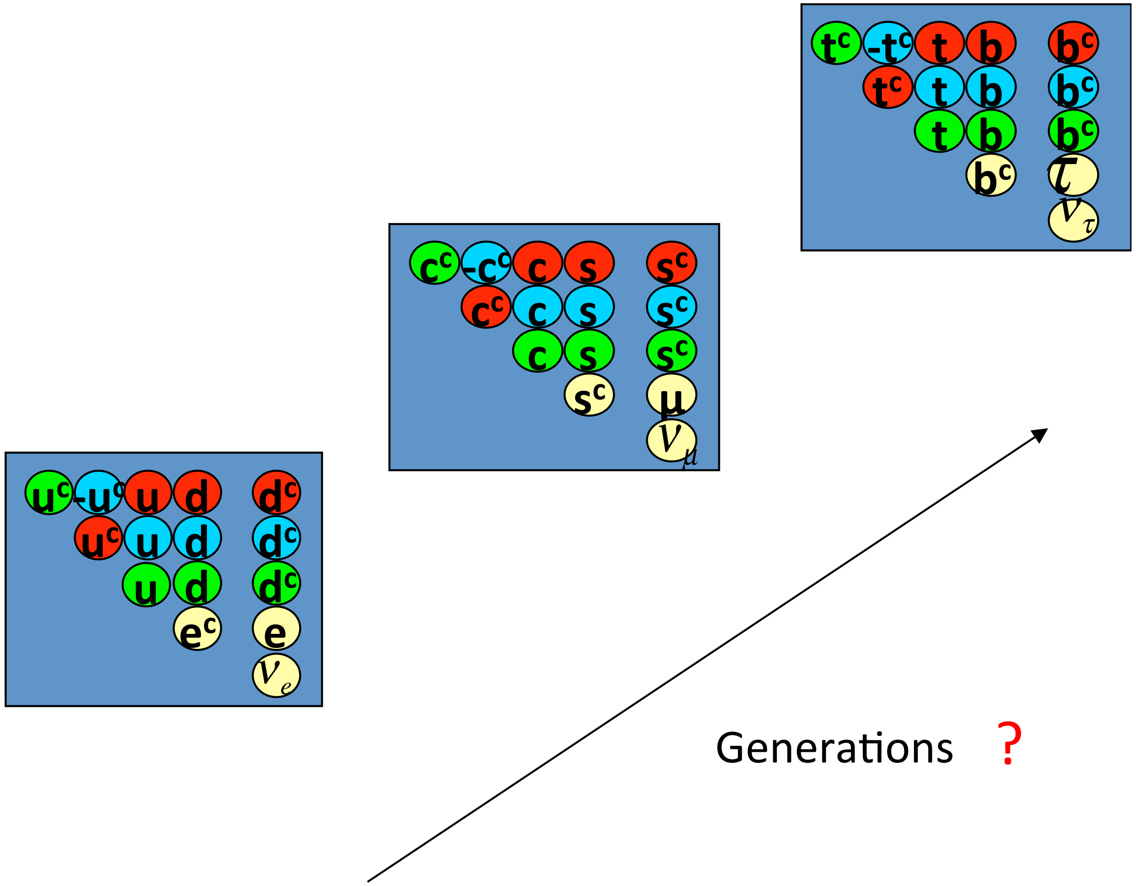
$$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$

$(\bar{5})_L$:



$(10)_L$:



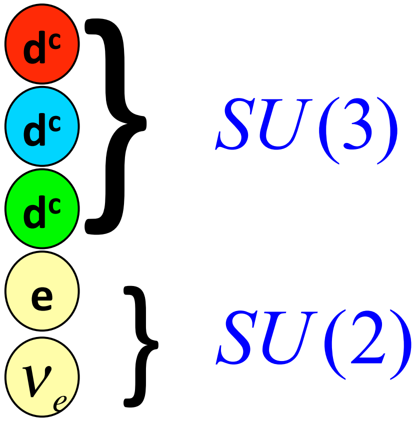


Grand Unification

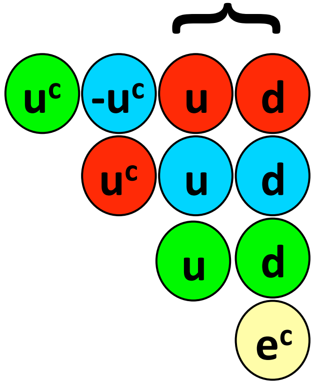
$SO(10) \supset SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$

Anomaly free

$(\bar{5})_L$:



$(10)_L$:



$\nu_{e,L}{}^{c} \equiv \nu_{e,R}$

$$(16)_L = (10)_L + (\bar{5})_L + (1)_L$$

$SO(10)$ **:** Group of matrices R † that leave invariant length of 10-dim vector

$$R^T R = RR^T = 1$$

†Adjoint representation $SO(n):\ n^2-(n^2+n)/2 = n(n-1)/2$

$SO(10)$ 45 gauge bosons

$SO(10)$ **:** Group of matrices R[†] that leave invariant length of 10-dim vector

$$R^T R = RR^T = 1$$

[†] Adjoint representation $SO(n):\ n^2-(n^2+n)/2=n(n-1)/2$

$SO(10)$ 45 gauge bosons

Rank 5

$$\begin{tikzcd}
 & SU(5)\times U(1) \\
SO(10) \arrow[ur] \arrow[dr] & \\
 & SU(4)\times SU(2)_L\times SU(2)_R
\end{tikzcd}$$

$45=24+1+10+\overline{10}$

$SO(10)$: Group of matrices R † that leave invariant length of 10-dim vector

$$R^T R = R R^T = 1$$

†Adjoint representation $O(n):\ n^2-(n^2+n)/2=n(n-1)/2$

$SO(10)$ 45 gauge bosons

Spinorial (16 dim) representation:

$c.f.\ SO(3) \sim SU(2)$ $\qquad \psi_{\alpha=1,2}, \quad R = e^{i\omega^{ab}\sigma_{ab}}, \quad \sigma_{ab} = \frac{1}{2}\varepsilon_{abc}\sigma_c \equiv \frac{i}{2}\left[\sigma_a, \sigma_b\right]$

$$SO(10) \qquad \underset{\nwarrow\ 2^4}{\chi_{16}^{\pm}} = \psi_1 \times \psi_2 \times \psi_3 \times \psi_4 \times \psi_5 \quad \text{with} \quad \sum_{i=1}^{5} \sigma_3^i = \pm 1$$

$SO(10)$: Group of matrices R† that leave invariant length of 10-dim vector

$R^T R = RR^T = 1$

Det R=1

†Adjoint representation $SO(n):\ n^2 - (n^2+n)/2 = n(n-1)/2$

$SO(10)$ 45 gauge bosons

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$$SO(10) \quad \chi_{16}^{\pm} = \psi_1 \times \psi_2 \times \psi_3 \times \psi_4 \times \psi_5 \quad \text{with} \quad \sum_{i=1}^{5} \sigma_3^i = \pm 1$$

Standard Model embedding:

$$SO(10) \supset SO(6) \times SO(4) \sim SU(4) \times SU(2) \times SU(2)$$

$$\supset SU(3) \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$$

$SO(10)$: Group of matrices R† that leave invariant length of 10-dim vector

$$R^T R = R R^T = 1$$

†Adjoint representation $SO(n):\ n^2-(n^2+n)/2=n(n-1)/2$

Det R=1

$SO(10)$ 45 gauge bosons

Spinorial (16 dim) representation:

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$$SO(10) \quad \chi^{\pm}_{16}=\psi_1\times\psi_2\times\psi_3\times\psi_4\times\psi_5 \quad \text{with} \quad \sum_{i=1}^{5}\sigma_3^i=\pm1$$

Standard Model embedding:

$$SO(10)\supset SO(6)\times SO(4)\sim SU(4)\times SU(2)\times SU(2)$$

$$\supset SU(3)\times U(1)_{B-L}\times SU(2)_L\times SU(2)_R \qquad \begin{cases}\tau^L{}_3=-\sigma_3\times1+1\times\sigma_3\\ \tau^L{}_\pm=\sqrt{2}\sigma_\mp\times\sigma_\pm\end{cases}$$

Identification of states of χ_{16}^{+} $\quad 16^{+}=10+\bar{5}+1$

$$\begin{bmatrix} u^i \\ d^i \end{bmatrix}_L = \begin{bmatrix} |++--+\rangle, |+-+-+\rangle, |-++-+\rangle \\ |++-+-\rangle, |+-++-\rangle, |-+++-\rangle \end{bmatrix} \qquad (3,2)$$

$$u_{iL}^c = \left(|+--++\rangle, |-+-++\rangle, |--+++\rangle \right) \qquad (\bar{3},1)$$

$$d_{iL}^c = \left(|+----\rangle, |-+---\rangle, |--+--\rangle \right) \qquad \left(\bar{3},1\right)$$

$$\begin{bmatrix} \nu \\ e^- \end{bmatrix}_L = \begin{bmatrix} |----+\rangle \\ |---+-\rangle \end{bmatrix} \qquad (1,2)$$

$$e_L^+ = |+++--\rangle \qquad (1,1)$$

$$N_L = |+++++\rangle \qquad (1,1)$$

$$(\tau^L{}_3 = -\sigma_3 \times 1 + 1 \times \sigma_3,\ \tau^L{}_{\pm} = \sqrt{2}\sigma_{\mp} \times \sigma_{\pm})$$

Alternative structures

$SU(5)$

$$SO(10)\xrightarrow[16]{M_X^{'}}SU(5)\xrightarrow[45]{M_X}SU(3)\times SU(2)\times U(1)\xrightarrow[10]{M_W}SU(3)\times SU(2)\times U(1)$$

$$10_{SO(10)}\equiv\left(5+\bar{5}\right)_{SU(5)}$$

Alternative structures

$SU(5)$

$Q = T_3 + \frac{Y}{2}$

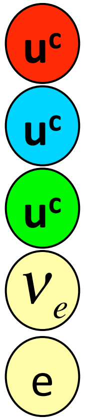
$$SO(10)\xrightarrow[16]{M'_X} SU(5)\xrightarrow[45]{M_X} SU(3)\times SU(2)\times U(1)\xrightarrow[10]{M_W} SU(3)\times U(1)$$

Flipped SU(5)

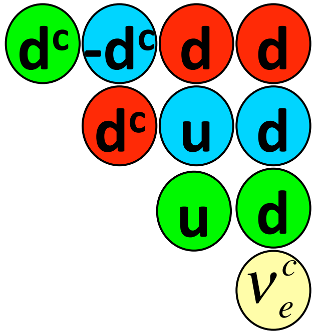
$Q = T_3 - \frac{1}{5}Y_Z + \frac{2}{5}\tilde{Y}_\chi$

$$SO(10)\xrightarrow[16]{M'_X} SU(5)\times U(1)_\chi \xrightarrow{M_X} SU(3)\times SU(2)\times U(1)_Z \times U(1)\chi$$

$(\bar{5})_L$:



$(10)_L$:



$(1)_L$: $\{e^c\}$

Alternative structures

$SU(5)$

$Q=T_3+\frac{Y}{2}$

$$SO(10)\xrightarrow[16]{M_X'}SU(5)\xrightarrow[45]{M_X}SU(3)\times SU(2)\times U(1)\xrightarrow[10]{M_W}SU(3)\times U(1)$$

Flipped SU(5)

$Q=T_3-\frac{1}{5}Y_Z+\frac{2}{5}\tilde{Y}_{\chi}$

$$SO(10)\xrightarrow[16]{M_X'}SU(5)\times U(1)_{\chi}\xrightarrow{M_X}SU(3)\times SU(2)\times U(1)_Z\times U(1)\chi$$

$10_H.10_H.5_h\rightarrow\left\langle\nu^c_H\right\rangle d^c_H D$ - simple doublet -triplet splitting

$10_f.\overline{10}_H.\phi\rightarrow\left\langle\nu^c_{\overline{H}}\right\rangle\nu^c\phi$ - right handed neutrino masses

Alternative structures

$SU(5)$

$$SO(10)\xrightarrow[16]{M'_X}SU(5)\xrightarrow[45]{M_X}SU(3)\times SU(2)\times U(1)\xrightarrow[10]{M_W}SU(3)\times U(1)$$

Pati-Salam

leptons - 4th colour $\begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}$

$$SO(10)\xrightarrow[54]{M_U}SU(4)\times SU(2)_L\times SU(2)_R\xrightarrow[45]{M_C}SU(3)\times U(1)_{B-L}\times SU(2)_L\times SU(2)_R$$

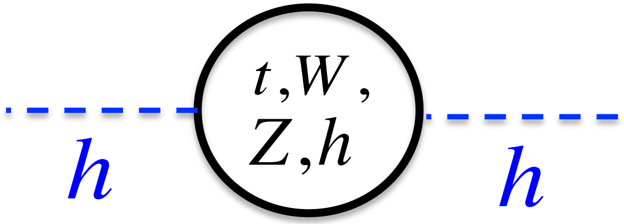
Parity restored

$$\xrightarrow[16]{M_R}SU(3)\times SU(2)\times U(1)\xrightarrow[10]{M_W}SU(3)\times U(1)$$

The Standard Model – unanswered questions

- Complicated choice of multiplets
- Fractional and integral charges?
- Neutrino masses?
- Only partial unification $A_\mu^\gamma = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$
- Many parameters 16 (25)
- The hierarchy problem
- Strong CP problem
- Dark matter, baryogenesis, inflation.....

Hierarchy problem?



$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2\right)\Lambda^2 = \left(\frac{\Lambda}{500GeV}\right)^2$$

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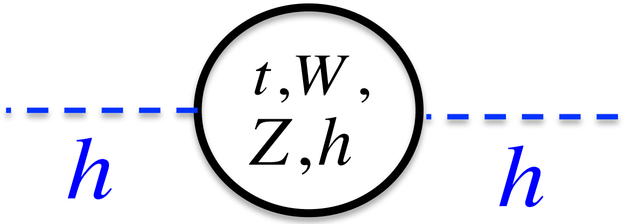

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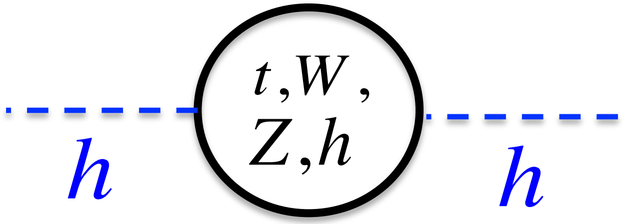
Field theory: δm^2 not measureable

...only $m^2 = m_0^2 + \delta m^2$ "physical"

Only $m^2 = 0$ special (classical scale invariance)

$$\Rightarrow \quad \frac{d\,m_H^2}{d\ln\mu} = \frac{3m_H^2}{8\pi^2}\left(2\lambda + y_t^2 - \frac{3g_2^2}{4} - \frac{3g_1^2}{20}\right)$$

Hierarchy problem?



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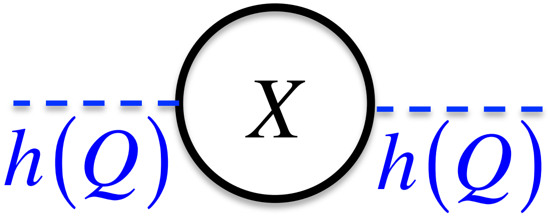
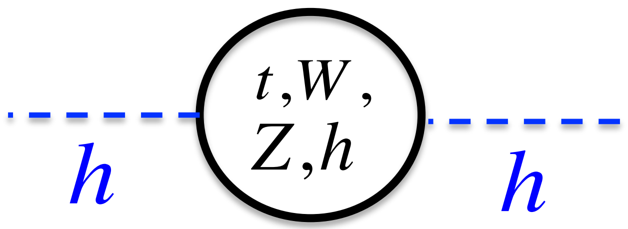
$c.f.$ **GUTS:**

$$\delta m_h^2 \propto M_X^2 \ln\left(\frac{Q^2 + M_X^2}{\Lambda^2}\right)$$



- "real hierarchy problem"

Hierarchy problem?



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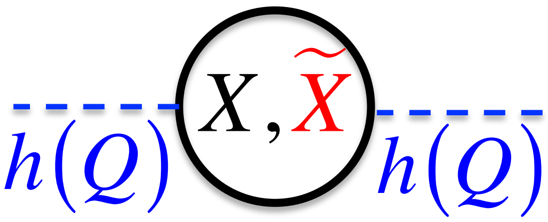

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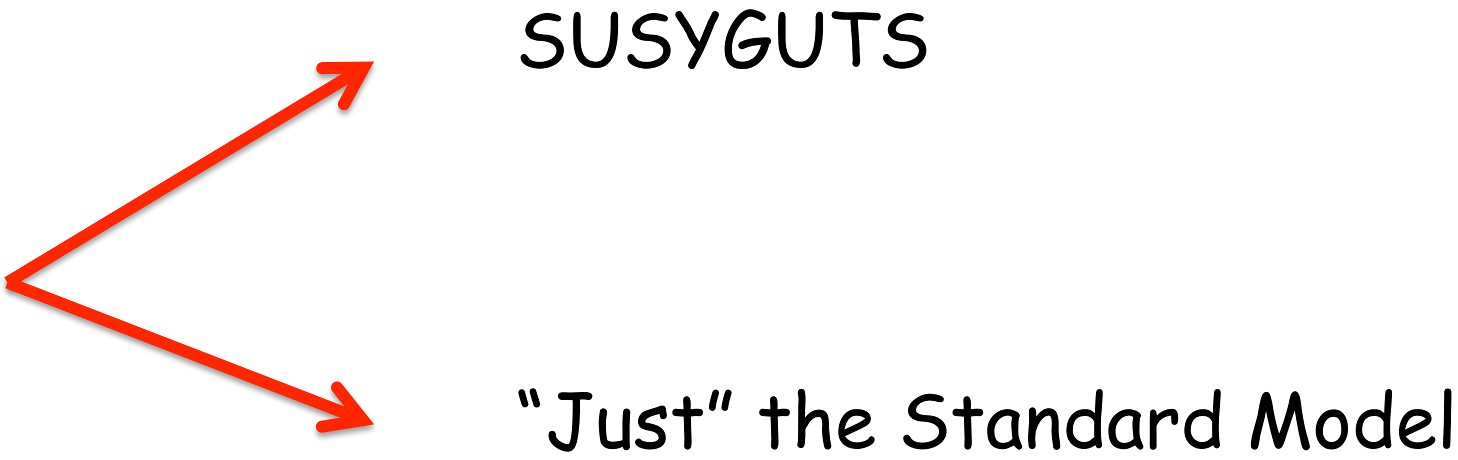
- "real hierarchy problem"

GUTS $\Longrightarrow$ SUSYGUTS

$$\delta m_h^2 \propto \left(M_{\widetilde{X}}^2 - M_X^2\right)$$

Hierarchy problem



SUSYGUTS

“Just” the Standard Model

Hierarchy problem

SUSYGUTS

“Just” the Standard Model

- no heavy GUT-like states
- any $m^2 = m_0^2 + \delta m^2$ possible
- $m^2 = 0$ … classical scale invariance
- Emergent symmetry?

$$m^2 = m_0^2 \left(\frac{\Lambda^2}{\Lambda_0^2} \right)^{\gamma_m} \qquad \left\{ \begin{array}{l} \text{D>4} \\ \text{Non-renormalisable ints} \end{array} \right.$$

III .JSM

No heavy thresholds?

- Neutrino masses?
- Baryogenesis?
- Strong CP problem?
- Gravity?

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Neutrino masses:

Add singlet neutrinos ν_{Ra}

$$L_{mass} = h_a \bar{l}_a \nu_{Ra} H + \frac{M_{ab}}{2} \nu_{Ra}^T C \nu_{Rb}$$

$$e.g. \quad h_A^2 = 5.10^{-14}, h_B^2 = 5.10^{-15}, M_a = 20GeV$$

$$m_A \simeq 0.1 eV, \quad m_B \simeq 0.01 eV$$

Ultra-weak:

Natural due to chiral symmetry

Baryogenesis

$$L_{mass} = h_a \bar{l}_a \nu_{Ra} H + \frac{M_{ab}}{2} \nu_{Ra}^T C \nu_{Rb}$$

- ν_{Ra} produced via Yukawa interactions $L_A = L_B = L_C = 0$

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Only

- ^ $\nu_{R\,A,B}$ in thermal equilibrium by t$_{EW}$ when sphalerons inoperative

$$\Delta_{L\,AB} = L_A + L_B \xrightarrow[Sphalerons]{} \Delta B = \Delta_{L\,AB} / 2 \quad ✓$$

Akhmedov, Rubakov, Smirnov

see also Shaposhnikov et al

Strong CP problem:

$$\frac{\theta}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}, \quad \theta \leq 10^{-10} ??$$

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Make θ a dynamical variable the axion, a….θ=0 at minimum of its potential

… complex scalar field, S

$$S = (|S| + f_a) e^{i\frac{a}{f_a}}, \qquad 10^{10} GeV \leq f_a \leq 10^{12} GeV$$

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DFSZ axion: 2 Higgs doublets $H_{1,2}$, complex singlet, S

$$\begin{aligned} V(H_1, H_2) &= \frac{\lambda_1}{2}|H_1|^4 + \frac{\lambda_2}{2}|H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 \\ &+ \lambda_4 |H_1^\dagger H_2|^2 + \zeta_1 |S|^2 |H_1|^2 + \zeta_2 |S|^2 |H_2|^2 \\ &+ \zeta_3 S^2 H_1 H_2 + h.c. + \zeta_4 \, |S|^4 \end{aligned}$$

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PQ symmetry: $H_1 \rightarrow H_1 e^{i\alpha}, \quad H_2 \rightarrow H_2 e^{i\beta}, \quad S \rightarrow S e^{-i(\alpha+\beta)/2}$

Axion, a : Pseudo Goldstone boson od spontaneously broken PQ

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Axion, a : Pseudo Goldstone boson od spontaneously broken PQ

Ultra weak sector: $\zeta_{1,2,3} \le 10^{-20} \left(\frac{10^{12} GeV}{f_a} \right)^2$

Ultra weak sector:

ζ_i multiplicatively renormalised

(Underlying shift symmetry $S \to S + \delta$)

Origin of large vev?

Start with $m = m_0 + \delta m = 0$ (Classical scale invariance)

Dimensional transmutation (Coleman Weinberg)

Coleman Weinberg in DFSZ model

$$V_{DFSZ}(H_1,H_2,S) \simeq \frac{\lambda_1}{2}\left(|H_1|^2+\frac{\zeta_1}{\lambda_1}|S|^2\right)^2+\frac{1}{64\pi^2}\left(\zeta_2|S|^2\right)^2\left(-\frac{1}{2}+\ln\frac{|S|^2}{f_a^2}\right)$$

$$+\frac{\lambda_2}{2}|H_2|^4+\zeta_3 S^2 H_1 H_2+h.c.$$

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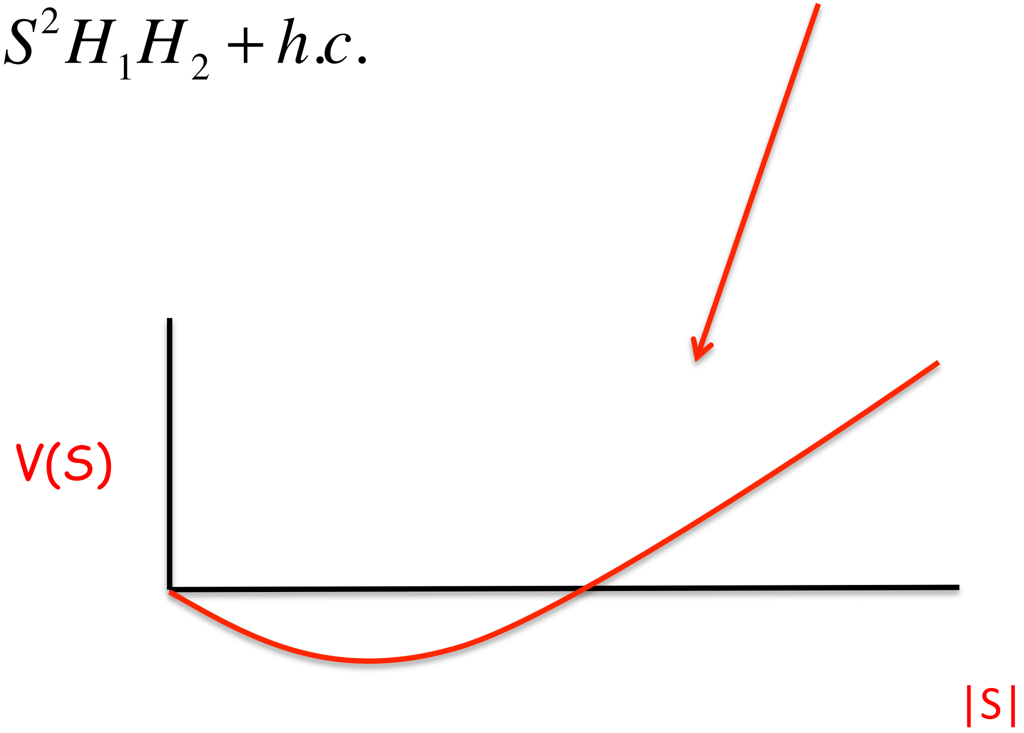

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$$+\frac{\lambda_2}{2}|H_2|^4 + \zeta_3 S^2 H_1 H_2 + h.c. \quad (\zeta_2 > \zeta_1 > \zeta_3 \text{ assumed})$$

$$\mathrm{v}_S = f_a, \quad \mathrm{v}_{H_1} = \frac{\zeta_1}{\lambda_1} f_a, \quad \mathrm{v}_{H_2} = \frac{\zeta_3}{2\zeta_2}\mathrm{v}_{H_1}$$

$$\boxed{m_{H_2^0}^2 = m_{H^\pm}^2 = m_X^2 = -\frac{\zeta_2}{2\zeta_1} m_h^2}$$

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$$\boxed{m^2_{H_2^0} = m^2_{H^\pm} = m^2_X = -\frac{\zeta_2}{2\zeta_1} m_h^2}$$

$$m^2_{|S|} = -\left(\frac{\zeta_2^2}{32\pi^2\zeta_1}\right)^2 m_h^2 \simeq 13\left(\frac{10^{12}\,GeV}{\mathrm{v}_S}\right)^2\left(\frac{m_{H_2}}{m_h}\right)^4 eV^2$$

$|S|$ Pseudo-dilaton

K. Allison, C.Hill, GGR

Phenomenology

Collider signals

Ultra weak couplings … just 2HD model with nearly degenerate heavy Higgs

Direct (axion-like) searches for pseudo-dilaton?

Cosmology

If inflation scale below PQ phase transition $\Delta_I < 10^5 \left(\frac{10^{12}GeV}{f_a}\right)^{1/2} \left(\frac{m_{H_2}}{m_h}\right) GeV$

…. no cosmological constraints

If inflation scale above PQ phase transition

…. potential Polonyi problem:

Coughlan et al

$$\boxed{V(S_I) \sim +\frac{1}{64\pi^2}\left(\zeta_2 |S_I|^2\right)^2 \left(-\frac{1}{2} + \ln\frac{|S_I|^2}{f_a^2}\right)}$$

(stored energy after inflation)

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S rolls when $9H_r^2 = m_S^2$ ….. coherent state (zero momentum)

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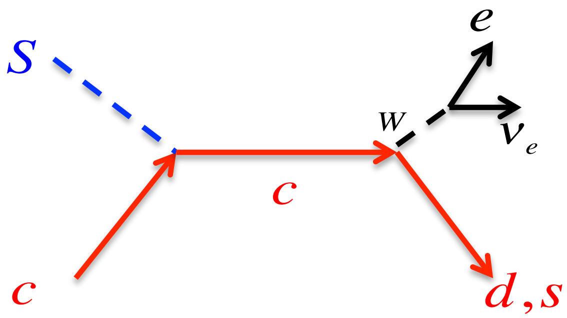
Resonant enhancement of annihilation:

$$\Gamma_s \simeq \frac{\sqrt{2}m_c^4}{\pi^{3/2}v_s^2\Gamma_c}\left(\frac{T}{m_c}\right)^{1/2} e^{-m_c/T}$$



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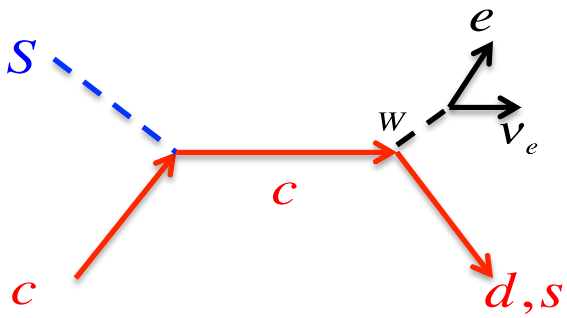

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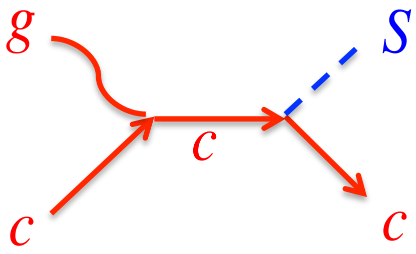


Inverse processes non resonant

$$\underline{\Omega_S \to 0} \qquad \Omega_a\,?$$

Summary - III

- "JSM" requires ultra-weak sectors – chiral and shift symmetries

- DFSZ axion + dimensional trasmutation $\Rightarrow f_a$

  ...consistent with classical scale invariance (not KSVZ model)

- Requires two Higgs doublets (type II couplings), light pseudo-dilaton

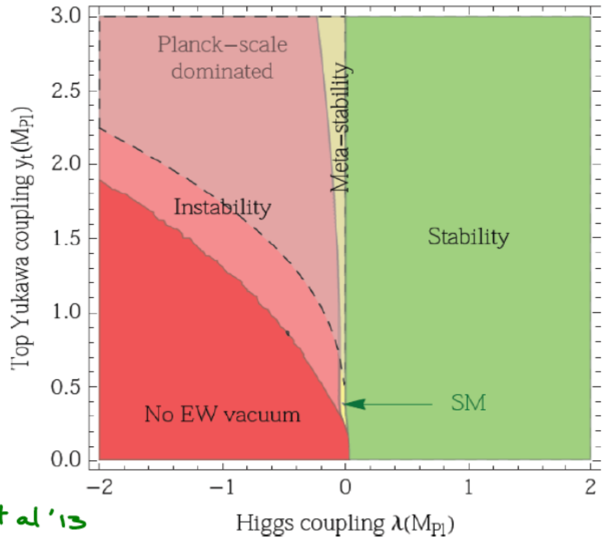
  $$m^2_{H^0_2} = m^2_{H^\pm} = m^2_X = R^2 m^2_h \qquad m_{|S|} \simeq 0.9\left(\frac{10^{12}\,GeV}{f_a}\right) R^2 eV$$

  h ≈ SM Higgs

  ...stable vacuum but loses simplicity of SM

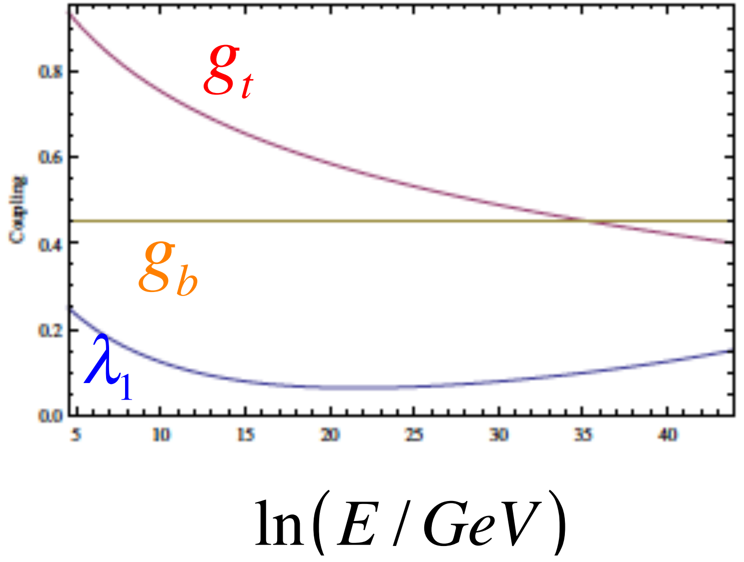
Energy dependence of couplings

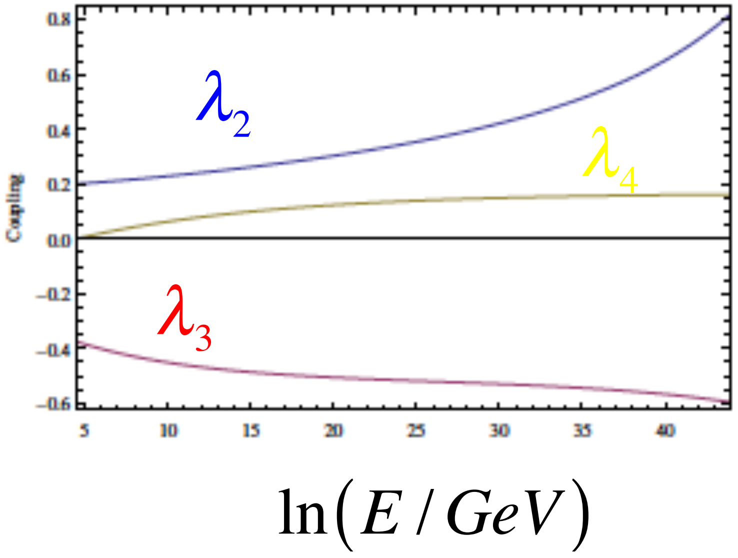
SM:



Buttazzo et al '13

DFSZ:





Summary - III

- "JSM" requires ultra-weak sectors – chiral and shift symmetries

- DFSZ axion + dimensional trasmutation $\Longrightarrow f_a$

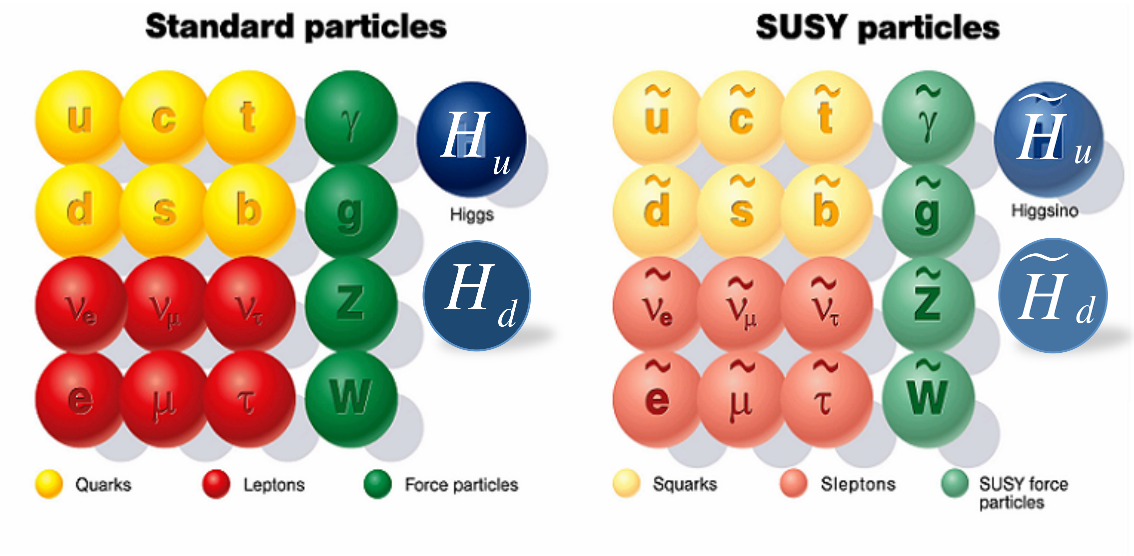
  ...consistent with classical scale invariance (not KSVZ model)

- Requires two Higgs doublets (type II couplings), light pseudo-dilaton

- But... no unification of forces and matter

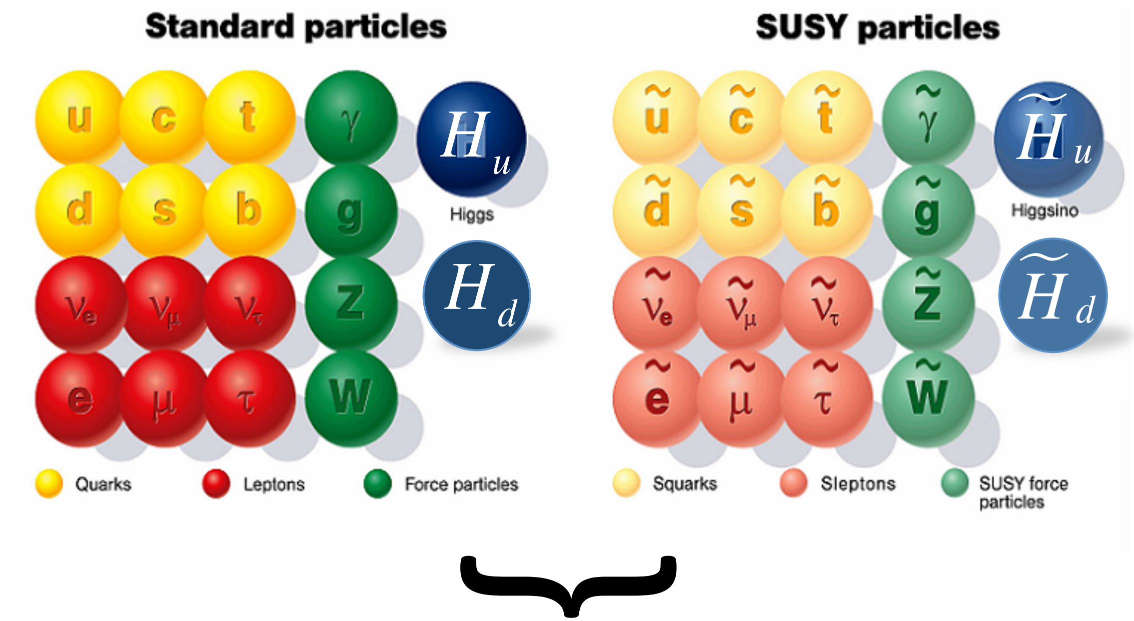
  speculative cutoff of quadratic divergence

IV. SUSY GUTS



$$G_{GUT} \times G_{Flavour} \times \left(N = 1\,\mathrm{SUSY}\right)$$

IV. SUSY GUTS



$$G_{GUT} \times G_{Flavour} \times \left(N = 1\,\mathrm{SUSY}\right)$$

Supermultiplets

SO(10): V_{45} Vector +3 φ_{16} chiral + H_{10} chiral +...

SUSY gauge coupling unification

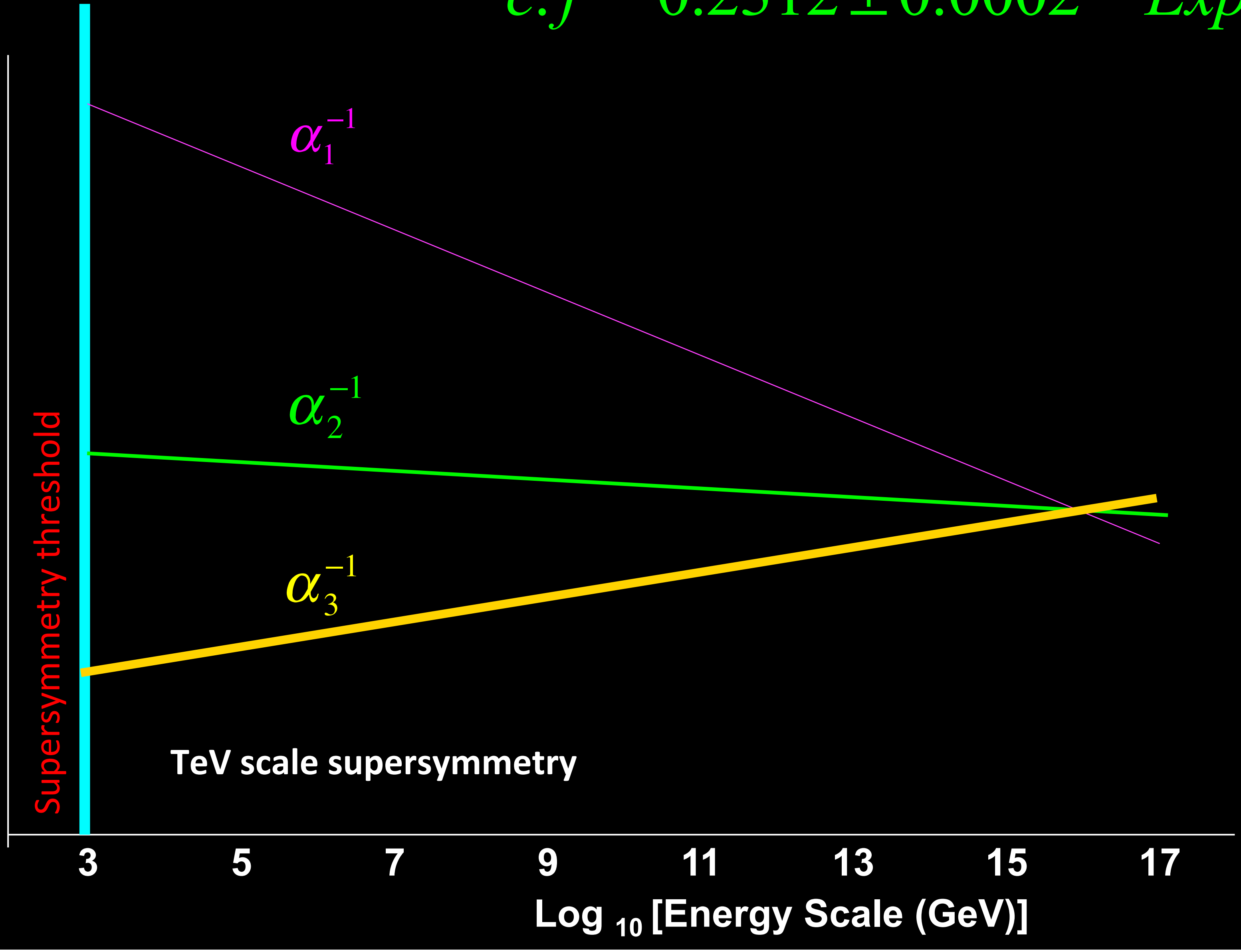
$$\alpha_i^{-1}(\mu) = \alpha^{-1}(M_X) + \frac{1}{2\pi} b_i \ln\left(\frac{M_X}{\mu}\right) + ..$$

$$b_i^{SM} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_g \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}$$

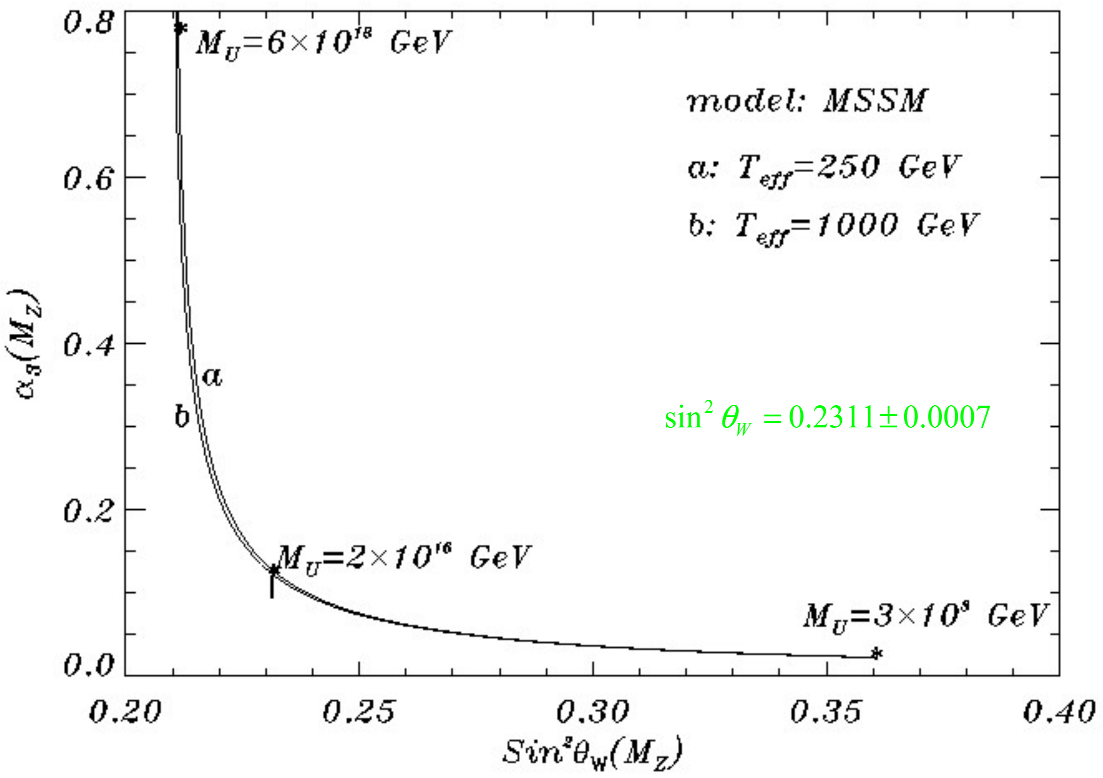
$$b_i^{MSSM} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_g \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + H \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

$$\sin^2\theta_W = 0.2337 \pm 0.0015$$

$$c.f \quad 0.2312 \pm 0.0002 \quad Expt$$



SUSY gauge coupling unification

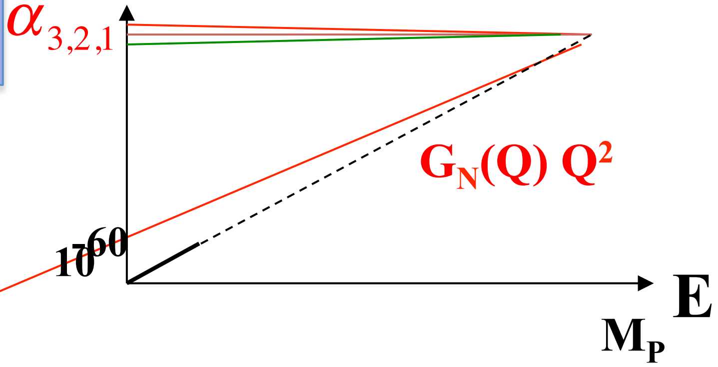


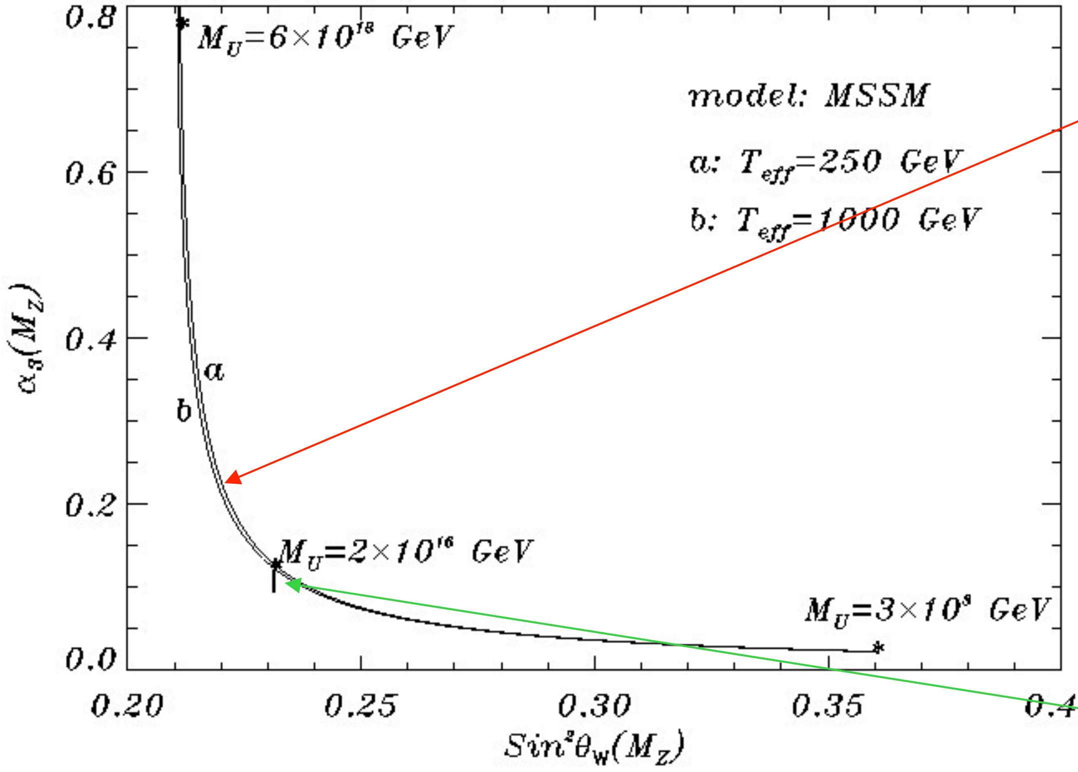
$$\sin^2\theta_W = 0.2334\pm0.0025-0.25(\alpha_s-0.119) \quad = 0.2311\pm0.0007 \quad (Expt)$$

$$\alpha_s = 0.134\pm0.01-4(\sin^2\theta_W-0.2334) \quad = 0.119\pm0.01 \quad (Expt)$$

SUSY gauge coupling unification

Unification with gravity?





$$M_U = (2.6 \pm 2).10^{16}\, GeV$$

$$\sin^2\theta_W = 0.23116(12) \quad (Expt)$$

$$\alpha_s = 0.134 \pm 0.01 - 4(\sin^2\theta_W - 0.23116) \qquad c.f. \quad 0.1184(7) \quad (Expt)$$

Gauge unification - Heterotic String

$$\boxed{L_{eff}^{HS} = \int \underbrace{d^{10}}_{\int d^4x\,V} x\sqrt{g}\,\underbrace{e^{-\phi}}_{\alpha_{10}^{-1}}\left(\frac{4}{\alpha'^4}R + \frac{k_i}{\alpha'^3}TrF_i^2 + \ldots\right)}$$

$\alpha' = 1/M_{string}^2$ only scale

$$G_N = \frac{\alpha_{10}\alpha'^4}{64\pi V}, \quad \alpha_{String} = \frac{\alpha_{10}\alpha'^3}{16\pi V} \quad \Rightarrow \quad G_N = \frac{\alpha_{String}\alpha'}{4}$$

$$\boxed{\frac{1}{g_i^2(M_Z)} = \frac{k_i}{g_{string}^2} + b_i \ln\left(\frac{M_{string}}{M_Z}\right) + \Delta_i}$$

$$\underline{M_{string} = g_{string}.M_{Planck} = 3.6\times 10^{17}\,GeV} \qquad c.f.M_U^{"\mathrm{expt}"} = (2.6\pm 2).10^{16}\,GeV$$

Gauge unification - Heterotic String

$$\boxed{L^{HS}_{eff} = \int \underbrace{d^{10}x}_{\int d^4x\, V} \sqrt{g}\, \underbrace{e^{-\phi}}_{\alpha_{10}^{-1}} \left(\frac{4}{\alpha'^4} R + \frac{k_i}{\alpha'^3} TrF_i^2 + \ldots\right)}$$

$\alpha' = 1/M^2_{string}$ only scale

$$G_N = \frac{\alpha_{10}\alpha'^4}{64\pi V}, \quad \alpha_{String} = \frac{\alpha_{10}\alpha'^3}{16\pi V} \quad \Rightarrow \quad G_N = \frac{\alpha_{String}\alpha'}{4}$$

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$\underline{M_{string} = g_{string}.M_{Planck} = 3.6\times10^{17}\,GeV}$ ....close...but not close enough!

..string threshold corrections, Δ_i?

Spontaneous symmetry breaking

$$SU(5) \xrightarrow[\Sigma_{24}]{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow[H_{\bar{5}}]{M_W} SU(3) \times U(1)$$

Spontaneous symmetry breaking

$$SU(5) \xrightarrow[\Sigma_{24}]{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow[H_{\bar{5}}]{M_W} SU(3) \times U(1)$$

$$P = \frac{\beta_2}{2} M \, Tr\left(\Sigma^2\right) + \frac{\beta_3}{3} Tr\left(\Sigma^3\right)$$

superpotential

Spontaneous symmetry breaking

$$SU(5)\xrightarrow[\Sigma_{24}]{M_X}SU(3)\times SU(2)\times U(1)\xrightarrow[H_{\bar{5}}]{M_W}SU(3)\times U(1)$$

$$P=\frac{\beta_2}{2}M\,Tr\left(\Sigma^2\right)+\frac{\beta_3}{3}Tr\left(\Sigma^3\right)$$

superpotential

$$V(\Sigma)=\sum_a\left|\frac{\partial P}{\partial\Sigma^a}\right|^2=Tr\left|\beta_3\Sigma^2+\beta_2M\Sigma-I\frac{\beta_3}{5}Tr\left(\Sigma^2\right)\right|^2$$

$$\left(\frac{\partial P}{\partial\Sigma^a}\to\frac{\partial P}{\partial\Sigma^i_j}-\frac{1}{N}\delta^i_jTr\left(\frac{\partial P}{\partial\Sigma}\right),\quad i,j=1..5,\quad a=1..24\right)$$

$$\left.\begin{array}{l}\langle\Sigma\rangle=0\\ \langle\Sigma\rangle=\mathrm{v}_4\mathrm{Diagonal}(1,1,1,1,-4)\\ \langle\Sigma\rangle=\mathrm{v}_3\mathrm{Diagonal}(2,2,2,-3,-3)\end{array}\right\}$$

Degenerate ↗ SUGRA

Degenerate ↘ Radiative corrections

Spontaneous symmetry breaking

$$SU(5) \xrightarrow[\Sigma_{24}]{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow[H_5]{M_W} SU(3) \times U(1)$$

Spontaneous symmetry breaking

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$$P_{5_M} = -\frac{1}{\sqrt{2}} M_{ij}^{d} \psi_{i\alpha} \chi_j^{\alpha\beta} H_{d\beta} - \frac{1}{4} M_{ij}^{u} \varepsilon_{\alpha\beta\gamma\delta\rho} \chi_i^{\alpha\beta} \chi_j^{\gamma\delta} H_u^{\rho}$$

Spontaneous symmetry breaking

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After diagonalising down quark mass matrix:

$$\left(\quad \boxed{\begin{array}{ll} m_d = m_e & ✗ \\ m_s = m_\mu & ✗ \\ m_b = m_\tau & ✓? \end{array}} \quad \right)$$

Spontaneous symmetry breaking

$$SU(5)\xrightarrow[\Sigma_{24}]{M_X}SU(3)\times SU(2)\times U(1)\xrightarrow[H_5]{M_W}SU(3)\times U(1)$$

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$$P_{Higgs}=\mu H_u H_d+\lambda H_u\Sigma H_d$$

$$V=\left(\left|\mu H_u+\lambda H_u\Sigma\right|^2+\left|\mu H_d+\lambda\Sigma H_d\right|^2\right)+\left|H_uH_d-\frac{1}{5}\left(H_uH_d\right)\right|^2$$

Spontaneous symmetry breaking

$$SU(5)\xrightarrow[\Sigma_{24}]{M_X}SU(3)\times SU(2)\times U(1)\xrightarrow[H_5]{M_W}SU(3)\times U(1)$$

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✗

$$V=\left(\left|\mu H_u+\lambda H_u\Sigma\right|^2+\left|\mu H_d+\lambda\Sigma H_d\right|^2\right)+\left|H_u H_d-\frac{1}{5}\left(H_u H_d\right)\right|^2$$

Must forbid these terms by symmetry

Spontaneous symmetry breaking

$$SU(5)\xrightarrow[\Sigma_{24}]{M_X}SU(3)\times SU(2)\times U(1)\xrightarrow[H_5]{M_W}SU(3)\times U(1)$$

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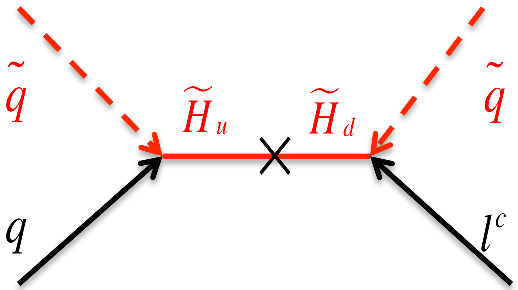
Must forbid these terms by symmetry

+ doublet- triplet splitting



D=5 proton decay amplitude

Doublet -triplet splitting

Missing doublet mechanism

No (1,2) component

$$\Theta_{50} = (8,2)+(6,3)+(\bar{6},1)+(3,2)+(\bar{3},1)+(1,1)$$

Doublet -triplet splitting

Missing doublet mechanism

No (1,2) component

$$\Theta_{50} = (8,2) + (6,3) + \left(\overline{6},1\right) + (3,2) + \left(\overline{3},1\right) + (1,1)$$

$$P_{MD} = b\,\Theta\Sigma_{75}H_u + b'\,\overline{\Theta}\Sigma_{75}H_d + \widetilde{M}\,\overline{\Theta}\Theta$$

$\left\langle \Sigma_{75} \right\rangle \propto M$ breaks SU(5) to SM

Doublet -triplet splitting

Missing doublet mechanism

No (1,2) component

$$\Theta_{50}=(8,2)+(6,3)+\left(\bar{6},1\right)+(3,2)+\left(\bar{3},1\right)+(1,1)$$

$$P_{MD}=b\Theta\Sigma_{75}H_u+b'\overline{\Theta}\Sigma_{75}H_d+\widetilde{M}\overline{\Theta}\Theta$$

$\left\langle\Sigma_{75}\right\rangle\propto M$ breaks SU(5) to SM

$$P_{MD}\supset bM\,\Theta_3H_{uT}+b'M\,\overline{\Theta}_3H_{dT}+\widetilde{M}\overline{\Theta}_3\Theta_3$$

Triplets get mass $\frac{M^2}{\widetilde{M}}$ (Still need to drive SSB – later)

Doublet -triplet splitting

Higher dimensions (String unification)

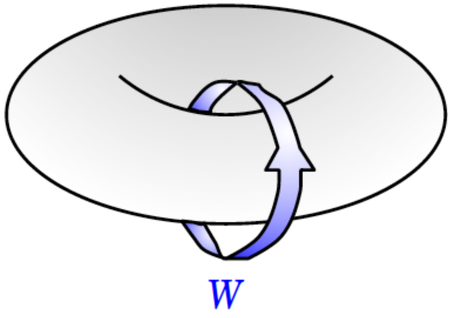
Compactification: $K = K_0 / H$

freely acting discrete group

Wilson line breaking: $W: \quad \overline{H} \subset G$

embedding of H into gauge group G

Massless states: $H \otimes \overline{H}$ singlets



$$W = P \exp\left(-i \int_{\gamma} T^a A_m^a \, dx^m \right)$$

Doublet -triplet splitting

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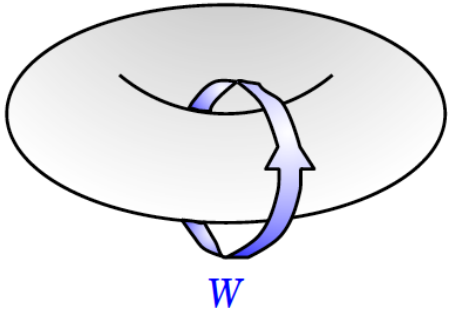


$$W = P\exp\left(-i\int_{\gamma} T^a A_m^a\, dx^m\right)$$

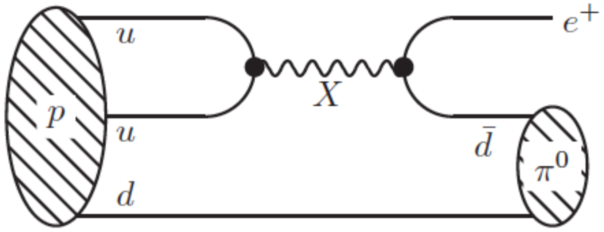
Breit, Ovrut, Segre

$$e.g. \quad SU(5): \quad H = Z_3, \quad \overline{H} = Diag(\alpha,\alpha,\alpha,1,1), \quad \alpha = e^{2i\pi/3}$$

$$\left(R \otimes \overline{R}\right): \quad (1 \otimes \overline{5}) \rightarrow \begin{pmatrix} H^- \\ \overline{H}^0 \end{pmatrix}_1, \quad \left(3,\overline{5}\right) \rightarrow \begin{pmatrix} e \\ \nu_e \end{pmatrix}_1 \oplus \begin{pmatrix} d^c \\ d^c \\ d^c \end{pmatrix}_{\alpha^2}, \quad \text{Matter} \quad \rightarrow \quad \left(3,\overline{5}+10\right)$$

SUSY GUTS - Nucleon decay

SUSY GUTS - Nucleon decay

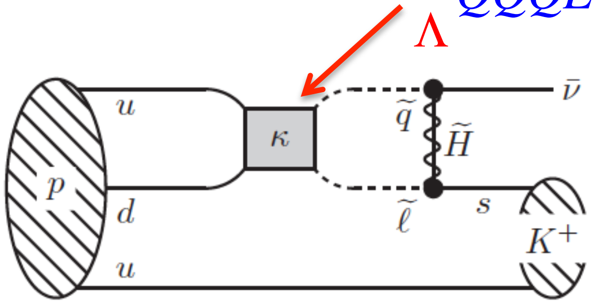


(a) Dimension 6.

$p \to \pi^0 + e^+$

$\tau_{p \to e^+ \pi^0} > 1 \times 10^{34}$ yrs, $M_X > 10^{16}\, GeV$

$\frac{1}{\Lambda} QQQL|_F$

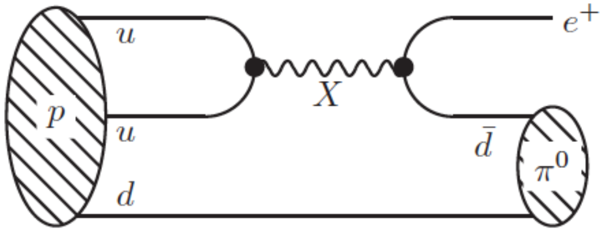


(b) Dimension 5.

$p \to K^+ + \bar{\nu}$

$\tau_{p \to K^+ \bar{\nu}} > 3.3 \times 10^{33}\, yrs$

SUSY GUTS - Nucleon decay



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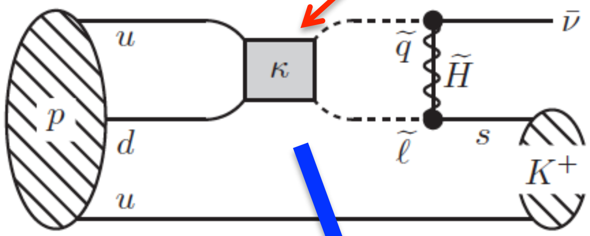

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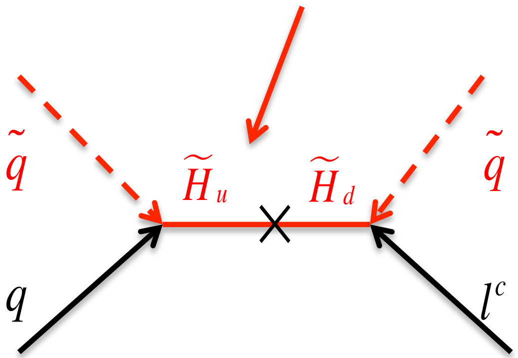
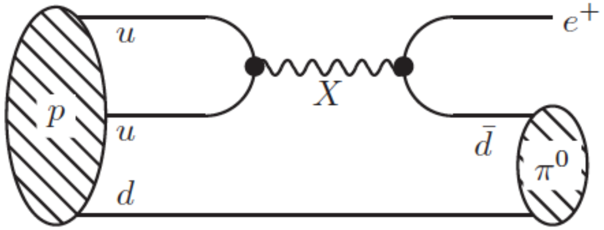


D=5 proton decay amplitude

SUSY GUTS - Nucleon decay

$$\frac{1}{\Lambda} QQQL\big|_F$$



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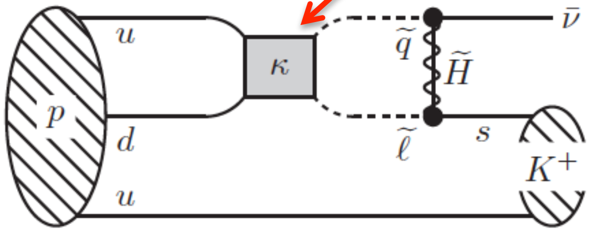

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$$p \to \pi^0 + e^+$$

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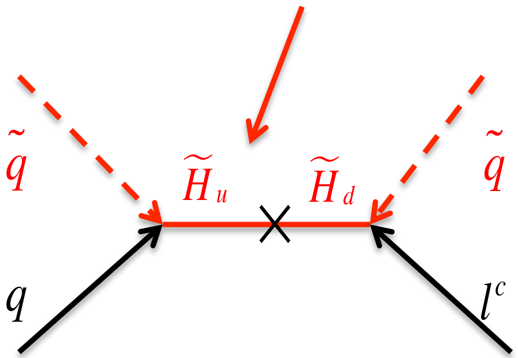
$$\frac{1}{M_{\widetilde{H}}} QQQL\big|_F$$

$$\Delta(B-L)=0$$



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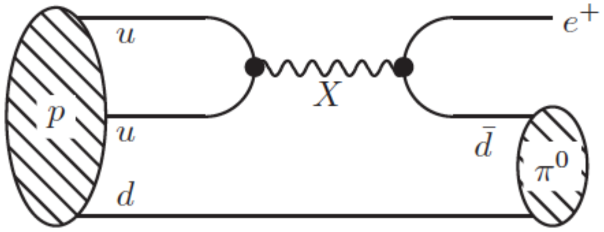

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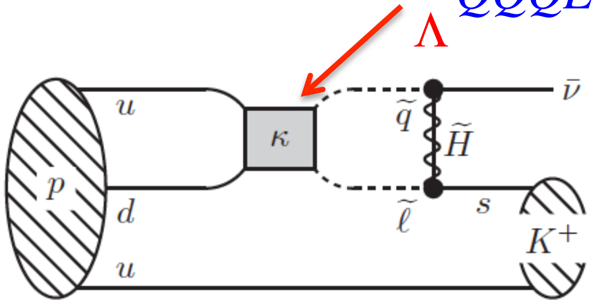

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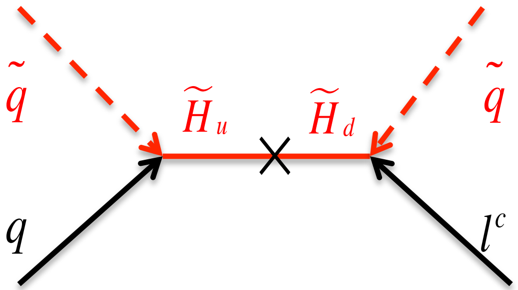
(b) Dimension 5.

$p \to K^+ + \bar{\nu}$

$\tau_{p\to K^+\bar{\nu}} > 3.3\times 10^{33}\, yrs$

$\Lambda > 10^{27}\, GeV,\ 10^9\, M_{Planck}$???

$\Delta(B-L)=0$



D=5 proton decay amplitude

SUSY extensions of the Standard Model

$$
\begin{aligned}
W = {}& h^E L H_d \overline{E} + h^D Q H_d \overline{D} + h^U Q H_u \overline{U} + \mu H_d H_u \\
& + \lambda L L \overline{E} + \lambda' L Q \overline{D} + \kappa L H_u + \lambda'' \overline{U}\,\overline{D}\,\overline{D} \\
& + \frac{1}{M}\left(QQQL + QQQH_d + Q\overline{U}\,\overline{E} H_d + \ldots(\not{L}) \right)
\end{aligned}
$$

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$$

$e.g.(LH_u)^2$

SUSY extensions of the Standard Model

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$$+ \frac{1}{M}\left(QQQL + QQQH_d + Q\overline{U}\,\overline{E} H_d + \ldots (\not{L}) \right)$$

R-parity: Z_2

$H_u, H_d \quad +1$

$L, \overline{E}, Q, \overline{D}, \overline{U}, \quad \theta \quad -1$

SUSY states odd

Weinberg, Sakai

SUSY extensions of the Standard Model

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R-parity: Z_2

SUSY states odd

Weinberg, Sakai

Baryon "parity": Z_3

Q	1
$\bar{D}, H_u$	α
$L, \bar{E}, \bar{U}, H_d$	α^2

LSP unstable

Discrete gauge symmetry
-anomaly free

Ibanez, GGR

SUSY extensions of the Standard Model

$$W = h^E L H_d \overline{E} + h^D Q H_d \overline{D} + h^U Q H_u \overline{U} + \mu H_d H_u$$

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Proton hexality: $Z_6 = Z_2^R \times Z_3^B$

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Dreiner, Luhn, Thormeier

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μ term, GUTs?

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R-parity:	Z_2	SUSY states odd
Baryon "parity":	Z_3	LSP unstable
Proton hexality:	$Z_6 = Z_2^R \times Z_3^B$	LSP stable
Z_N^R R-symmetry	N=4,6,8,12,24	LSP stable

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg, Vaudrevange
Babu, Gogoladze,Wang

A unique solution : Z_4^R discrete R symmetry

MSSM spectrum
No perturbative μ term
Commutes with SO(10)
Anomaly cancellation

N	q_{10}	$q_{\bar{5}}$	q_{H_u}	q_{H_d}	q_N
4	1	1	0	0	2

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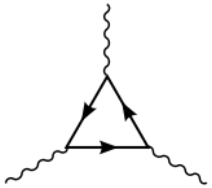
4	1	1	0	0	2

Green Schwarz term

$$A_{G-G-\mathbb{Z}_N} = \rho \mod \eta \quad \left\{ \begin{array}{l} N \\ N/2 \end{array} \right.$$



$$A_{\mathrm{SU}(3)-\mathrm{SU}(3)-\mathbb{Z}_N} = \frac{1}{2}\sum_i \left[3 \cdot q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 3R$$

$$A_{\mathrm{SU}(2)-\mathrm{SU}(2)-\mathbb{Z}_N} = \frac{1}{2}\sum_i \left[3 \cdot q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 2R + \frac{1}{2}\left(q_H + q_{\bar{H}} - 2R\right)$$

$$A_{\mathrm{U}(1)_Y\text{-}\mathrm{U}(1)_Y\text{-}\mathbb{Z}_N^R} = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^g + q_{\overline{\mathbf{5}}}^g\right) + \frac{3}{5}\left[\frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 11\right] \qquad (R = 1)$$

$$\Rightarrow N = 3, \boxed{4}, 6, 8, 12, 24$$

A unique solution : Z_4^R discrete R symmetry

MSSM spectrum
No perturbative μ term
Commutes with SO(10)
Anomaly cancellation

N	q_{10}	$q_{\bar{5}}$	q_{H_u}	q_{H_d}	q_N
4	1	1	0	0	2

D=5 operators

$\frac{1}{M}Q\cancel{Q}QL \qquad \frac{1}{M}LLH_uH_u$

Weinberg operator

SUSY breaking

$\langle W\rangle,\langle\lambda\lambda\rangle$ R=2 non=perturbative breaking

$Z_{4R}\to Z_2^R \quad R-parity$

Domain walls safe

$\mu\sim m_{3/2},\ O(\frac{m_{3/2}}{M^2}QQQL)$

$M_{higgs} \approx M_{SUSY}$

$\mu, \cancel{B}, \cancel{L}$

Nucleon decay outlook

- Nucleon decay D=6 operators

Operator renormalisation

$$\tau(p \to \pi^0 e^+) = \left(\frac{M_{\rm GUT}}{10^{16}\ {\rm GeV}}\right)^4 \left(\frac{1/35}{\alpha_{\rm GUT}}\right)^2 \left(\frac{0.015\ {\rm GeV}^3}{\alpha_N}\right)^2 \left(\frac{5}{A_L}\right)^2 4.4 \times 10^{34}\ {\rm yr}.$$

Hadronic matrix element

$$\tau^{SuperK}_{p\to e^+\pi^0} > 1\times 10^{34}\,{\rm yrs}$$

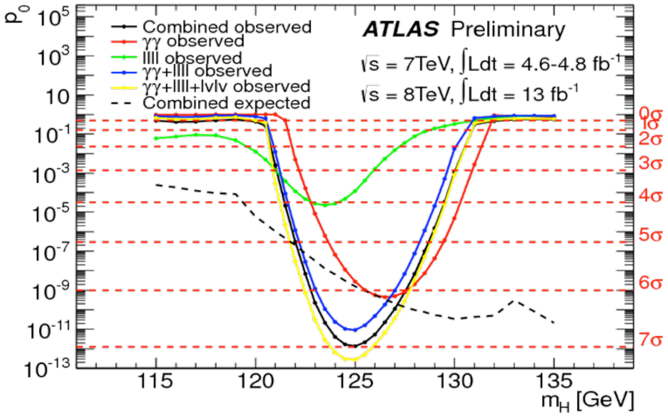
Giudice, Romanino

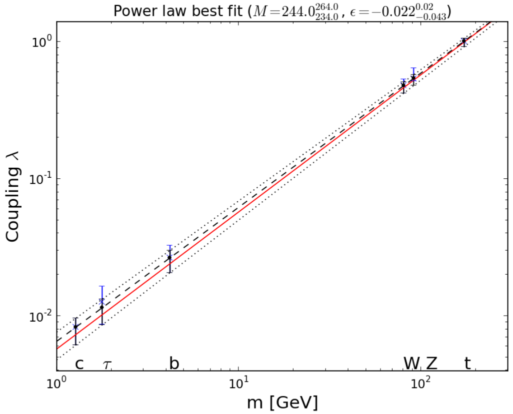
$$M_{\rm GUT} > \left(\frac{\alpha_{\rm GUT}}{1/35}\right)^{1/2} \left(\frac{\alpha_N}{0.015\ {\rm GeV}^3}\right)^{1/2} \left(\frac{A_L}{5}\right)^{1/2} 6 \times 10^{15}\ {\rm GeV}$$

$$c.f.\, M_U = (2.5 \pm 2).10^{16}\, GeV$$

V. Phenomenology of SUGRA

Higgs discovery**!** … completes the “Standard Model”





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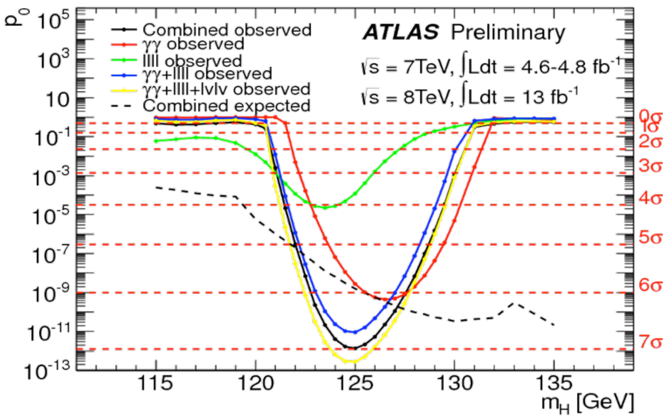
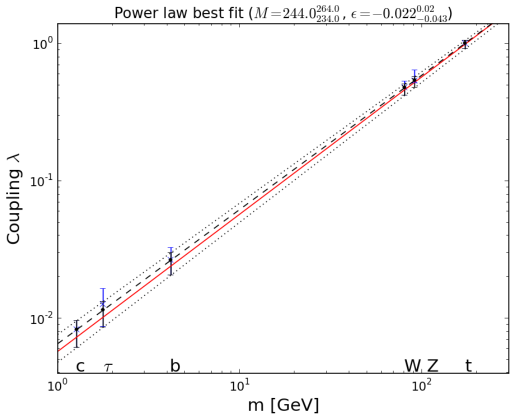
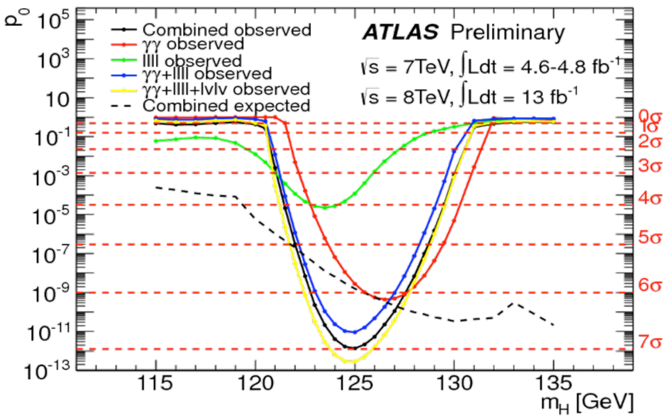
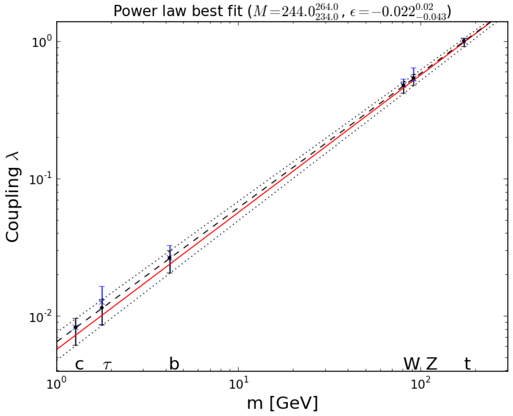




- "Light", weakly interacting SUSY ✔

Higgs discovery**!** … completes the "Standard Model"





- "Light", weakly interacting SUSY ✔
- "Heavy", no evidence for sparticles SUSY ✗

$$m_h^2 = M_Z^2 + \frac{3m_t^2 h_t^2}{4\pi^2}\left(\ln\left(\frac{M_s^2}{m_t^2}\right)+\delta_t\right)+\ldots \simeq 126\,GeV$$

$$\delta m_{H_u}^2 \simeq -\frac{3y_t^2}{4\pi^2}\left(m_{stop}^2 + \frac{g_s^2}{3\pi^2} m_{gluino}^2 \log\left(\frac{\Lambda}{m_{gluino}}\right)\right)\log\left(\frac{\Lambda}{m_{stop}}\right) \ ?$$

SUSY under pressure "Little hierarchy problem" breaking

Little hierarchy problem ⇒ definite SUSY structure

^

MSSM: 105 +(19) Parameters

$$M_Z^2 = \sum_{\tilde{q},\tilde{l}} a_i \widetilde{m}_i^{\,2} + \sum_{\tilde{g},\widetilde{W},\widetilde{B}} b_i \widetilde{M}_i^{\,2} + \ldots$$

$$m_{\tilde{q}} > 0.6 - 1TeV \quad \Rightarrow \quad \Delta > a\frac{\widetilde{m}^2}{M_Z^2} \sim 100$$

(Unless light stop $m_{\tilde{t},LHC} > 250\;GeV$)

⇒ Correlations between SUSY breaking parameters and/or additional low-scale states

Little hierarchy problem ⇒ definite SUSY breaking structure

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⇒ Correlations between SUSY breaking parameters and/or additional low-scale states

Fine Tuning measure:

$$\Delta(a_i) = \left|\frac{a_i}{M_Z}\frac{\partial M_Z}{\partial a_i}\right|,$$

$$\Delta_{\mathrm{m}} = Max_{a_i}\Delta(a_i), \quad \Delta_q = \left(\sum \Delta_{\gamma_i}^2\right)^{1/2}$$

Ellis, Enquist, Nanopoulos, Zwirner

Barbieri, Giudice

Fine tuning from a likelihood fit:

“Nuisance” variable

$$L\left(\text{data} \mid \gamma_i\right) \propto \int d\mathrm{v}\,\delta\left(m_Z - m_Z^0\right)\delta\left(\mathrm{v}\text{-}\left(-\frac{m^2}{\lambda}\right)^{1/2}\right)L\left(\text{data} \mid \gamma_i;\mathrm{v}\right)$$

$$= \frac{1}{\Delta_q}\delta\left(n_q(\ln\gamma_i - \ln\gamma_i^S)\right)L\left(\text{data} \mid \gamma_i;\mathrm{v}_0\right)$$

Fine tuning not optional!

Probabilistic interpretation:

$$\chi^2_{new} = \chi^2_{old} + 2\ln\Delta_q$$

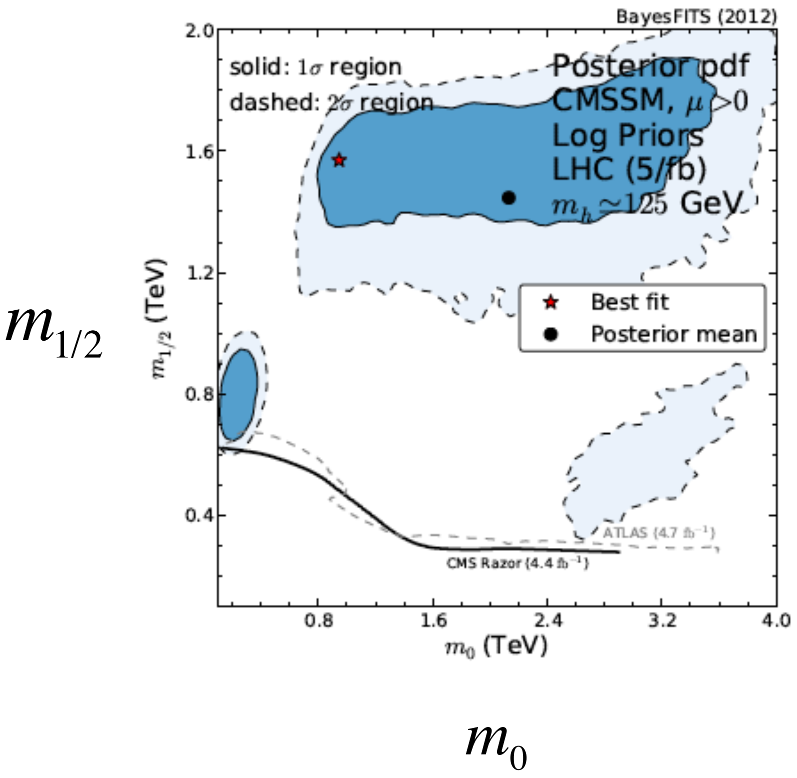
$$\Delta_q \ll 100$$

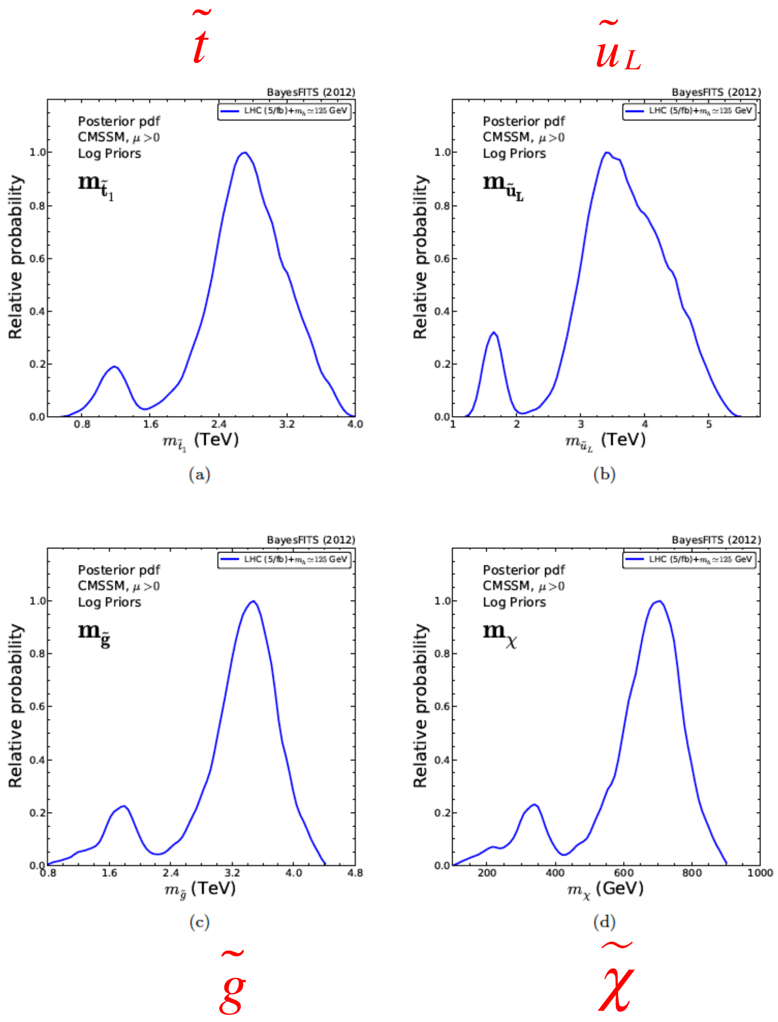
- The CMSSM

$$\gamma_i = \mu_0, m_0, m_{1/2}, A_0, B_0$$

assume correlation between SUSY breaking parameters

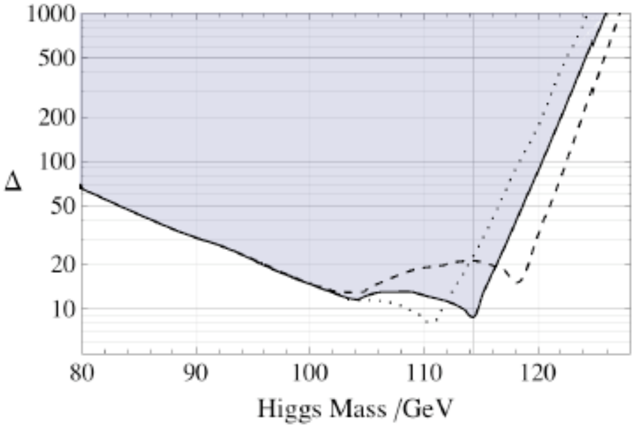
SUSY spectrum : CMSSM



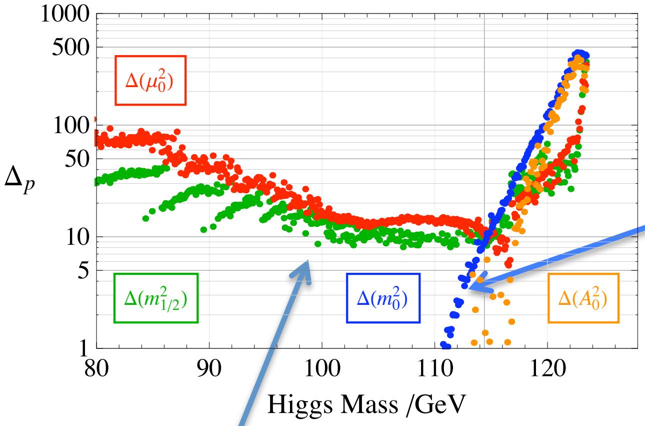


Fowlie et al

• The CMSSM - before LHC



$$\Delta_i,\ i = \mu_0, m_0, m_{1/2}, A_0, B_0$$



Limit of focus point

λ increase with m_H

$$v^2 = -\frac{m^2}{\lambda}$$

Constraints

SUSY particle masses

$3.20 < 10^4\ \mathrm{Br}(b \to s\gamma) < 3.84$

$\mathrm{Br}(b \to \mu\mu) < 1.8 \times 10^{-8}$

$\delta a_\mu < 292 \times 10^{-11}$

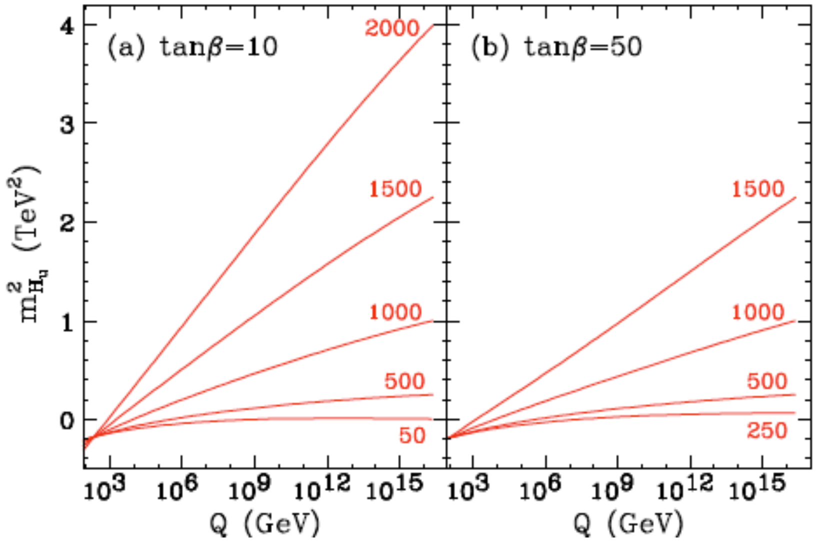
$-0.0007 < \delta\rho < 0.0012$

Radiative EW breaking

Relic density unrestricted

Guuge coupling unification

Focus Point



$$2|y_t|^2\,(m^2_{H_u}+m^2_{Q_3}+m^2_{\bar{u}_3})+2|a_t|^2$$

$$16\pi^2\frac{d}{dt}m^2_{H_u}=3X_t-6g_2^2|M_2|^2-\frac{6}{5}g_1^2|M_1|^2$$

$$16\pi^2\frac{d}{dt}m^2_{Q_3}=X_t+X_b-\frac{32}{3}g_3^2|M_3|^2-6g_2^2|M_2|^2-\frac{2}{15}g_1^2|M_1|^2$$

$$16\pi^2\frac{d}{dt}m^2_{\bar{u}3}=2X_t-\frac{32}{3}g_3^2|M_3|^2-\frac{32}{15}g_1^2|M_1|^2$$

$$\boxed{m^2_{H_u}\left(Q^2\right)=m^2_{H_u}\left(M_P^2\right)+\frac{1}{2}\left(m^2_{H_u}\left(M_P^2\right)+m^2_{Q_3}\left(M_P^2\right)+m^2_{\bar{u}3}\left(M_P^2\right)\right)\left[\left(\frac{Q^2}{M_P^2}\right)^{\frac{3y_t^2}{4\pi^2}}-1\right]}$$

$$m_0^2 \qquad 3m_0^2 \qquad \simeq -\frac{2}{3},\ Q^2\simeq M_Z^2$$

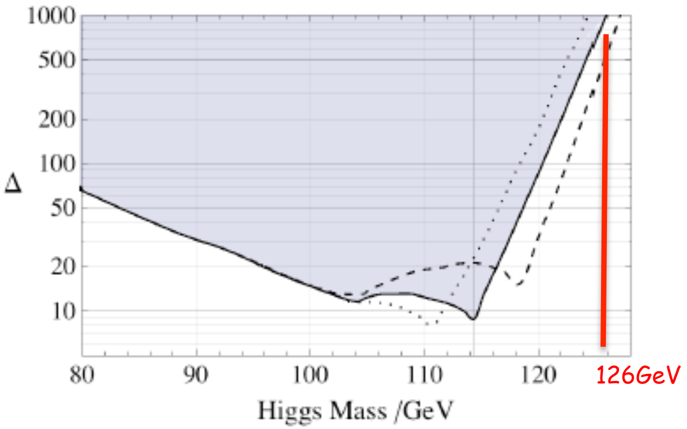
"Focus point": $m^2_{H_u}\left(0\right)=m^2_{Q_3}\left(0\right)=m^2_{\bar{u}3}\left(0\right)\equiv m^2$

$$\boxed{m^2_{H_u}\left(t_0\right)=a_0\ m^2+..,\ a_0\leq 0.1}$$

Natural choice

$i.e.\ m^3_{Q_3}, m^2_{\bar{u}3}\gg M_Z^2$ possible

The CMSSM - after Higgs discovery



$M_S^2 = m_{q_3} m_{U_3}$

$$M_{h^0}^2 = M_Z^2 \cos^2 2\beta + \frac{3M_t^2 h_t^2}{4\pi^2}\left(\ln(\frac{M_S^2}{M_t^2}) + \delta_t\right) + \cdots \qquad \simeq 126GeV$$

$$\boxed{\Delta_{Min} > 350\,, \quad m_h = 125.6 \pm 3GeV}$$

Reduced fine tuning (c.f. CMSSM)

- New focus points?

Gauginos: $M_{\tilde{g},\widetilde{W},\widetilde{B}}$ Non-universal gaugino correlations

- New degrees of freedom

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2\,|\,y_t\,|^2\,(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2\,|\,a_t\,|^2\right) - 6g_2^2\,|\,M_2\,|^2 - \frac{6}{5} g_1^2\,|\,M_1\,|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $\left|M_2\right|^2 \simeq \left|M_3\right|^2$ at M_{SUSY}

Abe, Kobayashi, Omura
Horton, GGR

(Also improves precision of gauge coupling unification)

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2\right) - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

Natural ratios?

$$\int d^2\theta\, f_{ab}\, Tr\, W^{a\alpha} W_\alpha^b + h.c. \qquad f_{ab} = \delta_{ab}\left[\frac{1}{g_a^2} + \frac{f_X X}{M_P} + \ldots\right]$$

$$\boxed{m_{1/2} = \frac{\sqrt{3}}{2}\mathrm{Re}\left(f_X\right) m_{3/2}}$$

Nonuniversal masses if X non-singlet – classify by representation of X

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2\,|\,y_t\,|^2\,(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}3}^2) + 2\,|\,a_t\,|^2\right) - 6g_2^2\,|\,M_2\,|^2 - \frac{6}{5}g_1^2\,|\,M_1\,|^2$$

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Natural ratios? e.g.:

GUT: $SU(5):\ \ \Phi^N \subset (24 \times 24)_{symm} = 1 + 24 + 75 + 200;\quad SO(10):\ \ (45 \times 45)_{symm} = 1 + 54 + 210 + 770$

$\eta_3 : 1 : \eta_1$ $\qquad$ $2.7\eta_3 : 1 : 0.5\eta_1$

Representation	$M_3 : M_2 : M_1$ at M_{GUT}	$M_3 : M_2 : M_1$ at M_{EWSB}
1	1:1:1	6:2:1
24	2:(-3):(-1)	12:(-6):(-1)
75	1:3:(-5)	6:6:(-5)
200	1:2:10	6:4:10

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2\,|\,y_t\,|^2\,(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}3}^2) + 2\,|\,a_t\,|^2\right) - 6g_2^2\,|\,M_2\,|^2 - \frac{6}{5} g_1^2\,|\,M_1\,|^2$$

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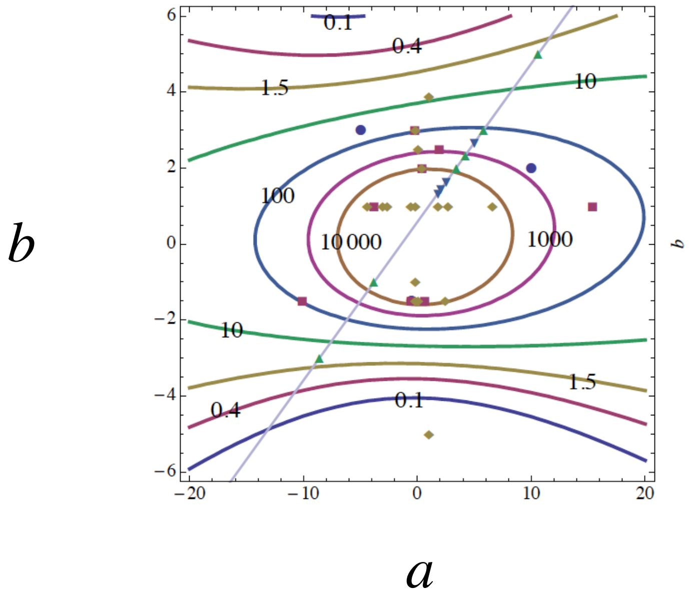
String: $\left(3+\delta_{GS}\right):\left(-1+\delta_{GS}\right):\left(-\frac{33}{5}+\delta_{GS}\right)$ (OII, also mixed moduli anomaly)

Reduced fine tuning : nonuniversal gaugino masses

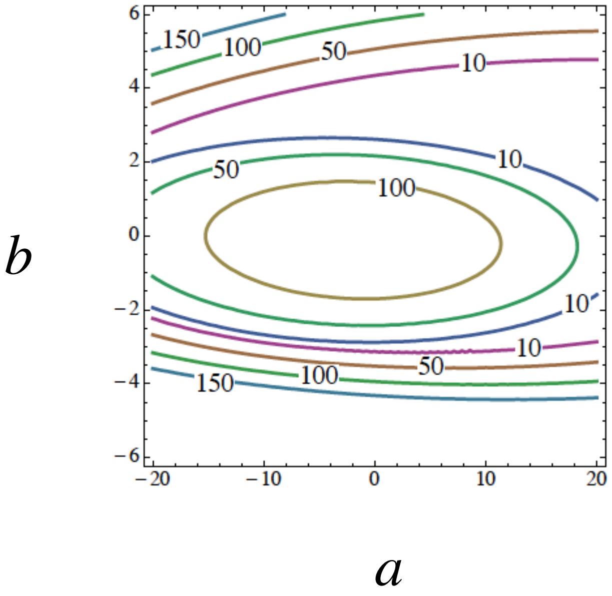
$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}3}^2) + 2|a_t|^2\right) - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

$$M_3 : M_2 : M_1 = 1 : b : a$$



Focus point scale



Fine tuning measure

GGR, Kaminska, Schmidt-Hoberg

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2\,|y_t|^2\,(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}3}^2) + 2\,|a_t|^2\right) - 6g_2^2\,|M_2|^2 - \frac{6}{5}g_1^2\,|M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

$$\boxed{\Delta_{Min}^{CMSSM} = 60\ (500), \quad m_h = 125.6 \pm 3GeV}$$

LHC8 SUSY bounds ✓

DM relic abundance ✓

DM searches ✗

Reduced fine tuning : Beyond the MSSM

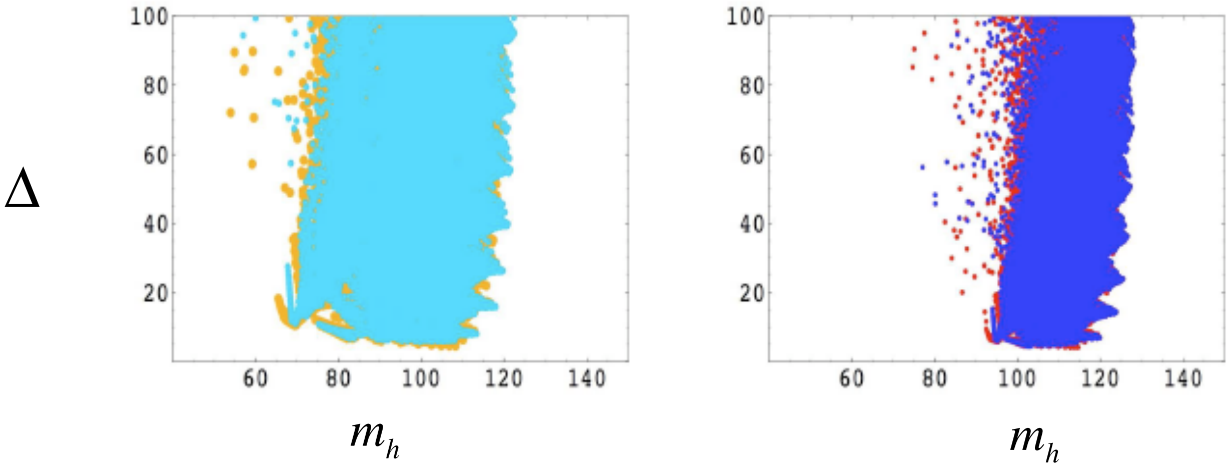
Reduced fine tuning : Beyond the MSSM

New heavy states – higher dimension operators

$$\delta L = \int d^2\theta \frac{1}{M_*}(\mu_0 + c_0 S)(H_u H_d)^2, \quad S = m_0\theta\theta$$

Dimension 5

$$\delta V = \varsigma_1\left(|h_u|^2 + |h_d|^2\right)h_u h_d + \varsigma_2 (h_u h_d)^2; \quad \varsigma_1 = \frac{\mu_0}{M_*},\ \varsigma_2 = \frac{c_0 m_0}{M_*}$$



MSSM + dim 5 operators

Even for M*=65 μ$_0$ a significant shift of m$_h$ for constant Δ

…effect mainly comes from ς_1 term … origin?

Reduced fine tuning : Beyond the MSSM

New heavy states – higher dimension operators

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Singlet extensions

$$W = W_{\rm Yukawa} + \lambda S H_u H_d + \frac{\kappa}{3} S^3$$

NMSSM

$$W = W_{\rm Yukawa} + (\mu + \lambda S) H_u H_d + \frac{\mu_S}{2} S^2 + \frac{\kappa}{3} S^3 + \xi S$$

GNMSSM

$$\mu_S >> m_{3/2} : \quad W_{eff}^{GNMSSM} = (H_u H_d)^2 / \mu_s + \mu H_u H_d$$

$$\delta V = \frac{\mu}{\mu_S} \left(|H_u|^2 + |H_d|^2 \right) H_u H_d$$

✓

Reduced fine tuning : Beyond the MSSM

New heavy states – higher dimension operators

$$\delta L = \int d^2\theta \frac{1}{M_*}(\mu_0 + c_0 S)(H_u H_d)^2 , \quad S = m_0 \theta\theta$$

Dimension 5

$$\delta V = \varsigma_1 \left(|h_u|^2 + |h_d|^2\right) h_u h_d + \varsigma_2 (h_u h_d)^2 ; \quad \varsigma_1 = \frac{\mu_0}{M_*}, \ \varsigma_2 = \frac{c_0 m_0}{M_*}$$

Z_N^R R-symmetry

N	q_{10}	$q_{\bar{5}}$	q_{H_u}	q_{H_d}	q_S
4	1	1	0	0	2
8	1	5	0	4	6

Singlet extensions

$$W = W_{\text{Yukawa}} + \lambda S H_u H_d + \frac{\kappa}{3} S^3$$

$$W = W_{\text{Yukawa}} + (\mu + \lambda S) H_u H_d + \frac{\mu_S}{2} S^2 + \frac{\kappa}{3} S^3 + \xi S$$

R-symmetry ensures singlets light

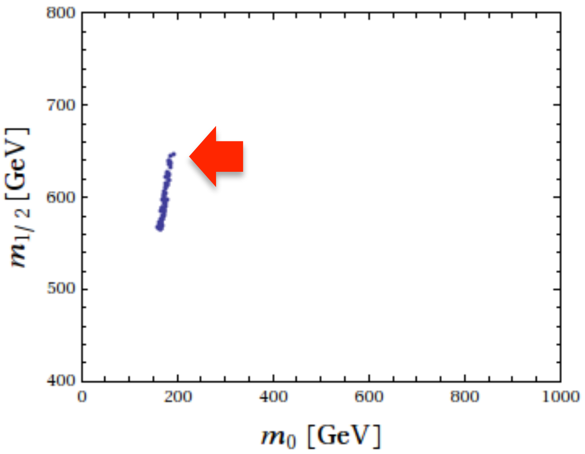
Fine tuning in the CGNMSSM $(\lambda \leq 0.7^{\dagger})$

$$\Delta_{Min} = 60\ (500), \quad m_h = 125.6 \pm 3GeV$$

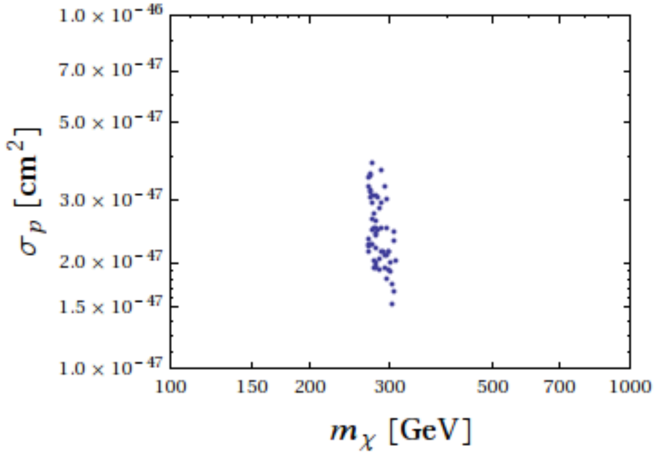
LHC8 SUSY bounds ✗

DM relic abundance ✔

DM searches ✔



Stau co-annihilation



DM searches insensitive

Fine tuning in the ©GNMSSM

$(\lambda \leq 0.7^{\dagger})$

Non-unversal gaugino masses

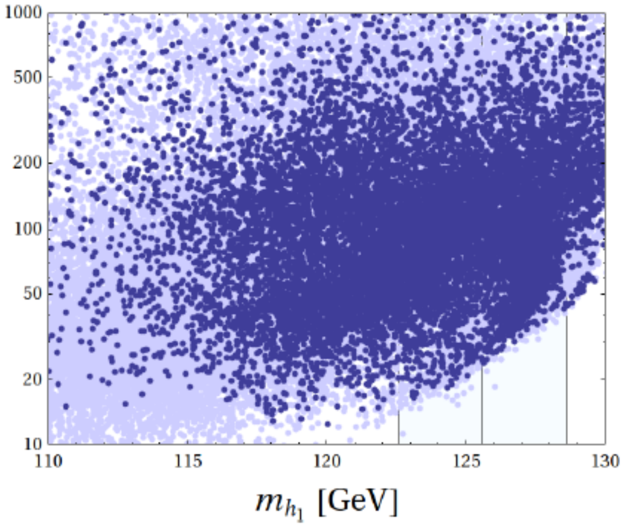
$$\Delta_{Min} = 20\,, \quad m_h = 125.6 \pm 3GeV$$

LHC8 SUSY bounds ✓

DM relic abundance ✓

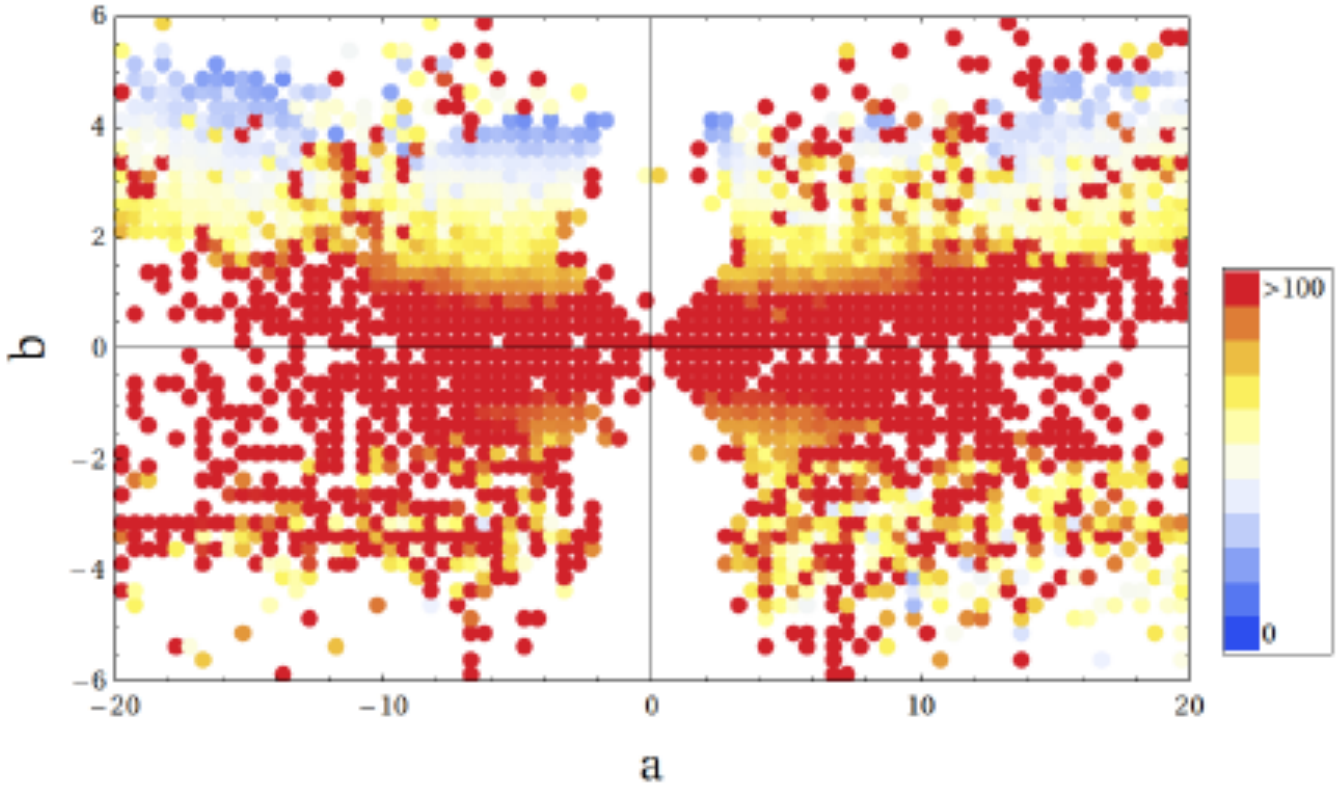
DM searches ✓

Δ



Fine tuning in the CGNMSSM

…fine tuning v/s gaugino mass ratios



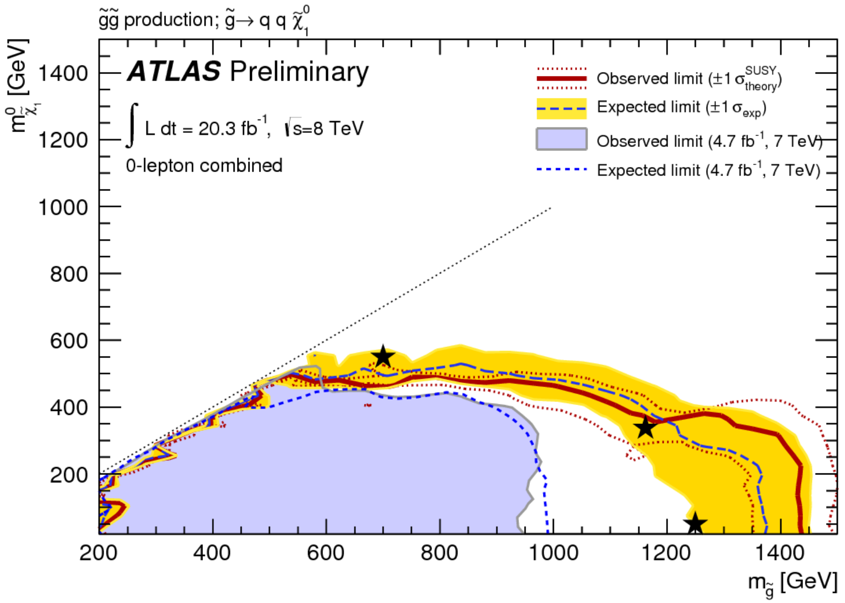
F.T.

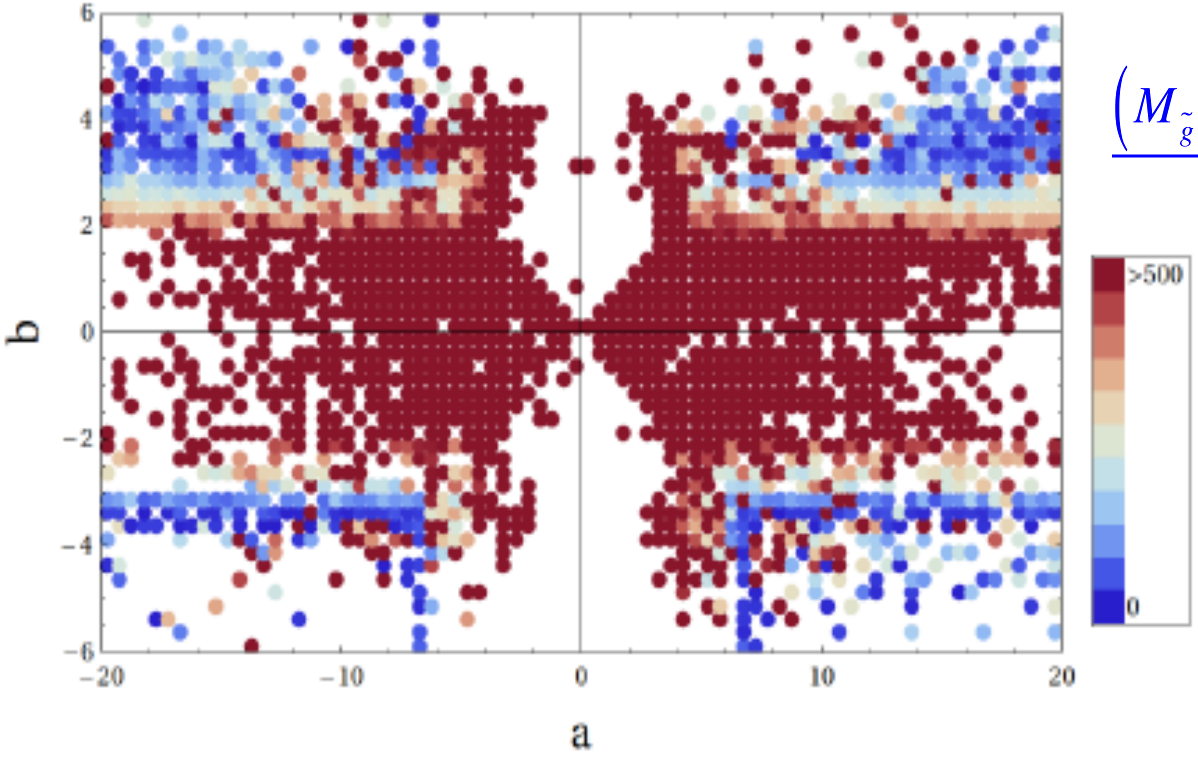
>100

0

$$M_3 = m_{1/2} \, , M_2 = b.m_{1/2} \, , M_1 = a.m_{1/2}$$

Compressed spectrum





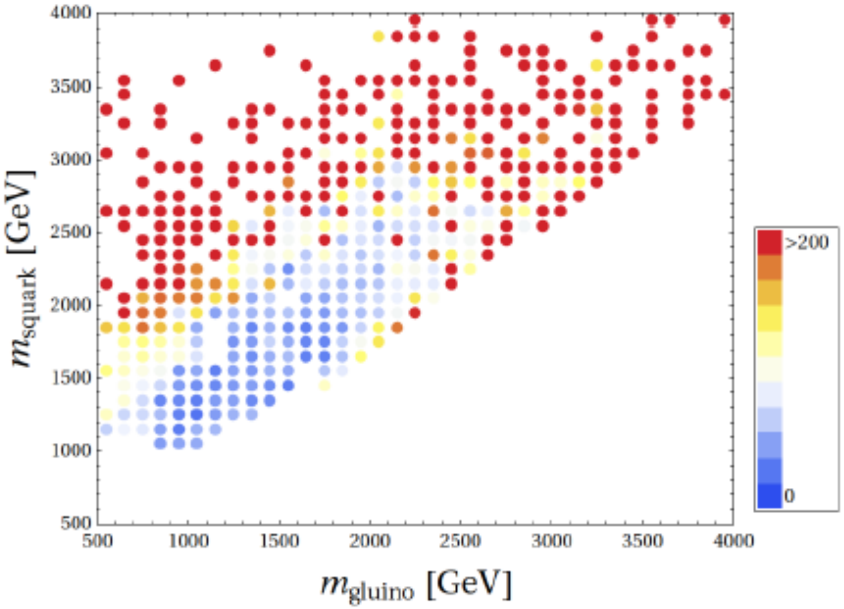
$$\frac{\left(M_{\tilde{g}} - M_{LSP}^{neutralino}\right)}{GeV}$$

> 500

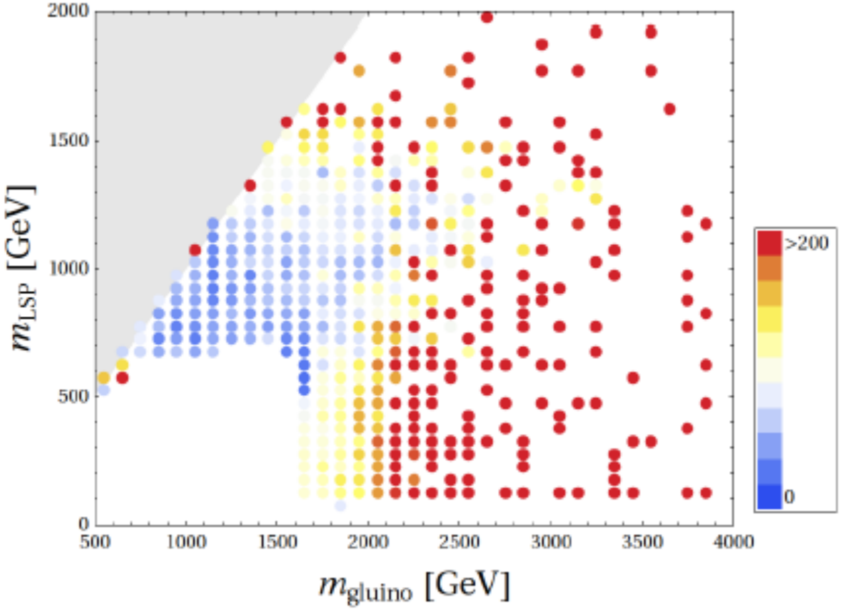
0

Masses v/s fine tuning

m_{squark}



m_{LSP}

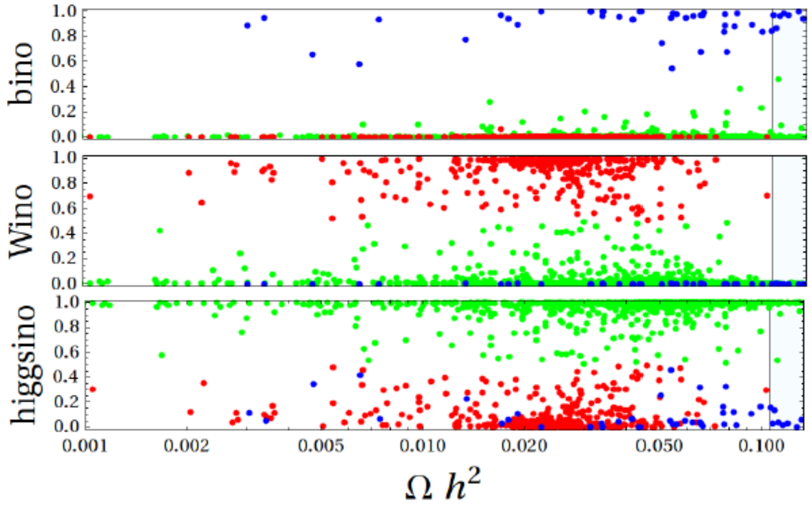


> 500

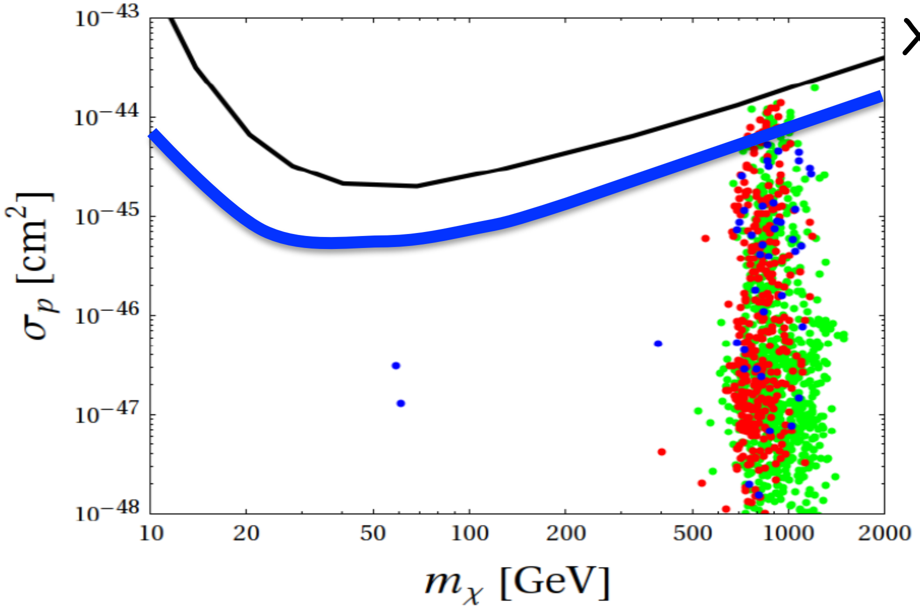
0

M_{gluino}

Dark matter



LSP composition



XENON 100

LUX

Direct DM searches

Summary

- GUTs $\Longrightarrow$ SUSY-GUTS (hierarchy problem)
- Low fine tuning not optional
- Fine tuning sensitive to SUSY spectrum

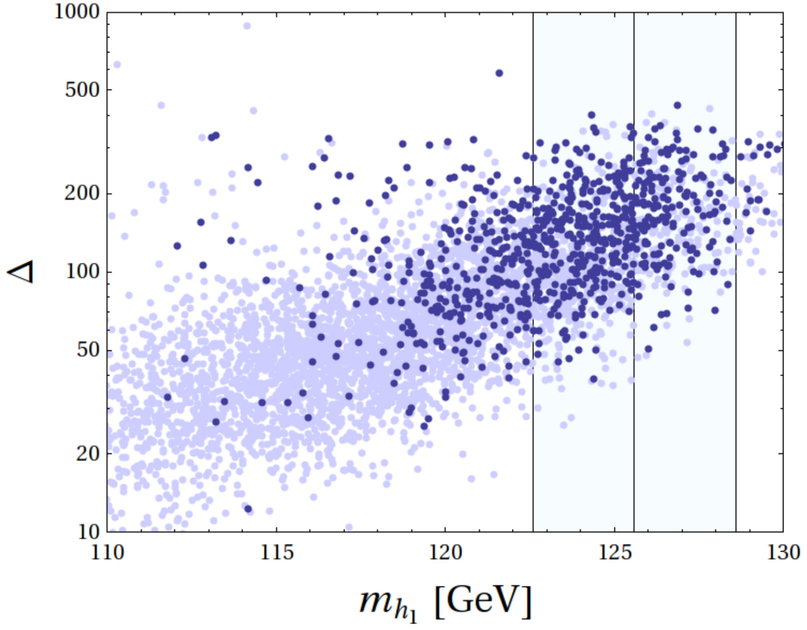
  …scalar and gaugino focus points
- $\Delta^{CMSSM} > 350$ ✗ $\Delta^{(C)MSSM} > 60$ ✗

  $\Delta^{CGMSSM} > 60$ ✗ $\Delta^{(C)GNMMS} > 20$ ✓

$$c.f. \quad \Delta^{CMSSM}_{Low\ scale} = (10-30), \quad m_{\tilde{t}} = (1-5)TeV$$

Barger et al

Summary



a, b not fixed

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Barger et al

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- $\Delta^{CMSSM} > 350$ $\Delta^{(C)MSSM} > 60$

  $\Delta^{CGMSSM} > 60$ $\Delta^{(C)GNMMS} > 20$
- Well motivated SUSY models remain to be tested

  LHC14?

  Compressed spectra, TeV squarks and gluinos