

Electroweak Standard Model

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History

- Fermi theory of β -decay (34):
contact interactions between two currents (prototype of modern effective theories)

$$\mathcal{L}_F = -\frac{G_\beta}{\sqrt{2}} J_{(h)\mu}^\dagger J_{(l)}^\mu + h.c. = -\frac{G_\beta}{\sqrt{2}} [\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma^\mu \nu(x)] + h.c.$$

- Parity nonconservation (56-57); V-A law (58); CVC hypothesis ($G_\beta \sim G_\mu$) (58)

$$\mathcal{L}_F = -\frac{G_\beta}{\sqrt{2}} [\bar{p}(x)\gamma_\mu(1 - \lambda\gamma_5)n(x)][\bar{e}(x)(\gamma^\mu(1 - \gamma_5)\nu(x)] + h.c. \quad (\lambda \simeq 1.27)$$

- Quark hypothesis (60); Cabibbo theory (63);

$$\mathcal{L}_{eff} = -\frac{G_\mu}{\sqrt{2}} J_\lambda^\dagger J^\lambda$$

$$J^\lambda = J_{(h)}^\lambda + J_{(l)}^\lambda$$

$$J_{(l)}^\lambda = \bar{\nu}_e \gamma^\lambda (1 - \gamma_5) e + \bar{\nu}_\mu \gamma^\lambda (1 - \gamma_5) \mu$$

$$J_{(h)}^\lambda = \cos \theta_c \bar{u} \gamma^\lambda (1 - \gamma_5) d + \sin \theta_c \bar{u} \gamma^\lambda (1 - \gamma_5) s$$

Θ_c : Cabibbo angle

$$G_\beta = G_\mu \cos \theta_c \simeq 0.98$$

Determination of G_μ from $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

$$\mathcal{M} = -i \frac{G_\mu}{\sqrt{2}} \bar{u}(e) \gamma^\lambda (1 - \gamma_5) v(\nu_e) \bar{u}(\nu_\mu) \gamma_\lambda (1 - \gamma_5) u(\mu)$$

$$\frac{1}{\tau} = \frac{G_\mu m_\mu^5}{192\pi^3}$$

Today:

$$\frac{1}{\tau} = \frac{G_\mu m_\mu^5}{192\pi^3} F[x] \left(1 + \frac{\alpha(m_\mu)}{\pi} H_1[x] + \frac{\alpha(m_\mu)^2}{\pi^2} H_2[x] \right) \quad x \equiv m_e^2/m_\mu^2$$

$$F[x] = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x = 0.99981295$$

$$H_1[x] = \frac{25}{8} - \frac{\pi^2}{2} + \mathcal{O}(x) = 1.80793 \quad (1959)$$

$$H_2[x] = \dots = 6.64 \quad (2000)$$

$$\alpha(m_\mu)^{-1} = \alpha^{-1} + \frac{1}{3\pi} \ln x + \frac{1}{6\pi} = 135.901$$

$$m_\mu = 105.6583715 \pm 0.0000035 \text{ MeV}$$

$$m_e = 0.510998928 \pm 0.000000011 \text{ MeV}$$

$$\tau = (2.1969811 \pm 0.0000022) \times 10^{-6} \text{ s}$$



$$G_\mu = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$$

Particle data book value

Fierz identity

$$e_I(1234) = [\bar{w}_1 \Gamma_I w_2] [\bar{w}_3 \Gamma_I w_4]$$

$$\Gamma_I \equiv \begin{cases} 1 & I=S \\ \gamma^\mu & I=V \\ \sigma^{\mu\nu} & I=T \\ \gamma^\mu \gamma_5 & I=A \\ \gamma_5 & I=P \end{cases}$$

$$e_I(1234) = \sum_J F_{IJ} e_J(1432)$$

$$(\Gamma_I)_{ab} (\Gamma_I)_{cd} = \sum_J F_{IJ} (\Gamma_J)_{ad} (\Gamma_J)_{cb}$$

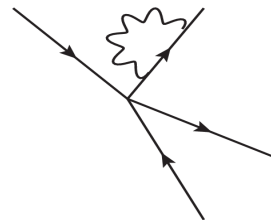
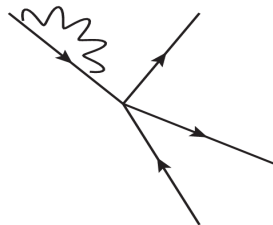
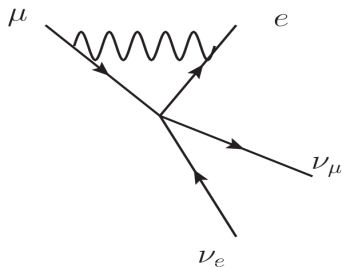
$$\sum_J (\Gamma_J)_{ad} (\Gamma_J)_{cb} = 4\delta_{ab} \delta_{cd} \quad \text{closure}$$

$$\Gamma_I \Gamma_J = c \Gamma_K$$

$$F = \frac{1}{4} \begin{pmatrix} 1 & 1 & \frac{1}{2} & -1 & 1 \\ 4 & -2 & 0 & -2 & -4 \\ 12 & 0 & -2 & 0 & 12 \\ -4 & -2 & 0 & -2 & 4 \\ 1 & -1 & \frac{1}{2} & 1 & 1 \end{pmatrix}$$

$$\mathcal{M} = -i \frac{G_\mu}{\sqrt{2}} \bar{u}(e) \gamma^\lambda (1 - \gamma_5) v(\nu_e) \bar{u}(\nu_\mu) \gamma_\lambda (1 - \gamma_5) u(\mu) \quad \text{charge exchange}$$

$$\rightarrow = -i \frac{G_\mu}{\sqrt{2}} \bar{u}(e) \gamma^\lambda (1 - \gamma_5) u(\mu) \bar{u}(\nu_\mu) \gamma_\lambda (1 - \gamma_5) v(\nu_e) \quad \text{charge retention}$$



W.I.

Energy-angle distribution of e^- (e^+) from the decay of a fully polarized μ^- (μ^+)

$$d\Gamma = \frac{d^3 p_e}{(2\pi)^4} \frac{m_\mu E_0}{3} G_\mu \left\{ 3 - 2x + \frac{\alpha}{2\pi} f(x) \pm \beta \cos \theta \left[1 - 2x + \frac{\alpha}{2\pi} g(x) \right] \right\}$$

$$f(x) = (6 - 4x)R(x) + \frac{1-x}{3x^2} (5 + 17x - 34x^2) \ln(m_\mu/m_e) + \dots$$

$$g(x) = (2 - 4x)R(x) + \dots$$

$$R(x) = \ln(m_\mu/m_e) \left[\frac{3}{2} + 2 \ln \left(\frac{1-x}{x} \right) \right] + \dots$$

$$3 - 2x = 6(1-x) + 4\rho \left[\frac{4}{3}x - 1 \right]; \quad 1 - 2x = -2(1-x) - 4\delta \left[\frac{4}{3}x - 1 \right]$$

Michel parameters; V-A, $\rho=\delta=3/4$

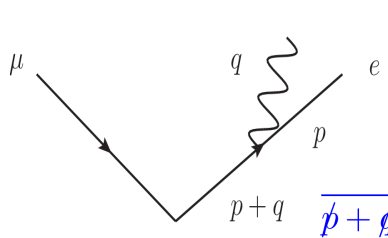
$$E_0 = (m_\mu^2 + m_e^2)/(2m_\mu)$$

$$x = E_e/E_0$$

$$\theta = \angle_{s_\mu}^{p_e}$$

Large radiative corrections because of $\ln(m_\mu/m_e) \rightarrow \infty$, $m_e \rightarrow 0$ (collinear singularity)

$\rho = 0.75080 \pm 0.00032(\text{stat}) \pm 0.00097(\text{syst}) \pm 0.00023$, without r.c. $\rho \approx 0.71$



$$q = (E_\gamma, 0, 0, E_\gamma)$$

$$p = (E_e, 0, \beta E_e \sin \theta, \beta E_e \cos \theta)$$

$$\frac{i}{\not{p} + \not{q} - m_e} = i \frac{\not{p} + \not{q} + m_e}{2p \cdot q} = i \frac{\not{p} + \not{q} + m_e}{2E_\gamma E_e (1 - \beta \cos \theta)}$$

$$|\mathcal{M}|^2 \sim \frac{1}{2p \cdot q}$$

$$\sigma \sim \int_{-1}^1 dz \frac{1}{1 - \beta z} = \frac{1}{\beta} \ln \frac{1 + \beta}{1 - \beta} = \frac{1}{\beta} \ln \frac{(1 + \beta)^2 E_e^2}{m_e^2}$$

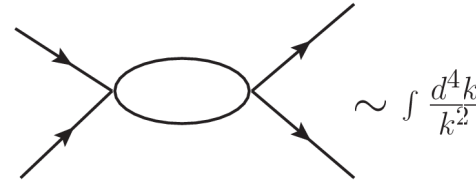
$$\rightarrow \ln \frac{4E_e^2}{m_e^2} \quad \text{as } \beta \rightarrow 1$$

But $\ln(m_\mu/m_e)$ absent in Γ (KLN theorem)

Fermi theory (or any effective):

- Not renormalizable

$$[\mathcal{L}] = 4, [\psi] = 3/2 \Rightarrow [G_\mu] = -2$$



- Violate unitarity :

$$\text{Ex.: } \nu_\mu(k_1) + e^-(p_1) \rightarrow \nu_e(p_2) + \mu^-(k_2)$$

$$\mathcal{M} = -i \frac{G_\mu}{\sqrt{2}} \bar{u}(\mu) \gamma^\lambda (1 - \gamma_5) u(\nu_\mu) \bar{u}(\nu_e) \gamma_\lambda (1 - \gamma_5) u(e)$$

$$|\bar{\mathcal{M}}|^2 = \frac{G_\mu^2}{2} \text{Tr} [k_2 \gamma_\mu (1 - \gamma_5) k_1 \gamma_\nu (1 - \gamma_5)] \frac{1}{2} \text{Tr} [p_2 \gamma^\mu (1 - \gamma_5) p_1 \gamma^\nu (1 - \gamma_5)] = \frac{G_\mu^2}{2} 32 s^2$$

$$d\sigma = |\bar{\mathcal{M}}|^2 \frac{1}{4s} \frac{1}{(4\pi)^2} d\Omega$$

$$\sigma = \frac{G_\mu^2 s}{\pi}$$

But optical theorem tells us the total cross section is related to the amplitude for elastic scattering in the forward direction

$$\sigma_T(\nu_\mu, e^- \rightarrow \text{anything}) = \frac{1}{s} \text{Im} \mathcal{A}(s, J, \theta = 0)$$

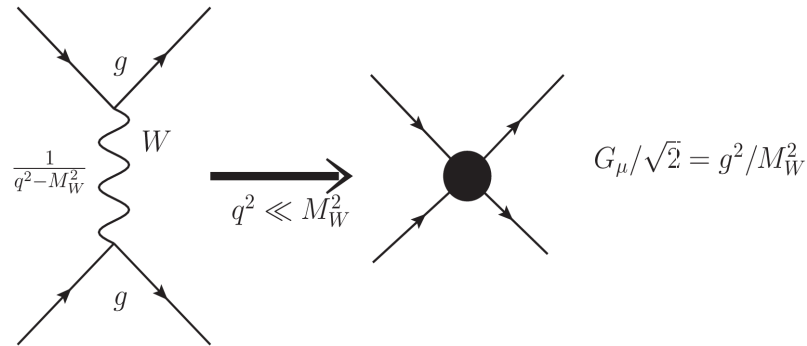
$$\mathcal{A}(s, l, \theta) = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) e^{i\delta_l} \sin \delta_l \quad \text{Spinless particle}$$

$$\sigma \leq \sigma_T \leq \frac{O(\pi)}{s} \Rightarrow s \leq \frac{O(\pi)}{G_\mu}$$

Intermediate Vector Boson theory (IVB)

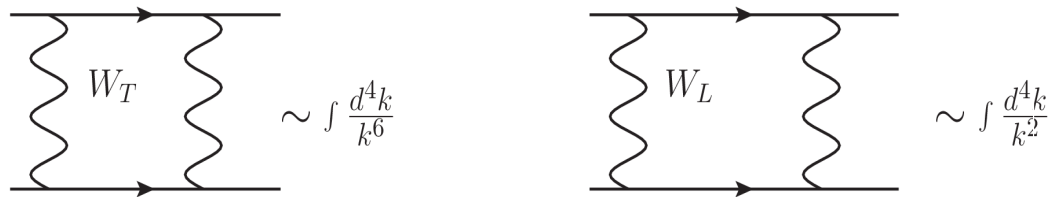
The contact interaction between currents is the result of the exchange of a heavy charged vector boson

$$\mathcal{L}_{int} = -gJ_{-}^{\mu}W_{\mu}^{+} + h.c. \quad J_{-}^{\mu} \equiv J_{(l)}^{\mu} + J_{(h)}^{\mu}$$



$[g]=0$ but theory not renormalizable; problem stays in the longitudinal part of the vector boson propagator

$$i\Delta_W^{\mu\nu}(k) = -i \frac{g^{\mu\nu} - k^{\mu}k^{\nu}/M_W^2}{k^2 - M_W^2 + i\epsilon}$$



Similarly we expect unitarity problem in processes with longitudinal W's like

$$e^{+} + e^{-} \rightarrow W^{+} + W^{-}$$

The Standard Electroweak Theory

Promote the IVB to be the carrier of a gauge interaction as described by a gauge Lagrangian \mathcal{L}_g

To any vector boson V_μ^A there is an associated generator T^A of the gauge group G forming a closed algebra

$$[T^A, T^B] = if^{ABC}T^C, \quad f^{ABC} \quad \text{Structure constants of } G$$

$$\mathcal{L}_g = -\frac{1}{4} \sum_{A=1}^N F_{\mu\nu}^A F^{A\mu\nu} + i\bar{\Psi}\not{D}\Psi + |D_\mu\phi|^2$$

Gauge symmetry dictates the
Interactions of V_μ^A

$$F_{\mu\nu}^A = \partial_\mu V_\nu^A - \partial_\nu V_\mu^A + gf^{ABC}V_\mu^B V_\nu^C$$

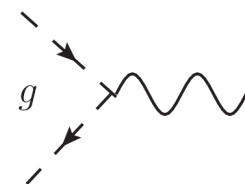
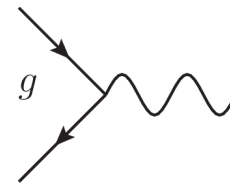
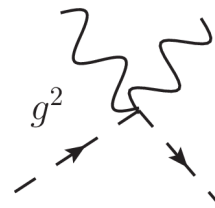
$$D_\mu = \partial_\mu - ig \sum_{A=1}^N V_\mu^A T^A$$

$$V_\mu^A \text{ interact with matter fields via currents } \sum_{A=1}^N J_\mu^A V^{A\mu}$$

$$J_\mu^A(\Psi) = \bar{\Psi}\gamma_\mu T^A \Psi, \quad J_\mu^A(\phi) = \phi^\dagger T^A \partial_\mu \phi - \partial_\mu \phi^\dagger T^A \phi$$

For scalars there is also a “sea-quill” term

$$\sum_{A,B=1}^N \phi^\dagger T^A T^B \phi V_\mu^{A\dagger} V^{B\mu}$$



Fermions and scalars are arranged in representation of G. For massless fermions the l.h. and r.h. components can be given different transformation properties under the Symmetry

$$\bar{\Psi} i \not{D} \Psi = \bar{\Psi}_L i \not{D} \Psi_L + \bar{\Psi}_R i \not{D} \Psi_R$$

$$\begin{aligned} \Psi_L &= \frac{1-\gamma_5}{2} \Psi & \bar{\Psi}_L &= \Psi_L^\dagger \gamma_0 = \bar{\Psi} \frac{1+\gamma_5}{2} \\ \Psi_R &= \frac{1+\gamma_5}{2} \Psi & \bar{\Psi}_R &= \Psi_R^\dagger \gamma_0 = \bar{\Psi} \frac{1-\gamma_5}{2} \end{aligned}$$

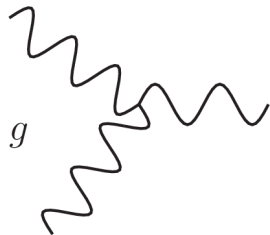
(Ψ Dirac field)

Mass terms break the symmetry if l.h. and r.h. fermions have different symmetry transformations

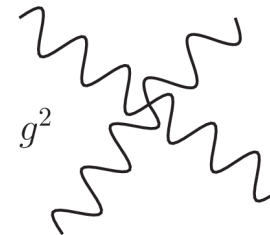
$$m \bar{\Psi} \Psi = m \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L$$

Non Abelian group: N generators, $f^{ABC} \neq 0$

Gauge symmetry gives trilinear and quadrilinear self- interactions of V_μ^A



derivative



contact

Abelian group: U(1) (N=1, $f^{ABC} = 0$)

QED: $T^1 = Q$, $g = e$, no self-interactions between photons

Gauge symmetry does not allow an explicit mass term $m V_\mu^A V^{A\mu}$

Getting the electroweak group (Glashow 61)

l.h. fermions enters into the weak charged current interactions.

l.h. and r.h. fermions enter into the e.m. Interactions.

Fermi charged current can be rewritten as a gauge current of an SU(2) group

$$J_\mu^F = \bar{\nu}\gamma_\mu(1 - \gamma_5)e + \bar{u}\gamma_\mu(1 - \gamma_5)d \Rightarrow J_\mu^- = \bar{l}_L\gamma_\mu\tau^-l_L + \bar{q}_L\gamma_\mu\tau^-q_L$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}; \quad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \quad \tau^\pm = \tau_1 \pm i\tau_2, \quad \tau_i \quad \text{Pauli matrices}$$

Algebra of SU(2)

$$T^A = \frac{\tau^A}{2} \quad (A = 1, 2, 3), \quad [T^A, T^B] = i\epsilon^{ABC}T^C \rightarrow [T^1, T^2] = iT^3 = i\frac{\tau_3}{2} \quad \text{Neutral current}$$

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}} (J_\mu^- W^{+\mu} + J_\mu^+ W^{-\mu}) + gJ_\mu^3 W_\mu^3$$

$$J_\mu^+ = (J_\mu^-)^\dagger, \quad W^\pm = \frac{W^1 \pm iW^2}{\sqrt{2}}, \quad J_\mu^3 = \bar{q}_L\gamma_\mu\frac{\tau_3}{2}q_L + \bar{l}_L\gamma_\mu\frac{\tau_3}{2}l_L$$

Q cannot identified with T_3 but $Q - T_3$ has the same value on the members of the SU(2) doublets

$$[Q - T_3, T_i] = 0$$

Electroweak Group: $SU(2) \times U(1)_Y$ where $Y = Q - T_3$

Fermions quantum numbers (one generation)

| ψ | u_L | d_L | u_R | d_R | ν_L | e_L | e_R | ν_R |
|--------|-------|--------|-------|--------|---------|--------|-------|---------|
| T_3 | $1/2$ | $-1/2$ | 0 | 0 | $1/2$ | $-1/2$ | 0 | 0 |
| Y | $1/6$ | $1/6$ | $2/3$ | $-1/3$ | $-1/2$ | $-1/2$ | -1 | 0 |

l.h. fermions are in SU(2) doublets, r.h. fermions in SU(2) singlets

Electroweak Lagrangian (gauge part, no mass terms)

$$\begin{aligned}
 \mathcal{L}_{symm} &= -\frac{1}{4} \sum_{A=1}^3 F_{\mu\nu}^A F^{A\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + i\bar{\psi}_L \not{D} \psi_L + i\bar{\psi}_R \not{D} \psi_R \\
 F_{\mu\nu}^A &= \partial_\mu W_\nu^A - \partial_\nu W_\mu^A + g\epsilon^{ABC} W_\mu^B W_\nu^C \\
 B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\
 D_\mu \psi_L &= \left[\partial_\mu - i\frac{g}{\sqrt{2}} (\tau^- W^+ - \tau^+ W^-) - igT^3 W_\mu^3 - ig'Y B_\mu \right] \psi_L \\
 D_\mu \psi_R &= [\partial_\mu - ig'Y B_\mu] \psi_R
 \end{aligned}$$

Neutral currents

$$gT^3 W_\mu^3 + g'Y B_\mu \quad l.h.$$

$$g'Y B_\mu \quad r.h.$$

Rotate the W^3 , B field to obtain a new field with vectorial couplings

$$W_\mu^3 = \sin \theta_W A_\mu + \cos \theta_W Z_\mu$$

$$B_\mu = \cos \theta_W A_\mu - \sin \theta_W Z_\mu$$

A_μ couplings:

$$\underbrace{g \sin \theta_W T^3}_e + \underbrace{g' \cos \theta_W Y}_e = e(T^3 + Y) = eQ; \quad \tan \theta_W = \frac{g'}{g}$$

$$e\bar{\psi}_L \gamma_\mu [T^3 + Y] \psi_L + e\bar{\psi}_R \gamma_\mu Y \psi_R = e\bar{\psi} \gamma_\mu Q \psi$$

Vectorial current

Z_μ couplings:


$$g \cos \theta_W T^3 - g' \sin \theta_W Y = g \cos \theta_W T^3 - g' \sin \theta_W (Q - T^3) = \frac{g}{\cos \theta_W} (T^3 - Q \sin^2 \theta_W)$$

$$\frac{g}{\cos \theta_W} \bar{\psi}_L \gamma_\mu [T^3 - Q \sin^2 \theta_W] \psi_L + \underbrace{g \tan \theta_W}_{g'} \sin \theta_W \bar{\psi}_R \gamma_\mu Y \psi_R = \frac{g}{\cos \theta_W} \bar{\psi} \gamma_\mu \left[T^3 \frac{1 - \gamma_5}{2} - Q \sin^2 \theta_W \right] \psi$$

Effective 4-fermion interactions at low energy:

Charged current: $\mathcal{L}_{eff}^{cc} = \frac{-g^2}{2M_W^2} J_\mu^+ J^{-\mu} \Rightarrow \frac{g^2}{8M_W^2} = \frac{G_\mu}{\sqrt{2}}$

Neutral current: $\mathcal{L}_{eff}^{nc} = \frac{-g^2}{2 \cos^2 \theta_W M_Z^2} J_\mu^Z J^{Z\mu} = 4 \frac{G_\mu}{\sqrt{2}} \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} J_\mu^Z J^{Z\mu} \equiv 4 \frac{G_\mu}{\sqrt{2}} \rho J_\mu^Z J^{Z\mu}$


 Symmetry factor for 2 identical currents

$$\frac{1}{4} \mathcal{L}_{eff} = \frac{1}{4} (\mathcal{L}_{eff}^{CC} + \mathcal{L}_{eff}^{NC}) = \frac{G_\mu}{\sqrt{2}} (J_\mu^+ J^{-\mu} + \rho J_\mu^Z J^{Z\mu}) \quad \rho \equiv \frac{M_W^2}{M_Z^2 \cos \theta_W}$$

Note: if I know $\sin \theta_W$, for example from N.C. experiments, I can predict M_W

$$M_W^2 = \frac{\sqrt{2} g^2}{8 G_\mu} = \frac{\sqrt{2} e^2}{8 G_\mu \sin^2 \theta_W} = \frac{\pi \alpha}{\sqrt{2} G_\mu \sin^2 \theta_W} \equiv \frac{A^2}{\sin^2 \theta_W}$$

$$A = \left(\frac{\pi \alpha}{\sqrt{2} G_\mu} \right)^{1/2} = 37.28039(1) \text{ GeV}$$

Neutrino-electron scattering

$$\nu_\mu(k_1) + e^-(p_1) \rightarrow \nu_\mu(k_2) + e^-(p_2)$$

$$\mathcal{M} = -i \frac{g^2}{4M_Z^2 \cos^2 \theta_W} g_L^\nu \bar{u}(k_2) \gamma^\lambda (1 - \gamma_5) u(k_1) [\bar{u}(p_2) [g_L^e \gamma_\lambda (1 - \gamma_5) + g_R^e \gamma_\lambda (1 + \gamma_5)] u(p_1)]$$

$$\frac{d\sigma}{dy} = \rho^2 \frac{8G_\mu^2}{\pi} m_e E_\nu (g_L^\nu)^2 \left[(g_L^e)^2 + (g_R^e)^2 (1 - y)^2 \right] \quad y \equiv E_e^{rec.} / E_\nu^{inc.}$$

$$\begin{aligned} \sigma(\nu_\mu e) &= \rho^2 \frac{8G_\mu^2}{\pi} m_e E_\nu (g_L^\nu)^2 \left[(g_L^e)^2 + \frac{1}{3} (g_R^e)^2 \right] \\ &= \rho^2 \frac{8G_\mu^2}{\pi} m_e E_\nu \left(\frac{1}{2} \right)^2 \left[\left(-\frac{1}{2} + \sin^2 \theta_W \right)^2 + \frac{1}{3} (\sin^2 \theta_W)^2 \right] \end{aligned}$$

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

$$\sigma(\bar{\nu}_\mu e) = \rho^2 \frac{8G_\mu^2}{\pi} m_e E_\nu (g_L^\nu)^2 \left[\frac{1}{3} (g_L^e)^2 + (g_R^e)^2 \right]$$

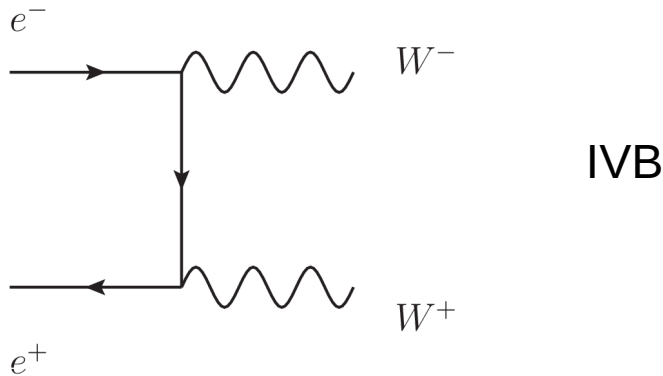
Sign ambiguities: *i)* $g_L \leftrightarrow -g_L, g_R \leftrightarrow -g_R$, *ii)* $g_L \leftrightarrow g_L, g_R \leftrightarrow -g_R$

Using also $\nu_e + e^- \rightarrow \nu_e + e^-$

$$\sigma(\nu_e e) = \frac{8G_\mu^2}{\pi} m_e E_\nu (g_L^\nu)^2 \left[(1 + \rho g_L^e)^2 + \frac{1}{3} (\rho g_R^e)^2 \right]$$

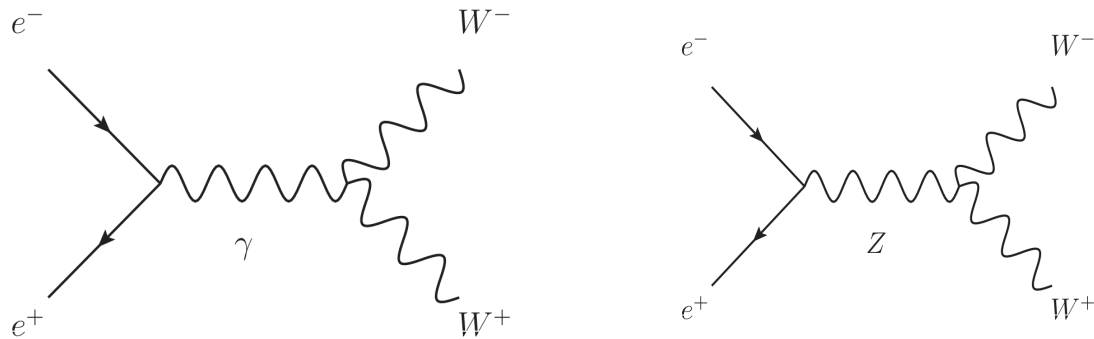
remove i)

In the IVB we expected the $e^+ + e^- \rightarrow W^+ + W^-$ cross-section to raise with s (the C.M. energy) when the W 's are longitudinally polarized

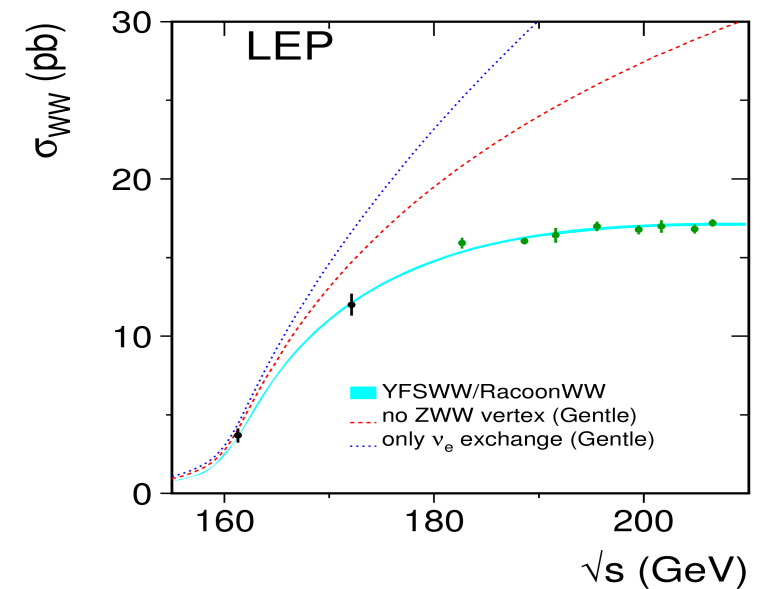


But in our gauge theory we have two extra contributions from

$$-\frac{1}{4} \sum_{A=1}^3 F_{\mu\nu}^A F^{A\mu\nu} \Rightarrow 2 \frac{1}{2} g \epsilon^{ABC} \partial_\mu W_\nu^A W_\mu^B W_\nu^C \Rightarrow g \epsilon^{123} \underbrace{\partial_\mu W_\nu^1 W_\mu^2 W_\nu^3}_{W^\pm} \underbrace{\gamma Z}_{\gamma Z}$$



These two diagrams cancel the bad high energy behavior of the neutrino exchange diagram



Getting the masses


U(1) model: $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2$

Two-point function (exact) $G_{\mu\nu}(p)$ satisfies the W.I.: $p^\mu p^\nu G_{\mu\nu}(p) = -i\xi$

Tree-level propagator: $G_{\mu\nu}^0 = \frac{-i}{p^2} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) - \frac{i\xi p_\mu p_\nu}{p^4}$

Longitudinal part not renormalized: $p^\mu p^\nu G_{\mu\nu}(p) = p^\mu p^\nu G_{\mu\nu}^0(p) = -i\xi$

Transverse part renormalization:

$-i$  $= \Pi_{\mu\nu} \equiv (p^2 g_{\mu\nu} - p_\mu p_\nu) \Pi(p^2)$

Renormalized propagator: $G_{\mu\nu} = \frac{-i}{p^2 [1 - \Pi(p^2)]} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) - \frac{i\xi p_\mu p_\nu}{p^4}$

Pole at $p^2=0$ not shifted unless $\Pi(p^2) \simeq \frac{e^2 v^2}{p^2}$ as $p^2 \rightarrow 0$

Exchange in the Π of a scalar massless state \rightarrow pole in $G_{\mu\nu}$ shifted at $m_\gamma^2 = e^2 v^2$

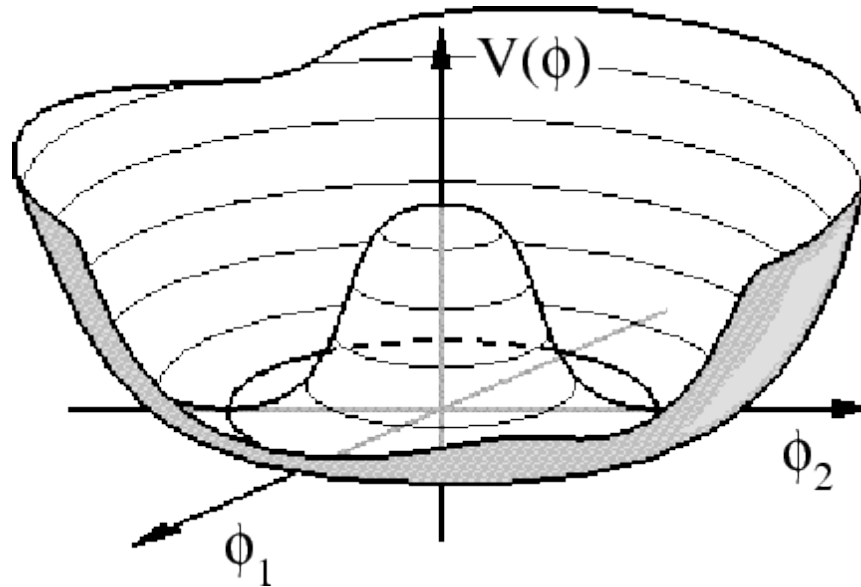
Getting the masses via spontaneous symmetry breaking

The Lagrangian of the theory respects a symmetry, but the vacuum state breaks it

Consider a single complex scalar with a “mexican hat” potential (*Goldstone model*)

$$\phi \equiv \frac{1}{\sqrt{2}} (\phi_1 + i \phi_2), \quad \mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi), \quad V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4$$

$$m^2 < 0, \quad \lambda > 0$$



The potential has an infinite number of equivalent minima for $|\phi|^2 = -\frac{m^2}{2\lambda}$

The system will choose one specific minimum, breaking the global rotational symmetry

We can expand the scalar field around a *real* vacuum expectation value (vev)

$$\phi \equiv \frac{1}{\sqrt{2}} [v + H(x) + iG(x)], \quad v = \sqrt{-\frac{m^2}{\lambda}}$$

At the minimum of the scalar potential (= the vacuum state) we have $\langle \phi \rangle = \frac{v}{\sqrt{2}}$

Up to an irrelevant constant, the scalar potential becomes

$$\begin{aligned} V = & (m^2 v + \lambda v^3) H + \frac{1}{2} (m^2 + 3\lambda v^2) H^2 + \frac{1}{2} (m^2 + \lambda v^2) G^2 \\ & + \lambda v H(H^2 + G^2) + \frac{\lambda}{4} (H^2 + G^2)^2 \end{aligned}$$

Inserting the value of v the linear term vanishes, and the masses of the scalars become

$$m_H^2 = -2m^2 = 2\lambda v^2, \quad m_G^2 = 0$$

G is the *Goldstone boson* associated with the spontaneous breaking of the global symmetry

In general: the number of Goldstone boson is related to the number of broken generators of the symmetry

Broken generator: it does not annihilate the vacuum

The Higgs mechanism

Simplest U(1) model

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \underbrace{|\left(\partial_\mu - ieA_\mu\right)\phi|^2}_{D_\mu} - (m^2)|\phi|^2 - \frac{\lambda}{4}|\phi|^4 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \quad m^2 < 0$$

$$\phi \equiv \frac{1}{\sqrt{2}}[v + h(x) + i\chi(x)], \quad v = \sqrt{-\frac{m^2}{\lambda}} \quad \langle \phi \rangle = \frac{v}{\sqrt{2}}$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\left[(\partial_\mu h)^2 + (\partial_\mu \chi)^2\right] + \frac{1}{2}(2\lambda v^2)h^2 + \frac{1}{2}e^2 v^2 A^\mu A_\mu + ev A^\mu \partial_\mu \chi + \dots$$

Vertices:

$$\text{wavy line with cross} \quad -ie^2 v^2 g_{\mu\nu}$$

$$\text{wavy line} \rightarrow - \quad ev p_\mu$$

$$\text{wavy line} \text{---} \text{1PI} \text{---} \text{wavy line} = \text{wavy line with cross} + \text{wavy line} \text{---} \text{---} \text{wavy line}$$

$$\Pi_{\mu\nu} = e^2 v^2 \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) = (p^2 g_{\mu\nu} - p_\mu p_\nu) \frac{e^2 v^2}{p^2}$$

Pole at $p^2=0$ due to the exchange of the massless χ field

X field is not physical, it can be eliminated via a (field-dependent) gauge transformation

$$\phi \rightarrow \phi' = \phi \exp[-i\epsilon(x)]$$

Original Lagrangian invariant under:

$$A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \epsilon(x)$$

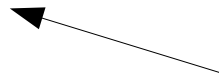
Shift the field Φ and write it in polar coordinates:

$$\phi(x) = \frac{1}{\sqrt{2}} (h(x) + v) \exp\left[+i \frac{\chi(x)}{v}\right]$$

via a gauge transformation I can eliminate χ

$$\left(\epsilon(x) = \frac{\chi(x)}{v} \right)$$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} e^2 v^2 A^\mu A_\mu + \frac{1}{2} e^2 h^2 A^\mu A_\mu + e^2 h v A^\mu A_\mu + \mathcal{L}(h)$$



No χ , A_μ massive (3 d.o.f.); χ eaten by A_μ

N.B. Elementary scalars are not essential to give mass to the vector bosons.

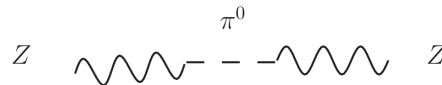
Essential: massless Goldstone bosons associated to the breaking of a global symmetry coupled to the gauge current

QCD con two massless quarks: $SU(2)_L \times SU(2)_R$ symmetry, ground state of the theory has an

indefinite number of massless quark pair due to strong interaction: $\langle 0 | \bar{q}q | 0 \rangle = \langle \bar{u}_L u_R + \bar{d}_L d_R + h.c. | 0 \rangle \neq 0$

$SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$ 3 massless pions

$$\langle 0 | J_\mu^Z | \pi^0 \rangle = i f p_\mu$$



$$\Pi_{\mu\nu}^Z = g_Z^2 f^2 \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

Current conservation

SU(2)xU(1): (Weinberg 67, Salam 68)

SSB via an Higgs doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_{1/2}$

$$\mathcal{L} = \mathcal{L}_{symm} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk}$$

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \underbrace{\left[(m^2) \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \right]}_{V(\Phi^\dagger \Phi)} \quad \text{Renormalizable interaction}$$

$$\mathcal{L}_{Yuk} = -Y_d \bar{\psi}_L \Phi \psi_R^d - Y_u \bar{\psi}_L \tilde{\Phi} \psi_R^u + h.c., \quad \psi_L = \begin{pmatrix} \psi^u \\ \psi^d \end{pmatrix}_L, \quad \tilde{\Phi} = i\tau_2 \Phi^*$$

if $\langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

Shift Φ and write it in terms of 4 real fields, $h, \chi_1, \chi_2, \chi_3$ as

$$\Phi(x) = \exp[i\tau \cdot \chi(x)/v] \begin{pmatrix} 0 \\ \frac{h(x)+v}{\sqrt{2}} \end{pmatrix} \quad \text{via gauge transformation I can eliminate } \chi \text{ (unitary gauge)}$$

$$Q = T_3 + Y \text{ annihilates the vacuum} \quad Q \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = 0$$

3 broken generators, 3 χ 's eaten: 3 massive vector boson, one massless:

$$\text{SU(2)xU(1)} \rightarrow \text{U(1)}_{em}$$

Gauge boson masses:

$$D_\mu \Phi = \left[\partial_\mu - i \frac{g}{\sqrt{2}} (\tau^- W^+ - \tau^+ W^-) - i g T^3 W_\mu^3 - i g' Y B_\mu \right] \Phi, \quad \Phi \rightarrow \langle \Phi \rangle$$

$$M_W^2 W_\mu^\dagger W^\mu = \frac{g^2}{2} |\tau_+ \langle \phi \rangle|^2 W_\mu^\dagger W^\mu = \frac{g^2}{2} \langle \Phi \rangle \tau_- \tau_+ \langle \Phi \rangle W_\mu^\dagger W^\mu = \frac{g^2 v^2}{4} W_\mu^\dagger W^\mu$$

$$\begin{aligned} \frac{1}{2} M_Z^2 Z^\mu Z_\mu &= \left| \left(g \cos \theta_W T^3 - g' \sin \theta_W \overbrace{(Q - T^3)}^Y \right) \langle \Phi \rangle \right|^2 Z^\mu Z_\mu, \quad (Q \langle \phi \rangle = 0) \\ &= (g \cos \theta_W + g' \sin \theta_W)^2 |T^3 \langle \phi \rangle|^2 Z^\mu Z_\mu = \frac{g^2}{\cos^2 \theta_W} |T^3 \langle \phi \rangle|^2 Z^\mu Z_\mu = \frac{1}{2} \frac{g^2 v^2}{4 \cos^2 \theta_W} Z^\mu Z_\mu \end{aligned}$$

$$M_W^2 = \frac{g^2 v^2}{4}; \quad M_Z^2 = \frac{g^2 v^2}{4 \cos^2 \theta_W} \Rightarrow \rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1 \quad \text{Only if the Higgs fields are singlets or doublets}$$

If there are several Higgses in generic representation (T, T₃)

$$\rho = \frac{\sum_{\Phi_a} \frac{1}{2} \langle T^+ T^- + T^- T^+ \rangle |_{\langle \Phi_a \rangle} v_{\Phi_a}^2}{\sum_{\Phi_a} 2 \langle (T^3)^2 \rangle |_{\langle \Phi_a \rangle} v_{\Phi_a}^2} = \frac{\sum_{\Phi_a} [T(T+1) - (T^3)^2]_{\Phi_a} v_{\Phi_a}^2}{\sum_{\Phi_a} 2 [(T^3)^2]_{\Phi_a} v_{\Phi_a}^2}$$

We must have at least one Higgs doublet to give mass to the fermions:

$$\begin{aligned}
 \mathcal{L}_{Yuk} &= -Y_d \bar{\psi}_L \Phi \psi_R^d - Y_u \bar{\psi}_L \tilde{\Phi} \Psi_R^u + h.c. \\
 &\quad - Y_d \frac{v}{\sqrt{2}} (\bar{\psi}_L^d \psi_R^d + \bar{\psi}_R^d \psi_L^d) - Y_u \frac{v}{\sqrt{2}} (\bar{\psi}_L^u \psi_R^u + \bar{\psi}_R^u \psi_L^u) \quad \Phi \Rightarrow \langle \Phi \rangle \\
 \Rightarrow m_u &= \frac{Y_u v}{\sqrt{2}}, \quad m_d = \frac{Y_d v}{\sqrt{2}}
 \end{aligned}$$

doublet, doublet, singlet

The ρ parameter fixes the relative strength of the charged-and neutral current interactions. Its experimental value is very close to 1, but the exact value depends on the experiments one is considering (radiative corrections enters into the game) . The value of ρ extracted in neutrino-hadron scattering is (slightly) different from that of neutrino-electron scattering.

$$\rho = 1 \Rightarrow \cos^2 \theta_W = \frac{M_W^2}{M_Z^2}$$

If I know M_Z I can predict M_W

$$M_W^2 = \frac{A^2}{\sin^2 \theta_W} = \frac{A^2}{1 - M_W^2/M_Z^2} \Rightarrow M_W^2 = \frac{M_Z^2}{2} \left\{ 1 + \left[1 - \frac{4A^2}{M_Z^2} \right]^{1/2} \right\}$$

The Standard Model

Strong, electromagnetic and weak interactions (not gravity) are described by a *renormalizable* Quantum Field Theory based on the principle of local gauge invariance with gauge symmetry group $SU(3)_c \times SU(2)_W \times U(1)_Y$ spontaneously broken to $SU(3)_c \times U(1)_{em}$. The quanta of the gauge fields (W,Z) acquire mass via the Higgs mechanism. The left-over of the EWSB process is (at least) a spin 0 particle, the Higgs particle, whose coupling to gauge bosons and to fermions is determined by their masses.

Elementary Particles

| | | | | |
|---------|------------------------------|----------------------------|----------------------------|--------------------|
| Quarks | u up | c charm | t top | γ photon |
| | d down | s strange | b bottom | |
| Leptons | ν_e electron neutrino | ν_μ muon neutrino | ν_τ tau neutrino | Z Z boson |
| | e electron | μ muon | τ tau | |
| | | | | W W boson |
| | I | II | III | |

Three Families of Matter

Force Carriers



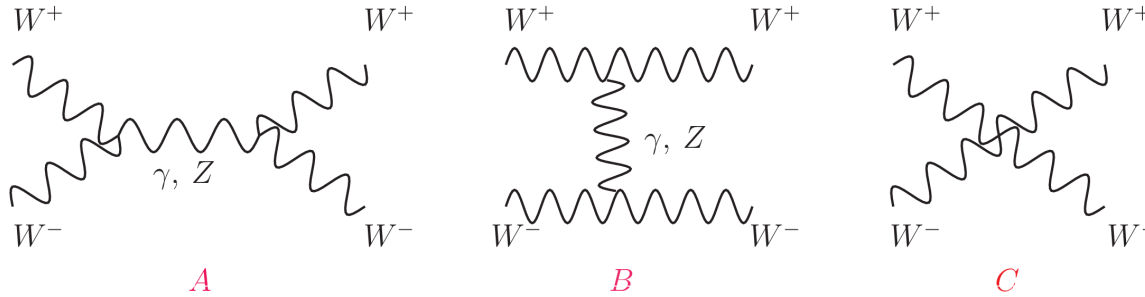
"The Higgs mechanism is just a reincarnation of the Communist Party: it controls the masses"

Anonymous

Tree-level unitarity

W-W scattering with longitudinal polarized W's:

$$W_L^+(p_1) + W_L^-(p_2) \rightarrow W_L^+(k_1) + W_L^-(k_2)$$



$$p^\mu = (E, \vec{p}), \quad \epsilon_L^\mu = \frac{1}{M_W} \left(p, \frac{E}{|\vec{p}|} \vec{p} \right) \rightarrow \epsilon_L^\mu = \frac{1}{M_W} p^\mu + v^\mu(p), \quad v(p) = -\frac{M_W}{E^2} (E, -\vec{p}) + \mathcal{O}(M_W^2/E^2)$$

\nwarrow
W 4-mom.

\nwarrow
Pol. Vec. Long. W

In the C.M. System: $p_1 = (E, \vec{p}), p_2 = (E, -\vec{p}) \rightarrow \epsilon_L(p_1) \simeq \frac{1}{M_W} p_1^\mu - \frac{M_W}{2E^2} p_2^\mu, \quad \epsilon_L(p_2) \simeq \frac{1}{M_W} p_2^\mu - \frac{M_W}{2E^2} p_1^\mu,$

At high-energy:

$$(A + B)_Z = i \frac{g^2 c_W^2}{M_W^4} \left\{ -\frac{t^2 + s^2 + 4st}{4} + 8 \frac{M_W^2}{s} (s^2 - t^2) - 3M_Z^2 (s + t) \right\}$$

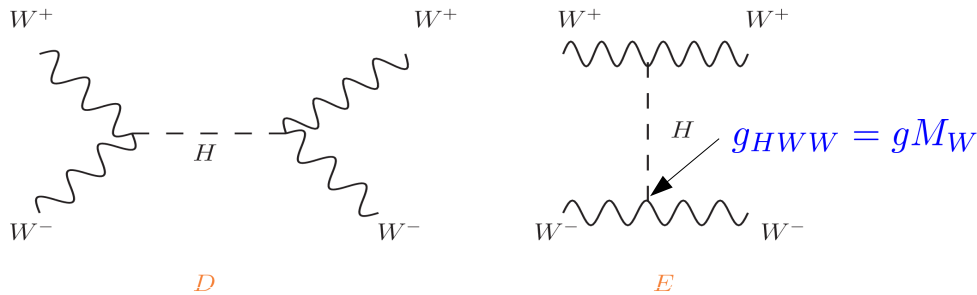
$$(A + B)_\gamma = (A + B)_Z (c_W^2 \rightarrow s_W^2, M_Z \rightarrow 0)$$

$$C = i \frac{g^2}{M_W^4} \left\{ \frac{t^2 + s^2 + 4st}{4} - 4 \frac{M_W^2}{s} (s^2 - st - 2t^2) \right\}$$

$$A + B + C = i \frac{g^2}{M_W^2} \frac{s + t}{4}$$

led but still it grows with energy

Not the end of the story: Higgs exchange (cancellation!)



$$D = -\frac{ig^2}{4M_W^2} \left(s + \frac{M_H^2 s}{s - M_H^2} \right)$$

$$E = -\frac{ig^2}{4M_W^2} \left(t + \frac{M_H^2 t}{t - M_H^2} \right)$$

$$A + \dots + E = -i\sqrt{2} G_\mu M_H^2 \left(\frac{t}{t - M_H^2} + \frac{s}{s - M_H^2} \right)$$

does not grow with energy if the Higgs is not too heavy

If the Higgs were very heavy, $W_L W_L$ scattering enjoys a period of rising with the energy up to M_H

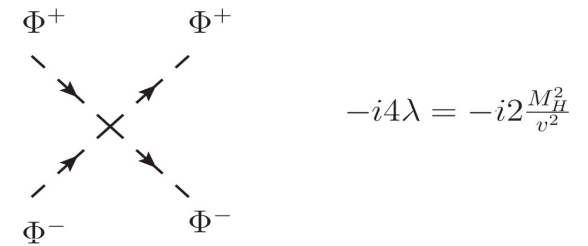
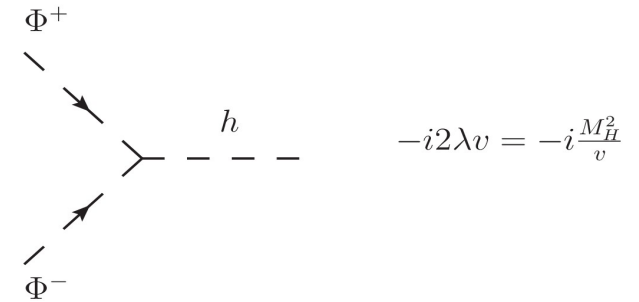
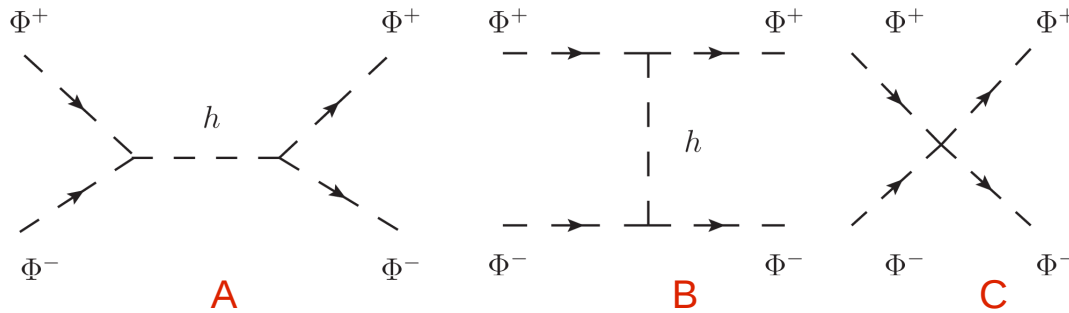
$$\mathcal{A}(s, J, \theta) = 16\pi \sum_{j=0}^{\infty} (2j+1) d^J(\theta) M^J \Rightarrow |M^J| \leq 1$$

$$M^{J=0} \sim -\frac{G_\mu s}{16\pi\sqrt{2}} \quad (M_H \rightarrow \infty) \Rightarrow s_c \sim \mathcal{O} \left(\frac{\pi\sqrt{2}}{G_\mu} \right) \sim (1 \text{ TeV})^2$$

At s_c something should enter in the game to restore unitarity

Equivalence Theorem

SM with gauge interaction switched off, $\Phi \Phi$ scattering: $\phi^+(p_1) + \phi^-(p_2) \rightarrow \phi^+(k_1) + \phi^-(k_2)$



$$A + \frac{1}{2}C = -i\frac{M_H^2}{v^2} - i\frac{M_H^2}{v^2} \frac{1}{s - M_H^2} = -i\frac{M_H^2}{v^2} \frac{s}{s - M_H^2}$$

$$B + \frac{1}{2}C = A + \frac{1}{2}C \quad (s \rightarrow t)$$

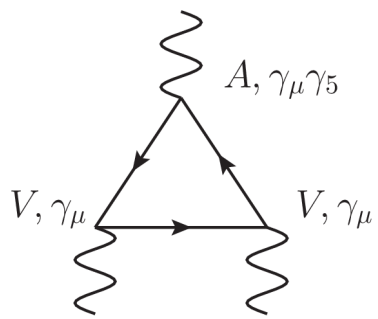
$$A + B + C = -i\sqrt{2}G_\mu M_H^2 \left(\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right)$$

At high energy

$$\mathcal{M}(W_L, Z_L \cdots \rightarrow W_L, Z_L \cdots) = \mathcal{M}(\Phi^+, \Phi^0 \cdots \rightarrow \Phi^+, \Phi^0 \cdots) + \mathcal{O}\left(\frac{M_W}{E}\right)$$

The S.M. is a renormalizable theory ('t Hooft, Veltman 71-72)

The theory is anomaly free



color

$$Tr(T^3 Q^2) = \underbrace{3 \cdot \frac{1}{2} \cdot \frac{4}{9}}_u + \underbrace{3 \cdot \left(-\frac{1}{2}\right) \cdot \frac{1}{9}}_d + \underbrace{\frac{1}{2} \cdot 0}_\nu + \underbrace{\left(-\frac{1}{2}\right) \cdot 1}_e = 0$$

However in the unitary gauge where only physical fields are present (the would be G.B. are eliminated) the propagator of the massive V.B. has a bad high energy behavior

$$i\Delta_W^{\mu\nu}(k) = -i \frac{g^{\mu\nu} - k^\mu k^\nu / M_W^2}{k^2 - M_W^2 + i\epsilon}$$

and the theory seems to be not renormalizable by power-counting arguments.

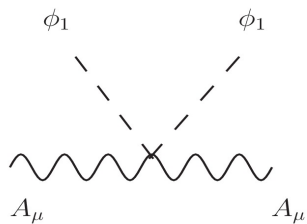
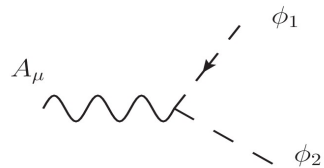
However, it is possible to choose a smart gauge (R_ξ gauge) where the V.B. Propagator has a good high energy behavior

$$i\Delta_W^{\mu\nu}(k) = \frac{-i}{k^2 - M_W^2 + i\epsilon} \left[g^{\mu\nu} + (\xi - 1) \frac{k^\mu k^\nu}{k^2 - \xi M_W^2} \right] = -i \left[\frac{g^{\mu\nu} - k^\mu k^\nu / M_W^2}{k^2 - M_W^2 + i\epsilon} + \frac{k^\mu k^\nu / M_W^2}{k^2 - \xi M_W^2} \right]$$

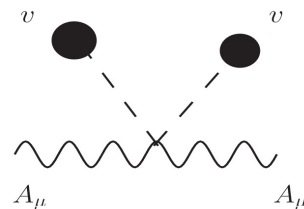
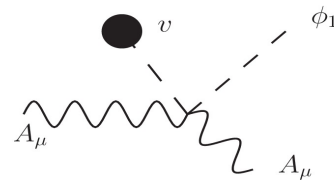
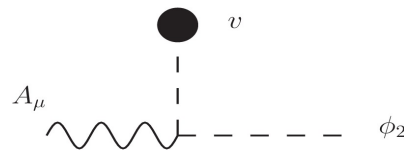
↖
Unphysical pole

U(1): $\phi = \phi_1 + \phi_2; \quad \langle \phi_1 \rangle = v, \langle \phi_2 \rangle = 0$

unbroken



broken



mixing eliminated via

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi} (\partial^\mu A_\mu + \xi \underbrace{ev}_{M_A} \phi_2)^2$$

Mass term for Φ_2 : ξM_A

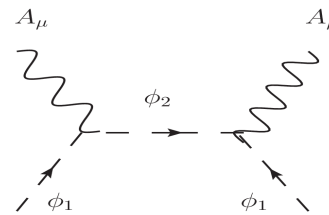
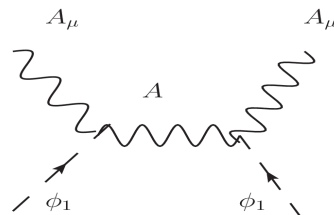
$$i\Delta_{\phi_2}(k) = \frac{i}{k^2 - \xi M_A^2}$$

Also the ghosts acquire a ξ -dependent mass term

interaction

mass

Cancellation of the unphysical pole



Renormalization of the S.M.

In the gauge sector there are 3 parameters: g , g' , $v \rightarrow$ 3 renormalization conditions. Whatever renormalization scheme we use we want to express our results in terms of the 3 best known parameters:

$$\begin{aligned}\alpha &= 1/137.035999174(35) \text{ [0.25ppb]} \quad (a_e) \\ G_\mu &= 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} \\ M_Z &= 91.1876(21) \text{ GeV}/c^2 \quad (\text{peak of } \sigma(e^+e^- \rightarrow \text{all}))\end{aligned}$$

$$a_e(QED) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n a_e^{(2n)}$$



Computed up to $n=5$


We need the radiative corrected relations between our renormalized parameters and α , G_μ and M_Z

Bare relations:

$$e_0 = g_0 \sin \theta_{W_0}, \quad g'_0 = g_0 \tan \theta_{W_0}, \quad M_{W_0} = \frac{g_0 v_0}{2}, \quad \rho_0 = \frac{M_{W_0}^2}{M_{Z_0}^2 \cos^2 \theta_{W_0}} = 1$$

Natural to identify:

$$g = \frac{e}{\sin \theta_W} \quad e = \sqrt{4\pi\alpha}$$



renormalized fine structure constant

Using the physical (pole) masses of the W and Z we have two possibility:

$$\cos^2 \theta_W = \frac{M_W^2}{M_Z^2} \quad \rho = 1 \quad (1)$$

$$\cos^2 \theta_W \neq \frac{M_W^2}{M_Z^2} \quad \rho = 1 + \mathcal{O}(\alpha) \quad (2)$$

Notation:

$$c_0^2 \equiv \cos^2 \theta_{W_0}, \quad s_0^2 \equiv \sin^2 \theta_{W_0}$$

$$c^2 = c_0^2 - \delta c^2, \quad s^2 = s_0^2 - \delta s^2$$

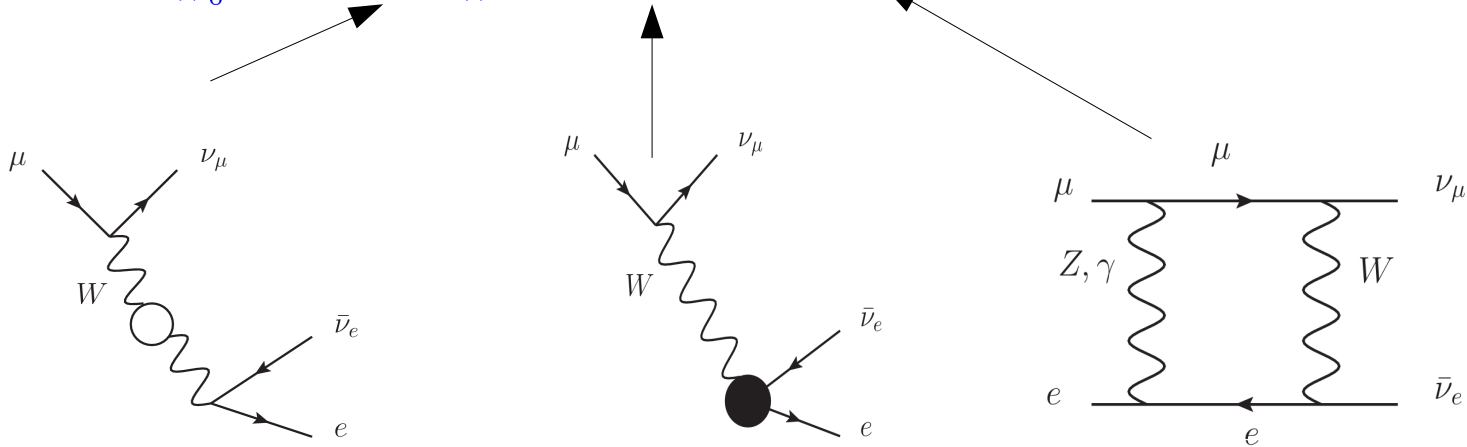
On-Shell scheme

$$(1) \quad \frac{\delta c^2}{c^2} \simeq \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} = \frac{\text{Re } A_{WW}(M_W^2)}{M_W^2} - \frac{\text{Re } A_{ZZ}(M_Z^2)}{M_Z^2}$$

$A_{WW}(q^2)$ Transverse part of the WW self-energy at momentum q^2

Relation with G_μ

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8M_{W_0}^2} \left\{ 1 - \frac{A_{WW}(0)}{M_W^2} + V_W + M_W^2 B_W \right\}$$



separate e.m.

$$\begin{aligned}
\frac{G_\mu}{\sqrt{2}} &= \frac{g_0^2}{8M_{W_0}^2} \left\{ 1 - \frac{A_{WW}(0)}{M_W^2} + V_W + M_W^2 B_W \right\} \\
&= \frac{e^2}{8s^2 M_W^2} \left\{ 1 + \frac{\delta M_W^2}{M_W^2} - \frac{A_{WW}(0)}{M_W^2} + V_W + M_W^2 B_W - \frac{\delta e^2}{e^2} + \frac{\delta s^2}{s^2} \right\} \\
&= \frac{e^2}{8s^2 M_W^2} \left\{ 1 + \frac{\text{Re } A_{WW}(M_W^2)}{M_W^2} - \frac{A_{WW}(0)}{M_W^2} + V_W + M_W^2 B_W \right. \\
&\quad \left. + \Pi_{\gamma\gamma}(0) + \frac{c^2}{s^2} \left[\frac{\text{Re } A_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\text{Re } A_{WW}(M_W^2)}{M_W^2} \right] \right\} \\
&= \frac{e^2}{8s^2 M_W^2} \{1 + \Delta r\}
\end{aligned}$$

$$\Pi_{\gamma\gamma}(q^2) = \frac{A_{\gamma\gamma}(q^2)}{-q^2}$$

$$M_W^2 = \frac{M_Z^2}{2} \left\{ 1 + \left[1 - \frac{4A^2}{M_Z^2(1 - \Delta r)} \right]^{1/2} \right\} \quad \text{Radiatively corrected}$$

Large contributions in Δr :

$$\Pi_{\gamma\gamma}(0), \quad \frac{c^2}{s^2} \left[\frac{\text{Re } A_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\text{Re } A_{WW}(M_W^2)}{M_W^2} \right]$$

$$\Pi_{\gamma\gamma}(0)$$

Contains hadronic contributions at low energy that cannot be computed in perturbation theory

$\Pi_{\gamma\gamma}(q^2)$ is an analytic function in the q^2 complex plane with a cut on the real axis starting at $4m_\pi^2$

$$\Pi_{\gamma\gamma}^h(0) = \underbrace{\Pi_{\gamma\gamma}^h(0) - \text{Re} \Pi_{\gamma\gamma}^h(M_Z^2)}_{\text{perturbative}} + \text{Re} \Pi_{\gamma\gamma}^h(M_Z^2)$$

$$= -\frac{M_Z^2}{\pi} \text{Re} \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im} \Pi_{\gamma\gamma}^h(s)}{s(s - M_Z^2 - i\epsilon)}$$

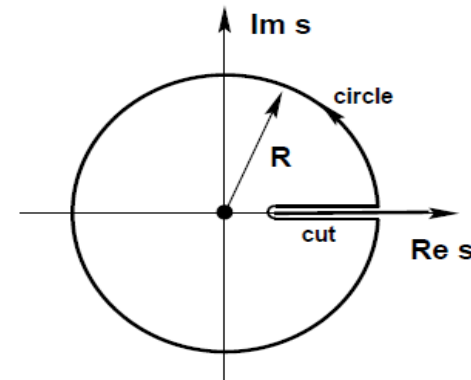
Subtracted dispersion relation (Cauchy)

but, from optical theorem

$$\text{Im} \Pi_{\gamma\gamma}^h = -\frac{s}{e^2} \sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})(s)$$

using

$$R(s) = \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma_{\text{tot}}(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)}$$



$$\begin{aligned} \Delta\alpha_{\text{hadrons}}^{(5)} &\equiv \text{Re} \Pi_{\gamma\gamma}^h(M_Z^2) - \Pi_{\gamma\gamma}^h(0) = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s - M_Z^2 - i\epsilon)} \\ &= -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{4m_\pi^2}^{s_0} ds \frac{R(s)}{s(s - M_Z^2 - i\epsilon)} - \frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{s_0}^{\infty} ds \frac{R(s)}{s(s - M_Z^2 - i\epsilon)} \end{aligned}$$

Using experimental data

Using perturbative QCD

Table 10.1: Recent evaluations of the on-shell $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$. For better comparison we adjusted central values and errors to correspond to a common and fixed value of $\alpha_s(M_Z) = 0.120$. References quoting results without the top quark decoupled are converted to the five flavor definition. Ref. [33] uses $\Lambda_{\text{QCD}} = 380 \pm 60$ MeV; for the conversion we assumed $\alpha_s(M_Z) = 0.118 \pm 0.003$.

| Reference | Result | Comment |
|----------------------------|-----------------------|--|
| Martin, Zeppenfeld [23] | 0.02744 ± 0.00036 | PQCD for $\sqrt{s} > 3$ GeV |
| Eidelman, Jegerlehner [24] | 0.02803 ± 0.00065 | PQCD for $\sqrt{s} > 40$ GeV |
| Geshkenbein, Morgunov [25] | 0.02780 ± 0.00006 | $\mathcal{O}(\alpha_s)$ resonance model |
| Burkhardt, Pietrzyk [26] | 0.0280 ± 0.0007 | PQCD for $\sqrt{s} > 40$ GeV |
| Swartz [27] | 0.02754 ± 0.00046 | use of fitting function |
| Alemany et al. [28] | 0.02816 ± 0.00062 | incl. τ decay data |
| Krasnikov, Rodenberg [29] | 0.02737 ± 0.00039 | PQCD for $\sqrt{s} > 2.3$ GeV |
| Davier & Höcker [30] | 0.02784 ± 0.00022 | PQCD for $\sqrt{s} > 1.8$ GeV |
| Kühn & Steinhauser [31] | 0.02778 ± 0.00016 | complete $\mathcal{O}(\alpha_s^2)$ |
| Erlar [19] | 0.02779 ± 0.00020 | conv. from $\overline{\text{MS}}$ scheme |
| Davier & Höcker [32] | 0.02770 ± 0.00015 | use of QCD sum rules |
| Groote et al. [33] | 0.02787 ± 0.00032 | use of QCD sum rules |
| Martin et al. [34] | 0.02741 ± 0.00019 | incl. new BES data |
| Burkhardt, Pietrzyk [35] | 0.02763 ± 0.00036 | PQCD for $\sqrt{s} > 12$ GeV |
| de Troconiz, Yndurain [36] | 0.02754 ± 0.00010 | PQCD for $s > 2$ GeV ² |
| Jegerlehner [37] | 0.02765 ± 0.00013 | conv. from MOM scheme |
| Hagiwara et al. [38] | 0.02757 ± 0.00023 | PQCD for $\sqrt{s} > 11.09$ GeV |
| Burkhardt, Pietrzyk [39] | 0.02760 ± 0.00035 | incl. KLOE data |
| Hagiwara et al. [40] | 0.02770 ± 0.00022 | incl. selected KLOE data |
| Jegerlehner [41] | 0.02755 ± 0.00013 | Adler function approach |
| Davier et al. [20] | 0.02750 ± 0.00010 | e^+e^- data |
| Davier et al. [20] | 0.02762 ± 0.00011 | incl. τ decay data |
| Hagiwara et al. [42] | 0.02764 ± 0.00014 | e^+e^- data |

Differences:

- Treatment of data and errors
- Integration
- Threshold for PQCD

$$\Delta\alpha^{(l)} = 0.031421$$

$$\Delta\alpha = \Delta\alpha^{(l)} + \Delta\alpha^{(h)} \simeq 0.06$$

$$\alpha(M_Z^2) = \frac{\alpha}{1 - \Delta\alpha} \simeq \frac{1}{128}$$

The other “large” term

$$\frac{c^2}{s^2} \left[\frac{\text{Re } A_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\text{Re } A_{WW}(M_W^2)}{M_W^2} \right] \simeq -\frac{c^2}{s^2} \delta\rho$$



heavy particles ($M \gg M_Z$)

Heavy particles contribute to the W-Z mass difference correction in the same way as in the ρ parameter (relative strength between NC and CC interactions)

$$\rho = \frac{\left(1 - \frac{A_{ZZ}(0)}{M_Z^2} + V_Z + M_Z^2 B_Z\right)}{\left(1 - \frac{A_{WW}(0)}{M_W^2} + V_W + M_W^2 B_W\right)} \simeq 1 + \frac{A_{WW}(0)}{M_W^2} - \frac{A_{ZZ}(0)}{M_Z^2} + \dots = 1 + \delta\rho + \dots$$


 $\sim q^2, \quad m_f^2$

Quadratic terms in m_f are going to survive in the difference?

M_w and M_z are degenerate in the limit $g' \rightarrow 0$.

Higgs potential is a function of $\Phi^\dagger \Phi = (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$
custodial symmetry $SU(2)_L \times SU(2)_R$

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix}, \quad \tilde{\Phi} = i\tau_2 \Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi_- \end{pmatrix}; \quad H = (\tilde{\Phi}, \Phi) = \begin{pmatrix} \phi_0^* & \phi_+ \\ -\phi_- & \phi_0 \end{pmatrix}$$

$$\mathcal{L}_{Higgs} = \frac{1}{2} \text{Tr}(D_\mu H)^\dagger (D^\mu H) - V(H)$$

$$D_\mu H = \partial_\mu H + \frac{i}{2} g \vec{W}_\mu \cdot \vec{\tau} H - \frac{i}{2} g' B_\mu H \tau_3$$

$$V(H) = \lambda \left(\frac{1}{2} \text{Tr}(H^\dagger H) - \frac{v^2}{2} \right)^2$$

If $g'=0$ \mathcal{L}_{Higgs} has global $SU(2)_L \times SU(2)_R$ invariance
 $(U_{L,R} = \exp[i\vec{\tau} \cdot \vec{\epsilon}_{L,R}])$

$$H \rightarrow U_L H U_R^\dagger$$

$$\vec{W}_\mu \cdot \vec{\tau} \rightarrow U_L (\vec{W}_\mu \cdot \vec{\tau}) U_L^\dagger$$

$SU(2)_L$ survives when $\langle \phi_4 \rangle = v \Rightarrow \langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow M_{W_3} = M_W$

$$H \rightarrow U_L H U_L^\dagger$$

$$\vec{W}_\mu \cdot \vec{\tau} \rightarrow U_L (\vec{W}_\mu \cdot \vec{\tau}) U_L^\dagger$$

$$\begin{aligned}
\mathcal{L}_{Yuk} &= -Y_d \bar{\psi}_L \Phi \psi_R^d - Y_u \bar{\psi}_L \tilde{\Phi} \psi_R^u + h.c. \\
&= Y \bar{\psi}_L H \psi_R + \Delta Y \bar{\psi}_L H \tau_3 \psi_R + h.c. \quad \psi_R = \begin{pmatrix} \psi_R^u \\ \psi_R^d \end{pmatrix}
\end{aligned}$$

First term invariant under: $H \rightarrow U_L H U_L^\dagger, \psi_L \rightarrow U_L \psi_L, \psi_R \rightarrow U_L \psi_R$

Corrections to the W-Z mass difference are due to hypercharge effects ($g' \neq 0$) or to mass splitting within isospin multiplets

$$\begin{aligned}
\delta \rho^{top} &= N_c \frac{G_\mu m_t^2}{8 \sqrt{2} \pi^2} \quad (m_b = 0) \\
\delta \rho^{Higgs} &\simeq -\frac{3 G_\mu M_W^2}{8 \sqrt{2} \pi^2} \tan \theta_W \ln \frac{M_H^2}{M_W^2} \quad (M_H \gg M_W)
\end{aligned}$$

Corrections proportional to M_H^2 appear at two-loop but are too small to be important

Heavy particles do not decouple in $\delta\rho$. In a diagram if couplings do not grow with mass heavy particles decouple, running of α or α_s not affected by heavy quarks.

$$\delta\rho = \frac{A_{WW}(0)}{M_W^2} - \frac{A_{ZZ}(0)}{M_Z^2} \neq 0 \quad g, g' \rightarrow 0$$



$$M_W^2 = \frac{g^2 v^2}{4}$$

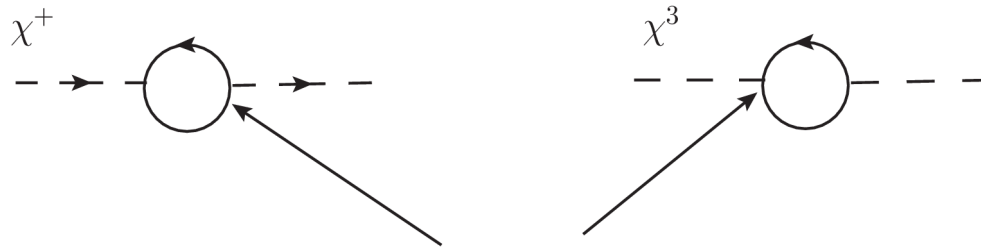
Corrections to $\delta\rho$ can be computed in the gauge-less limit of the SM, a Yukawa theory with gauge boson as external non propagating fields.

$$\mathcal{L}_{g.l.} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi) - (Y_t \bar{\psi}_L \tilde{\Phi} t_R + h.c.)$$

W, Z (no kinetic term)

$$\mathcal{L}_{kin} = Z_2^+ \left| \partial_\mu \chi^+ - \frac{g v}{2} W^+ \right|^2 + \frac{1}{2} Z_2^0 \left(\partial_\mu \chi^3 - \frac{g v}{2 \cos \theta_W} Z \right)^2 \Rightarrow \rho = \frac{Z_2^+}{Z_2^0}$$

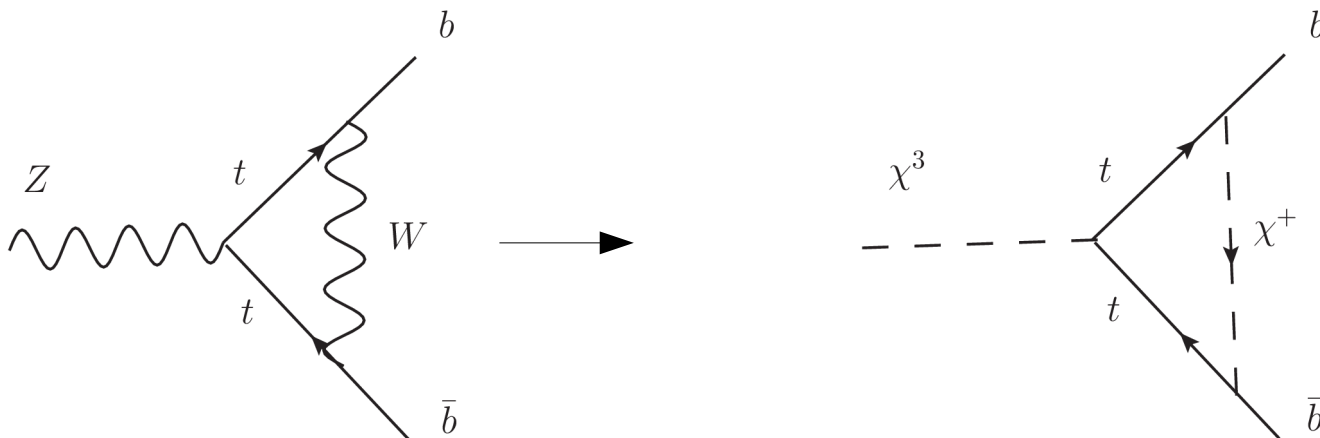
w.f.r, of the Goldstone bosons, $Z_2 = 1 + O(\alpha)$



grows with the mass

X's are the longitudinal modes of the W,Z

Other non decoupling effect in $Z \rightarrow b \bar{b}$



$$\simeq G_\mu m_t^2$$

Large counterterm contribution associated with δs^2 can be eliminated using a \overline{MS}

definition of θ_w $\hat{s}^2 \equiv \sin^2 \hat{\theta}_W(M_Z)$

$$\hat{\rho} = \frac{M_W^2}{M_Z^2 \hat{c}^2} \simeq 1 + \delta\rho$$

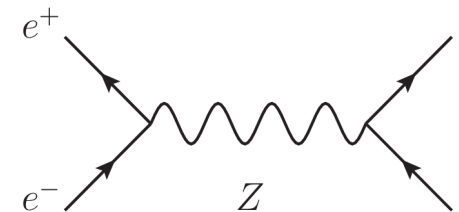
$$M_W^2 = \frac{\hat{\rho} M_Z^2}{2} \left\{ 1 + \left[1 - \frac{4A^2}{\hat{\rho} M_Z^2 (1 - \Delta \hat{r}_W)} \right]^{1/2} \right\}$$

$$\hat{s}^2 = \frac{1}{2} \left\{ 1 + \left[1 - \frac{4A^2}{\hat{\rho} M_Z^2 (1 - \Delta \hat{r}_W)} \right]^{1/2} \right\}$$

no m_t^2

Z-pole physics:

Dominant contribution is the resonant Z exchange diagram



The bulk of the corrections can be absorbed into effective couplings

$$\frac{g}{2c} J_Z^\mu = \frac{g}{2c} \bar{\psi} (\gamma_\mu (g_v - g_a \gamma_5)) \psi$$

$$g_v^f = T_f^3 - 2Q_f \sin^2 \theta_W \Rightarrow \sqrt{\rho} (T_f^3 - 2Q_f \sin^2 \theta_{eff}^f)$$

$$g_a^f = T_f^3 \Rightarrow \sqrt{\rho} (T_f^3)$$

$$\sin^2 \theta_{eff}^f = \hat{k}(M_Z^2) \sin^2 \hat{\theta}_W(M_Z) \simeq \sin^2 \hat{\theta}_W(M_Z) \quad \hat{k}(M_Z^2) = 1 + \dots$$

$\sin^2 \theta_{eff}^f$ can be obtained from asymmetries

$$\frac{d\sigma_{f\bar{f}}}{d\cos\theta} = \frac{3}{8}\sigma_{f\bar{f}}^{tot} [(1 - P_e A_e)(1 + \cos^2 \theta) + 2(A_e - P_e)A_f \cos \theta]$$

$$A_f \equiv \frac{2g_v^f g_a^f}{(g_v^f)^2 + (g_a^f)^2} = \frac{2x_f}{1+x_f^2},$$

$$x_f \equiv \frac{g_v^f}{g_a^f} = 1 - 4|Q_f| \sin^2 \theta_{eff}^f$$

e^- polarization

Forward-Backward

$$A_{FB}^f(e^+e^- \rightarrow f\bar{f}) = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \simeq \frac{3}{4}A_e A_f$$

f = l one measures $A_e^2 \simeq 4x_f^2$ ($g_v^e \ll g_a^e$) $A_e \sim 0.15$

f = q one measures $A_e A_q$ but $g_v^q \sim g_a^q$ one measures mainly A_e

τ polarization

$$A_{pol}^\tau(e^+e^- \rightarrow \tau^+\tau^-) = \frac{\sigma(\tau_L) - \sigma(\tau_R)}{\sigma(\tau_L) + \sigma(\tau_R)} \sim -A_\tau$$

Left-Right

$$A_{LR}^f(e^+e^- \rightarrow f\bar{f}) = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \frac{1}{<|P_e|>} \simeq A_e \quad \sigma_L \equiv \sigma(e^- l.h.)$$

Left-Right Forward-Backward

$$A_{LRFB}^f(e^+e^- \rightarrow f\bar{f}) = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F - \sigma_B)_L + (\sigma_F - \sigma_B)_R} \frac{1}{<|P_e|>} \simeq \frac{3}{4}A_f$$

$$a_\mu = \frac{(g-2)_\mu}{2}$$

$$a_\mu(exp) = 116\,592\,089(63) \times 10^{-11} \quad [0.5ppm]$$

$$a_\mu(th.) = a_\mu(QED) + a_\mu(hadronic) + a_\mu(weak)$$

$$a_\mu(QED) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n a_e^{(2n)}$$

TABLE III. Contributions to muon $g-2$ from QED perturbation term $a_\mu^{(2n)}(\alpha/\pi)^n \times 10^{11}$. They are evaluated with two values of the fine-structure constant determined by the Rb experiment and by the electron $g-2$ (a_e).

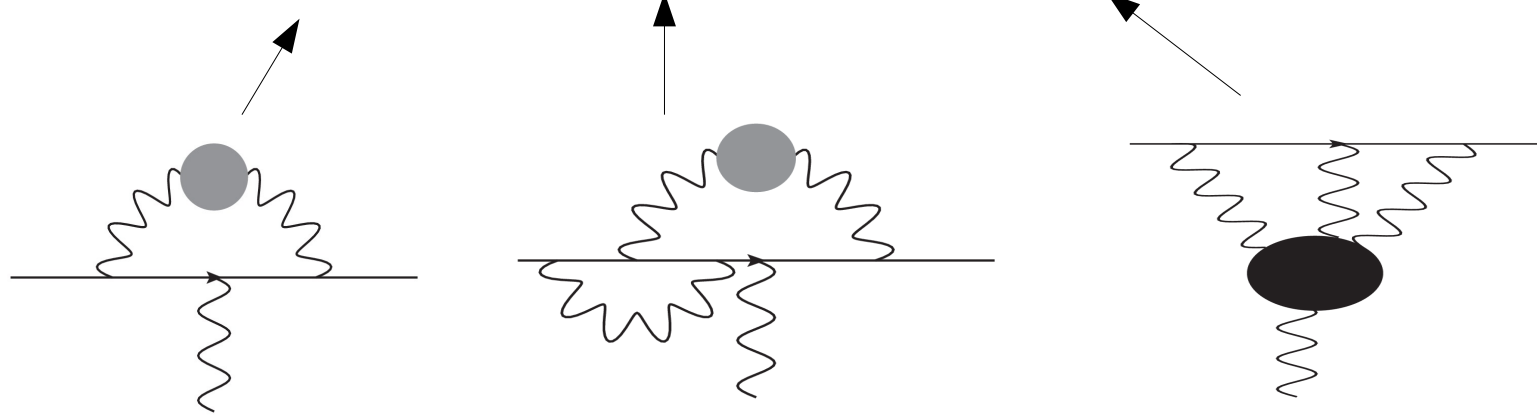
| order | with $\alpha^{-1}(\text{Rb})$ | with $\alpha^{-1}(a_e)$ |
|---------------------|-------------------------------|-------------------------|
| 2 | 116 140 973.318 (77) | 116 140 973.212 (30) |
| 4 | 413 217.6291 (90) | 413 217.6284 (89) |
| 6 | 30 141.902 48 (41) | 30 141.902 39 (40) |
| 8 | 381.008 (19) | 381.008 (19) |
| 10 | 5.0938 (70) | 5.0938 (70) |
| $a_\mu(\text{QED})$ | 116 584 718.951 (80) | 116 584 718.845 (37) |

$$\alpha^2 = \frac{2R_\infty}{c} \frac{m_{\text{Rb}}}{m_e} \frac{h}{m_{\text{Rb}}},$$

$$\alpha(\text{Rb}) = 1/137.035999049(90) \quad [0.66ppb]$$

$$a_\mu(weak) = 154(2) \times 10^{-11}$$

$$a_\mu(hadronic) = a_\mu(had. v.p.) + a_\mu(had. NLO v.p.) + a_\mu(had. l.l.)$$



$$a_\mu(had. v.p.) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s) = 6949.1(37.2)_{exp}(21.0)_{rad} \times 10^{-11}$$

$K(s)$ monotonically decreasing function for increasing s

$$a_\mu(had. NLO v.p.) = -98.4(06)_{exp}(0.4)_{rad} \times 10^{-11}$$

$$a_\mu(had. l.l.) = 116(40) \times 10^{-11}$$

Models of low-energy hadronic interactions with e.m. current

$$a_\mu(th) = 116\,591\,840(59) \times 10^{-11}$$

$$a_\mu(exp) - a_\mu(th.) = 249(87) \times 10^{-11}$$

The main source of error in $a_\mu(\text{th.})$ comes from $a_\mu(\text{hadronic})$ where in the dispersion relation enters the same experimental data that are employed in the calculation of $\Delta\alpha_{had.}^{(5)}$.

If I change $a_\mu(\text{hadronic})$ I get a too light Higgs.

New Physics explanations:

One needs a relatively light not colored particle with couplings to the down fermion enhanced with respect to the SM

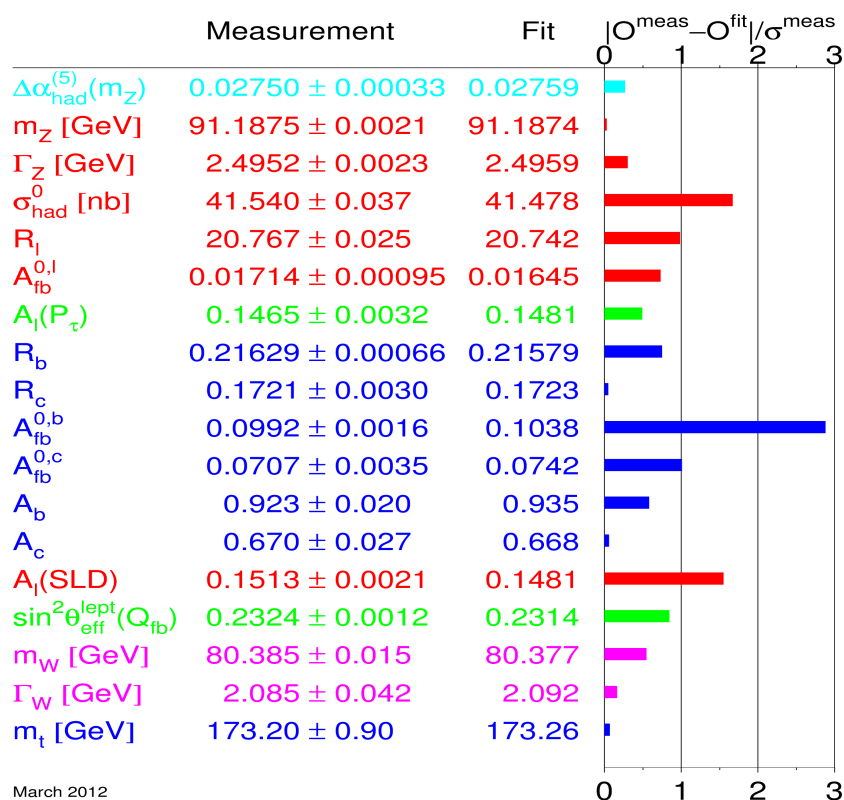
SUSY

$$a_\mu^{\text{SUSY}} \simeq (\text{sgn } \mu) \times (130 \times 10^{-11}) \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \tan \beta$$

SM Fit

One can make a global fit including “all” possible measurements and using the radiatively corrected predictions for the various observable. The latter, besides α , G_μ , M_Z and lepton masses depend upon:

$$m_t, \Delta\alpha_{had}^{(5)}, \alpha_s(M_Z), M_H$$

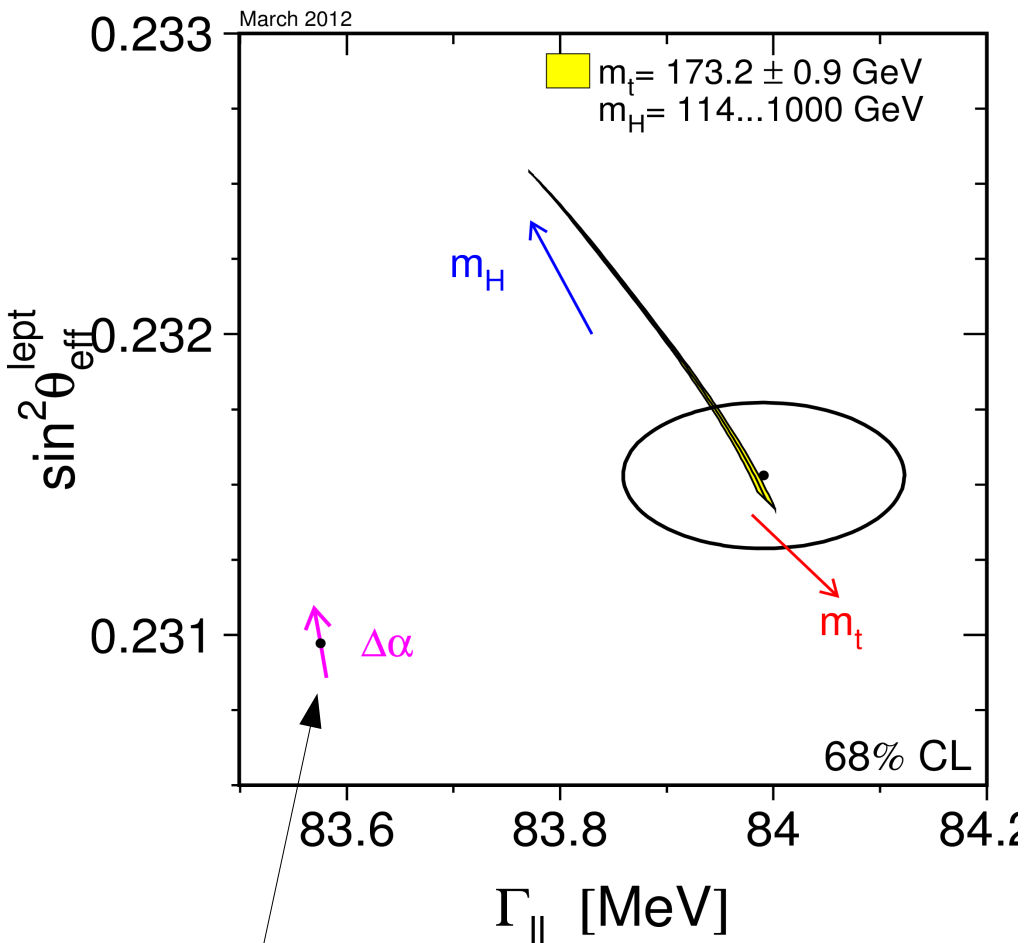


Predictions for m_t , M_W , M_H

Very weak sensitivity to M_H , without the Value of m_T we cannot predict it.

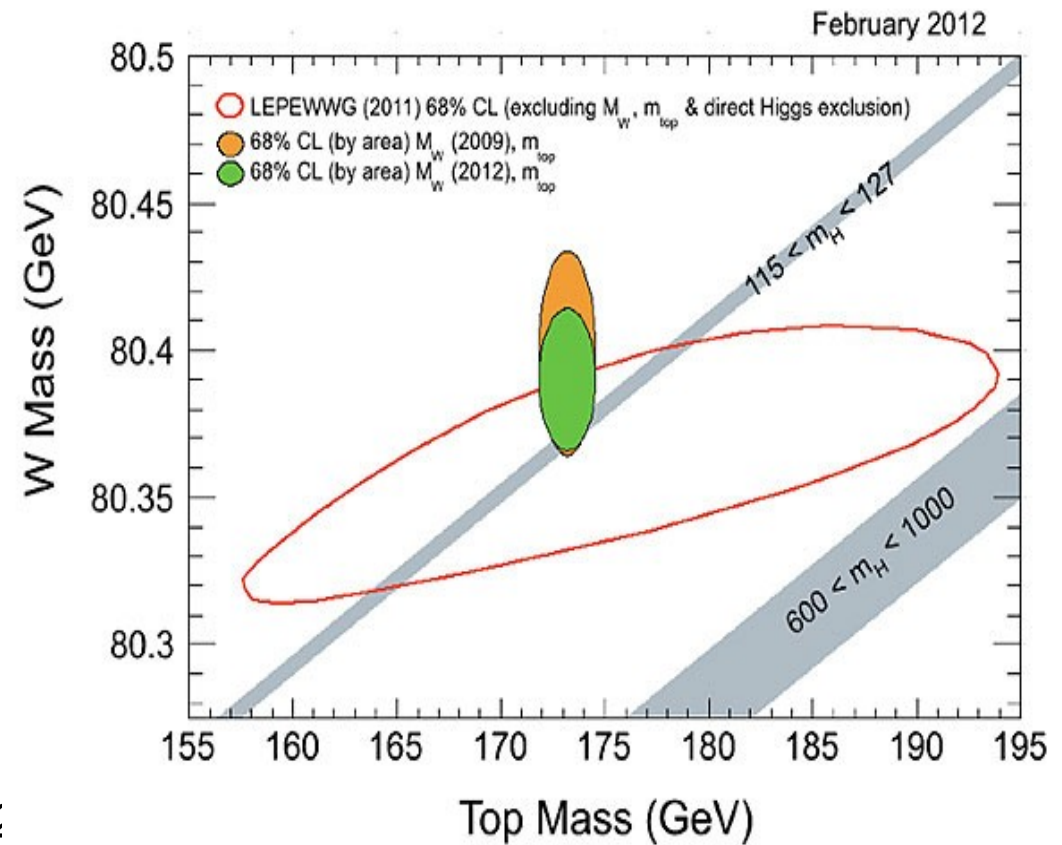
$$R_l = \frac{\Gamma_{had}}{\Gamma_l}, \quad R_q = \frac{\Gamma_q}{\Gamma_{had}}$$

Purely EW corrections
established

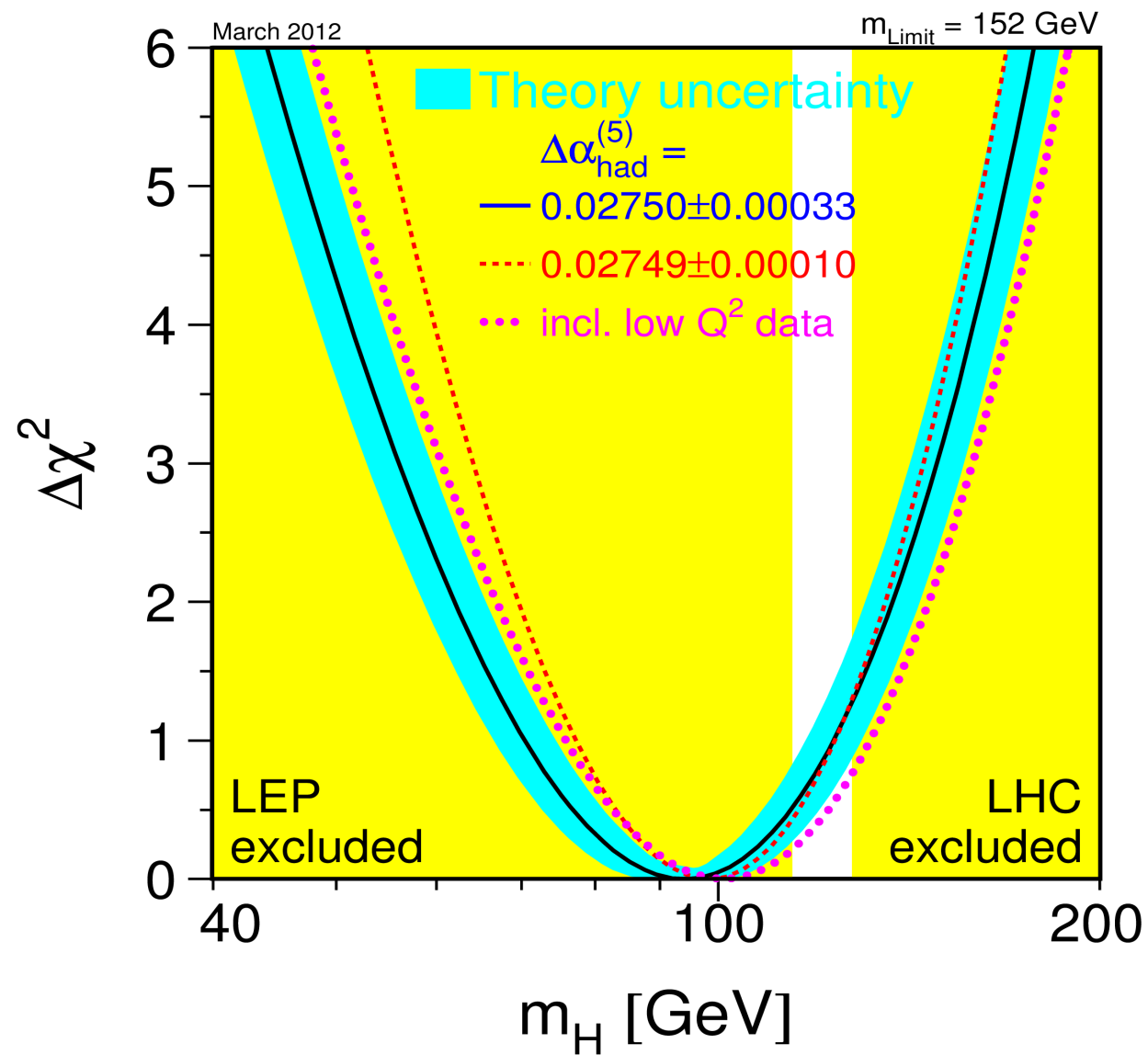


only QED corrections

indirect vs. direct
 M_t, M_W determination



Global fit to M_H



SM prediction for M_H

I want to get a probability density function for M_H in the SM using all the available information, from precision physics and from direct searches (obviously excluding LHC results) to see if the particle that has been discovered at LHC has a mass compatible with the SM prediction (p.d.f $\neq 0$)

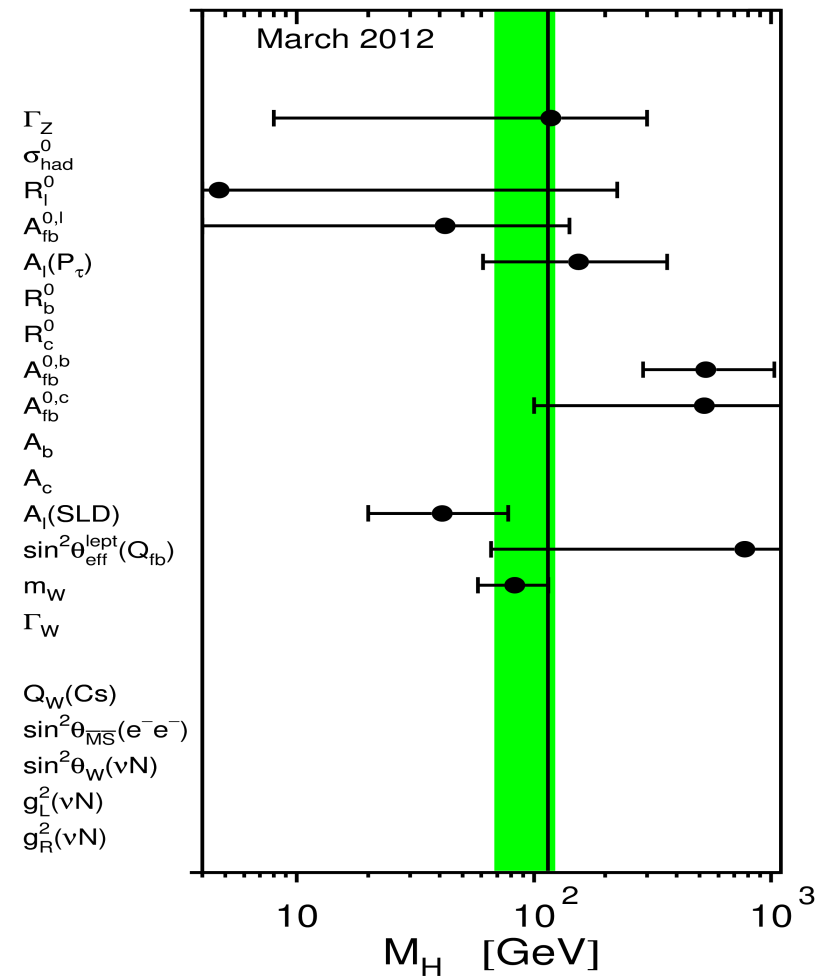
Few observables are really sensitive to the Higgs

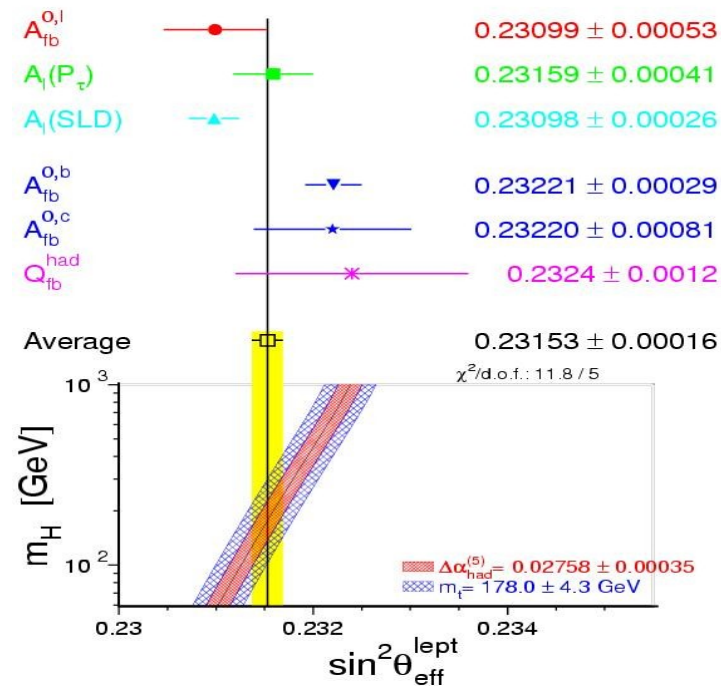
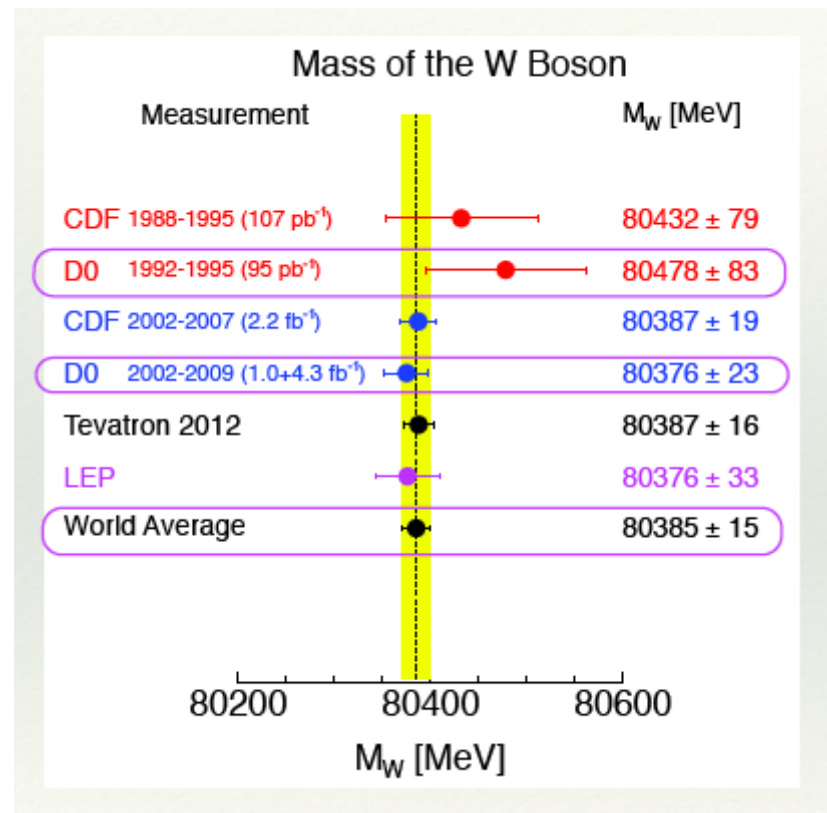
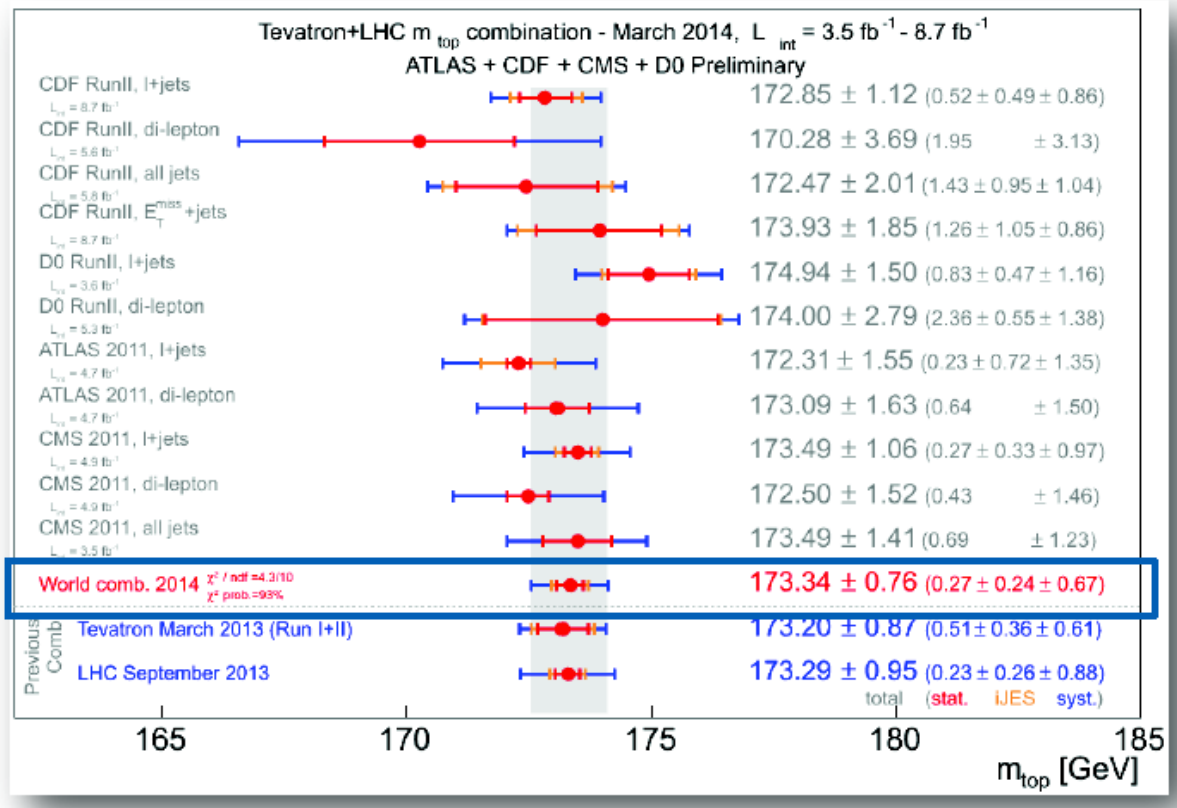
Simplified analysis using

$$M_W, \sin^2 \theta_{eff}^{lept.}$$



asymmetries





- Parametrization:

$$\sin^2 \theta_{eff}^{lept} = (\sin^2 \theta_{eff}^{lept})^\circ + c_1 A_1 + c_5 A_1^2 + c_2 A_2 - c_3 A_3 + c_4 A_4,$$

$$M_W = M_W^\circ - d_1 A_1 - d_5 A_1^2 - d_2 A_2 + d_3 A_3 - d_4 A_4,$$

where

$$A_1 = \ln \frac{M_H}{100 \text{ GeV}}, \quad A_2 = \frac{\Delta \alpha_{had}^{(5)}}{0.02761} - 1,$$

$$A_3 = \left(\frac{m_t}{175 \text{ GeV}} \right)^2 - 1, \quad A_4 = \frac{\alpha_s(M_Z)}{0.118} - 1,$$

$c_i, d_i > 0$ theoretical coefficients (depend upon the RS)

- Two quantities normally distributed

$$W = \sin^2 \theta_{eff}^{lept} - (\sin^2 \theta_{eff}^{lept})^\circ - c_2 A_2 + c_3 A_3 - c_4 A_4,$$

$$Y = M_W^\circ - M_W - d_2 A_2 + d_3 A_3 - d_4 A_4$$

- Likelihood of our indirect measurements $\Theta = \{W, Y\}$ is a two-dimensional correlated normal

$$f(\underline{\theta} | \ln(m_H)) \propto e^{-\chi^2/2}$$

$$\chi^2 = \underline{\Delta}^T \mathbf{V}^{-1} \underline{\Delta}, \quad V_{ij} = \sum_l \frac{\partial \Theta_i}{\partial X_l} \cdot \frac{\partial \Theta_j}{\partial X_l} \cdot \sigma^2(X_l), \quad \underline{\Delta} = \begin{pmatrix} w - c_1 \ln(/100) - c_5 \ln^2(/100) \\ y - d_1 \ln(/100) - d_5 \ln^2(/100) \end{pmatrix}$$

- Using Bayes' theorem the likelihood is turned into a p.d.f. of M_H via a uniform prior in $\ln(M_H)$

$$f(M_H | ind.) = \frac{M_H^{-1} e^{-(x^2/2)}}{\int_0^\infty M_H^{-1} e^{-(x^2/2)} dM_H}.$$

Bayes' theorem: $f(\mu|x) = \frac{f(x|\mu) \cdot f(\mu)}{\int f(x|\mu) \cdot f(\mu) d\mu}$

Likelihood

prior

How $f(M_H | ind.)$ is going to be modified by the results of the direct search experiments?

Ideal experiment (sharp kinematical limit, M_K) with outcome no candidate:

- $f(M_H)$ must vanish below M_K (we did not observe)
- Above M_K the relative probabilities cannot change (experiment is not sensitive there)

$$f(M_H | dir. \& ind.) = \begin{cases} 0 & M_H < M_K \\ \frac{f(M_H | ind.)}{\int_{M_K}^\infty f(M_H | ind.) dM_H} & M_H \geq M_K, \end{cases}$$

Just Bayes theorem:

$$f(M_H | \text{dir. \& ind.}) \propto f(\text{dir.} | M_H) \cdot f(M_H | \text{ind.})$$

Likelihood for the ideal experiment:

$$f(\text{dir.} | M_H) = f(\text{"zero cand."} | M_H) = \begin{cases} 0 & M_H < M_K \\ 1 & M_H \geq M_K \end{cases} \quad \text{Step function}$$

Real life:

no sharp kinematical limit, step function should be replaced by a smooth curve that goes to zero for low masses and to 1 for $M_H \rightarrow M_{Keff}$

Normalize the likelihood to the no signal case (pure background)

(Constant factors do not play any role in Bayes' theorem)

$$\mathcal{R}(M_H) = \frac{L(M_H)}{L(M_H \rightarrow \infty)}$$

Likelihood ratio
(should be provided by the experiments)

$$f(M_H | \text{dir. \& ind.}) = \frac{\mathcal{R}(M_H) f(M_H | \text{ind.})}{\int_0^\infty \mathcal{R}(M_H) f(M_H | \text{ind.}) dM_H}$$

Role of $\mathcal{R}(M_H)$

$\mathcal{R} = 1$

Region where the experiment is not sensitive;
shape of $f(M_H | \text{ind.})$ does not change

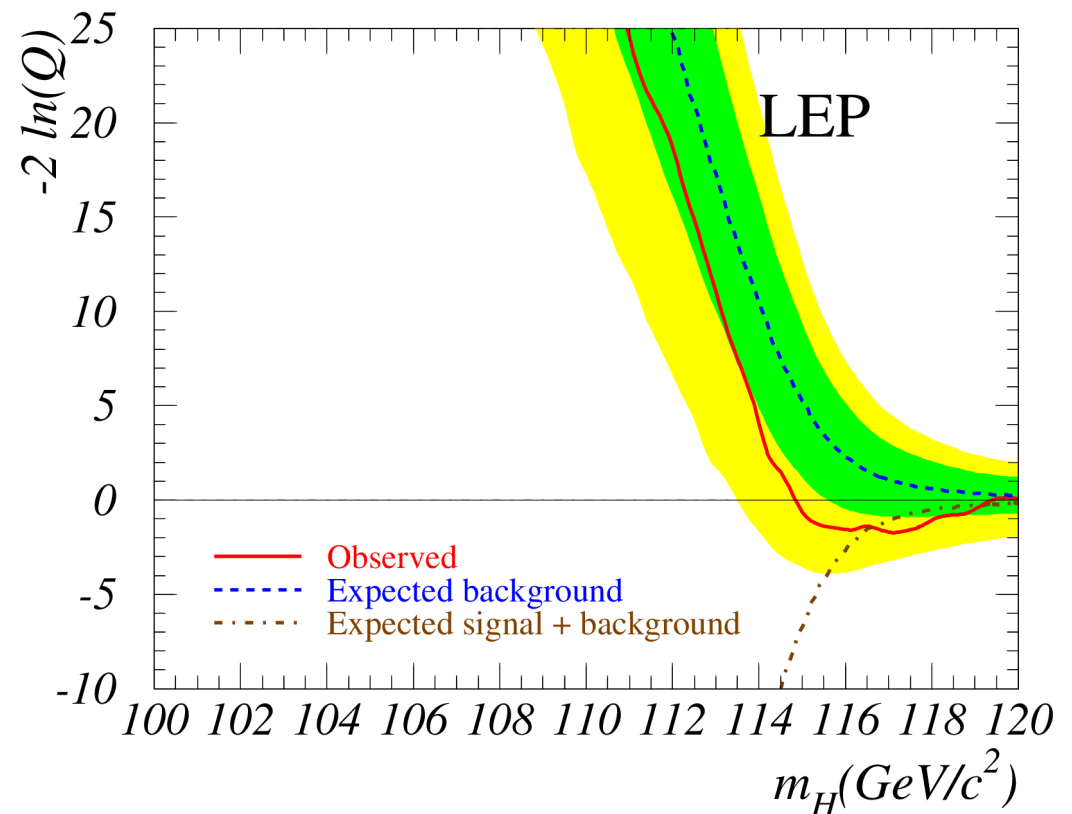
$\mathcal{R} < 1$

Probability is decreased,
p.d.f. is pushed above M_K
 $\mathcal{R}(M_H) \rightarrow 0$ cuts the region

$\mathcal{R} > 1$

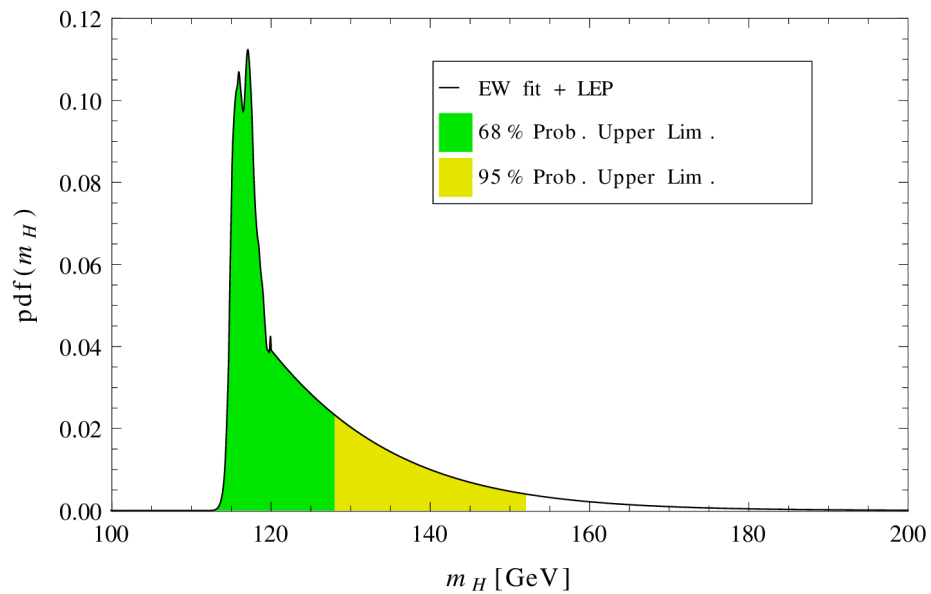
Probability is increased,
p.d.f. is stretched below M_K ,
very large $\mathcal{R}(M_H)$ prompt
a discovery

$\mathcal{Q} = \mathcal{R}$

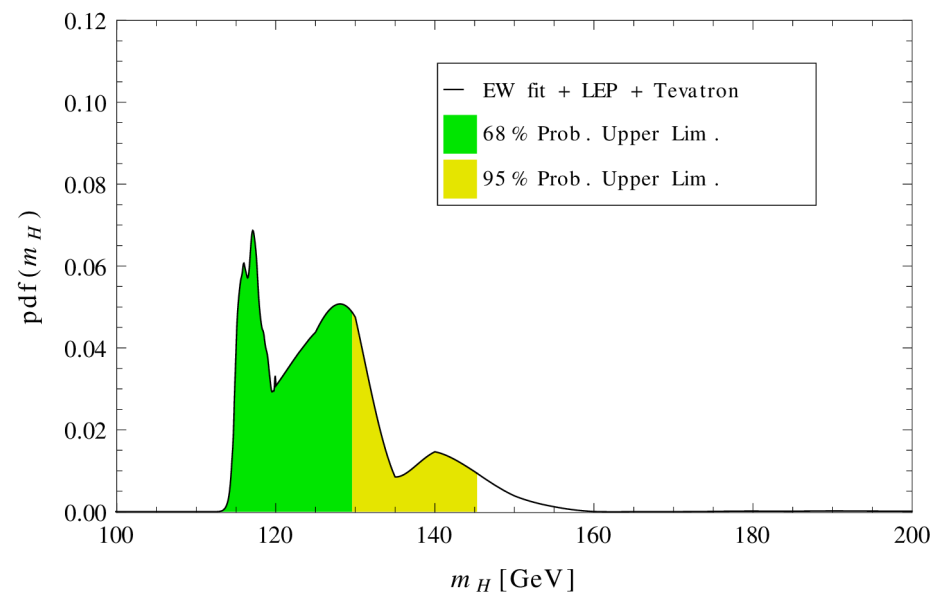


Combining direct and indirect information:

LEP

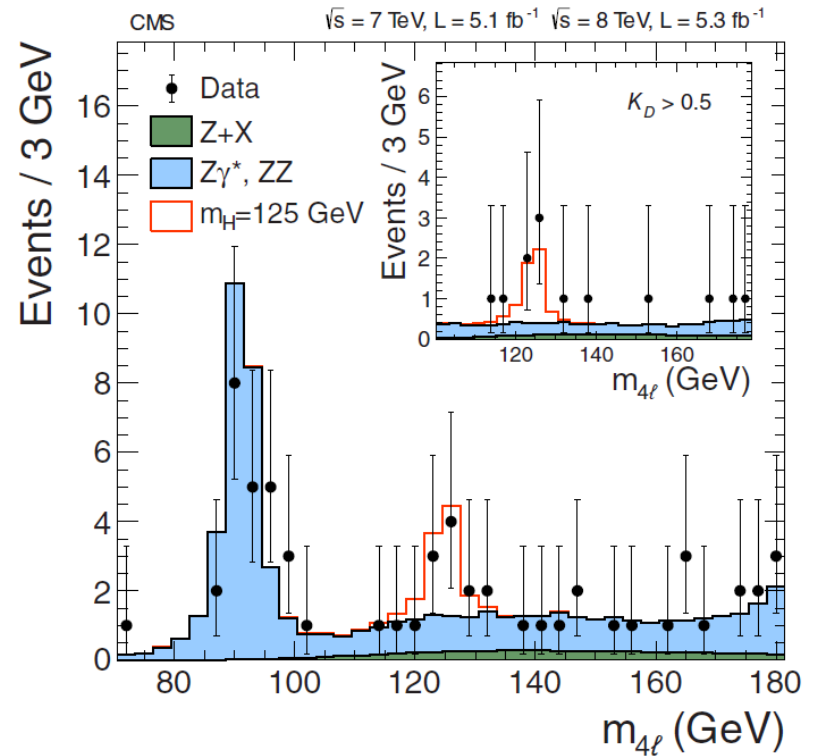
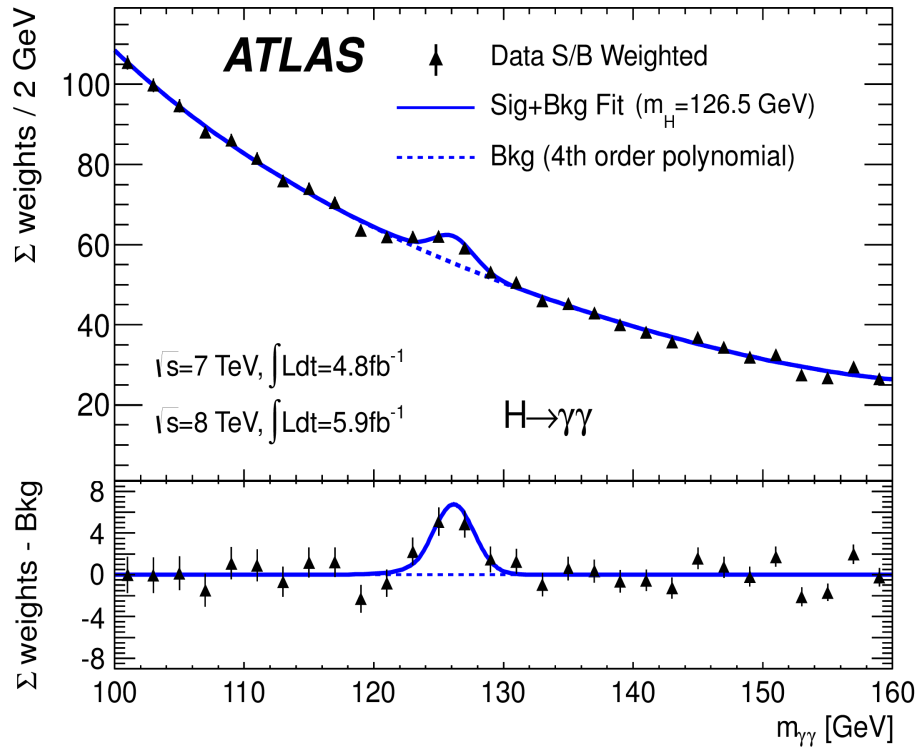


LEP+ TEVATRON



SM: M_H between 114 and 160 GeV with 95% probability below 145 GeV

It is where the SM predicts it should be:
LHC 4th of July 2012 news



$$m_H = 125.36 \pm 0.37 (\text{stat.}) \pm 0.18 (\text{syst.}) \text{ GeV} \quad m_H = 125.03 \pm 0.26 (\text{stat.}) \pm 0.13 (\text{syst.}) \text{ GeV}$$

New Physics effects, where they could be?

New particles are going to contribute to the W,Z self-energies (process-independent contributions) and to vertices (for specific processes). With $M_{\text{NP}} \gg M_Z$ where and what kind of “large” effects can we expect?

Self-energy: 3 types of NP contributions

$$\star \quad \alpha(M_Z)T \equiv \frac{A_{WW}(0)}{M_W^2} - \frac{A_{ZZ}(0)}{M_Z^2} \propto \Pi_{11}(0) - \Pi_{33}(0)$$

isospin violation

Isospin split particles: effects grow as the difference in the mass squared between partners of multiplet. Top contributes quadratically, Higgs logarithmically

$$\star \quad \frac{\alpha(M_Z)}{4s^2c^2}S \equiv \frac{1}{M_Z^2} \left\{ A_{ZZ}(M_Z^2) - A_{ZZ}(0) - \frac{c^2 - s^2}{cs} [A_{\gamma Z}(M_Z^2) - A_{\gamma Z}(0)] - A_{\gamma\gamma}(M_Z^2) \right\}$$

$$\propto \Pi_{3Y}(M_Z^2) - \Pi_{3Y}(0)$$

No-effects that grow quadratically with the masses, but constant terms possible ($\neq 0$, $M_{\text{NP}} \rightarrow \infty$)

Top and Higgs logarithmically

$$\star \quad \frac{\alpha(M_Z)}{4s^2c^2}U \equiv \frac{A_{WW}(M_W^2) - A_{WW}(0)}{M_W^2} - c^2 \frac{A_{ZZ}(M_Z^2) - A_{ZZ}(0)}{M_Z^2}$$

$$- \frac{1}{M_Z^2} [2cs (A_{\gamma Z}(M_Z^2) - A_{\gamma Z}(0)) - s^2 A_{\gamma\gamma}(M_Z^2)]$$

$$\propto \frac{1}{M_W^2} [\Pi_{11}(M_W^2) - \Pi_{11}(0)] - \frac{1}{M_Z^2} [\Pi_{33}(M_Z^2) - \Pi_{33}(0)]$$

Isospin violation in the derivatives

U in many models is usually very small, $U=0$

SM is constrained

At the time of LEP we could envisage specific type of NP that could allow a heavy Higgs in the EW fit (“conspiracy”).

$$\begin{aligned}\hat{\rho} &= \rho_0 + \delta\rho \quad (\rho_0^{\text{SM}} = 1, \delta\rho \leftrightarrow (\epsilon_1, T)) \\ \Delta\hat{r}_W &\leftrightarrow (\epsilon_3, S)\end{aligned}$$

$$\begin{aligned}\sin^2 \theta_{eff}^{lept} &\sim \frac{1}{2} \left\{ 1 - \left[1 - \frac{4A^2}{M_Z^2 \hat{\rho} (1 - \Delta\hat{r}_W)} \right]^{1/2} \right\} \\ &\sim (\sin^2 \theta_{eff}^{lept})^\circ + c_1 \ln \left(\frac{M_H}{M_H^\circ} \right) + c_2 \left[\frac{(\Delta\alpha)_h}{(\Delta\alpha)_h^\circ} - 1 \right] - c_3 \left[\left(\frac{M_t}{M_t^\circ} \right)^2 - 1 \right] + \dots\end{aligned}$$

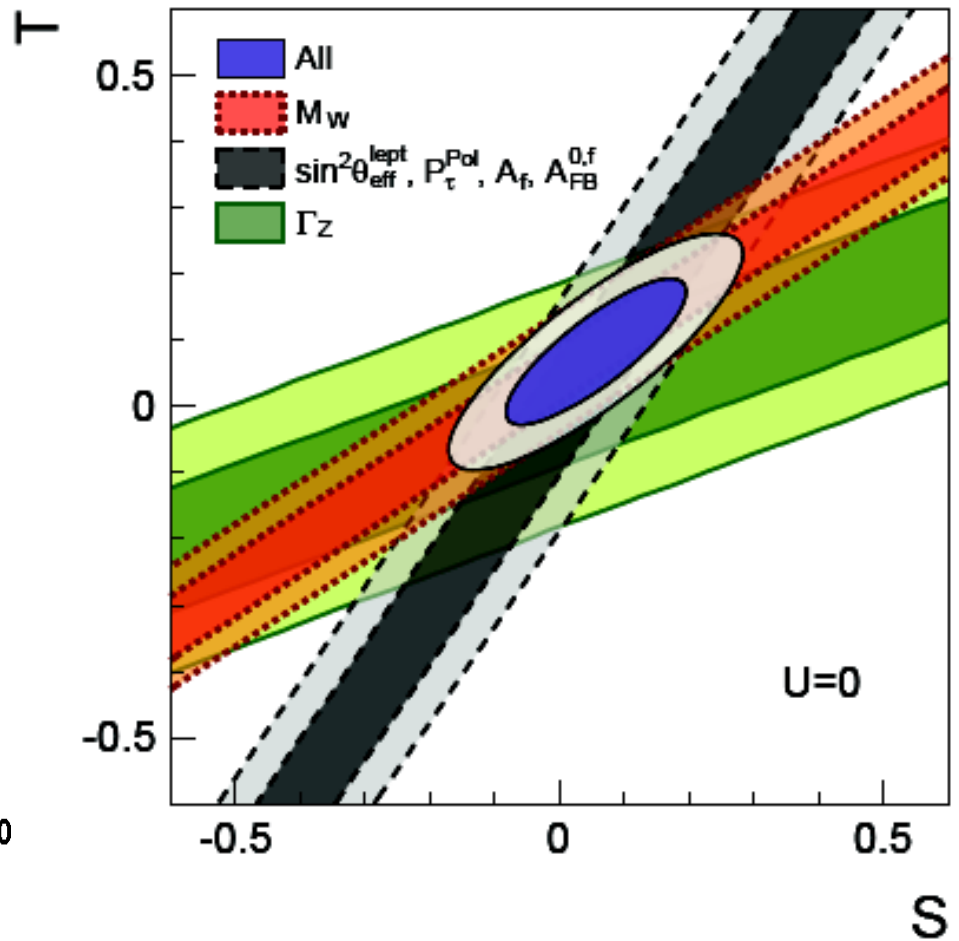
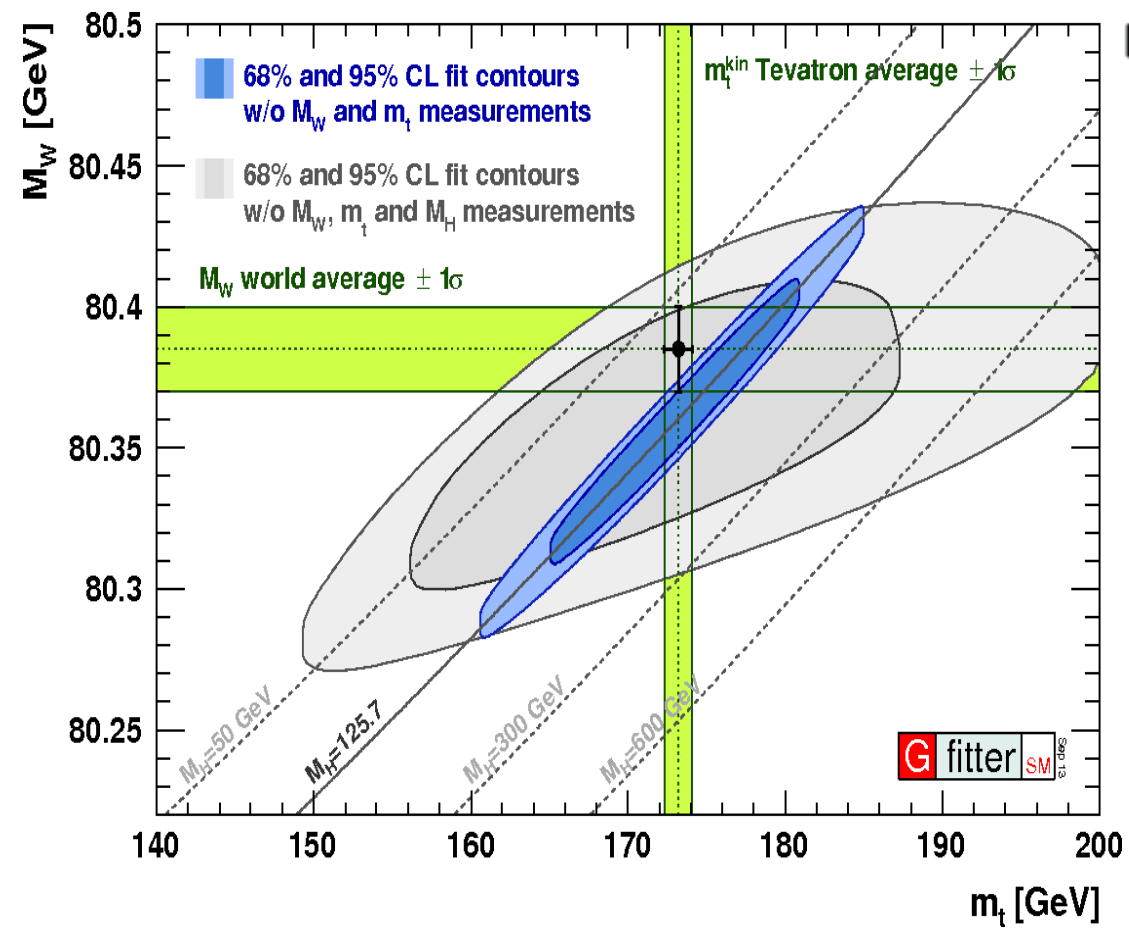
$$c_i > 0$$

To increase the fitted M_H :

$$\left\{ \begin{array}{l} \hat{\rho} > 1 \rightarrow \\ \Delta\hat{r}_W < 0 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \rho_0 > 1 \leftarrow \\ \delta\rho > 0 \leftarrow \end{array} \right\} \quad \begin{array}{l} \text{Extra Z} \\ \text{Isospin (s)fermions,} \\ \text{Multi Higgs models,} \end{array}$$

↖ Light sleptons

NP (if there) seems to be of the decoupling type



Ciuchini, Franco, Mishima, Silvestrini (13)

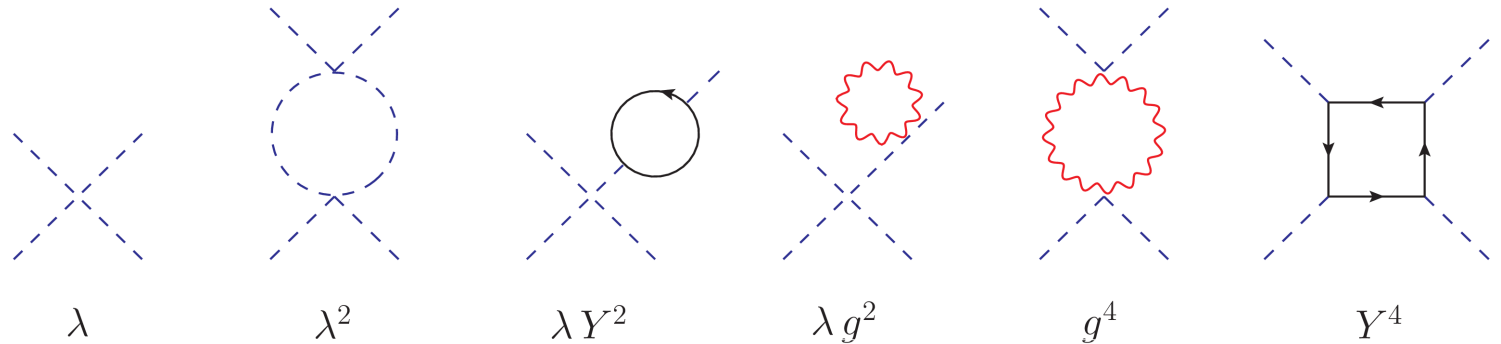
Vacuum Stability bound

Quantum corrections to the classical Higgs potential can modify its shape

$$V^{class}(\phi) = -\frac{1}{2}m^2\phi^2 + \lambda\phi^4 \longrightarrow V^{eff} \approx -\frac{1}{2}m^2(\mu)\phi^2(\mu) + \lambda(\mu)\phi^4(\mu) \sim \lambda(\mu)\phi^4(\mu)$$

$\phi \sim \mu \gg v$

λ runs



$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2} \left[+24\lambda^2 + \lambda (4N_c Y_t - 9g^2 - 3g'^2) - \underbrace{2N_c Y_t^4 + \frac{9}{8}g^4 + \frac{3}{8}g'^4 + \frac{3}{4}g^2 g'^2 + \dots}_{B < 0 \text{ at the weak scale}} \right]$$

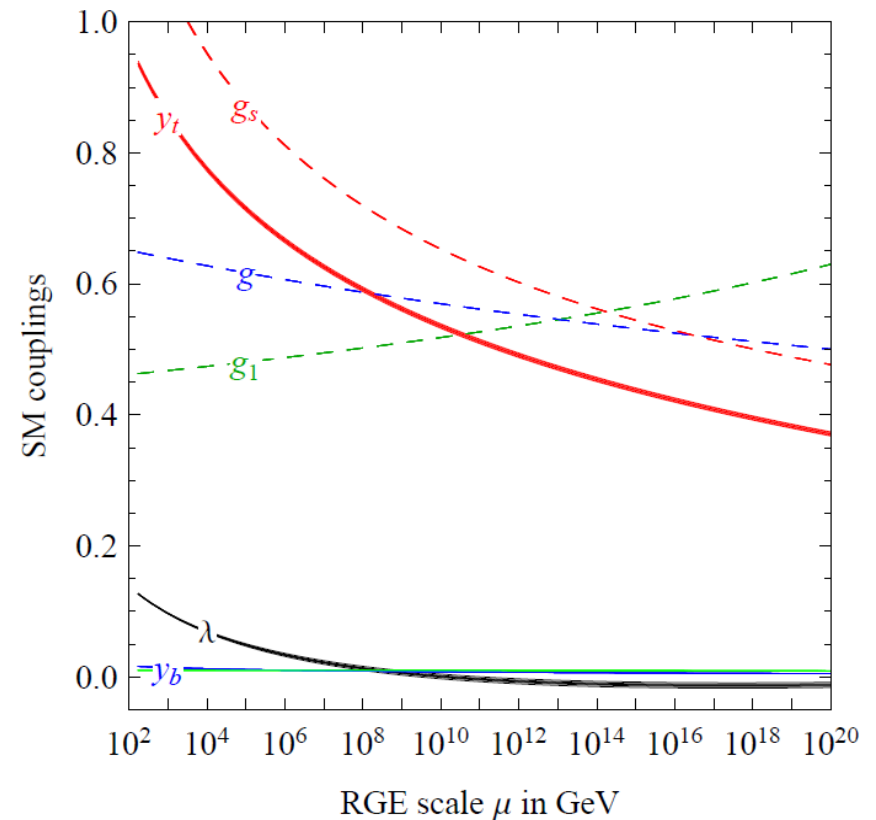
First case: $\lambda \sim 0$ ($M_H \sim 0$)

$$V_{eff}^{1l} = -\frac{1}{2}m^2(M)\phi^2 + \lambda(M)\phi^4 + \frac{B}{32\pi^2}\phi^4 \ln \frac{\phi^2}{M^2}$$

If B were constant at large values of Φ the potential would become negative and unbounded. But B runs

Various possibilities:

- B is negative at the weak scale but not large enough to make B negative at a large scale such that the potential can become negative.
SM vacuum is stable
- B is very negative at the weak scale and stays negative till the Planck scale
SM vacuum is unstable
N.P. should appear below the Planck scale to rescue our lives
- B is sufficient negative at the weak scale that the potential will become negative at a certain scale. However, increasing more the scale B turns positive. The potential develops a second deeper minimum at a large scale
SM is unstable, but



Other case: $B \sim 0$, M_H large

$$V_{eff}^{1l} \sim \lambda(M)\phi^4 + \frac{3\lambda^2}{4\pi^2}\phi^4 \ln \frac{\phi^2}{M^2} \Rightarrow V_{eff}^{RGE} = \frac{\lambda\phi^4}{1 - \frac{3\lambda}{4\pi^2} \ln \frac{\phi^2}{M^2}}$$

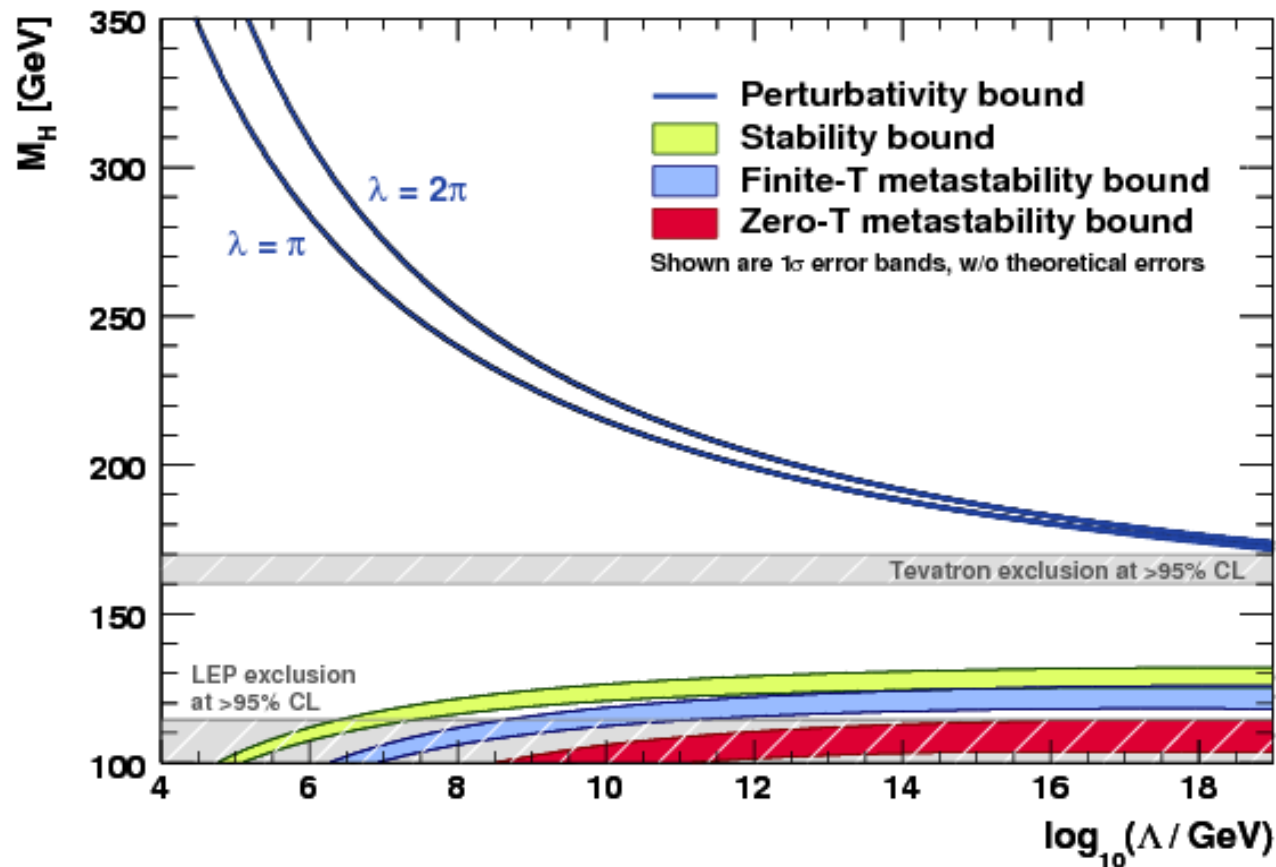
Landau pole
At large Φ perturbativity is lost

Which values of the Higgs mass ensure vacuum stability and perturbativity up to the Planck scale ?

Given the initial values for the couplings obtained from the experimental results we look for:

Vacuum stability $\rightarrow V_{\text{eff}} = 0$ ($\sim \lambda = 0$)

Perturbativity \rightarrow when λ becomes large

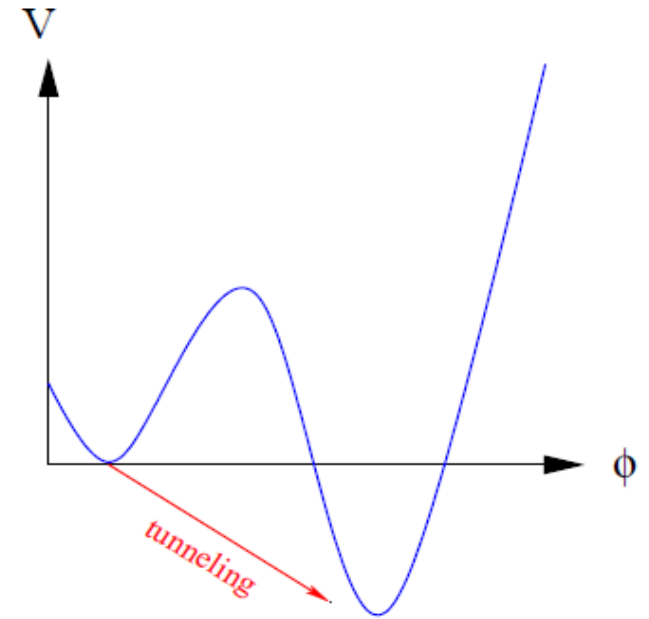


Ellis et al. 09

$M_H \sim 125\text{-}126$ GeV: $-\gamma_t^4$ wins: $\lambda(M_t) \sim 0.14$ runs towards smaller values and can eventually become negative. If so the potential is either unbounded from below or can develop a second (deeper) minimum at large field values

The problem

There is a transition probability between the false and true vacua



It is really a problem ?

It is a problem that must be cured via the appearance of New Physics at a scale below that where the potential become unstable ONLY if the transition probability is smaller than the life of the universe.

Metastability condition: if λ becomes negative provided it remains small in absolute magnitude the SM vacuum is unstable but sufficiently long-lived compared to the age of the Universe

If our vacuum is only a local minimum of the potential, quantum tunneling towards the true minimum can happen. Bubbles of true vacuum can form in the false vacuum and possibly expand throughout the universe converting false vacuum to true. These bubbles are nothing but the solution of the e.o.m. that interpolate between the two vacua (bounces)

Coleman 79

Transition probability $p \sim \tau_U^4 \Lambda_B^4 e^{-S(\Lambda_B)}$ $S(\Lambda_B) = \frac{8\pi^2}{3|\lambda(\Lambda_B)|}$

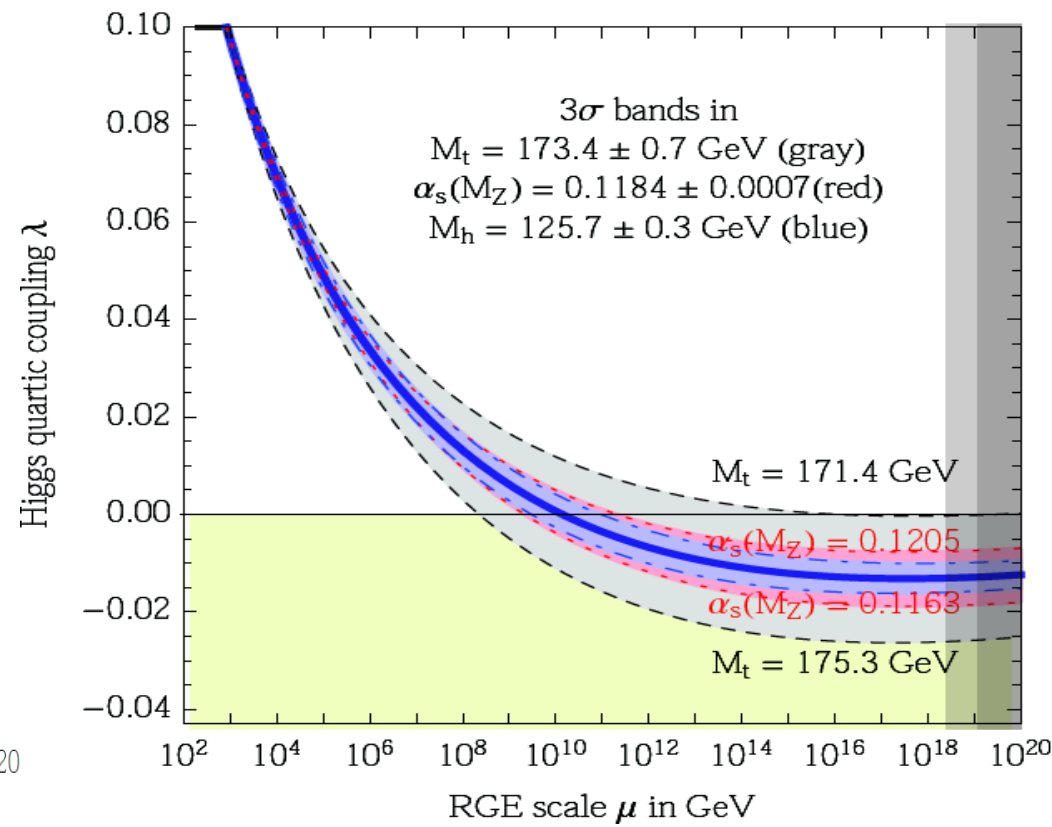
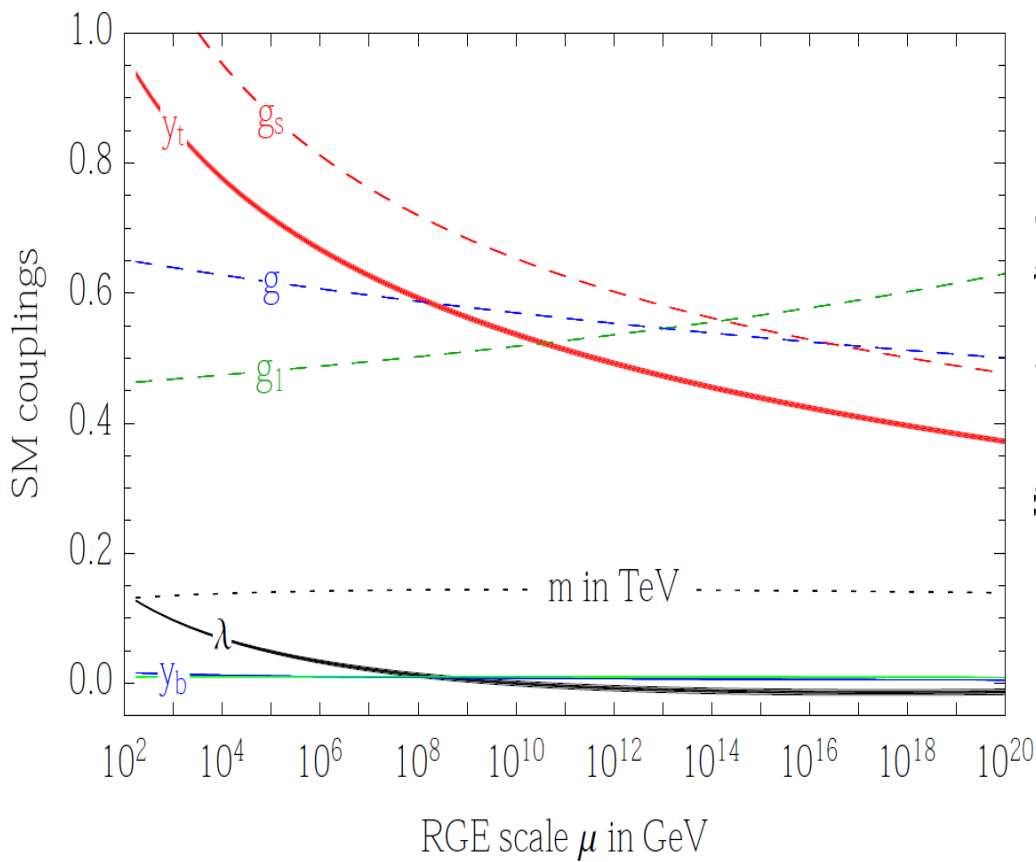
$S(\Lambda_B)$ action of the bounce of size $R = \Lambda_B^{-1}$

Tunneling is dominated by the bounce of size R such that $\lambda(\Lambda_B)$ is minimized, i.e. $\beta_\lambda(\Lambda_B) = 0$.

$$p \sim \left(e^{140} \frac{\Lambda_B}{M_P} \right)^4 e^{-\frac{2600}{|\lambda/0.01|}}$$

Caveat: unknown Planckian dynamics could affect the tunneling rate.

Branchina, Messina (13)



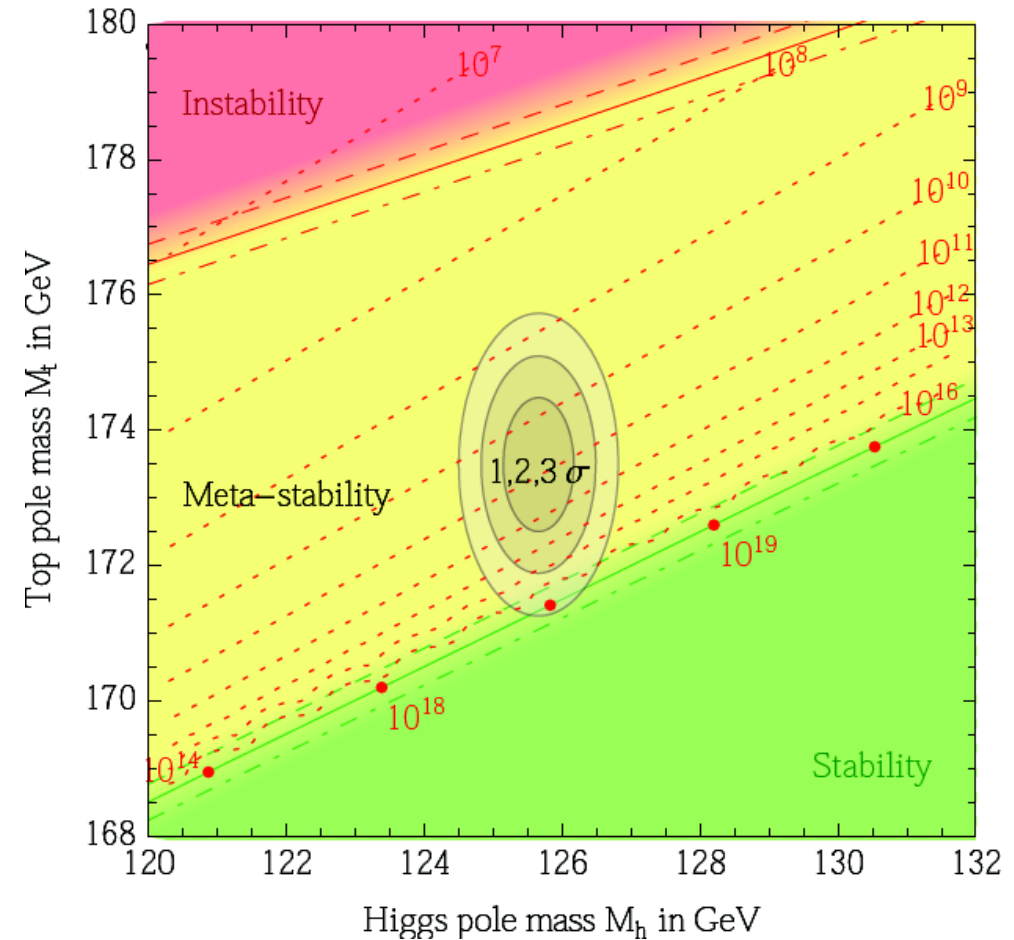
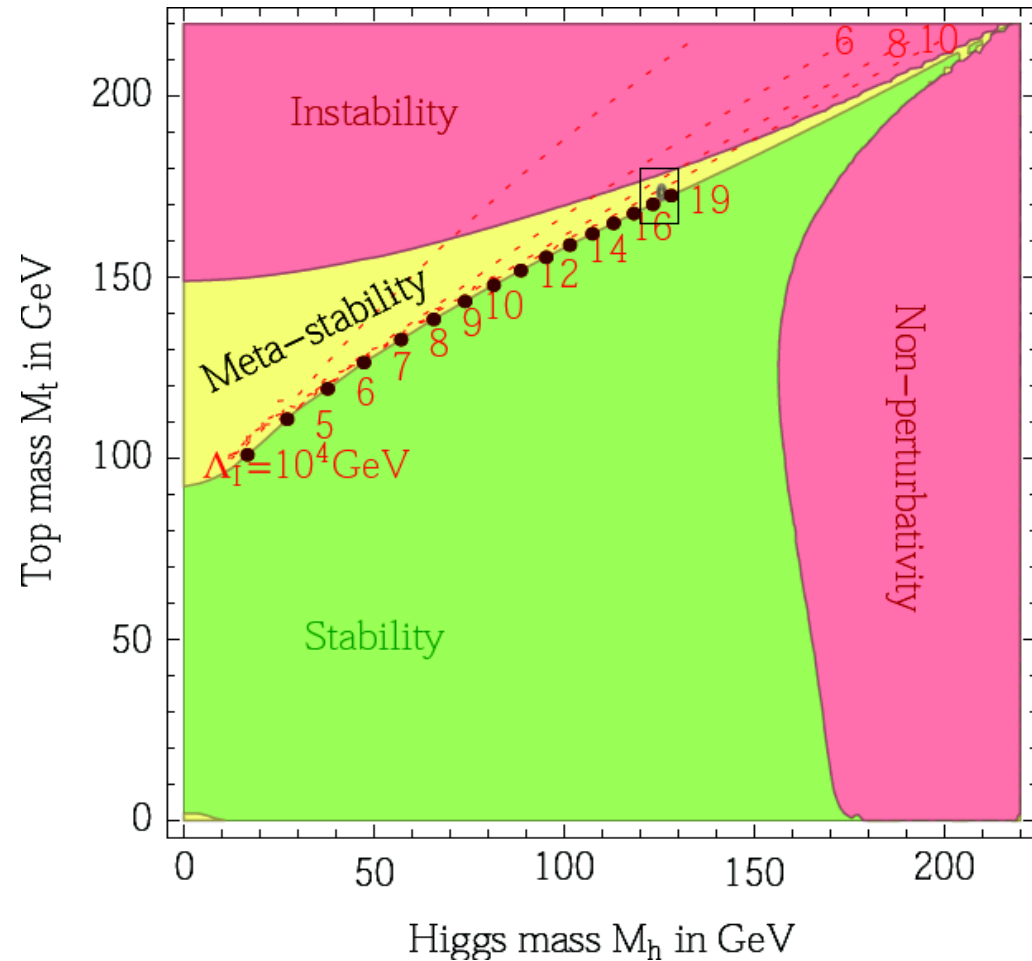
$\lambda(M_{Pl}) > 0$ or $\lambda(M_{Pl}) < 0$ depends on the assumed value of M_t ($\rightarrow Y_t(\mu_W)$)

λ never becomes too negative at M_{Pl} . Both λ and β_λ are very close to zero around M_{Pl}

$$\lambda(M_{Pl.}) = -0.0128 + 0.0010 \left(\frac{M_h - 125.66 \text{ GeV}}{0.34 \text{ GeV}} \right) - 0.0043 \left(\frac{M_t - 173.35 \text{ GeV}}{0.65 \text{ GeV}} \right) + 0.0018 \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right)$$

Stability requires $Y_t(M_t) = 0.927$, present value (including 3 loop QCD) $Y_t(M_t) = 0.937$
difference as large as the full (QCD + EW) two-loop contribution

SM phase diagram



We live in a metastable universe close to the border with the stability region.

Stability condition:

$$\frac{M_h}{\text{GeV}} > 129.6 + 1.3 \left(\frac{M_t - 173.35 \text{ GeV}}{0.65 \text{ GeV}} \right) - 0.5 \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.3_{\text{pert.}} \pm 0.6_{\text{non-pert.}}$$

$$M_t < (171.36 \pm 0.15_{\text{pert.}} \pm 0.30_{\text{non-pert.}} \pm 0.25_{\alpha_s} \pm 0.17_{M_h}) \text{ GeV}$$