Applications of Anisotropic Gauge/Gravity Dualities

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Review Talk on parts of Recent Work.

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Outline

Introduction

- Since the initial AdS/CFT correspondence was found, there has been a lot of effort to understand better the gauge/gravity duality. Integrability, scattering amplitudes, computations of various correlators in both sides...
- Another direction for research is to construct gauge/gravity dualities that can be though as toy models to describe realistic systems and theories, with the hope of some universal behaviors.
- Lot of effort to find more realistic gauge/gravity dualities. For example:

✓ Less Supersymmetry. Example: $\mathcal{N} = 1$ β deformed theories.

✓ Broken conformal symmetry, confinement. Example: D4 Witten model.

✓ Finite temperature. Example: Black hole in AdS.

✓ Inclusion of dynamical quarks in Quenched and Unquenhed theories. Needed for: Meson Spectrum, Static potential screening.

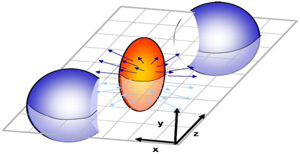
✓ Inclusion of Anisotropy. Example: In this talk

...

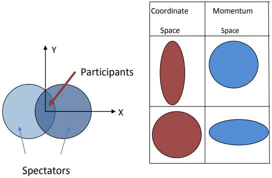
Why do we need anisotropic theories?

- It is interesting theoretically to study thermodynamics and observables in anisotropic IIB SUGRA solutions.
- Challenging calculations for certain observables, since isotropic symmetry is broken. New methods need to be developed for analytic calculations.
- Several Systems are anisotropic.
  •In early stages the QGP has anisotropies! From this phase important information is extracted for the QGP.
  •...
- Hope for new universal properties of the theories, depending on generic characteristic of the new geometries.

An example of anisotropy due to initial bc:Elliptic flow



Pressure gradients for non-central collisions along the short axis of the elliptic flow are higher than the long axis. Therefore the expansion along the short axis is more rapid leading to anisotropic momentum distribution.



Longitudinal expansion

- By considering completely central collisions we focus more on the longitudinal anisotropic expansion.
- At $\tau = \tau_0$ the partonic momentum distributions can be assumed isotropic. Directly after that rapid longitudinal expansion along the beam line which leads to local momentum and pressure anisotropy $P_L < P_T$.
- Finally the plasma becomes and remains isotropic for $\tau \geq \tau_{iso}$.
- Lots of work in gauge/gravity duality for $\tau \geq \tau_{iso}$. Here we study certain quantities in the earlier time where the plasma is anisotropic.

Motivation–Things to Do

- Several physical systems are anisotropic. Eg: The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- But regarding the QGP, the main question we answer here is: How the inclusion of anisotropy modifies the results on several observables in our dual QGP compared to the isotropic theory?
- There exist several results for observables in weakly coupled theories. Is there any relevance with the strongly coupled limit models?
- Properties of the supergravity solutions, that are dual to the anisotropic plasmas.
- Theoretical interest for the deformed theory, since it is a consistent top-down model. Several Universality Relations are violated! New universal propertied depending on the shape of the geometry.

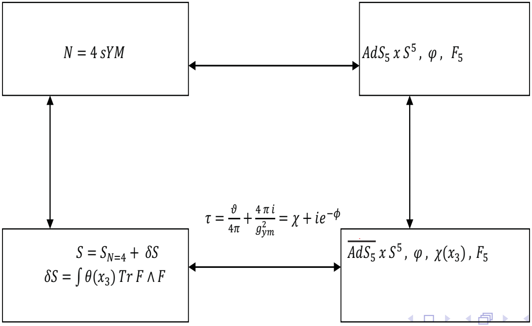
An Example:How does Anisotropy is introduced?

- Introduction of additional branes: Lifhsitz-like Supergravity solutions.

(Azeyanagi, Li, Takayanagi, 0905.0688 jhep)

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

- Which equivalently leads to the following deformation diagram.



An anisotropic background

The metric in string frame *(Mateos, Trancanelli, 1105.3472 prl, 1106.1637 jhep)*

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}\,dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3{}^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}\,d\Omega_{S^5}^2\,.$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy. The anisotropic parameter is α with units of inverse length ($\chi = \alpha x_3$).
In sufficiently high temperatures, $T \gg \alpha$, and imposed boundary conditions the Einstein equations can be solved analytically:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2\frac{1}{24u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4\log 2 + (3u_h^4 + 7u^4)\log\left(1 + \frac{u^2}{u_h^2}\right)\right]$$

$$\mathcal{B}(u) = 1 - \alpha^2\frac{u_h^2}{24}\left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right)\right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}$$

The isotropic limit $\alpha \to 0$ reproduce the well know result of the isotropic D3-brane solution (dual to $\mathcal{N} = 4$ sYM finite T solution).

The metric can be expressed in α, T parameters through

$$u_h = \frac{1}{\pi T} + \alpha^2 \frac{5log2 - 2}{48\pi^3 T^3} \ .$$

The pressures can be found from the expectation value of the stress tensor, where the elements $\langle T_{11} \rangle = \langle T_{22} \rangle = P_{x_1 x_2}$ denote the pressure along the x_1 and x_2 directions and $\langle T_{33} \rangle = P_{x_3}$ is the pressure along the anisotropic direction.

$$P_{x_3} < P_{x_1 x_2} \ ,$$

resembling the plasma pressure anisotropies.

But let us work with generic anisotropic theories!

Write the anisotropic metric as

$$ds^2 = g_{00}dx_0^2 + g_{11}dx_1^2 + g_{22}dx_2^2 + g_{33}dx_3^2 + g_{uu}du^2 + \text{internal space}$$

$x_{1,2} =: x_\perp$ transverse direction to anisotropy, $g_{11} = g_{22}$
$x_3 =: x_\parallel$ parallel direction to anisotropy.
$Q_\parallel := Q_{x_3} = Q_{anisotropic}$
$Q_\perp := Q_{x_1 \ or \ x_2}$

$$\downarrow \quad \downarrow \quad \downarrow$$

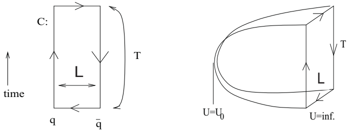
$$\frac{Q_\parallel}{Q_\perp} = ?, \quad \frac{Q_\parallel}{Q_{iso}} = ?, \ldots$$

Warm up: Static Potential and Force in the anisotropic background

- We consider a string world-sheet (τ, σ) of the following form.

String Configuration

$$x_0 = \tau, \qquad x_3 = \sigma, \qquad u = u(\sigma) \text{ , along aniso direction}$$
$$x_0 = \tau, \qquad x_2 = \sigma, \qquad u = u(\sigma) \text{ , along transverse direction}$$



Lets name x_p the direction where the pair is aligned.The solution to Nambu-Goto action

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\tilde{g}}$$

is a catenary shape w-s with u_0 being the turning point.

Results depend on x_p. In general the length of the two endpoints of the string on the boundary is given by

$$L_p = 2\int_{\infty}^{u_0} \frac{du}{u'} = 2\int_{u_0}^{\infty} du \sqrt{\frac{-g_{uu} c_0^2}{(g_{00} g_{pp} + c_0^2) g_{pp}}}\ .$$

Which should be inverted as $u_0(L)$. The normalized energy of the string is

$$2\pi\alpha' V_p \quad = \quad c_0 L + 2\left[\int_{u_0}^{\infty} du \sqrt{-g_{uu} g_{00}}\left(\sqrt{1+\frac{c_0^2}{g_{pp} g_{00}}} - 1\right) - \int_{u_h}^{u_0} du \sqrt{-g_{00} g_{uu}}\right]\ .$$

(Sonnenschein, hep-th 0003032, review;D.G 1202.4436 jhep;)

Applied in the axion deformed anisotropic theory:

• $V_{\parallel} < V_{\perp} < V_{iso}$.
  Screening!

(D.G 1202.4436 jhep; Rebhan, Steineder 1205.4684 jhep; Chernicoff, Fernandez, Mateos, Trancanelli, 1208.2672 jhep, D.G 1306.1404 review;)

• Note: Analysis of the Imaginary Potential and the Thermal Width can be made by fluctuating the same string configuration.
(Bitaghsir, D.G, Soltanpanahi, 1306.2929 jhep)

Another observable: The Jet Quenching

• The parameter is a measure of energy loss of the quark, and can be measured as the expectation value of WL with light-like lines.

(Liu,Rajagopal,Wiedermann,hep-ph/0605178 prl)

• Canceling the divergences and applying approximations we obtain

$$\hat{q}_{p\ (k)} = \frac{\sqrt{2}}{\pi\alpha'}\left(\int_0^{u_h}\frac{1}{g_{kk}}\sqrt{\frac{g_{uu}}{g_{--}}}\right)^{-1}.$$

(D.G 1202.4436 jhep)

where $g_{--} = 1/2(g_{00} + g_{pp})$.

The index p denotes the direction along the motion of the quark and k the direction along which the momentum broadening happen.

Eg:Motion along the anisotropy: $p = 3$ and momentum broadening in the transverse space $k = 2$.

(Chernicoff, Fernandez, Mateos, Trancanelli, 1203.056 jhep; Rebhan, Steineder 1205.4684 jhep)

Generic Remarks that need further attention:

• Observables along the anisotropic direction depend stronger on the anisotropy. The degree of modification of the anisotropic geometry passes to the observables.

• Certain qualitative features of the observables depend on the shape of the geometry (prolate, oblate) than its exact details. Universal Properties?

Universality Relations

Shear Viscosity over Entropy density ratio universal low value prediction
(Policastro, Son, Starinets, hep-th/0104066 prl; Buchel, Liu, hep-th/0311175 prl;)

$$\frac{\eta}{s} \gtrsim \frac{1}{4\pi} .$$

In anisotropic theories has been found to be clearly violated! (Rebhan, Steineder 1110.6825 prl; Jain, Kundu, Sen, Sinha, Trivedi 1406.4874)

Reason:

$$\frac{\eta}{s} \propto \frac{g_{11}(u_h)}{g_{33}(u_h)} \frac{1}{4\pi} .$$

All prolate deformed geometries violate the bound!

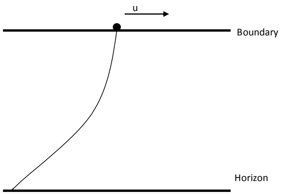
• Another universality relation which is violated is the inequality between the Langevin coefficients: $\kappa_L > \kappa_T$.
(Gursoy, Kiritsis, Mazzanti, Nitti 1006.3261 jhep;D.G, Soltanpanahi, 1310.6725 prd, 1312.7474 jhep)

Drag Force

The dynamics of the quark can be described by

$$\frac{dp}{dt} = F_{drag} + F(t) \ .$$

In AdS/CFT the drag force of a single quark moving in the anisotropic plasma can be represented by a trailing string from the boundary where the probe quark moves with the constant speed, to the horizon of the black hole. *(Herzog, Karch, Kovtun, Kozcaz, Yaffe hep-th/0605158 jhep; Gubser, hep- th/0605182 prd)*



At $u = u_0$ there is horizon of the induced worldsheet metric, given by

$$\left(g_{uu}(g_{00} + g_{pp} v^2)\right) \big|_{u=u_0} = 0 \ .$$

Calculating the momentum flowing from the boundary to the bulk, the drag force of a quark moving along the p direction for any background, is found to be

$$F_{drag,p} = -\sqrt{\lambda}\frac{\sqrt{-g_{00}g_{pp}}}{(2\pi)}\Big|_{u=u_0}$$

The 'effective world-sheet temperature' is

$$T_{ws}^2 = \left|\frac{1}{16\pi^2}\frac{1}{g_{00}g_{uu}}(g_{00}\,g_{pp})'\left(\frac{g_{00}}{g_{pp}}\right)'\right|\Bigg|_{u=u_0} \ .$$

In near horizon Dp black brane geometries $T_{ws} < T$. *(Nakamura, Ooguri 1309.4089, prd)*

Momentum Broadening

The $F(t)$ is the factor that causes the momentum broadening, which leads to

$$\frac{\left\langle p_{L,T}^2 \right\rangle}{\mathcal{T}} = 2\kappa_{L,T}$$

$\kappa =$ Mean Squared Momentum Transfer per Time.

• The index L refers to the direction along the motion of quark, the index T is the direction transverse to the velocity of quark.

• In strong coupling limit for a quark moving along p direction, these fluctuations are introduced to the Wilson line

$$t = \tau, \quad u = \sigma, \quad x_p = v\, t + \xi(\sigma) + \delta x_p(\tau, \sigma), \quad x_2 = \delta x_2(\tau, \sigma), \quad x_3 = \delta x_3(\tau, \sigma).$$

$\delta x_p(\tau, \sigma)$: Longitudinal fluctuation.
$\delta x_2(\tau, \sigma)$: Transverse fluctuation .

Their ratio can be simplified to

$$\frac{\kappa_L}{\kappa_T} = \frac{1}{g_{pp} g_{kk}} \frac{(g_{00} g_{pp})'}{(g_{00}/g_{pp})'}\Bigg|_{u=u_0}$$

Example: $p = 3$ and $k = 1$: Quark moves along the anisotropic direction x_3 and the transverse direction is x_1.

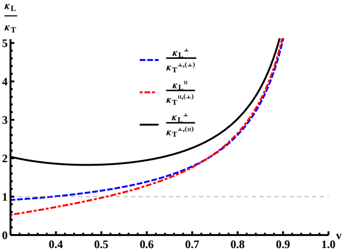
• This is a Universal Inequality independent of the isotropic background used! *(Gursoy, Kiritsis, Mazzanti, Nitti 1006.3261 jhep; D.G, Soltanpanahi 1310.6725 prd, 1312.7474 jhep)*

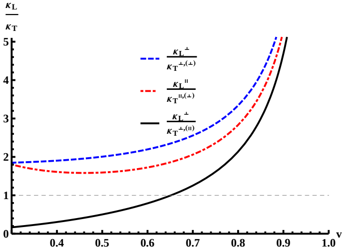
• Only possibility to have it violated is in the anisotropic theories!

• Anisotropic theories allow negative κ_L coefficient! ...?

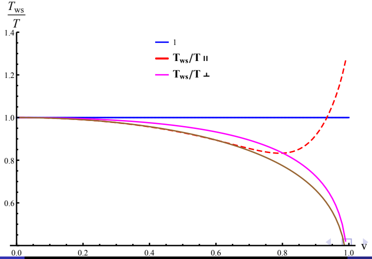
Consider the space dependent axion anisotropic background.

- $\kappa_L < \kappa_T$ depending on shape of the geometry. Oblate vs Prolate Geometries.





- $T_{ws} \lessgtr T$ in aniso theories in contrast to iso.



Conclusions

Working with generic anisotropic theories:

- Several observables have been studied Static Potential, the Drag Force, the Jet Quenching...
- Universal Relations are violated in the Anisotropic Theories. The Langevin coefficients inequality $\kappa_L > \kappa_T$ proved to hold for isotropic backgrounds is violated for the anisotropic theories!

  Related progress:
- Non-Integrability of the anisotropic spaces and possible appearance of chaos. *(D.G, Sfetsos 1403.2703 jhep)*

  Work in progress:
- Anisotropic k-string configurations. Challenging due to broken isotropy.
- Thermalization!

Thank you

AdS/CFT correspondence

- The AdS/CFT correspondence, in its initial form is a duality between the $\mathcal{N} = 4$ supersymmetric Yang-Mills and type IIB superstring theory on $AdS_5 \times S^5$.
- In this correspondence there exist a map between gauge invariant operators in field theory and states in string theory.
- It is a strong/weak duality:

$$g_s \propto g_{YM}^2 \ , \qquad \frac{R^4}{l_s^4} \propto g_{YM}^2 N_c := \lambda$$

- Example: The Wilson loop, which is related to the static potential of the heavy bound $\mathrm{Q}\bar{\mathrm{Q}}$ state. In $\mathcal{N} = 4$ *sYM* defined as

$$W_c \propto Tr_R P \exp \oint_C d\sigma \left(iA_\mu \dot{x}^\mu + \Phi_i \dot{y}^i \right) \ .$$

- The expectation values of Wilson loop in the fundamental representation corresponds to a string worldsheet extending in the $AdS_5 \times S^5$ with boundary the loop C placed on the AdS boundary.

$$< W[C] >= e^{-S_{string[C]}} \qquad \textit{(Maldacena; Rey, Yee, 1998)}$$

Anisotropic momentum distribution function in weakly coupled plasmas

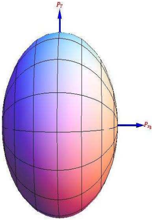
The anisotropic distribution function that can be written as

$$f_{aniso} = c_{norm}(\xi) f_{iso}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

where *(Romatschke, Strickland, hep-ph/0304092)*

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$

and **n** the unit vector along the anisotropic direction.



To relate ξ and α we use the pressures

$$\Delta := \frac{P_T}{P_L} - 1 = \frac{P_{x_1 x_2}}{P_{x_3}} - 1 \,.$$

Using the anisotropic distribution function: *(Martinez, Strickland, 0902.3834 prc)*

$$\Delta = \frac{1}{2}(\xi - 3) + \xi \left((1+\xi) \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}} - 1 \right)^{-1}$$

Using the supergravity model

$$\Delta = \frac{\alpha^2}{2\pi^2 T^2} \,.$$

For

$$T \gg \alpha \Rightarrow \xi \ll 1 \Rightarrow \xi \backsimeq \frac{5\alpha^2}{8\pi^2 T^2} \,.$$

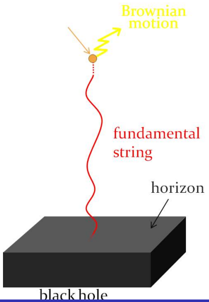
The Nambu-Goto action in fluctuations around the solution to quadratic order become

$$S_2 = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\,\frac{\tilde{G}^{\alpha\beta}}{2}\left[N(u)\,\partial_\alpha\delta x_p\,\partial_\beta\delta x_p + \sum_{i=2,3} g_{ii}\partial_\alpha\delta x_i\,\partial_\beta\delta x_i\right]$$

where

$$\tilde{G}^{\alpha\beta} = \sqrt{-\tilde{g}}\,\tilde{g}^{\alpha\beta}, \qquad N(u) = \frac{g_{00}\,g_{pp} + C^2}{g_{00} + g_{pp}\,v^2},$$

$\tilde{g}^{\alpha\beta}$ is the world-sheet metric, expressed in metric elements of the background.



The Langevin coefficients for a quark moving along the p direction and the transverse direction is taken to be k:

$$\kappa_T = \frac{1}{\pi\alpha'}\,g_{kk}\Big|_{u=u_0} T_{ws}\,,$$

$$\kappa_L = \frac{16\,\pi}{\alpha'}\,\frac{|g_{00}|\,g_{uu}}{g_{pp}\left(\frac{g_{00}}{g_{pp}}\right)'^2}\Bigg|_{u=u_0} T_{ws}^3\,.$$

The world-sheet metric for the string configuration is

$$\tilde{g}_{\alpha\beta} = \begin{pmatrix} g_{00} + v^2 g_{pp} & g_{pp} v \xi' \\ g_{pp} v \xi' & g_{uu} + \xi'^2 g_{pp} \end{pmatrix} .$$

The equations of motion lead to

$$\xi'^2 = -G_{uu} C^2 \frac{G_{00} + G_{pp}\, v^2}{G_{00}\, G_{pp}(C^2 + G_{00}\, G_{pp})}$$

here $C = 2\pi\, \alpha' \Pi_u^p$. ξ'^2 needs to be real quantity $\rightarrow$ solution of the problem.

$$P(k_\perp) = \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp)$$

$$\mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d\,(\mathcal{R})} \left\langle \mathrm{Tr} \left[W_\mathcal{R}^\dagger[0, x_\perp] \, W_\mathcal{R}[0,0] \right] \right\rangle$$

$$W_\mathcal{R}\left[x^+, x_\perp\right] \equiv P \left\{ \exp \left[ig \int_0^{L^-} dx^- \, A_\mathcal{R}^+(x^+, x^-, x_\perp) \right] \right\}$$

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{1}{L} \int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp)$$