Summer School and Workshop on the Standard Model and Beyond

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Flavour Physics (THEORY)

Ferruccio Feruglio
University of Padova

Premise

several aspects

1 flavour puzzle

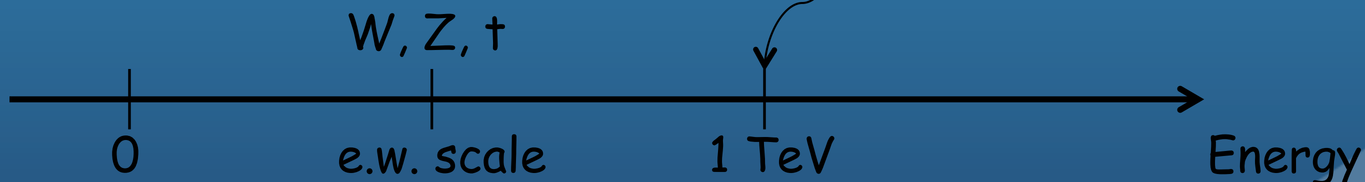
origin of $\mathcal{Y}$

origin of the 22 (20 if B-L is conserved) parameters $\mathcal{Y}$ needed to describe fermion masses and mixing angles in the SM, minimally extended to accommodate massive neutrinos.

2 flavour problem

how to keep at an acceptable level the contributions to FCNC and CP violation from new flavoured particles at the TeV scale

new particle threshold favored by known solutions to the gauge hierarchy problem



less urgent? no evidence for a new threshold so far this two lectures: main focus on 1 + few comments on 2

-- not a review of the existing models, but rather a

-- reappraisal of old ideas, by stressing the conceptual points by means of simple examples

Approaches to the flavour puzzle

fundamental theory

↓

[symmetry and/or dynamical principle]

↓

$\mathcal{Y}$

1 $\mathcal{Y}$ should be deduced from first principles

most striking fact: nothing approaching a standard theory of $\mathcal{Y}$, despite decades of experimental progress and theoretical efforts

2 $\mathcal{Y}$ are due to chance

many variants

bottom-up: anarchy, FN models, fermions in ED, partial compositness

top-down: fundamental theory with a landscape of ground states

observed $\mathcal{Y}$ are environmental and cannot be fully predicted ↔ relative sizes of solar planetary orbits

assumptions

knowledge of statistical distribution of $\mathcal{Y}$ in the fundamental theory

the observed $\mathcal{Y}$ are typical

[any anthropic selection?]

relevant questions

how typical are the $\mathcal{Y}$ we observe?

which is the statistical distribution of $\mathcal{Y}$ in the fundamental theory?

Lecture I
broken flavour symmetries

short review of SM and surroundings

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\,\overline{\Psi}\gamma^{\mu} D_{\mu}\Psi$$ gauge sector

$$+ D_{\mu}\Phi^{+} D^{\mu}\Phi - V(\Phi)$$ symmetry breaking sector

$$+ (\overline{\Psi} Y \Phi \Psi + h.c.)$$ flavour sector

$$+ L_{NP}$$

3 copies of fermions with the same quantum numbers distinguishable only by their masses. Each copy, or generations, consists of 5 independent multiplets of the gauge group G=SU(3)xSU(2)xU(1)

$$q_L = (3,2,+1/6) \quad u_R = (3,1,+2/3) \quad d_R = (3,1,-1/3)$$

$$l_L = (1,2,-1/2) \quad e_R = (1,1,-1)$$

fermion masses and mixing angles described by

$$L_Y = -\overline{d}_R y_d (\Phi^{+} q_L) - \overline{u}_R y_u (\tilde{\Phi}^{+} q_L) - \overline{e}_R y_e (\Phi^{+} l_L) + h.c.$$

y_f are diagonalized by

$f_L \to V_f f_L$
$f_R \to U_f f_R$

U_f and V_f unitary

$$U_f^+ y_f V_f = y_f^{diag}$$

y_f^{diag} diagonal, real, positive

$$m_f = \frac{y_f^{diag}}{\sqrt{2}} v \qquad f = u, d, e$$

the charged current interaction becomes

$$-\frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{u}_L \gamma^\mu V_u^+ V_d d_L + \ldots \right) + h.c.$$

$$V_{CKM} = V_u^+ V_d$$

mixing matrix
3 angles and 1 phase

so far neutrinos are massless
an appealing extension of the SM to accommodate massive neutrinos is obtained by adding 3 copies of right-handed neutrinos, full singlets under G=SU(3)xSU(2)xU(1)

[much more on this in Guido Altarelli lectures]

$$\nu^c \equiv (1,1,0)$$

$$L_Y \to L_Y - \left[\nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2} \nu^c M \nu^c + h.c. \right]$$

mass term for right-handed neutrinos: G invariant, violates (B-L) by two units.

the new mass parameter M is independent from the electroweak breaking scale v. If M>>v, we might be interested in an effective description valid for energies much smaller than M. This is obtained by "integrating out'' the field ν^c [exercise]

$$L_{eff}(l) = \frac{1}{2}(\tilde{\Phi}^+ l)\left[y_\nu^T M^{-1} y_\nu\right](\tilde{\Phi}^+ l) + h.c. + \ldots$$

terms suppressed by more powers of M^{-1}

[This particular mechanism is called (type I) see-saw.]

$$m_\nu = m_D^T\, M^{-1}\, m_D \qquad m_D = \frac{y_\nu}{\sqrt{2}} v$$

smallness of neutrino masses due to largeness of M

m_ν is diagonalized by

$\nu_L \to U_\nu \nu_L$

U_ν unitary

$$U_\nu^T m_\nu U_\nu = m_\nu^{diag}$$

the charged current interaction becomes

$$-\frac{g}{\sqrt{2}} W_\mu^- \left(\bar{e}_L \gamma^\mu V_e^+ U_\nu \nu_L + \ldots\right) + h.c.$$

$$U_{PMNS} = V_e^+ U_\nu$$

mixing matrix, depending on 3 mixing angles and 3 phases [exercise] count the number of physical parameters in the type I see-saw model distinguish between moduli and phases

y_e, y_ν and M depend on (18+18+12)=48 parameters, 24 moduli and 24 phases

we are free to choose any basis leaving the kinetic terms canonical (and the gauge interactions unchange)

$$e^c \to \Omega_{e^c} e^c \qquad \nu^c \to \Omega_{\nu^c} \nu^c \qquad l \to \Omega_l l \qquad [U(3)^3]$$

these transformations contain 27 parameters (9 angles and 18 phases) and effectively modify y_e, y_ν and M

$$y_e \to \Omega^T_{e^c} y_e \Omega_l \qquad y_\nu \to \Omega^T_{\nu^c} y_\nu \Omega_l \qquad M \to \Omega^T_{\nu^c} M \Omega_{\nu^c}$$

so that we can remove 27 parameters from y_e, y_ν and M

we remain with 21 parameters: 15 moduli and 6 phases
the moduli are 9 physical masses and 6 mixing angles

the same count in the quark sector would give a total of 9 moduli (6 masses amd 3 mixing angles) and 0 phases <- wrong
how the above argument should be modified, in general?

a look to the data

charged lepton masses

$$m_e = 0.510\,998\,928 \pm 0.000\,000\,011 \quad MeV$$

$$m_\mu = 105.658\,3715 \pm 0.000\,0035 \quad MeV$$

$$m_\tau = 1776.82 \pm 0.16 \quad MeV$$

[$\overline{\text{MS}}$ masses except for the top quark
μ=2 GeV for u, d , s
μ=m_f for f=c,b]

quark masses

$$m_u = 2.3^{+0.7}_{-0.5} \quad MeV$$

$$m_d = 4.8^{+0.5}_{-0.3} \quad MeV$$

$$m_s = 95 \pm 5 \quad MeV$$

$$m_c = 1.275 \pm 0.025 \quad GeV$$

$$m_b = 4.18 \pm 0.03 \quad GeV$$

$$m_t = 173.2 \pm 0.9 \quad GeV$$

quark mixing

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$$\lambda = 0.225\,35 \pm 0.000\,65 \qquad \overline{\rho} = 0.131^{+0.026}_{-0.013} \approx \rho$$

$$A = 0.811^{+0.022}_{-0.012} \qquad \overline{\eta} = 0.345^{+0.013}_{-0.014} \approx \eta$$

neutrino conventions

$$[\Delta m^2_{ij} \equiv m_i^2 - m_j^2]$$

$$m_1 < m_2$$

$$\Delta m^2_{21} < \left|\Delta m^2_{32}\right|, \left|\Delta m^2_{31}\right|$$

i.e. 1 and 2 are, by definition, the closest levels

two possibilities:

3

normal hierarchy

2
1

2
1

inverted hierarchy

3

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ -c_{12}s_{13}c_{23}e^{-i\delta} + s_{12}s_{23} & -s_{12}s_{13}c_{23}e^{-i\delta} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$c_{12} \equiv \cos\vartheta_{12}, \ldots$$

three mixing angles $\vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23}$

three phases (in the most general case) δ $\qquad \underbrace{\alpha, \quad \beta}_{\text{do not enter}}$ $P_{ff'} = P(\nu_f \to \nu_{f'})$

oscillations can only test 6 combinations

$\Delta m^2_{21}, \Delta m^2_{32},$ $\vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23}$ δ

neutrinos

Summary of unkowns

$m_\nu < 2.2\ eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

absolute neutrino mass scale is unknown [but well-constrained!]

$$\Delta m^2_{atm} \equiv \begin{cases} \Delta m^2_{31} = (2.462 \pm 0.033) \times 10^{-3}\ \mathrm{eV}^2 & \mathrm{NO} \\ \Delta m^2_{32} = -(2.453 \pm 0.047) \times 10^{-3}\ \mathrm{eV}^2 & \mathrm{IO} \end{cases}$$

sign $[\Delta m^2_{atm}]$ unknown

[ordering (either normal or inverted hierarchy) not known]

$$\Delta m^2_{sol} \equiv \Delta m^2_{21} = (7.55^{+0.18}_{-0.17}) \times 10^{-5}\ \mathrm{eV}^2$$

$$\sin^2 \vartheta_{13} = 0.0223^{+0.0011}_{-0.0010} \qquad \delta_{CP} = (259^{+76}_{-69})$$

δ, α, β unknown

[CP violation in lepton sector not yet established]

$$\sin^2 \vartheta_{23} = [0.451^{+0.026}_{-0.020}] \oplus [0.580^{+0.024}_{-0.039}]$$

$$\sin^2 \vartheta_{12} = 0.311^{+0.013}_{-0.012}$$

[G.-Garcia, Maltoni, Salvado, Schwetz 1209.3023 http://www.nu-fit.org]

violation of individual lepton number implied by neutrino oscillations

violation of total lepton number not yet established

comments

masses span several order of magnitudes: 5-6 in the charged sector
12 if we also include neutrinos

leaving neutrino aside, the mass spread within each generation is much smaller

useful book-keeping
(masses renormalized at the common scale m_Z)

$$\frac{m_e}{m_\tau} \approx \lambda^{5.4} \qquad \frac{m_\mu}{m_\tau} \approx \lambda^{1.9}$$

$$\frac{m_d}{m_b} \approx \lambda^{4.3} \qquad \frac{m_s}{m_b} \approx \lambda^{2.3}$$

$$\frac{m_u}{m_t} \approx \lambda^{7.4} \qquad \frac{m_c}{m_t} \approx \lambda^{3.6}$$

$$|V_{ud}| \approx 1 \quad |V_{us}| \approx \lambda \quad |V_{cb}| \approx \lambda^2 \quad |V_{ub}| \approx \lambda^4 \div \lambda^3$$

neutrino sector

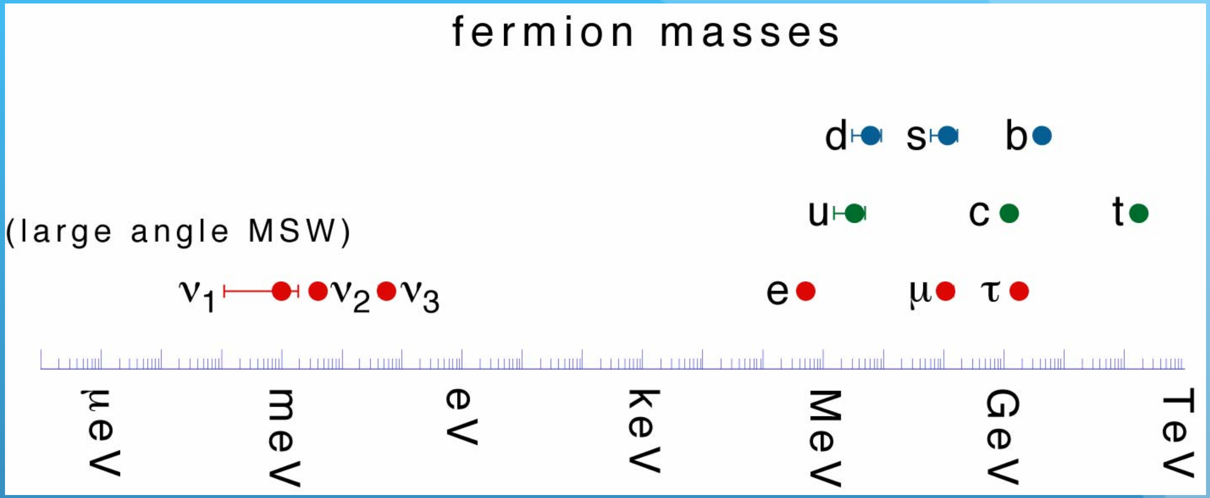
$$|U_{fi}| \approx O(1)$$

except

$$|U_{e3}| \approx O(\lambda)$$

$$\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \approx \lambda^{2.3}$$

questions



- why N_g=3?
- origin of the hierarchies in the charged fermion sector
- origin of mixing angles
- why neutrino masses are much smaller than charged fermion masses ?
- why lepton mixing is different from quark mixing?

[see Guido Altarelli lectures]

spontaneously broken flavour symmetries [Froggatt-Nielsen 1979]

tailored to explain small parameters

example:
QED

why $\xi = \frac{m_e}{\Lambda} << 1$?

some cut-off scale
e.g. M_{Pl}

idea: easier to explain why $\xi = 0$
the $\xi = 0$ limit corresponds to
the $U(1)_A$ axial symmetry

$$U(1)_A \begin{cases} e_L \to e^{i\alpha} e_L \\ e_R \to e^{-i\alpha} e_R \end{cases}$$

$U(1)_A$ approximate symmetry of the real world, broken by $\xi \neq 0$

- ξ small is natural in the 't Hooft sense:
  by sending ξ to zero the symmetry of the theory is enhanced

- bonus:
  stability under radiative corrections $\quad \delta\xi \propto \xi \quad$ since in the symmetry limit $\delta\xi=0$

SM [the quark sector]

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

mass ratios and mixing angles are small breaking terms of an approximate symmetry G_f

simplest candidate G_f=U(1)$_{FN}$

[change of notation such that all fermions are left-handed]

$$\psi_R \to (\psi_R)^c = (\psi^c)_L$$

introduce a new abelian symmetry U(1)$_{FN}$

assume all charges p, q, r non-negative and ordered such that $p_1 \geq p_2 \geq p_3$, etc…

	q	u^c	d^c
1st gen	p_1	q_1	r_1
2nd gen	p_2	q_2	r_2
3rd gen	p_3	q_3	r_3

FN charges

take FN(Φ)=0 and consider the choice: p_3 = q_3 = 0 and (other charges) > 0

only one term is allowed
in the Yukawa lagrangian

$$-t^c\, Y_u^{33}\, \tilde{\Phi}^+ \begin{pmatrix} t \\ b \end{pmatrix} + h.c.$$

$$m_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y_u^{33} \end{pmatrix} \frac{v}{\sqrt{2}}$$

$$m_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{v}{\sqrt{2}}$$

$$m_t = Y_u^{33} \frac{v}{\sqrt{2}} \qquad m_u = m_c = m_d = m_s = m_b = 0$$

zeros can be replaced by small quantities if $U(1)_{FN}$ is broken, for instance spontaneously through the VEV of a scalar field θ such that FN(θ)=-1

$$L_Y = -u_i^c Y_u^{ij} \tilde{\Phi}^+ q_j \left(\frac{\theta}{\Lambda}\right)^{q_i+p_j} - d_i^c Y_d^{ij} \Phi^+ q_j \left(\frac{\theta}{\Lambda}\right)^{r_i+p_j} + h.c.$$

when θ takes a VEV, the vanishing entries in m_u and m_d are replaced by powers of the symmetry breaking parameter

$$\lambda = \frac{\langle \theta \rangle}{\Lambda}$$

$$(y_u)_{ij} = \lambda^{q_i + p_j} Y_u^{ij} \qquad (y_d)_{ij} = \lambda^{r_i + p_j} Y_d^{ij}$$

if $\lambda < 1$, for instance $\lambda \approx 0.2$, then we do not need hierarchical Yukawa couplings $Y_{u,d}$. We can take $Y_{u,d} = O(1)$ and appropriate charges p, q, r

at the LO in an expansion in powers of λ we have [omitting all O(1) coefficients $Y_{u,d}$] [exercise]

$$\begin{aligned}
m_u &\approx \lambda^{q_1+p_1} v & m_d &\approx \lambda^{r_1+p_1} v \\
m_c &\approx \lambda^{q_2+p_2} v & m_s &\approx \lambda^{r_2+p_2} v \\
m_t &\approx \lambda^{q_3+p_3} v & m_b &\approx \lambda^{r_3+p_3} v
\end{aligned}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & \lambda^{p_1-p_2} & \lambda^{p_1-p_3} \\ \lambda^{p_1-p_2} & 1 & \lambda^{p_2-p_3} \\ \lambda^{p_1-p_3} & \lambda^{p_2-p_3} & 1 \end{pmatrix}$$

CKM angles only depend on p_i

one prediction
independent from the
specific charge choice

$$V_{ud} \approx V_{cs} \approx V_{tb} \approx O(1)$$
$$V_{ub} \approx V_{td} \approx V_{us} \times V_{cb}$$

[O.K. within a factor of 2]

correct orders of magnitude of V_{ij} reproduced by e.g.

$(p_1, p_2, p_3) = (3,2,0)$

correct orders of magnitude of quark mass ratios reproduced by e.g.

$(q_1, q_2, q_3) = (4,2,0)$

$(r_1, r_2, r_3) = (1+r_3, r_3, r_3)$

to match $m_b \approx \lambda^{r_3} v$

we would need $r_3 \approx 2$, unless there are two Higgs doublets such that $m_u \approx v_u$ and $m_d \approx v_d$
if $v_d/v_u \approx \lambda^2$ then we can take $r_3 = 0$

results [exercise]

$$\frac{m_d}{m_b} \approx \lambda^4 \qquad \frac{m_s}{m_b} \approx \lambda^2$$

$$\frac{m_u}{m_t} \approx \lambda^7 \qquad \frac{m_c}{m_t} \approx \lambda^4$$

$$|V_{ud}| \approx 1 \quad |V_{us}| \approx \lambda \quad |V_{cb}| \approx \lambda^2 \quad |V_{ub}| \approx \lambda^3$$

successful:
compare with previous empirical bookeeping $\lambda \approx 0.22$

extension to the lepton sector

	l	e^c	ν^c
1st gen	s_1	t_1	u_1
2nd gen	s_2	t_2	u_2
3rd gen	s_3	t_3	u_3

FN charges

assume all charges s, t, u non-negative and ordered such that $s_1 \geq s_2 \geq s_3$, etc…

$$L_Y = -e_i^c Y_d^{ij} \Phi^+ l_j \left(\frac{\theta}{\Lambda}\right)^{t_i+s_j} + \frac{1}{2}(\tilde{\Phi}^+ l_i)\left[y_\nu^T M^{-1} y_\nu\right]_{ij}(\tilde{\Phi}^+ l_j)\left(\frac{\theta}{\Lambda}\right)^{s_i+s_j} + h.c.$$

no dependence on right-handed neutrino charges u_i at low energies [for $u_i \geq 0$]

at the LO in an expansion in powers of λ we have
[omitting all O(1) coefficients; M_0 is the typical scale in M]

$$m_e \approx \lambda^{t_1+s_1} v \qquad m_1 \approx \lambda^{2s_1} v^2 / M_0$$
$$m_\mu \approx \lambda^{t_2+s_2} v \qquad m_2 \approx \lambda^{2s_2} v^2 / M_0$$
$$m_\tau \approx \lambda^{t_3+s_3} v \qquad m_3 \approx \lambda^{2s_3} v^2 / M_0$$

$$U_{PMNS} \approx \begin{pmatrix} 1 & \lambda^{s_1-s_2} & \lambda^{s_1-s_3} \\ \lambda^{s_1-s_2} & 1 & \lambda^{s_2-s_3} \\ \lambda^{s_1-s_3} & \lambda^{s_2-s_3} & 1 \end{pmatrix}$$

PMNS angles and neutrino masses only depend on s_i

charge choice [example]

$(s_1, s_2, s_3) = (1+s_3, s_3, s_3)$

$(t_1, t_2, t_3) = (4,2,0)$

leads to [exercise]

$$\frac{m_e}{m_\tau} \approx \lambda^5 \qquad \frac{m_\mu}{m_\tau} \approx \lambda^2$$

$$\frac{m_1}{m_3} \approx \lambda \qquad \frac{m_2}{m_3} \approx O(1)$$

$$\vartheta_{23} \approx O(1) \qquad \vartheta_{13} \approx \lambda \qquad \vartheta_{12} \approx \lambda$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx O(1) \qquad \vartheta_{12} \approx \lambda$$

if accidentally
det(23) ≈ O(λ) in m_ν

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \lambda^2 \qquad \vartheta_{12} \approx 1$$

almost all mass ratios and mixing angles correctly reproduced
at the level of orders of magnitudes
quarks and leptons treated on the same footing

weak point: large number of unknown O(1) parameters $Y_{u,d,e}$, (M/M_0)
not constrained by the $U(1)_{FN}$ symmetry
impossible to go beyond the order-of-magnitude level
impossible to make a precision test of the idea

embedding in [SUSY] SU(5) GUT

$$\left.\begin{array}{l} q = (3,2,+1/6) \\ u^c = (\bar{3},1,-2/3) \\ e^c = (1,1,1) \end{array}\right\} \sim 10 \text{ of } SU(5) \qquad \left.\begin{array}{l} d^c = (\bar{3},1,1/3) \\ l = (1,2,-1/2) \end{array}\right\} \sim \bar{5} \text{ of } SU(5)$$

constraint $\boxed{p = q = t \qquad r = s}$

we found

$(p_1, p_2, p_3) = (3,2,0)$

$(q_1, q_2, q_3) = (4,2,0)$

$(t_1, t_2, t_3) = (4,2,0)$

$(r_1, r_2, r_3) = (1+r_3, r_3, r_3)$

$(s_1, s_2, s_3) = (1+s_3, s_3, s_3)$

not bad. If we take $r_3=s_3$ and $p_1=q_1=t_1=(3 \text{ or } 4)$ we can reproduce the data up to another moderate tuning in the O(1) parameters

two new predictions $\quad \boxed{\dfrac{m_c}{m_t} \approx V_{cb}^2 \qquad \dfrac{m_u}{m_t} \approx V_{ub}^2} \quad$ close to the data

[a "minimal" Yukawa coupling to Higgses in 5 and 5 of SU(5)
would imply $\quad m_e = m_d \qquad m_\mu = m_s \qquad m_\tau = m_b$
the 1st two relations are wrong but remedies are well known]

embedding in [SUSY] SO(10) GUT ?

$$16 = (q, u^c, e^c, l, d^c, \nu^c)$$

a whole SM fermion generation plus a right-handed neutrino in a unique multiplet

⇒ constraint $p = q = t = r = s$

it does not seem possible….

previous framework cannot explain special features

1 ϑ_{23} maximal ?

today most precise single determination of ϑ_{23} is from T2K ($P_{\mu\mu}$)
[1403.1532]

$$\sin^2\vartheta_{23} = \begin{cases} 0.514^{+0.055}_{-0.056} & \text{(NH)} \\ 0.511^{+0.055}_{-0.055} & \text{(IH)} \end{cases}$$

well compatible with ϑ_{23} maximal

ϑ_{23} maximal difficult to attribute to pure chance …

ϑ_{23} cannot be made maximal by RGE evolution
[barring tuning of b.c. and/or thresold corrections]

when a flavour symmetry is present, ϑ_{23} is determined entirely by breaking effects [no maximal ϑ_{23} from an exact symmetry]

broken abelian symmetries do not work
[not a theorem but no counterexamples]

⇒ we are left with broken non-abelian symmetries

2 $\delta_{CP} = -\pi/2$?

3 U_{PMNS} close to TB (BM,…) ?

discrete non-abelian symmetries?
see G. Altarelli lectures

generalizations

[more on this later on…]

largest possible flavour symmetry G_f is obtained in the limit $\mathcal{Y} = 0$

$$G_{MFV} = U(3)^5$$

[symmetry of kinetic terms with SM particle content]

observed fermion masses and mixing angles (and the anomaly) break G_{MFV} completely (up to the hypercharge and, possibly, B-L)

$$G_{MFV} \supseteq G_f \to H_f$$

for any realistic flavour symmetry

in most predictive models the breaking is spontaneous, by a set of <scalar fields>

$$\theta \to \theta_g = \rho(g)\theta$$

under G_f

$\langle\theta\rangle$ determined by minimizing an energy functional $V(\theta)$ invariant under G_f

$$V\left(\theta_g\right) = V\left(\theta\right)$$

$\langle\theta\rangle$, absolute minimum of $V(\theta)$, breaks G_f down to H_f

Yukawas promoted to dynamical variables

$$\mathsf{y}\left(\theta / \Lambda_f\right)$$

observed Yukawa couplings

$$\mathsf{y}\left(<\theta> / \Lambda_f\right)$$

huge number of possibilities: choice of G_f (global, local, continuous, discrete,…)
choice of representations for scalars φ and fermions

Lecture II

hierarchies without symmetries

previous result

$$(y_u)_{ij} = \lambda^{q_i+p_j} Y_u^{ij} \qquad (y_d)_{ij} = \lambda^{r_i+p_j} Y_d^{ij}$$

can be rewritten as

$$y_u = F_{u^c} Y_u F_q$$

$$y_d = F_{d^c} Y_d F_q$$

$$F_X = \begin{pmatrix} \lambda^{FN(X_1)} & 0 & 0 \\ 0 & \lambda^{FN(X_2)} & 0 \\ 0 & 0 & \lambda^{FN(X_3)} \end{pmatrix}$$

$FN(q) = (p_1, p_2, p_3)$

$FN(u^c) = (q_1, q_2, q_3)$

$FN(d^c) = (r_1, r_2, r_3)$

$Y_{u,d} \approx O(1)$

undetermined by $U(1)_{FN}$

$FN(X_i)$ are $U(1)_{FN}$ charges

$(X = q, u^c, d^c)$

can we obtain this from other frameworks ?

hierarchies from Extra Dimensions [see Antoniadis lectures]

original hope [Kaluza 1921, Klein 1926]: unification of all - gravity and electromagnetism - fundamental interactions

in 5D the metric tensor contains the electromagnetic field

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} + \ldots & -\varphi A_\nu \\ -\varphi A_\mu & -\varphi \end{pmatrix}$$

if $y=x^4$ is compactified on a circle of radius R the U(1) gauge invariance is part of the invariance under general coordinate transformations

$$y \to y + \xi(x)$$ induces $$A_\mu(x) \to A_\mu(x) + \partial_\mu \xi(x)$$ $$x^M = (x, y)$$

the original KK idea
[gauge group=isometry group of the compact space K]
does not work since in the reduction from D>4 to 4D it is not possible to generate chiral fermions

the chirality problem

in a vector-like theory all fermions can be given gauge invariant mass terms

in a chiral theory there are fermions that cannot be given gauge invariant mass terms

vector-like theories are automatically free from gauge anomalies.
In chiral theories the requirement of gauge anomaly cancellation is, in general, a constraint on the representation content in the fermion sector
the SM is an anomaly-free chiral gauge theory

in 1985 Witten proved that in the dimensional reduction [a truncation of the full theory where only the zero modes are kept] of any KK theory where the gauge fields are embedded in the metric g_{MN} there are no 4D chiral fermions

ways out

- in string theory there are also ``external '' gauge fields, that cannot be identified with component of the metric
- in QFT with no gravity + some special ingredient [here a toy model in 5D]

ED can be relevant to the flavour puzzle

$$\mathcal{L} = i\bar{\Psi}\Gamma^M D_M \Psi = i\bar{\Psi}(\Gamma^\mu D_\mu + \Gamma^i D_i)\Psi$$

(index i running over the EDs)

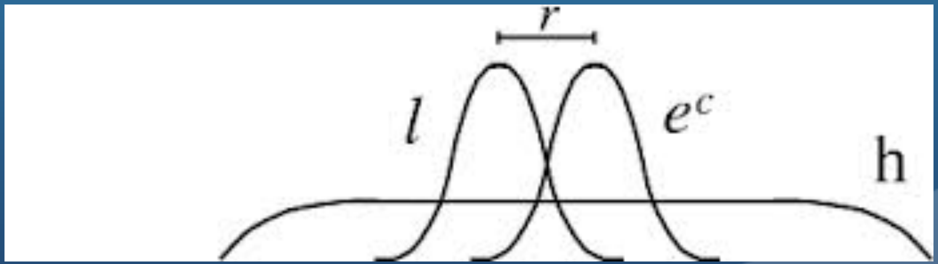
if the ED are compactified on a space K with characteristic length R, with 1/R>> EW scale, in first approximation SM fermions can be thought as 4D zero modes

$$\Gamma^\mu D_\mu \Psi = 0 \quad \Rightarrow \quad \Gamma^i D_i \Psi = 0$$

fermions are also zero modes of the Dirac operator ($\Gamma^i D_i$) in the compact space

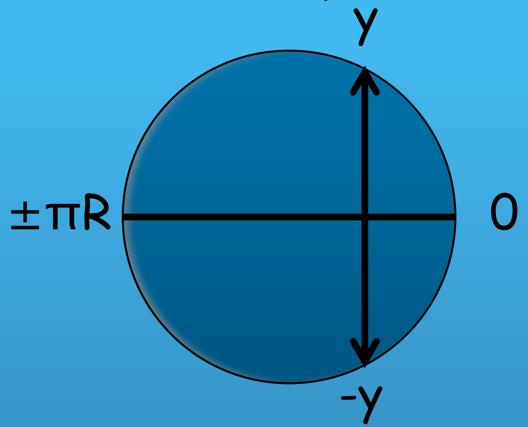
- the number of zero modes of ($\Gamma^i D_i$) depends on the features of the compact space K, opening the possibility of predicting N_g

- the zero modes of ($\Gamma^i D_i$) can be localized in specific region of the compact space. Since 4D Yukawa couplings are determined by the overlaps of the zero-mode profiles in K, their size can be explained in terms of geometrical properties



a toy model in 5D [D.E. Kaplan, Tait 0110126; Arkani-Hamed, Schmaltz 9902417]

consider a 5D space-time where the 5th dimension is an orbifold S^1/Z_2



$$Z_2 \qquad y \leftrightarrow -y$$

this poses some restriction on the type of allowed fields φ(x,y)

$$\varphi(x, y+2\pi R) = \varphi(x,y) \qquad \varphi(x,-y) = \pm\varphi(x,y)$$

free field expansion

$$\varphi(x,-y) = +\varphi(x,y) \qquad \varphi(x,y) = \sum_{n=0}^{+\infty} \frac{1}{\sqrt{(1+\delta_{n0})\pi R}} \varphi^{(n)}(x) \cos\frac{ny}{R}$$

$$m_n^2 = \left(\frac{n}{R}\right)^2$$

$$\varphi(x,-y) = -\varphi(x,y) \qquad \varphi(x,y) = \sum_{n=1}^{+\infty} \frac{1}{\sqrt{\pi R}} \varphi^{(n)}(x) \sin\frac{ny}{R}$$

only even fields have zero modes

consider the SM with $g=g_s=0$ and $g'\neq 0$ and focus on the charge lepton sector

$$\Psi = \begin{pmatrix} e \\ \bar{E}^c \end{pmatrix} \qquad \Psi' = \begin{pmatrix} E \\ \bar{e}^c \end{pmatrix}$$

$$Y(e,e^c) = (-1/2,+1)$$ SM states

$$Y(E,E^c) = (-1,+1/2)$$ additional states

smallest spinor in 5D has 4 components

4D chiral components

$$\Psi_L = \begin{pmatrix} e \\ 0 \end{pmatrix} \qquad \Psi_R = \begin{pmatrix} 0 \\ \bar{E}^c \end{pmatrix}$$

and similarly for Ψ'

unwanted zero-modes eliminated by Z_2 parity assignment

$$\Psi_L(x,-y) = \Psi_L(x,y) \qquad \Psi_R(x,-y) = -\Psi_R(x,y)$$

$$\Psi'_L(x,-y) = -\Psi'_L(x,y) \qquad \Psi'_R(x,-y) = \Psi'_R(x,y)$$

(e,e^c) only have zero-modes

the reduction from 5D to 4D is a chiral theory with the desired particle content

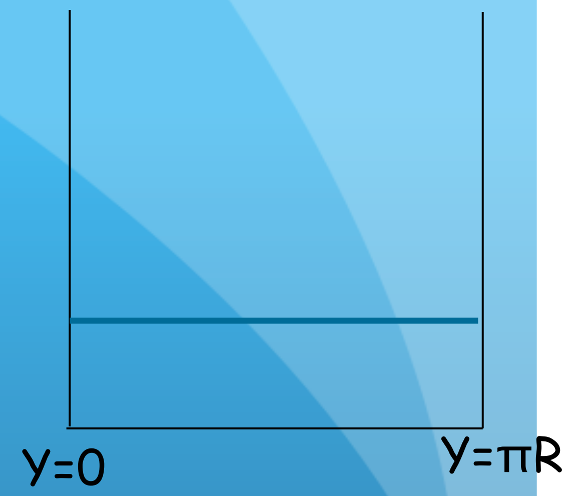
localization of fermion zero modes

$$\mathcal{L} = i\bar{\Psi}\Gamma^M \partial_M \Psi + \ldots = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - \bar{\Psi}\gamma_5 \partial_y \Psi + \ldots$$

equation for the zero mode

$$\partial_y e = 0 \qquad e(x,y) = \frac{1}{\sqrt{2\pi R}} e(x) + \ldots$$

zero mode of e is constant in y



the profile of the zero mode along y can be modified by

$$\mathcal{L} = i\bar{\Psi}\Gamma^M \partial_M \Psi + m\bar{\Psi}\Psi + \ldots = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - \bar{\Psi}\gamma_5 \partial_y \Psi + m\bar{\Psi}\Psi + \ldots$$

$m = M\,\varepsilon(y)$ to respect the parity Z_2 [M constant]

the equation for the zero mode becomes

$$\partial_y e + M\,\varepsilon(y) e = 0$$

[exercise]

$$e(x,y) = \sqrt{\frac{2M}{1 - e^{-2M\pi R}}}\, e^{-M|y|} e(x) + \ldots$$

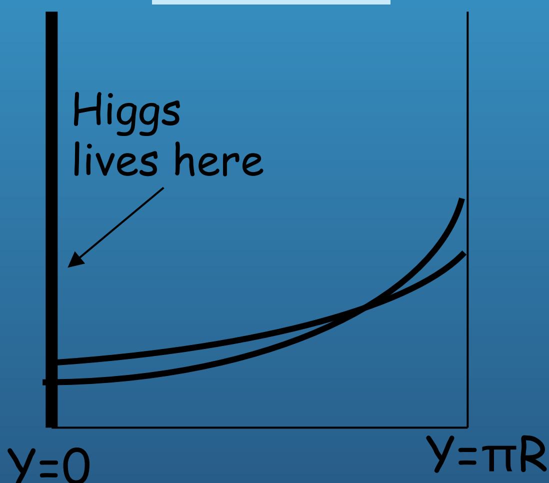
M > 0 zero mode localized at y=0
M < 0 zero mode localized at y=πR

adding mass terms M_j and M'_i to our lagrangian, we get zero modes

$$\Psi_j(x,y) = \sqrt{\frac{2M_j}{1-e^{-2M_j \pi R}}}\, e^{-M_j|y|} \begin{pmatrix} e_j(x) \\ 0 \end{pmatrix} + \dots$$

$$\Psi'_i(x,y) = \sqrt{\frac{2M'_i}{1-e^{-2M'_i \pi R}}}\, e^{-M'_i|y|} \begin{pmatrix} 0 \\ \bar{e}^c_i(x) \end{pmatrix} + \dots$$

assuming



after EW symmetry breaking
the Yukawa interaction reads

$$\mathcal{L}_Y = -\frac{\delta(y)}{\Lambda} \bar{\Psi}'_i Y_e^{ij} \Psi_j \frac{v}{\sqrt{2}} + \dots$$

$$\mathcal{L}^{(4)} = \int_{y=0}^{2\pi R} \mathcal{L}_Y dy = -\frac{1}{\Lambda} \bar{\Psi}'_i(x,0) Y_e^{ij} \Psi_j(x,0) \frac{v}{\sqrt{2}} + \dots$$

after integrating over y

[exercise]

$$y_e = F_{e^c} Y_e F_l$$

same pattern as from $U(1)_{FN}$

$$F_{e_i^c} = \sqrt{\frac{2M'_i/\Lambda}{1-e^{-2M'_i \pi R}}}$$

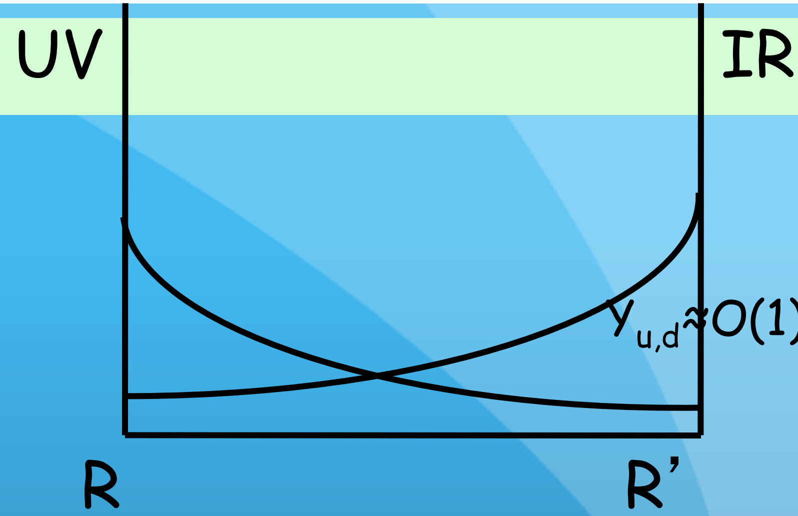
$$F_{l_j} = \sqrt{\frac{2M_j/\Lambda}{1-e^{-2M_j \pi R}}}$$

$$F_i \propto \sqrt{\frac{\xi_i}{1-e^{-\xi_i}}} \approx \begin{cases} \sqrt{\xi_i} & \xi_i >> 1 \\ 1 & \xi_i \approx 0 \\ e^{\xi_i/2} & \xi_i << -1 \end{cases}$$

$$\xi_i = 2M_i \pi R$$

no symmetry:
now hierarchy comes from geometry

it works also in a warped space



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$M_{Pl}^{-1} \approx R \le z \le R' \approx (TeV)^{-1}$$

same suppression factors

$$F_{X_i} = \sqrt{\frac{2\mu_i}{1-e^{-2\mu_i \rho}}}$$

dictionary

ED	μ_i	ρ
Flat $[0,\pi R]$	M_i / Λ	$\Lambda \pi R$
Warped $[R,R']$	$1/2 - M_i R$	$\log R'/R$

M_i = bulk mass of fermion X_i
$Y_{u,d}$ = O(1) Yukawa couplings between bulk fermions and a Higgs localized at one brane

can be extended to neutrinos and to the quark sector

compatible with SU(5) and SO(10) GUTs

[Kitano-Li Phys. Rev. D67 (2003) 116004]
[F, Patel, Vicino 1407.2913]

Lecture II (continue)
the flavour problem

New Physics: effective lagrangian approach

$$L = L_{SM} + \sum_i c_i^5 \frac{O_i^5}{\Lambda} + \sum_i c_i^6 \frac{O_i^6}{\Lambda^2} + \ldots$$

O^d_i gauge invariant operators of dimension d

here: constraints from flavour physics on $|\Delta F|=2$ d=6 operators

FLAVOUR PROBLEM

Operator	Bounds on Λ in TeV ($c_{ij}=1$) Re	Bounds on Λ in TeV ($c_{ij}=1$) Im	Bounds on c_{ij} ($\Lambda = 1$ TeV) Re	Bounds on c_{ij} ($\Lambda = 1$ TeV) Im	Observables
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	$\Delta m_K;\ \epsilon_K$
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	$\Delta m_K;\ \epsilon_K$
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	$\Delta m_D;\ \lvert q/p\rvert, \phi_D$
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	$\Delta m_D;\ \lvert q/p\rvert, \phi_D$
$(\bar{b}_L\gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	$\Delta m_{B_d};\ S_{\psi K_S}$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	$\Delta m_{B_d};\ S_{\psi K_S}$
$(\bar{b}_L\gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

TABLE I: Bounds on representative dimension-six $\Delta F = 2$ operators. Bounds on Λ are quoted assuming an effective coupling $1/\Lambda^2$, or, alternatively, the bounds on the respective c_{ij}'s assuming $\Lambda = 1$ TeV. Observables related to CPV are separated from the CP conserving ones with semicolons. In the B_s system we only quote a bound on the modulo of the NP amplitude derived from Δm_{B_s} (see text). For the definition of the CPV observables in the D system see Ref. [15].

[Isidori, Nir, Perez, 2010]

Minimal Flavour Violation

either the scale of new physics is very large or flavour violation from New Physics is highly non-generic. Useful benchmark: a framework where the only source of flavour violation beyond the SM are the Yukawa coupling

in the limit $y_u = y_d = 0$, the SM lagrangian is invariant under a $U(3)^3$ flavour symmetry

$$G_q = SU(3)_{u^c} \times SU(3)_{d^c} \times SU(3)_q \times \ldots$$

$$q = (1,1,3) \quad u^c = (\bar{3},1,1) \quad d^c = (1,\bar{3},1)$$

if the Yukawa couplings y_u and y_d are promoted to non-dynamical fields (spurions) transforming conveniently, the SM lagrangian remains formally invariant under the flavour group G_q

$$L_{SM} = \ldots - d^c y_d (\Phi^+ q) - u^c y_u (\tilde{\Phi}^+ q) + h.c.$$

$$y_u = (3,1,\bar{3}) \qquad y_d = (1,3,\bar{3})$$

MFV assumes that new operators coming from New Physics do not involve any additional field/spurions and that they are still invariant under G_q
[additional assumption: no additional sources of CPV other than those in $y_{u,d}$]

Example: leading operator with ΔF=2 in MFV

choose, e.g. the basis where

$$y_d = y_d^{Diag} \qquad y_u = y_u^{Diag} V_{CKM}$$

$y_{u,d}^{Diag}$ diagonal

we can form the MFV invariant [exercise]

$$\bar{q}_{Li}\gamma^{\mu}(y_u^+ y_u)_{ij} q_{Lj}\, \bar{q}_{Lk}\gamma_{\mu}(y_u^+ y_u)_{kl} q_{Ll}$$

looking at the down quark sector and selecting i=k=d,s and j=l=b
we get the MFV operator contributing to ΔB=2

$$O_{MFV}(|\Delta B| = 2) = \frac{c}{\Lambda_{NP}^2} y_t^4 (V_{tb} V_{tq}^*)^2\, \bar{q}_L \gamma^{\mu} b_L\, \bar{q}_L \gamma_{\mu} b_L \qquad (q = d,s)$$

where we used
$y_u^{Diag} \approx \mathrm{diag}(0,0,y_t)$

same CKM suppression as in the SM. Now the bound on the scale of New Physics reads

$$\Lambda_{NP} > 5.9\ TeV$$

$$\Lambda_{NP} \leftrightarrow \frac{\Lambda_{NP}}{4\pi} \leftrightarrow \frac{4\pi}{g}\Lambda_{NP}$$

$$\Delta_q \equiv \frac{M_{12}^q}{M_{12}^{q,SM}}$$

(q=d,s)

[O_{MFV} modify M_{12} for B_d and B_s in the same way:
i.e Δ_d and Δ_s are identical and real in MFV]

bound on the scale of New Physics in MFV

Operator	Bound on Λ	Observables
$H^\dagger \left(\overline{D}_R Y^{d\dagger} Y^u Y^{u\dagger} \sigma_{\mu\nu} Q_L\right) (e F_{\mu\nu})$	6.1 TeV	$B \to X_s \gamma$, $B \to X_s \ell^+ \ell^-$
$\frac{1}{2}(\overline{Q}_L Y^u Y^{u\dagger} \gamma_\mu Q_L)^2$	5.9 TeV	ϵ_K, Δm_{B_d}, Δm_{B_s}
$H_D^\dagger \left(\overline{D}_R Y^{d\dagger} Y^u Y^{u\dagger} \sigma_{\mu\nu} T^a Q_L\right) (g_s G^a_{\mu\nu})$	3.4 TeV	$B \to X_s \gamma$, $B \to X_s \ell^+ \ell^-$
$\left(\overline{Q}_L Y^u Y^{u\dagger} \gamma_\mu Q_L\right) \left(\overline{E}_R \gamma_\mu E_R\right)$	2.7 TeV	$B \to X_s \ell^+ \ell^-$, $B_s \to \mu^+ \mu^-$
$i \left(\overline{Q}_L Y^u Y^{u\dagger} \gamma_\mu Q_L\right) H_U^\dagger D_\mu H_U$	2.3 TeV	$B \to X_s \ell^+ \ell^-$, $B_s \to \mu^+ \mu^-$
$\left(\overline{Q}_L Y^u Y^{u\dagger} \gamma_\mu Q_L\right) \left(\overline{L}_L \gamma_\mu L_L\right)$	1.7 TeV	$B \to X_s \ell^+ \ell^-$, $B_s \to \mu^+ \mu^-$
$\left(\overline{Q}_L Y^u Y^{u\dagger} \gamma_\mu Q_L\right) (e D_\mu F_{\mu\nu})$	1.5 TeV	$B \to X_s \ell^+ \ell^-$

TABLE II: Bounds on the scale of new physics (at 95% C.L.) for some representative $\Delta F = 1$ [27] and $\Delta F = 2$ [12] MFV operators (assuming effective coupling $\pm 1/\Lambda^2$), and corresponding observables used to set the bounds.

[Isidori, Nir, Perez, 2010]

Lepton Flavour Violation and MFV

relevant operators

$$i\frac{e}{\Lambda^2}e^c\left(\sigma^{\mu\nu}F_{\mu\nu}\right)\mathcal{Z}(\Phi^+ l) + \frac{1}{\Lambda^2}[\text{4-fermion}] + h.c. + \ldots$$

$\mathcal{Z}_{ij}$ a matrix in flavour space

$$L_Y = -e^c\, y_e(\Phi^+ l) + \; h.c. + \ldots$$

$$BR(\mu \to e\gamma) < 5.7\times 10^{-13}$$

$$\frac{\mathcal{Z}_{\mu e}}{\Lambda^2} < 2\times 10^{-9}\ \text{TeV}^{-2}$$

$$\Lambda > 2\times 10^4\left[\sqrt{\mathcal{Z}_{\mu e}}\right] TeV$$

extension of MFV to leptons is ambiguous:
we can describe neutrino masses in several ways

one neat prediction:
in the limit of vanishing neutrino masses, the only available spurion is

$$y_e = y_e^{Diag}$$

no LFV when $m_\nu = 0$

LFV in the limit of vanishing neutrino masses

previous conclusion can be evaded in several models of fermion masses
e.g. in partial compositeness where elementary fermions acquire a mass through their mixing with a composite sector

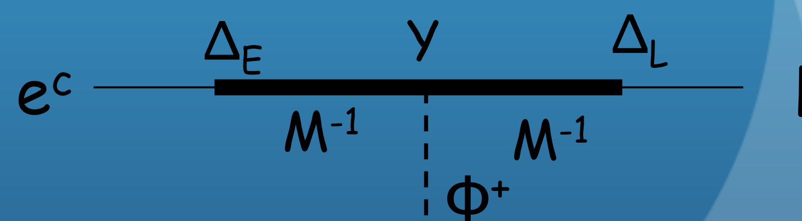
a toy model

$$
\begin{aligned}
L_Y = & -e^c \Delta_E E - L^c \Delta_L l && \Leftrightarrow \text{ elementary-composite mixing} \\
& - E^c M E - L^c M L && \Leftrightarrow \text{ Dirac masses for composite fermions} \\
& - E^c Y (\Phi^+ L) - (L^c \tilde{\Phi}^+) \tilde{Y} E + h.c. && \Leftrightarrow \text{ Yukawa coupling of composite fermions}
\end{aligned}
$$

by integrating out the composite sector [exercise]

$$L_Y = -e^c y_e (\Phi^+ l) + h.c.$$

$$y_e = (\Delta_E M^{-1}) Y (M^{-1} \Delta_L) + \ldots$$



higher-orders in (Φ/M)

at the LO

$$y_e = F_{E^c} Y F_L$$

$$F_{E^c} = \Delta_E M^{-1} \qquad F_L = M^{-1} \Delta_L$$

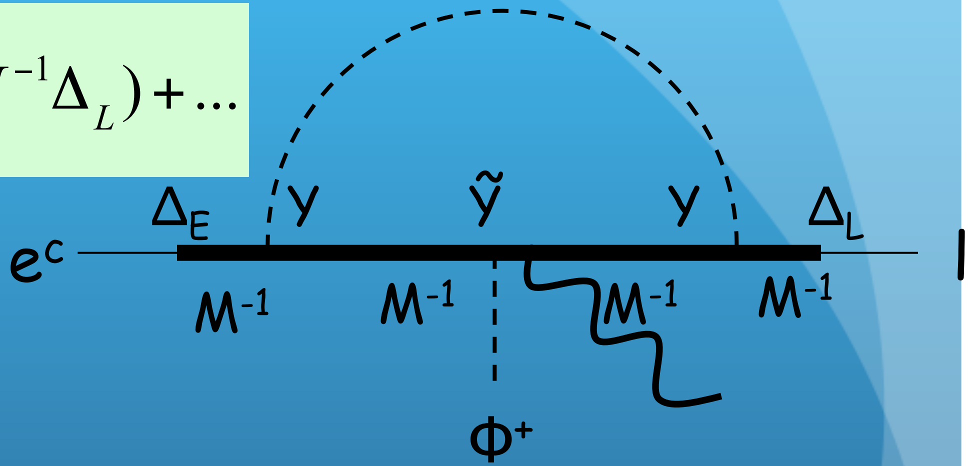
so far neutrino are massless
do we expect LFV in our toy model?

one-loop contribution to lepton dipole operator from Higgs exchange
(assuming M proportional to identity)

$$\frac{Z}{\Lambda^2} \approx \frac{1}{16\pi^2 M^2}(\Delta_E M^{-1}) Y \tilde{Y} Y (M^{-1}\Delta_L) + \ldots$$

in general these combinations
not diagonal in the same basis

$$y_e = (\Delta_E M^{-1}) Y (M^{-1}\Delta_L) + \ldots$$



LFV not suppressed by neutrino masses and unrelated to (B-L) breaking scale

rough estimate

$$\frac{Z_{\mu e}}{\Lambda^2} < 2\times 10^{-9}\ \text{TeV}^{-2}$$

$$M > 10\ \text{TeV}$$

$$\Delta_E \approx \Delta_L$$

$$\frac{\Delta_f}{M} \approx \sqrt{\frac{m_f}{v}}$$

$$Y \approx \tilde{Y} \approx O(1)$$