Black Holes and Q.M

Based on work with

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(1) Preliminary Comments:

"Elementary" particle of mass M

Two length scales:

Quantum: $r_c = \frac{\hbar}{M}$

Classical: $r_g = 2 G_N M$

Black Hole if: $\boxed{2 G_N M \geq \frac{\hbar}{M}}$

- In classical GR a BH is simply defined as a distribution of mass M localized in a region smaller than $2 G_N M$

Defining:

$$G_N = \frac{L_{P\ell}^2}{\hbar} \qquad G_N = \frac{\hbar}{M_{P\ell}^2}$$

The quantum Mechanical characterization of B+H

$$\frac{2\hbar M}{M_{P\ell}^2} \geqslant \frac{\hbar}{M} \Rightarrow \boxed{\frac{2M^2}{M_{P\ell}^2} \geqslant 1}$$

B.H's $\leftrightarrow$ Ultra-Planckian

Let us introduce a parameter N to characterize q. mechanically any mass M :

$$\boxed{N \equiv \frac{M^2 L_{pe}^2}{\hbar^2}}$$

we are simply measuring mass in Planck units.

In these units the two length scales r_c, r_g become :

$$\boxed{\begin{aligned} r_g &= \sqrt{N}\, L_{pe} \\ r_c &= \frac{\hbar}{M} = \frac{L_{pe}}{\sqrt{N}} \end{aligned}}$$

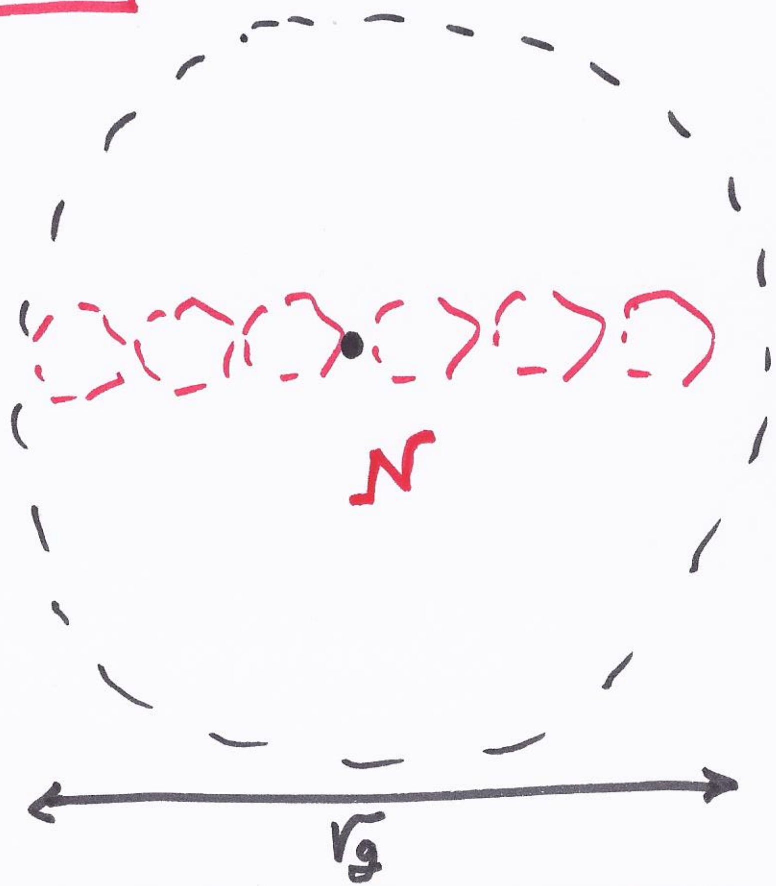
Note the suggestive relation:

$$\boxed{r_c = \frac{L_{pe}^2}{r_g}}$$

Similar to T-duality

Moreover:

$$\boxed{r_g = N r_c}$$



2 Classical / Semi-Classical / Quantum.

(i) Classical Limit: (G.R)

$$\hbar = 0 \qquad G_N \neq 0 \qquad M \;,\; r_g = 2 G_N M$$

In this limit: $\boxed{L_P = M_P = 0}$

Therefore this is the limit with:

$$\boxed{\begin{array}{l} N = \infty \\ \sqrt{N}\, M_P = \text{finite} \\ \sqrt{N}\, L_P = \text{finite} \end{array}}$$

(ii) Semi-Classical limit:

How to characterize this limit?

(-1) We keep $\hbar \neq 0$

(-2) We keep the classical space-time BH geometry i.e $r_g \neq 0$ (horizon)

(-3) We ignore gravitational Back-reaction.

This means:

(-1)	$\hbar \neq 0$
(-2)	r_g - finite
(-3)	$M = \infty$

In "quantum units" this corresponds to:

$$N = \infty \;, \; \underbrace{L_P = 0 \;, \; M_P = \infty}_{G_N = 0}$$

with

$$\boxed{\sqrt{N}\, L_P = r_g = \text{finite}}$$

This is the limit where we can define QM on classical BH geometry.

The key point we tend to forget is that in this semi-classical limit we are necessarily working in the

$$\boxed{N = \infty}$$

limit!

(iii) Full QM limit:

$$\hbar \neq 0 \quad G_N \neq 0 \quad r_g = \text{finite} \quad M = \text{finite}$$

i.e

$$\boxed{N = \text{finite}}$$

In other words:

$$N\text{-finite} \quad \text{iff}: \begin{cases} \hbar \neq 0 \\ \text{grav back reaction} \end{cases}$$

$$N = \infty \quad \text{iff}: \begin{cases} \hbar \neq 0 \\ \underline{\text{No}} \text{ grav back reaction.} \end{cases}$$

At this point we can easily identify the reason underlying many puzzles:

It lies in trying to implement some form of gr back reaction (mostly BH evaporation) Keeping $\boxed{N=\infty}$!!

Indeed:

In the limit $N = \infty$ we can describe Hawking's Thermality (eternal BH)

BUT

We cannot even define Hawking's evaporation!

- So what do we need to start talking about BH evaporation (and in that sense about information paradox and related puzzles)?

- Quantitative control over

$$\boxed{1/N \text{ effects}}$$

In other words we need to distinguish between the quantum effects (due to the special features of the background geometry) That depend on

$$(\hbar, r_g)$$

for instance thermality

$$T = \frac{\hbar}{r_g}$$

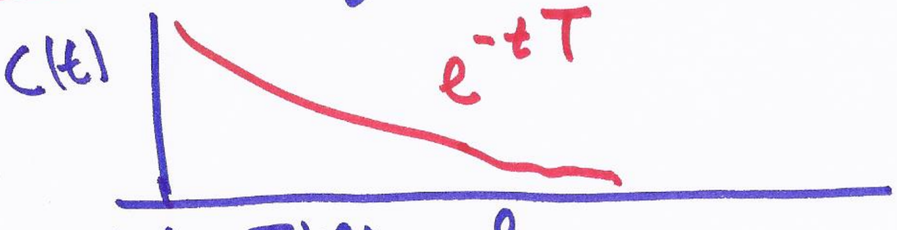
and real <u>Quantum Gravity</u> effects.

Quantum Gravity effects are of two main types.

$\boxed{\begin{array}{ll} \text{(I)} & 1/N\text{ - effects} \\ \text{(II)} & e^{-N}\text{ - effects.} \end{array}}$

Effects of type (II) are easier to figure out and we can formally account for them using a functional integral definition of q. gravity and performing saddle point approximation.

Example: long time decay of correlators (Maldacena ...)



pure Thermal.

"finite N-effect"

Effects of type (I) are more difficult to control and generically require a

microscopic model of BH's.

Before presenting a microscopic model of BH's let us say few words about AdS/CFT.

Two main problems:

1) To define the BH quantum state in the Hilbert space of the dual CFT.

2) To use this state and the dual CFT Hamiltonian to describe real unitary evaporation processes.

Note:

$$|\Psi_{BH}\rangle = |\Psi_{BH}(r_g, N)\rangle$$

$$\lim_{N\to\infty} |\Psi_{BH}(r_g, N)\rangle \equiv |\tilde{\Psi}_{BH}(r_g)\rangle$$

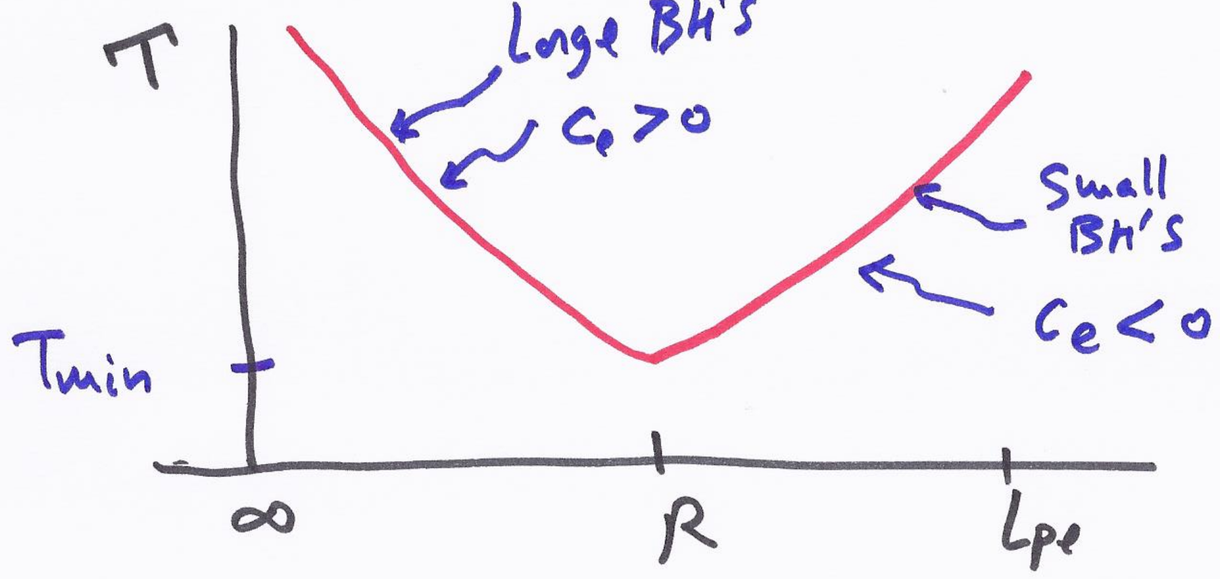
"Thermal Double"

(only describes semi-classical physics)

In standard AdS_5 / CFT_4 we have:

$$\boxed{\frac{R^3}{L_{Pl_5}^3} = N_{YM}^2}$$

For BH's in AdS:



So for Page BH's $R_g \geq R$ $\Rightarrow$

$$\boxed{N \geq N_{YM}^2}$$

which means that $1/N$ BH effects require to work with <u>finite N_{YM}^2</u>.

<u>Equivalently</u>: In the lage $N_{YM}^2 = \infty$ limit we cannot use AdS/CFT to desribe BH evaporation.

For small BH's $\boxed{N < N_{YM}^2}$

In principle No natural relation between:

$$\frac{1}{N_{YM}} \quad \text{and} \quad \frac{1}{N} \quad \text{effects!}$$

Our Proposal:

BH's for $r_g \gg L_P$ are:

Bound States of weakly interacting gravitons at a quantum Critical Point.

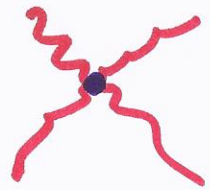
Some Remarks:

(1) $|BH(r_g; N)\rangle$ can be described using the graviton Fock space with $|0\rangle$ the Minkowski vacuum.

(2) We look for a bound state with typical mass M and width r satisfying the relation

$$r = 2 G_N M$$

(3) The corresponding N-body hamiltonian is at weak coupling.

$H_{int} =$  $\approx 1/N$

- What makes this simple-minded Bound State a Black Hole ??

- The special features of the spectrum of collective modes !!

What is specific of a BH is to have a large number $O(N)$ of gapless collective excitations.

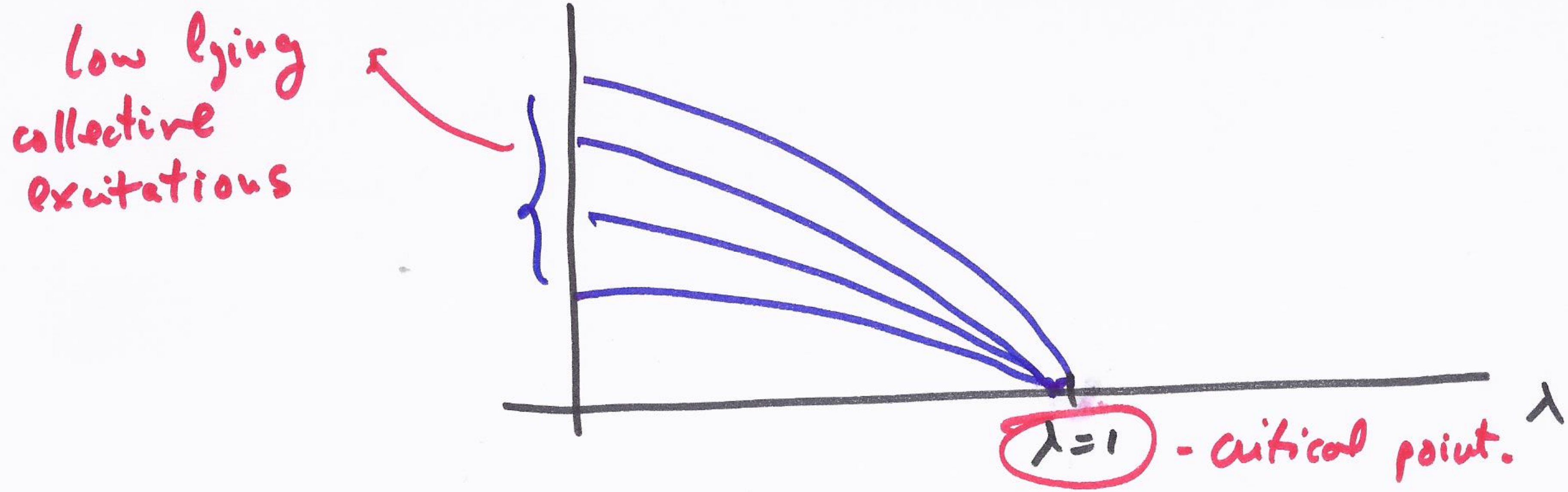
⊛ These gapless collective excitation appear at the **quantum Critical Point**

More precisely:

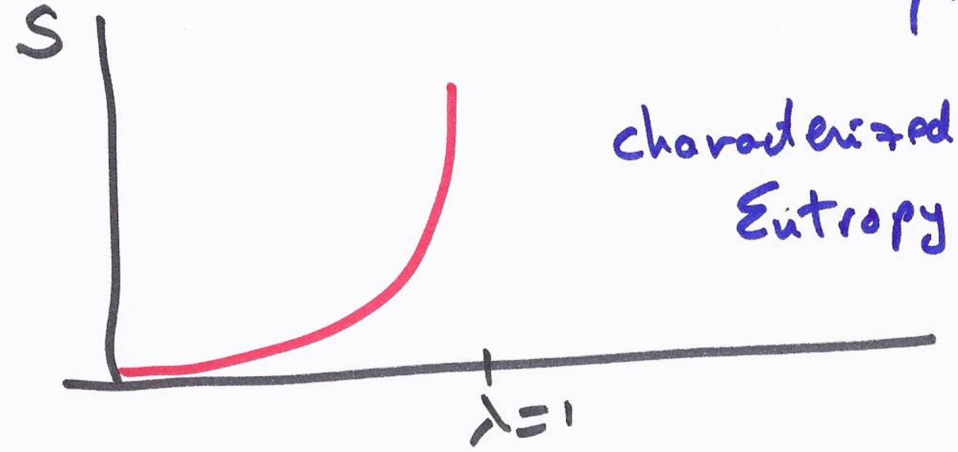
N-gravitons of w-length $\sim \sqrt{N} L_{pe}$ with interaction $\alpha = \sim O\left(\frac{1}{N}\right)$ define a self-sustained Bose-Einstein condensate.

In the mean-field approximation is characterized by the coupling

$$\boxed{\lambda \equiv \alpha \cdot N}$$



At $\lambda = 1$ the system undergoes a quantum critical phase transition.



characterized by increase of Entropy.

Hawking evaporation becomes standard emission due to interaction among constituents (quantum depletion)

In mean field the emission rate is easy to compute

$$\boxed{\frac{dN}{dt} \simeq \frac{1}{\sqrt{N} L_{pl}}}$$

In the s. classical limit this leads to S.B.H law.

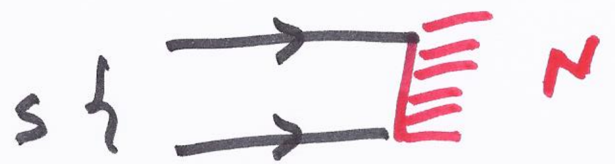
This emission process becomes thermal in $N \to \infty$ limit where the difference between emissions of energetically different quanta is purely <u>combinatorial</u>

$$N \to N+1$$
$$N \to N+n \quad (N = \infty)$$

Two final comments on work in progress with
Gia Dvali, Dieter Lust, Reinke Isermann and Stephan Stieberger.

- Can perturbation theory teach us something about the microscopic structure of Black Holes?.

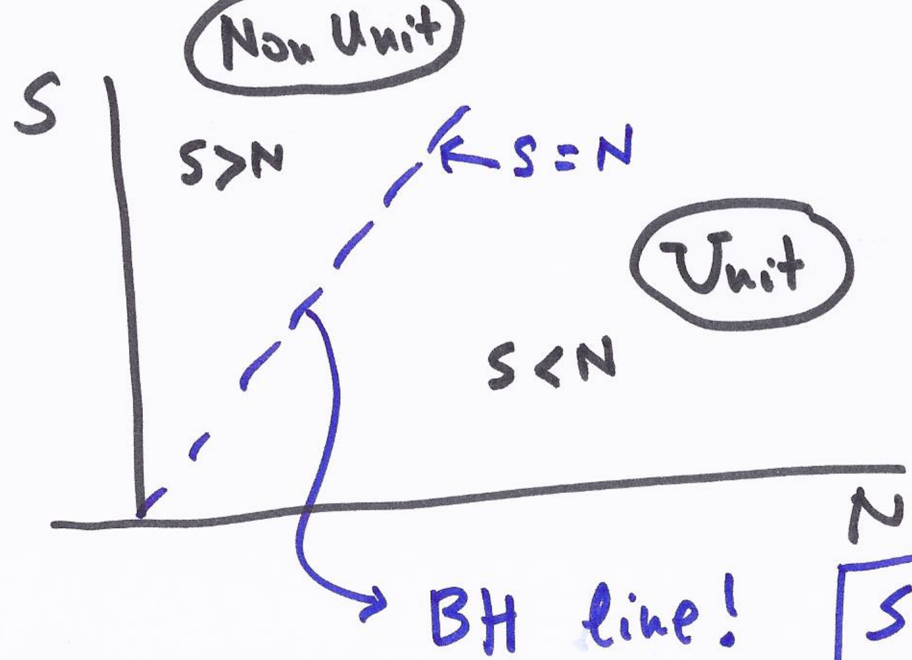
- Yes. (If we focus on the right Kinematics)



$$\boxed{P(2 \to N) \sim \left(\frac{s}{N^2}\right)^N N!}$$

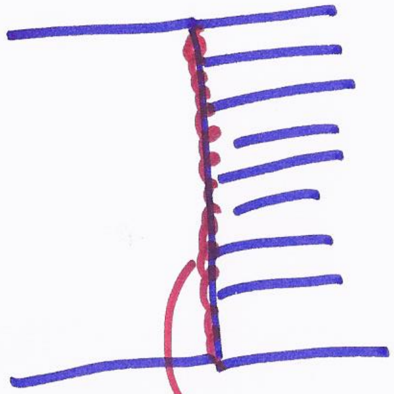
"Unitarity" $\Rightarrow$ $P(2 \to N) \geq P(2 \to N+1)$

$$\boxed{s < N}$$



$$\boxed{S = M^2 = N}$$

What about String theory Effects?



They correspond to Reggeization

$$\boxed{\frac{\sqrt{s}}{N} = \frac{1}{L_s}}$$

$$\boxed{g_s = \frac{1}{\sqrt{N}}}$$

String–BH correspondence!

B.H threshold: $\frac{\sqrt{s}}{N} = \frac{1}{\sqrt{N}\, L_{Pl}}$

Incidentally if $g_s = g_{YM}^2$

$$\boxed{N = N_{YM}^2}$$

reminiscent of "Holography"

In summary:

- For understanding BH evaporation (information puzzle) you need to work with N-finite
  i.e you need a microscopic model.
- A graviton condensate at critical point works well
- Perturbative input provides strong support to this picture.