

# SUSY fine-tuning without prejudice

D. Ghilencea\*

CERN TH and IFIN-HH Bucharest

---

Workshop on the Standard Model and Beyond, 3-14 Sep 2014.

9 Sep - Corfu, Greece.

\* sponsored by PN-II-ID-PCE-2011-3-0607, contract 118/2011.

## (I). Fine-tuning and Naturalness:

L. Susskind PRD 20 (1979), 2619; K. Wilson.

- why EW scale  $v \sim M_Z \ll M_P$  stable under quantum corrections?,  $\left[ \frac{G_f h^2}{G_N c^2} = 1.7 \times 10^{33} \right]$

$$\delta v^2 \sim \delta m_h^2 \sim f(\alpha_j) \Lambda^2, \text{ if } \Lambda \sim M_P: \text{ tune coupling: } 1 : 10^{33} (!).$$

Hierarchy Problem  $\Leftrightarrow$  Fine tuning. Unnatural. Ways out: symmetries:

"Naturalness dogma": 't Hooft (1979)

- Solutions:

1. **scale (conformal) symmetry....** see for example Bardeen 1995;
2. **SUSY:**  $\delta m_h^2 \sim m_S^2 \ln \Lambda / m_S$ ,  $m_S \sim \text{TeV} \dots$  no SUSY seen,  $m_S \gg \text{TeV} \rightarrow$  back to SM fine-tuning  
 ....worse fine tunings: cosmological const.  $\left[ \frac{\rho_v}{\rho} \approx \frac{(2.3 \times 10^{-12} \text{GeV})^4}{(10^{19} \text{GeV})^4} \right]$   
 ....but softly broken TeV-scale SUSY solves hierarchy problem
3. **EFT approach, enforce consistency at every loop order.**

- General potential, SUSY models:

$$\begin{aligned} V = & m_1^2 |h_1|^2 + m_2^2 |h_2|^2 - (B_0 \mu_0 h_1 \cdot h_2 + h.c.) + \lambda_1 |h_1|^4 + \lambda_2 |h_2|^4 + \lambda_3 |h_1|^2 |h_2|^2 \\ & + \lambda_4 |h_1 \cdot h_2|^2 + [\lambda_5/2 (h_1 \cdot h_2)^2 + \lambda_6 |h_1|^2 (h_1 \cdot h_2) + \lambda_7 |h_2|^2 (h_1 \cdot h_2) + h.c.] \end{aligned}$$

$$\begin{aligned} m^2 &\equiv m_1^2 \cos^2 \beta + m_2^2 \sin^2 \beta - B_0 \mu_0 \sin 2\beta, \quad \text{UV : } m_{1,2}^2 = m_0^2 + \mu_0^2 \\ \lambda &\equiv \lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \lambda_{345}/4 \sin^2 2\beta + \sin 2\beta (\lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta) \end{aligned}$$

- The Problem: scales vs. couplings “tension” (“fine-tuning”):

$$v^2 = -m^2/\lambda, \text{ with } v = \mathcal{O}(100 \text{ GeV}), \quad \lambda < 1, \quad \text{but} \quad m_{1,2}, B_0 \text{ and } m \sim \mathcal{O}(1 \text{ TeV}).$$

-  $m_h > m_Z \leftrightarrow$  large loop effects  $\leftrightarrow$  large  $m_{1,2}, B_0 \dots$ ; also a problem of couplings ( $\lambda$  small)

$\Rightarrow$  Solution: - increase  $\lambda$  by 1.- quantum corrections.

2.- (susy) corrections from “new physics” beyond MSSM.

- A closer look: Lagrangian  $\mathcal{L}$  of UV parameters  $\gamma_i : m_0, \mu_0, A_0, B_0, m_{1/2}, \dots$

$$\begin{aligned} \frac{(g_1^2 + g_2^2) v^2}{8} &= -\mu^2 + \frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1} + \dots, \\ 2 m_3^2 &= (m_1^2 + m_2^2 + 2\mu^2) \sin 2\beta + \dots, \end{aligned}$$

so  $v = v(\gamma, \beta)$ ,  $\beta = \beta_0(\gamma) \Rightarrow m_Z = m_Z(\gamma, \beta_0(\gamma))$ . Taylor expansion

$$\begin{aligned} m_Z &= m_Z^0 + \left( \frac{\partial m_Z}{\partial \gamma_i} \right)_{\gamma_i=\gamma_i^0} (\gamma_i - \gamma_i^0) + \dots, \quad m_Z^0 = 91.187 \text{ GeV} \\ \Rightarrow \frac{\delta m_Z}{m_Z^0} &= \Delta_q n^i \frac{\delta \gamma_i}{\gamma_i^0} + \mathcal{O}((\delta \gamma_i)^2), \quad \vec{n} \text{ normal to } m_Z(\gamma_i^0, \beta_0(\gamma_i^0)) = m_Z^0. \end{aligned}$$

with notation  $\Delta_q \equiv \left\{ \sum_i \left( \frac{\partial \ln m_Z}{\partial \ln \gamma_i} \right)_{\gamma_i=\gamma_i^0}^2 \right\}^{1/2}$  I shall call this  $\Delta_q$  “fine tuning” from now on.

$$\frac{\delta m_Z}{m_Z^0} = 4.6 \times 10^{-5} \text{ (2}\sigma\text{)}, \text{ if } \Delta_q \approx 1000 \rightarrow \frac{\delta \gamma_i}{\gamma_i^0} \approx 4.6 \times 10^{-8} \rightarrow \gamma_i = 1 \text{ TeV} \rightarrow \delta \gamma_i = 46 \text{ keV!}$$

(II). Fixing EW scale ( $m_Z$ ) and emergent fine-tuning.

D.G., G. Ross, 2012, 2013.

- Max of likelihood to fit EW observables  $O_j$  ( $\gamma_i$  UV parameters), factorization:

$$L(\text{data}|\gamma_i; v, \beta) = \prod_{j \geq 1} L(O_j|\gamma_i; v, \beta), \quad \gamma_i = m_0, m_{1/2}, A_0, B_0, \mu_0, \dots (\text{susy}) \quad (y_t, y_b, \dots)$$

- Unlike SM, SUSY **predicts**  $v = v(\gamma_i) \Rightarrow$  Regard  $m_Z$  as an observable with  $m_Z^0 = 91.187 \text{ GeV}$ .

$$f_1 \equiv v - (-m^2/\lambda)^{1/2} = 0, \quad f_2 \equiv \tan \beta - \tan \beta_0(\gamma_i) = 0$$

$$\begin{aligned} L_{\text{total}}(\text{data}, m_Z|\gamma_i) &= \int dv \, d(\tan \beta) \, \delta(f_1(\gamma_i; v, \beta)) \, \delta(f_2(\gamma_i; v, \beta)) \, L(\text{data}|\gamma_i; v, \beta) \, \delta(1 - m_Z/m_Z^0) \\ &= v_0 \left[ L(\text{data}|\gamma_i; v_0, \beta) \, \delta[f_1(\gamma_i; v_0, \beta)] \right]_{\beta=\beta_0(\gamma_i)} \quad v_0 = 246 \text{ GeV} \\ &= L(\text{data}|\gamma_i; v_0, \beta_0(\gamma_i)) \, \delta\left[1 - \frac{m_Z(\gamma_i, \beta_0(\gamma_i))}{m_Z^0}\right], \quad \begin{aligned} m_Z &= gv/2 \\ m_Z^0 &= gv_0/2 \\ g^2 &= g_1^2 + g_2^2 \end{aligned} \end{aligned}$$

- Note: factorization assumes no correlations (see later)

All  $\gamma_i$  vary simultaneously:

$$\delta \left[ 1 - \frac{m_Z(\gamma_i, \beta_0(\gamma_i))}{m_Z^0} \right] = \frac{1}{\Delta_q} \delta \left[ n^j \left( 1 - \frac{\gamma_j}{\gamma_j^0} \right) \right], \quad \Delta_q \equiv \left[ \sum_i \left( \frac{\partial \ln m_Z(\gamma_i, \beta_0(\gamma_i))}{\partial \ln \gamma_i} \right) \right]_{\gamma_i=\gamma_i^0}^{1/2},$$

$$\Rightarrow L_{\text{total}}(\text{data}, m_Z | \gamma_i^0) = \frac{1}{\Delta_q} L(\text{data} | \gamma_i; v_0, \beta_0(\gamma_i)) \Big|_{\gamma_i=\gamma_i^0}$$

$\Rightarrow$  Emergent fine-tuning  $\Delta_q$ , as part of total likelihood (not put in by hand!).

$\Rightarrow$  should maximize the ratio  $L/\Delta_q$ . This shows how to compare models with:

good fit but large  $\Delta_q$  and less-good fit but smaller fine tuning!

Bayesian case: see Casas et al 2008, Allanach et al 2007, 2009, Fichet 2012, D.G., H.M. Lee, M. Park 2012

$$\delta(f(\vec{z})) = \frac{1}{|\nabla_z f|_o} \delta \left[ \vec{n} \cdot (\vec{z} - \vec{z}^0) \right], \quad n_i = \frac{\partial_{z_i} f}{|\nabla f|_o}$$

With  $\chi^2 = -2 \ln L$ :

D.G., G.G. Ross, 2012, 2013.

$$\chi_{\text{total}}^2(\gamma_i) = \left[ \underbrace{\chi^2(\gamma_i) + 2 \ln \Delta_q(\gamma_i)}_{\chi_z^2} \right]_{f_1=0, f_2=0}$$

- Good fit:  $\chi_{\text{total}}^2 / \text{ndf} \approx 1$ .  $\Rightarrow$  Naturalness bound:  $\Delta_q < \exp(\text{ndf}/2) \sim 100$ .

$\Rightarrow$  Good fit and fixing EW scale ( $m_Z$ ) demands small EW fine tuning.

$\Rightarrow$  Implications for SUSY models:  $\chi_{\min}^2 \geq \chi_z^2$ :

$$\Delta_q \approx 10 \quad \Rightarrow \quad \chi_z^2 / \text{ndf} \approx 0.5 \quad (\text{ndf} = 9)$$

$$\Delta_q \approx 100 \quad \Rightarrow \quad \chi_z^2 / \text{ndf} \approx 1$$

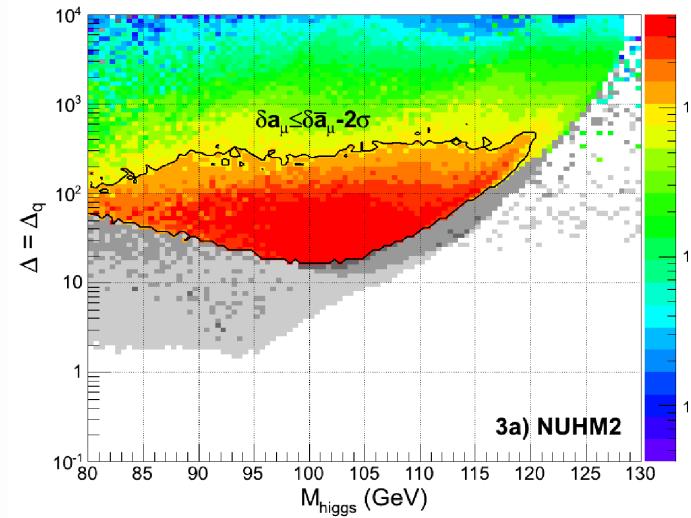
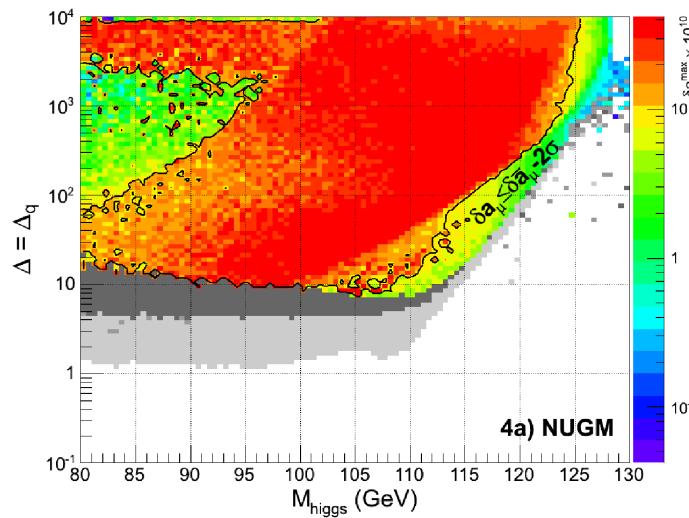
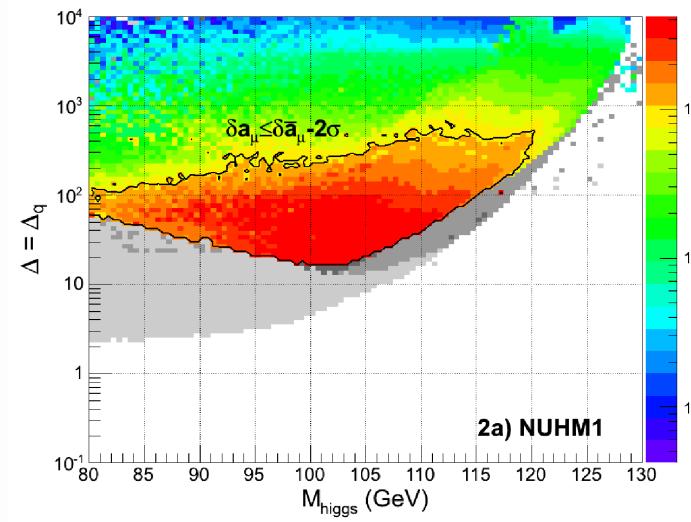
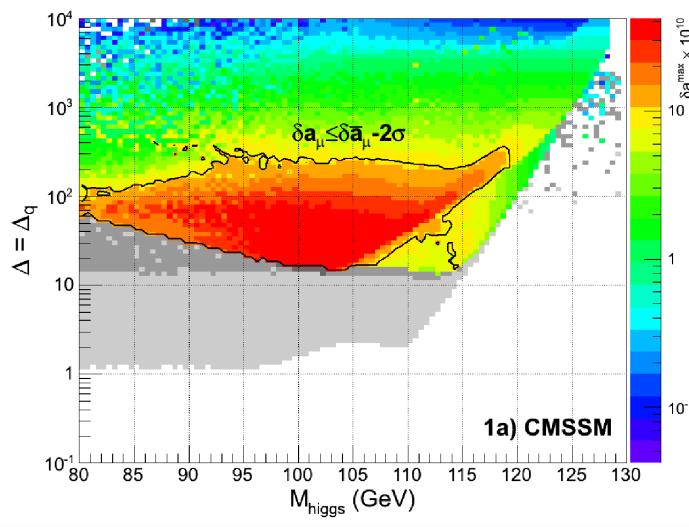
$$\Delta_q \approx 1000 \quad \Rightarrow \quad \chi_z^2 / \text{ndf} \approx 1.5$$

$$\Delta_q \approx 2000 \quad \Rightarrow \quad \chi_z^2 / \text{ndf} \approx 1.7$$

- for  $m_h \sim 125 - 126$  GeV:  $\Delta_q \sim 500 - 2000$  (see next). Numerical studies  $\chi_{\min}^2 / \text{ndf} > 1 (\sim 2)$ .

- $\Delta$  in SUSY:  $\Delta_q$  vs  $m_h$  [2-loop, all  $\{\gamma, \tan \beta\}$  values]

D.G., H. M. Lee, M. Park



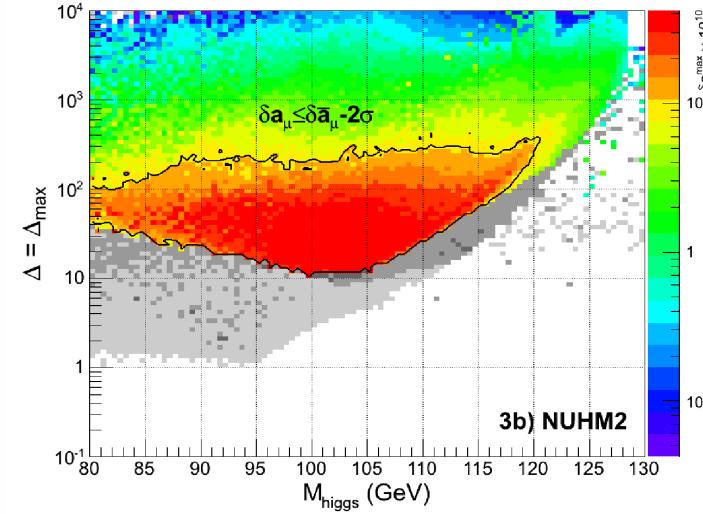
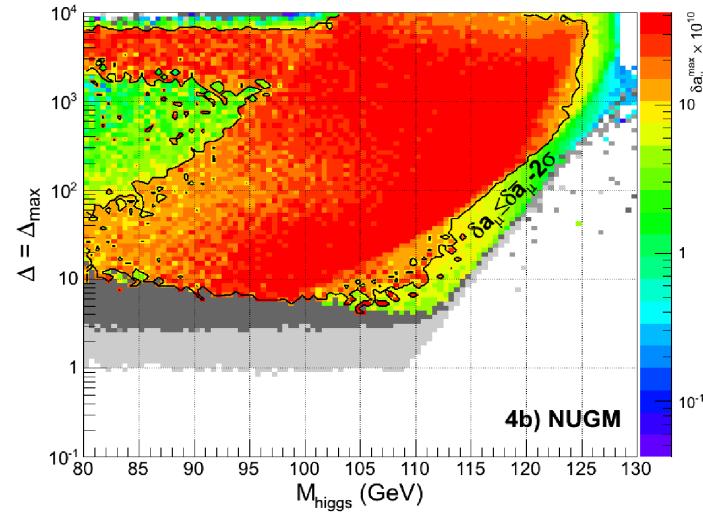
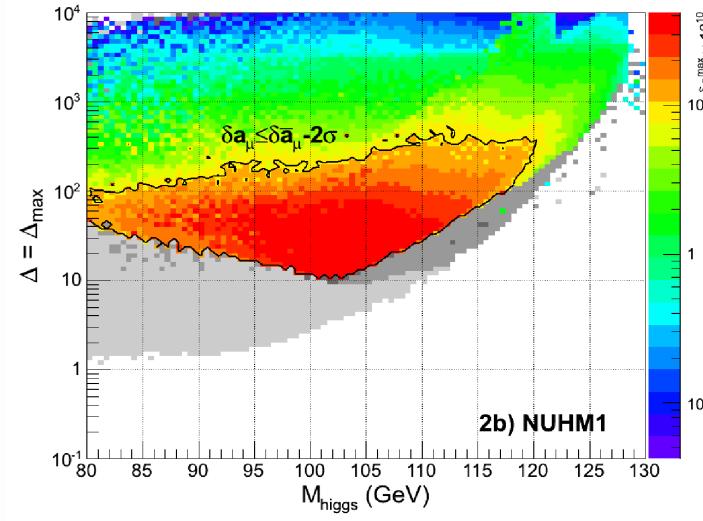
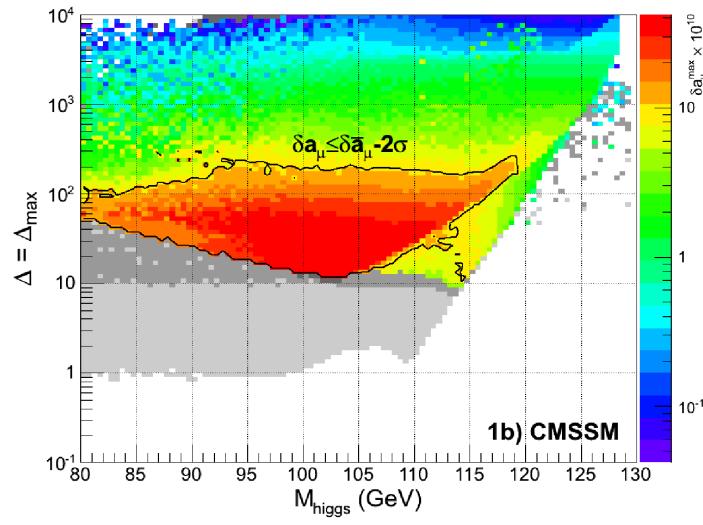
$\Rightarrow m_h$  strongest impact:  $\Delta_q \sim \exp(m_h)$ .  $\Delta_q \sim 1000$  at 125 GeV.

• grey 0: excluded by SUSY; grey 1:  $b \rightarrow s\gamma$ ,  $B_s \rightarrow \mu^+\mu^-$ ,  $\delta\rho$ ; grey 2: excluded by  $\delta a_\mu > 0$ .

$\delta a_\mu$ :  $2\sigma$  contour (red) [smaller  $\delta a_\mu$  outside]

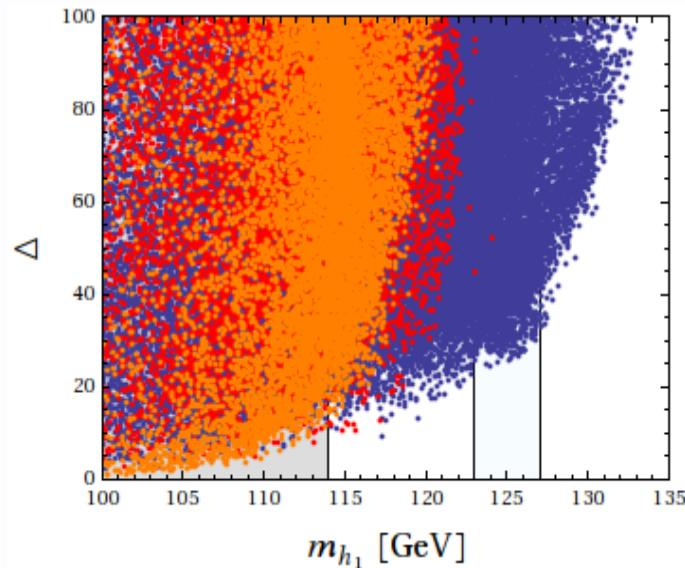
- $\Delta$  in SUSY:  $\Delta_{max}$  vs  $m_h$  [2-loop, all  $\{\gamma, \tan \beta\}$  values]

D.G., H. M. Lee, M. Park



$\Rightarrow \min \Delta_{max}$  v. similar to  $\Delta_q$  where  $\Delta_{max} \equiv \max_\gamma |\Delta_\gamma|$ ,  $\Delta_\gamma \equiv \left( \frac{\partial \ln v^2}{\partial \ln \gamma^2} \right)_o$ ,  $\gamma : m_0, m_{1/2}, \mu_0, A_0 \dots$

- $\Delta$  in SUSY:  $\Delta_{max}$  vs  $m_h$  in **MSSM**, **NMSSM**, **GNMSSM**. (1-loop).



**MSSM**

**NMSSM:**  $W = W_Y + \lambda S H_1 H_2 + \kappa S^3$ ,  $\Delta \sim 200$ .

**GNMSSM:**  $W = W_{NMSSM} + m_{3/2}^2 S + m_{3/2} S^2 + m_{3/2} H_1 H_2$ .  
from underlying  $Z_4^R$  symmetry.  
 $W = W_Y + (\mu + \lambda S) H_1 H_2 + M_* S^2 + \kappa S^3$   
 $\Delta < 30$  for  $m_h \leq 126 \text{ GeV}$ .

G.G. Ross, Schmidt-Hoberg

U. Ellwanger et al; D.G., G.Ross, S. Cassel

- Summary:

Model	$\Delta_q$	$\chi_Z^2(123);$	$\Delta_q$	$\chi_Z^2(125);$	$\Delta_q$	$\chi_Z^2(126);$	$\Delta_q$	$\chi_Z^2(127);$	ndf	$\chi_Z^2/\text{ndf}$ (126)
CMSSM	380	11.88	1100	14.01	1800	14.99	3100	16.08	9	1.66
NUHM1	500	12.43	1000	13.82	1500	14.63	2100	15.29	8	1.83
NUHM2	470	12.31	1000	13.82	1300	14.34	2000	15.20	7	2.05
NUGM	230	10.88	700	13.10	1000	13.82	1300	14.34	7	1.97
NUGMd	200	10.59	530	12.55	850	13.49	1300	14.34	9	1.5
NMSSM	>100	9.21	>200	10.59	>200	10.59	>200	10.59	8	1.32
GNMSSM	22	6.18	25	6.43	27	6.59	31	6.87	7	0.95

- error 2-loop  $m_h$ : 2-3 GeV;  $\Delta_q \sim \exp(m_h/\text{GeV}) \Rightarrow \exp(2) \sim 7.4 \Rightarrow \Delta_q = 20$  or  $150$  equally “good”

(III). So far we discussed  $\chi_{min}^2 \sim \chi_z^2 = 2 \ln \Delta_q$ . What about the deviation  $\delta\chi^2$  from  $\chi_{min}^2$ ?

- Many observables depend on  $v = v(\gamma)$ , so  $\mathcal{O}_i = \mathcal{O}_i(\gamma, v(\gamma))$ , total  $\chi_{\text{total}}^2 = -2 \ln L_{\text{tot}}$ :

D.G. arXiv:1311.6144

$$\begin{aligned} L_{tot}(\mathcal{O}|\gamma) &\sim \frac{1}{(\det M)^{1/2}} \exp \left\{ -1/2 (\mathcal{O}_i - \mathcal{O}_i^0) (M^{-1})_{ij} (\mathcal{O}_j - \mathcal{O}_j^0) \right\} \\ &\sim \frac{1}{\Delta} \frac{1}{(\det \tilde{M})^{1/2}} \exp \left\{ -1/2 (\gamma_\alpha - \gamma_\alpha^0) \tilde{M}_{\alpha\beta}^{-1} (\gamma_\beta - \gamma_\beta^0) \right\}. \end{aligned}$$

$$\mathcal{O}_i(\gamma) = \mathcal{O}_i(\gamma^0) + (\gamma_\alpha - \gamma_\alpha^0) \left( \frac{d\mathcal{O}_i}{d\gamma_\alpha} \right)_{\gamma=\gamma^0} + \dots, \quad \tilde{M}^{-1} \equiv \mathcal{J}^T M^{-1} \mathcal{J}, \quad \mathcal{J}_{i\alpha} \equiv \frac{1}{\mathcal{O}_i^0} \left[ \frac{d\mathcal{O}_i}{d \ln \gamma_\alpha} \right]_{\gamma=\gamma^0}$$

where  $\Delta \equiv [\det M \det \tilde{M}^{-1}]^{1/2}$ , and covariance matrix, if  $M_{ij} = \sigma_i^2 \delta_{ij}$  (Δ: “differential entropy”)

$$\tilde{M}_{\alpha\beta}^{-1} = \left\{ \left( \frac{d(\mathcal{O}_i/\sigma_i)}{d \ln \gamma_\alpha} \right) \left( \frac{d(\mathcal{O}_j/\sigma_j)}{d \ln \gamma_\beta} \right) \right\}_{\gamma=\gamma^0}, \quad \alpha, \beta = 1, 2, \dots, s.$$

- With  $v = v(\gamma)$ , covariance matrix contains info about  $\Delta_q$  from all  $\mathcal{O}_i$ !

D.G. arXiv:1311.6144

$$\begin{aligned}\tilde{M}_{\alpha\beta}^{-1} &= \tilde{M}_{\alpha\beta}^{-1}\Big|_{v=const} + \sum_{i=1}^s \left\{ \left( \frac{\partial \mathcal{O}_i / \sigma_i}{\partial \ln v} \right)^2 \left( \frac{\partial \ln v}{\partial \ln \gamma_\alpha} \right) \left( \frac{\partial \ln v}{\partial \ln \gamma_\beta} \right) \right\}_{\gamma=\gamma^0} \\ &\quad + \sum_{i=1}^s \left\{ \left( \frac{\partial \mathcal{O}_i / \sigma_i}{\partial \ln v} \right) \left( \frac{\partial \ln v}{\partial \ln \gamma_\alpha} \right) \left( \frac{\partial \mathcal{O}_i / \sigma_i}{\partial \ln \gamma_\beta} \right) + (\alpha \leftrightarrow \beta) \right\}_{\gamma=\gamma^0}\end{aligned}$$

$\Rightarrow$  includes individual fine-tunings  $\Delta_\gamma = \partial \ln v / \partial \ln \gamma$  from all observables.

- Also

$$\text{Tr } \tilde{M}^{-1} = \sum_{i=1}^n \sum_{\alpha=1}^s \left( \frac{d \mathcal{O}_i / \sigma_i}{d \ln \gamma_\alpha} \right)_{\gamma=\gamma^0}^2 = \sum_{i=1}^n \left( \frac{\partial \mathcal{O}_i / \sigma_i}{\partial \ln v} \right)_{\gamma=\gamma^0}^2 \times \underbrace{\sum_{\alpha=1}^s \left( \frac{\partial \ln v}{\partial \ln \gamma_\alpha} \right)_{\gamma=\gamma^0}^2}_{= \Delta_q^2} + \dots,$$

$\Rightarrow \tilde{M}$  more fundamental, includes fine-tuning effects in a first approximation!

- Precision data fits either miss this effect ( $v=\text{constant}$ ) or include it in a less manifest way ( $\mu = \mu(\gamma)$ )

- The s-standard deviation confidence interval:

$$-2 \ln L(\gamma') \leq -2 \ln L_{max}(\gamma^0) + s^2, \quad \Rightarrow \quad \sum_{i=1}^n \left\{ \left( \frac{d\mathcal{O}_i/\sigma_i}{d \ln \gamma_\alpha} \right)_{\gamma=\gamma^0} (\gamma'_\alpha / \gamma_\alpha^0 - 1) \right\}^2 \leq s^2$$

$$\Delta_q \leq \frac{s \sigma_z}{m_Z(\gamma^0)} \left| \frac{n^\alpha (\gamma'_\alpha - \gamma_\alpha^0)}{\gamma_\alpha^0} \right|^{-1} \leq \frac{s \sigma_z}{m_Z(\gamma^0)} \left| \frac{n^\alpha \sigma_{th,\alpha}}{\gamma_\alpha^0} \right|^{-1}$$

$\Rightarrow \Delta_q$  upper bound: byproduct of a good fit condition.  $|\gamma'_\alpha - \gamma_\alpha^0| > \sigma_{th,\alpha}$ .

- Other possible measures of tunings related to  $\tilde{M}$ :

$$\rho_\alpha = \sqrt{1 - \tilde{M}_{\alpha\alpha} (\tilde{M}^{-1})_{\alpha\alpha}}, \quad 0 \leq \rho_\alpha \leq 1.$$

- $\rho_\alpha = 0$  then  $\gamma_\alpha$ : independent of the rest.  $\rho_\alpha = 1$ : combination of the rest.
- v. useful to determine the **independent** UV parameters (soft masses, tri/bi-linear couplings)

$$\rho_{\alpha\beta} = \frac{\tilde{M}_{\alpha\beta}}{\sigma_\alpha \sigma_\beta}$$

(IV). For model building: ways to achieve smaller  $\Delta_q$ :

$$v^2 = -\frac{m^2}{\lambda}, \quad v = \mathcal{O}(100 \text{ GeV}), \quad m \sim \mathcal{O}(\text{TeV}). \quad \lambda < 1.$$

1.- increase  $\lambda$  (effective higgs coupling) by **quantum** corrections. ( $m$ : soft masses combination)

2.- increase  $\lambda$  (and  $m_h$ ) by **classical** (susy) corrections from “new physics” beyond MSSM higgs,  
parametrised by d=5, 6 operators:

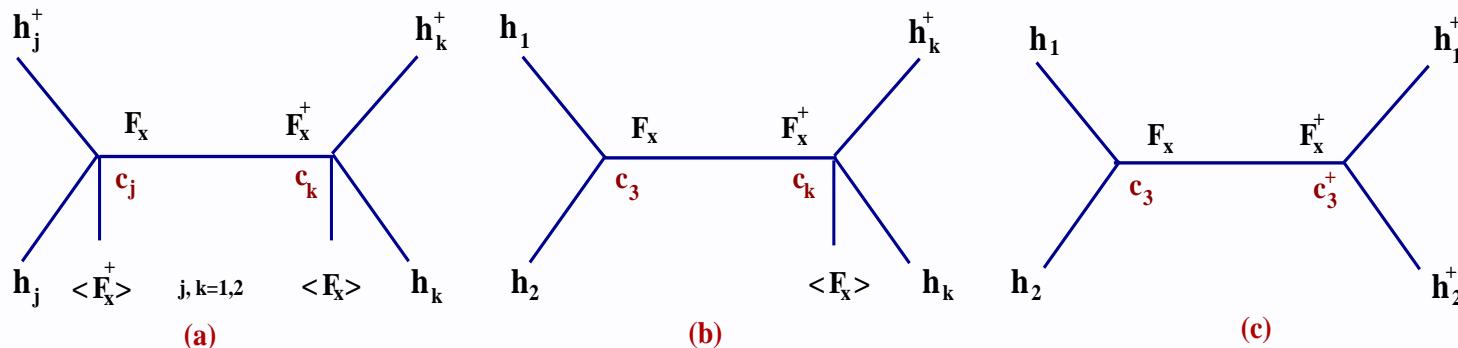
$$\Delta_q(m_h) \approx \exp(-\delta m_h/\text{GeV}) \Delta_q(m_h)|_{\text{CMSSM}}$$

Carena et al 2009, D.G., Dudas, Antoniadis 2009, 2010.

3.- increase  $\lambda$  by reducing SUSY breaking scale to  $\sqrt{F_X} = 1 \text{ TeV}$  (from “usual”  $10^{10} \text{ GeV}$ ).

Corrections to  $\lambda$  from:

Casas et al 2003, D.G., Dudas, Antoniadis 2010, 2014.



No new states added! downside: v. light gravitino, the LSP (mili-eV). Dark matter?

## (V) Conclusions:

- $\chi^2$  and fine-tuning often studied separately. We showed they are related.
- $\Delta_q$  emerges from total likelihood to fit the data **including**  $m_Z(\gamma)$  which thus accounts for naturalness.

large  $L_{\text{total}}(\text{data}, m_Z | \gamma) = L(\text{data} | \gamma) / \Delta_q$  required or small  $\chi^2_{\text{total}} = \chi^2 + 2 \ln \Delta_q$ .

- Mathematical support for  $\Delta_q$  as a fine tuning measure
- Bound on  $\Delta_q$ : good fit:  $\chi^2_z / \text{ndf} \approx 1$  then  $\Delta_q < \exp(\text{ndf}/2) \sim 100$ .
- Covariance matrix  $\tilde{M}$  automatically includes EW fine-tuning effects if  $v = v(\gamma)$ .
- $\Delta_q$  less fundamental than  $\tilde{M}$ ....
- Small  $\Delta_q$  still possible:

$\Rightarrow \Delta_q, \Delta_{\max} \sim \exp(m_h/\text{GeV}) \sim 500 - 1000$ , for  $m_h \approx 126$  GeV. GNMSSM has  $\Delta_q \sim \mathcal{O}(20)$ .

- Significant error factor:  $\sim e^{\sigma_{m_h}}$  ( $= 7.5$  to  $20$ ) of  $\Delta_q$  due to theoretical  $\sigma_{m_h} = 2 - 3$  GeV.

- Naturalness in the Bayesian approach.

$$p(a|b) p(b) = p(a \cap b) = p(b|a) p(a)$$

Bayes theorem:

[ initial belief + data  $\rightarrow$  updated belief ].

Thomas Bayes (1761), Laplace (1812)

$$p(\gamma|\text{data}) = \frac{L(\text{data}|\gamma) p(\gamma)}{p(\text{data})}, \quad p(\text{data}) = \int L(\text{data}|\gamma) p(\gamma) d\gamma, \quad \gamma: \{m_0, m_{1/2}, \mu_0, A_0, B_0; y_t, y_b, \dots\}.$$

- $p(\text{data})$ : “evidence”. Models 1, 2:  $p_1(\text{data})/p_2(\text{data})$ .  $p(\gamma)$ =priors.
- EW constraints:  $f_1(\gamma; v, \beta) = f_2(\gamma; v, \beta) = 0, \Rightarrow v(\gamma, \dots), \tan \beta_0(\gamma \dots)$ .

$$\begin{aligned} p(\text{data}) &= \int d\gamma p(\gamma) dv d(\tan \beta) \delta(m_Z - m_Z^0) \delta(f_1(\gamma; v, \beta)) \delta(f_2(\gamma; v, \beta)) L(\text{data}|\gamma; \beta, v), \\ &= \int_{f_{1,2}=0} dS_\gamma \frac{L(\text{data}|\gamma)}{\Delta_q(\gamma)} p(\gamma), \quad \Delta_q \equiv \left[ \sum_\gamma \Delta_\gamma^2 \right]^{1/2}, \quad d\gamma \equiv \prod_i d\gamma_i, \quad p(\gamma) = p(\gamma_1, \dots, \gamma_i). \end{aligned}$$

D.G., H. M. Lee, M. Park, arXiv:1203:0569

$\Rightarrow \Delta_q$  due to fixing EW scale, in addition to/independent of priors  $p(\gamma)$ ! large  $L/\Delta_q$  needed.

answers e)?

$$\int_{R^n} h(z_1, \dots, z_n) \delta(g(z_1, \dots, z_n)) dz_1 \dots dz_n = \int_{S_{n-1}} dS_{n-1} h(z_1, \dots, z_n) \frac{1}{|\nabla_{z_i} g|},$$

- Reducing  $\Delta_q$  by physics beyond MSSM higgs sector: MSSM + d=6 operators

$$\begin{aligned}\mathcal{O}_j &= \int d^4\theta \mathcal{Z}_j (H_j^\dagger e^{V_j} H_j)^2, \quad (j = 1, 2). & \mathcal{O}_3 &= \int d^4\theta \mathcal{Z}_3 (H_1^\dagger e^{V_1} H_1) (H_2^\dagger e^{V_2} H_2) \\ \mathcal{O}_4 &= \int d^4\theta \mathcal{Z}_4 (H_2 H_1) (H_2 H_1)^\dagger, & \mathcal{O}_k &= \int d^4\theta \mathcal{Z}_k (H_k^\dagger e^{V_k} H_k) H_2 H_1 + h.c., \quad (k=5,6) \\ \mathcal{O}_7 &= \int d^2\theta \mathcal{Z}_7 \text{Tr} W_i^\alpha W_{i,\alpha} (H_2 H_1) + h.c.,\end{aligned}$$

where  $\mathcal{Z}_j(S, S^\dagger) = \alpha_{j0} + \alpha_{j1} S + \alpha_{j1}^* S^\dagger + \alpha_{j2} m_0^2 S S^\dagger$ ,  $\alpha_{jk} \sim 1/M_*^2$ ,  $S = m_0 \theta \theta$

$\mathcal{O}_{1,2,3}$ : generated by massive T, U(1);  $\mathcal{O}_4$ : singlet, T.  $\mathcal{O}_{5,6}$ : 2 D, singlet.

$$\begin{aligned}\Rightarrow \delta m_h^2 &= -2v^2 [(\alpha_{30} + \alpha_{40})\mu_0^2 - \alpha_{20} m_Z^2] - \frac{(2\zeta_0 \mu_0)^2 v^4}{m_A^2 - m_Z^2} + \frac{v^2 \cot \beta}{m_A^2 - m_Z^2} [4m_A^2 \mu_0^2 (2\alpha_{50} + \alpha_{60}) \\ &\quad - (2\alpha_{60} - 3\alpha_{70}) m_A^2 m_Z^2 - (2\alpha_{60} + \alpha_{70}) m_Z^4] + \mathcal{O}(1/(M_*^2 \tan^2 \beta))\end{aligned}$$

$\Rightarrow \alpha_{j0}$  (choice?)  $\Rightarrow$  increase  $m_h$ , reduce fine-tuning by:

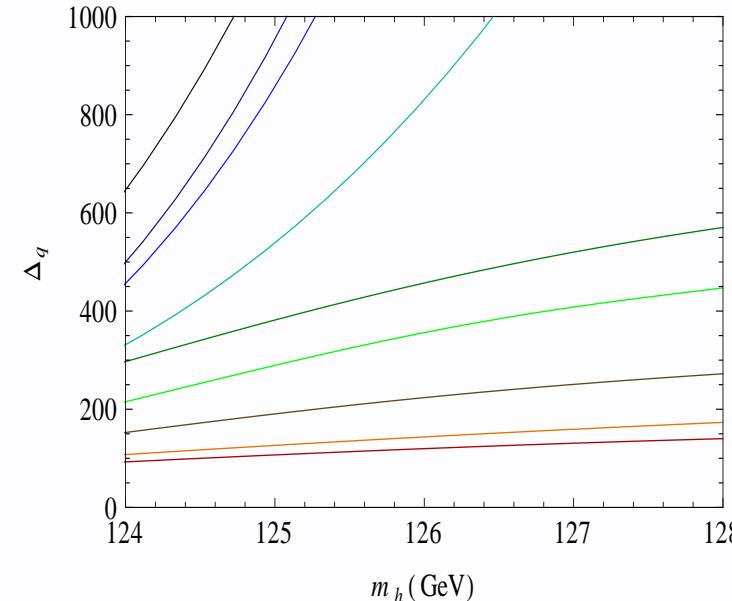
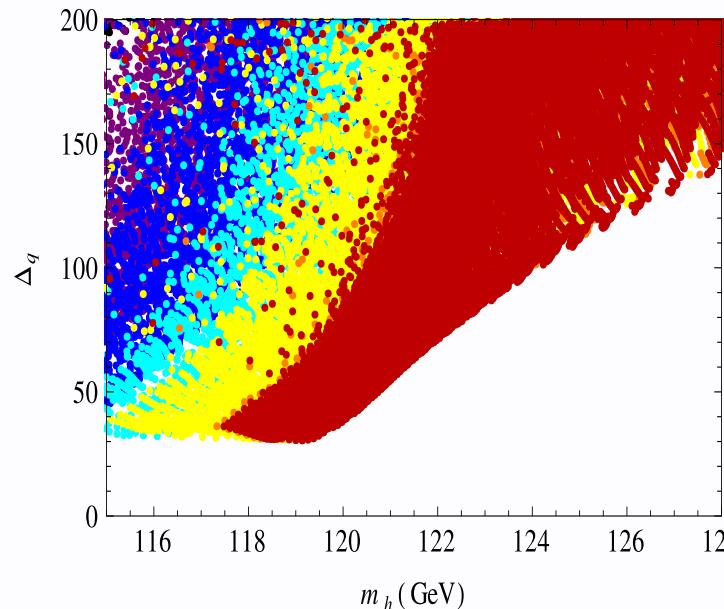
D.G. et al, NPB 848(2011), NPB 831(2010),

M. Carena et al, PRD 85(2012), PRD 81, 82(2010),

F. Boudjema et al, PRD 85 (2012)

$$\Delta_q(m_h) \approx \exp(-\delta m_h/\text{GeV}) \Delta_q(m_h) \Big|_{\text{CMSSM}}$$

- “Non-linear” MSSM: MSSM with low-scale of SUSY breaking:  $\sqrt{F} \sim \mathcal{O}(1)$  TeV.

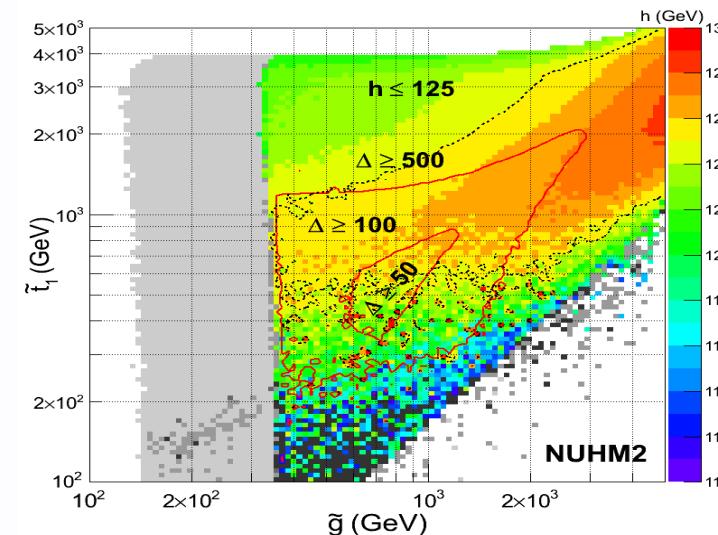
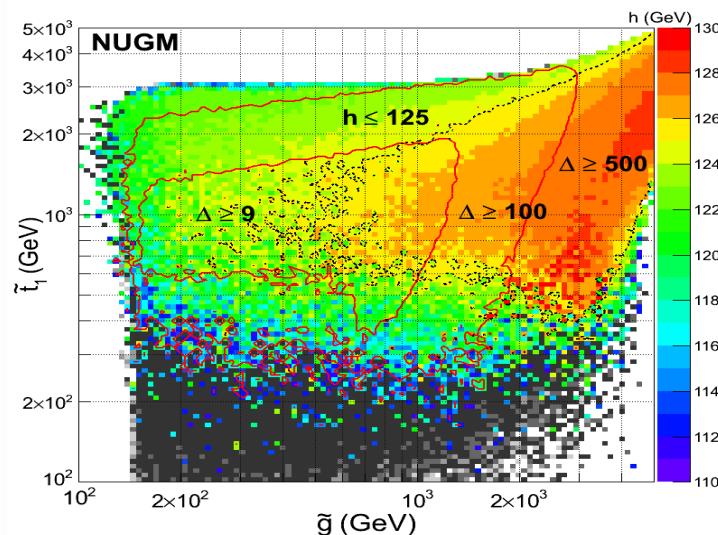
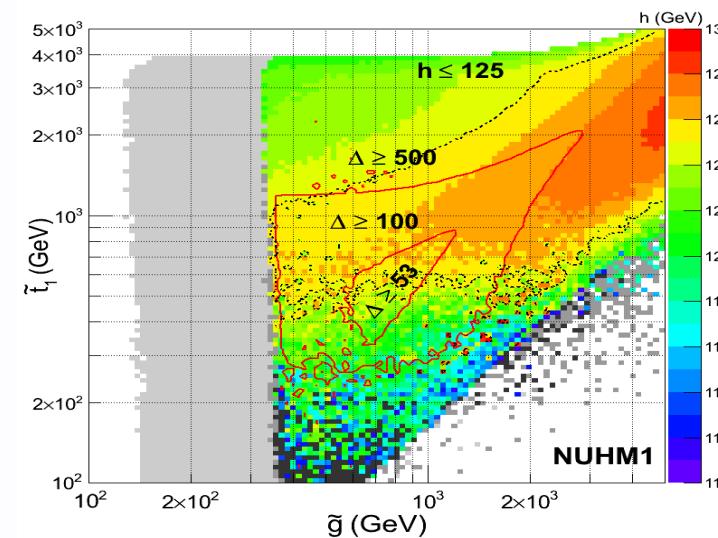
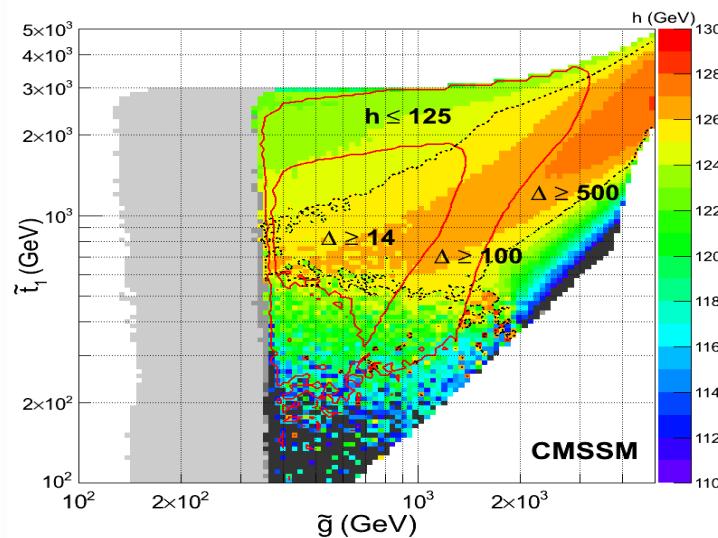


D.G., Dudas, Antoniadis 2011, 2014.

- left plot:  $\Delta_q$  vs.  $m_h$  for  $\sqrt{F} = 3.2$  TeV.  $\Delta_q \sim 100$ .
- right plot:  $\Delta_q$  vs  $m_h$  for  $\sqrt{F} = 3.2$  TeV (lower curve) to 9 TeV (top curve).
- MSSM recovered at large  $\sqrt{F}$ .

- Stop vs Gluino with largest  $m_h$  and min  $\Delta_q$ .  $\{\gamma, \tan \beta\}$  all values]

D.G., H. M. Lee, M. Park



⇒ constraints on  $m_h$  strongly reduce the viable regions.