Corfu, September 2014

INTRODUCTION TO STRING THEORY

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SUPERFIELDS
European Research Council
Adv. Grant no. 226455

I OBVIOUSLY CANNOT IN TWO LECTURES COVER THE WHOLE OF SUPERSTRING THEORY. GIVEN TIME LIMITATIONS, AND GIVEN THE BROAD RANGE OF INTERESTS AMONG THE PARTICIPANTS, I WILL TRY TO FOCUS ON GENERAL PRINCIPLES.

LACES 2014

Lezioni Avanzate di Campi E Stringhe

Galileo Galilei Institute for Theoretical Physics, Arcetri - Italy

November 24 - December 12, 2014

The aim of the school is to bring together PhD students with interests in Gauge and String Theory, and help them building a solid and specialised background on recent developments in Theoretical Physics.

The school is open to about 20 selected PhD students. Application to the School and for financial support is open until September 30th, and should be done through the registration page. Later applications might also be considered.

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Why String Theory?

In the 20th century Physics has undergone incredible successes!

On the one hand Quantum Mechanics has changed our approach towards processes at very short length scales. Its history has been marked by incredible successes, culminating in our understanding of Particle Physics via Quantum Field Theory and the formulation of the Standard Model describing matter and gauge interactions

On the other hand gravitational phenomena at large scales are successfully described by General Relativity, a classical theory of space and time. It has passed a long series of experimental tests, starting from the bending of light from massive objects and Mercury's perihelion, to our everyday experience with GPS localisation

What happens when the gravitational field becomes stronger and stronger, and gravitational phenomena are important even at small scales?

One needs a quantum theory of Gravity at scales around the Planck scale

$$\ell_P = \sqrt{G_N \hbar / c^3} \sim 1.6 \times 10^{-25} A$$

Unfortunately, the strength of the interaction grows with energy and the theory becomes non-renormalisable

(at least perturbatively around flat background)

String Theory provides a consistent framework to unify Particle Physics and Gravity

What is String Theory?

It is fare to say that we don't have a complete understanding of the theory

We are still missing (and seeking) a general principle behind its (non-perturbative) formulation

The mechanical model of a vibrating strings involves an infinity tower of massive gauge fields of increasing spin

Who gives mass to these fields?
Is there a formulation where the infinite gauge symmetries are unbroken and manifest?

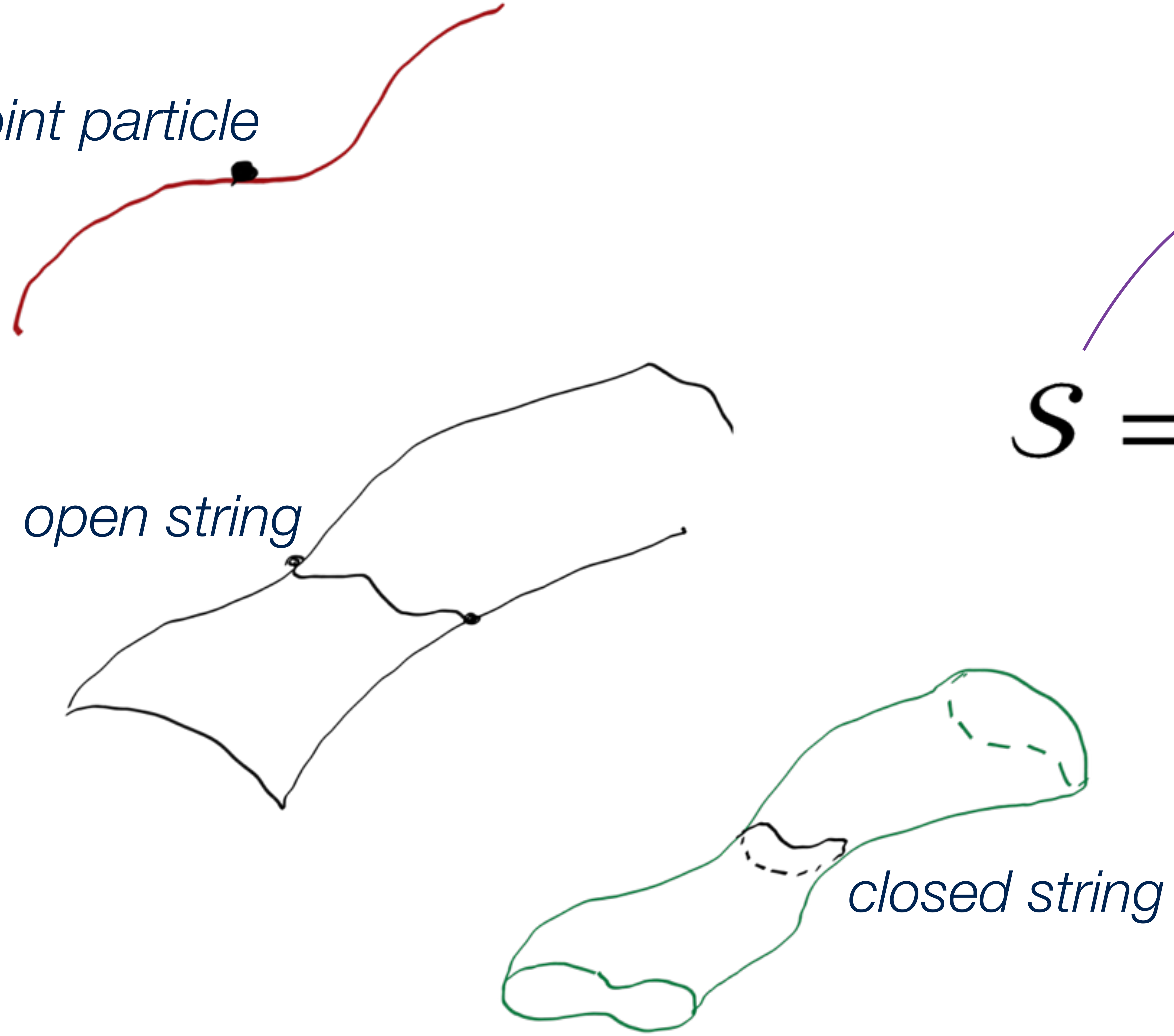
Despite the lack of a general principle,
String Theory has produced a number of
astonishing results in Particle Physics and
Quantum Gravity

Outline

Lecture 1: the mechanical model of a vibrating string

Lecture 2: the many uses of D-branes

In analogy with the point particle case, we demand that, during their motion, strings sweep minimal area surfaces

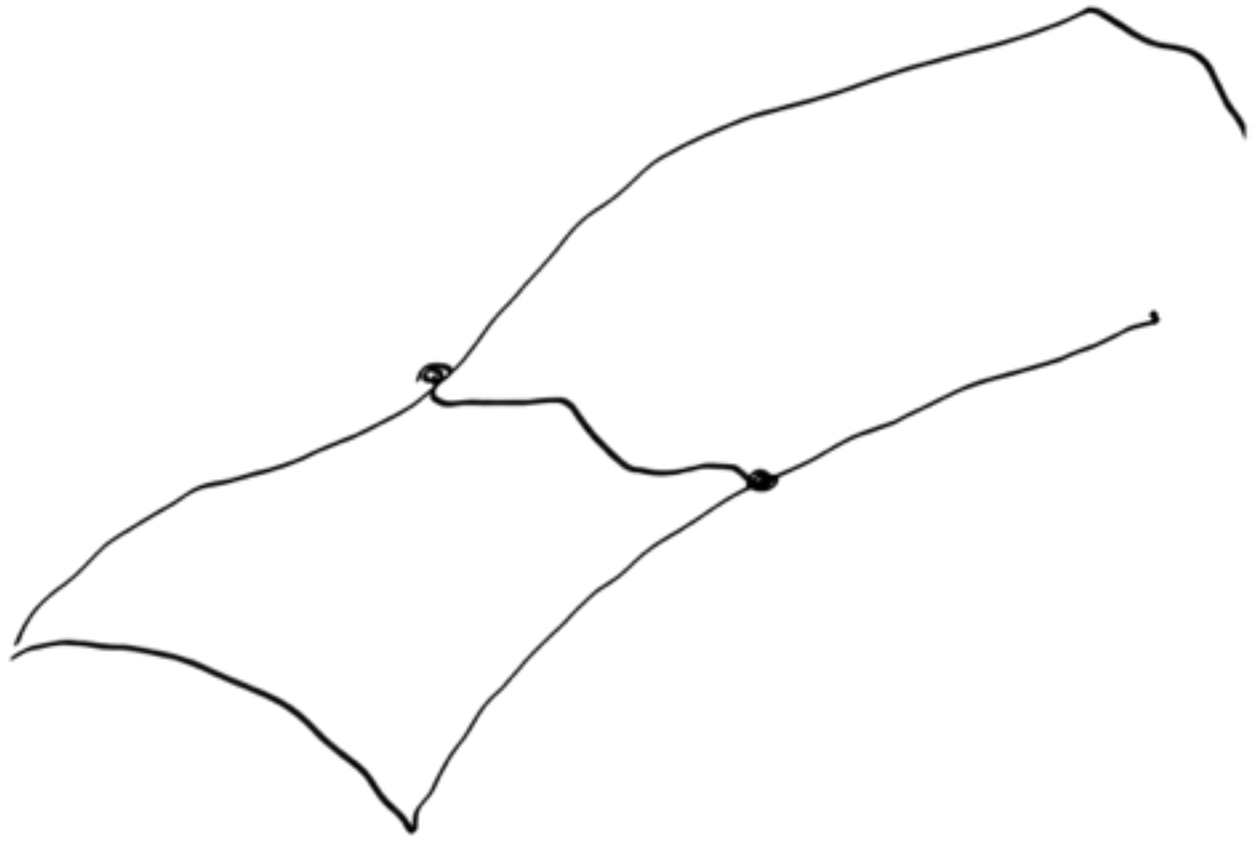


Nambu-Goto action

$$S = T \int d^2\xi \sqrt{\det G_{\alpha\beta}}$$

area of the world sheet

In analogy with the point particle case, we demand that, during their motion, strings sweep minimal area surfaces



$$\xi^\alpha = (\xi^0\,,\,\xi^1)$$ *coordinates on the w.s.*

$$X^\mu = (X^0\,,\,\dots\,,\,X^{D-1})$$ *embedding of the w.s. into space time*

$$G_{\alpha\beta} = g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X$$ *induced metric on the w.s.*

$$g_{\mu\nu}(X)$$ *metric on the space time*

(for the purpose of these lectures we shall always assume it to be the flat metric)

$$S = T\int d^2\xi\,\sqrt{\det G_{\alpha\beta}}$$

area of the world sheet

string tension

The Nambu-Goto action

$$S = T \int d^2\xi \, \sqrt{\det G_{\alpha\beta}}$$

is Lorentz invariant (in the flat ambient space)

is invariant under reparametrisation of the world-sheet

depends on a single parameter: the string tension $T = \dfrac{1}{2\pi\alpha'}$

The Polyakov-Brink-Di Vecchia-Howe action

$$S = -T \int d^2\xi \sqrt{G}\, G^{\alpha\beta}\, \partial_\alpha X^\mu\, \partial_\beta X_\mu$$

after solving the equations of motion for the two-dimensional metric

$$S_{NG} \quad \Leftrightarrow \quad S_{PBDVH}$$

the two actions are (classically) equivalent

The Polyakov-Brink-Di Vecchia-Howe action

$$S = -T \int d^2\xi \, \sqrt{G} \, G^{\alpha\beta} \, \partial_\alpha X^\mu \, \partial_\beta X_\mu$$

is Lorentz invariant (in the flat ambient space)

is invariant under reparametrisation of the world-sheet

is invariant under Weyl rescalings of the world-sheet metric

Two-dimensional reparametrisation invariance allows one to cast the metric in the form of the two-dimensional flat metric, so that the equations of motion become

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \sigma^2} \right) X^\mu = 0 \qquad \xi^\alpha = (\tau\,,\,\sigma)$$

supplemented by suitable boundary conditions

closed strings

open strings

Two-dimensional reparametrisation invariance allows one to cast the metric in the form of the two-dimensional flat metric, so that the equations of motion become

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \sigma^2} \right) X^\mu = 0 \qquad\qquad \xi^\alpha = (\tau\,,\ \sigma)$$

In addition the coordinates obey the constraint

$$\left[(\partial_\tau \pm \partial_\sigma)\, X^\mu \right]^2 = 0$$

The solution of the two-dimensional wave equation decomposes as the sum of left-moving and right-moving waves

$$X^\mu(\tau,\sigma) = X_L^\mu(\tau+\sigma) + X_R^\mu(\tau-\sigma)$$

closed strings: the two waves are completely independent

open strings: standing waves

$$X_L^\mu = \pm X_R^\mu$$

free end-points

fixed end-points

The story for the bosonic string continues with the mode expansion of the solution, the interpretation of the Fourier coefficients in terms of operators, first quantisation, construction of the spectrum, ...

At the end of the story one realises that the bosonic string cannot describe the real world since does not contains space-time fermions, and is unstable ...

The Superstring

two-dimensional Dirac matrices

world sheet fermions

$$S = \int d^2\xi \left((\partial X)^2 - i\bar{\psi}\,\rho^\alpha \partial_\alpha \psi \right)$$

This two-dimensional action is invariant under super-reparametrisations, super Weyl rescalings, and target space Lorentz transformations.

The Superstring

two-dimensional Dirac matrices

world sheet fermions

$$S = \int d^2\xi \left((\partial X)^2 - i\bar{\psi}\,\rho^\alpha \partial_\alpha \psi \right)$$

Again, the solution to the equations of motion both for the bosons and for the fermions decomposes as the sum of left-moving and right-moving waves.

The solution

$$X_R^\mu = \frac{1}{2}\, x^\mu + \alpha' p^\mu (\tau - \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \alpha_n^\mu \, e^{2i\pi n(\tau-\sigma)}$$

$$X_L^\mu = \frac{1}{2}\, x^\mu + \alpha' p^\mu (\tau + \sigma) + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \tilde{\alpha}_n^\mu \, e^{2i\pi n(\tau+\sigma)}$$

The solution

The world-sheet fermions only enter the action via bilinear terms. Hence, one has the possibility of imposing period or anti-periodic boundary conditions. One has

Ramond sector: periodic b.c.

Neveu-Schwarz sector: anti-periodic b.c.

The solution

$$\psi_R^\mu = \frac{1}{\sqrt{2}} \sum_{n\in\mathbb{Z}} d_n^\mu \, e^{-2i\pi n(\tau-\sigma)}$$

R sector

$$\psi_R^\mu = \frac{1}{\sqrt{2}} \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^\mu \, e^{-2i\pi r(\tau-\sigma)}$$

NS sector

Similar expressions exist for the left-moving modes

Actually, these solutions are not independent.
The choice of the flat two-dimensional metric does not fix all symmetries.
One can still go to the light-cone gauge where the oscillators of the "+" directions are not excited.
Moreover, the super-Virasoro constraints

energy-momentum tensor *super-current*

$$T_{\alpha\beta} = 0\,, \qquad J_\alpha = 0$$

express the "-" oscillators in terms of the transverse ones. For instance,

$$\alpha' p^+ \partial_+ X^- = (\partial_+ X^i)^2 + \frac{i}{2}\psi^i \partial_+ \psi^i \qquad p^+ \psi^- = 2\,\psi^i \partial_+ X^i$$

The quantisation proceeds as usual by identifying the Fourier coefficients with (annihilation and creation) operators.

One imposes the following (anti-)commutation relations

$$[\alpha_n^\mu \, , \, \alpha_m^\nu] = n \, \delta_{m+n} \, \eta^{\mu\nu}$$

$$\{b_r^\mu \, , \, b_s^\nu\} = \delta_{r+s} \, \eta^{\mu\nu}$$

$$\{d_n^\mu \, , \, d_m^\nu\} = \delta_{m+n} \, \eta^{\mu\nu}$$

The vacuum

in the NS sector:

$$|0\rangle$$

it is a Lorentz singlet

in the R sector:

$$|a\rangle\,, \quad |\dot{a}\rangle$$

it is a Lorentz spinor

(Majorana-Weyl spinors)

The (super-)Virasoro constraints include ...

$$M^2 = \frac{2}{\alpha'} \left(N_X + \widetilde{N}_X + N_\psi + \widetilde{N}_\psi - \tfrac{1}{2}\,\Delta - \tfrac{1}{2}\,\widetilde{\Delta} \right)$$

mass-shell condition

$$N_X + N_\psi - \tfrac{1}{2}\,\Delta = \widetilde{N}_X + \widetilde{N}_\psi - \tfrac{1}{2}\,\widetilde{\Delta}$$

level matching condition

normal-ordering constants

The normal-ordering constants ...

one periodic boson: $$\Delta = -\frac{1}{12}$$

one periodic fermion: $$\Delta = +\frac{1}{12}$$

one anti-periodic fermion: $$\Delta = -\frac{1}{24}$$

in the NS sector: $$\Delta_{NS} = -\frac{1}{8}$$

in the R sector: $$\Delta_R = 0$$

The light spectrum: NS-NS sector

$$M^2 = \frac{2}{\alpha'}\left(N_X + \tilde{N}_X + N_\psi + \tilde{N}_\psi - \tfrac{1}{8}(D-2)\right)$$

$$N_X + N_\psi = \tilde{N}_X + \tilde{N}_\psi$$

$$|0\rangle\, |\tilde{0}\rangle$$

Lorentz scalar - tachyonic

$$b^i_{-1/2}|0\rangle\, \tilde{b}^j_{-1/2}|\tilde{0}\rangle$$

Metric, scalar (dilaton), antisymmetric rank-two tensor

for consistency these excitations must be massless

$$D = 10$$

The light spectrum: NS-R sector

$$M^2 = \frac{2}{\alpha'}\left(N_X + \tilde{N}_X + N_\psi + \tilde{N}_\psi - \tfrac{1}{2}\right)$$

$$N_X + N_\psi - \tfrac{1}{2} = \tilde{N}_X + \tilde{N}_\psi$$

$$b^i_{-1/2}|0\rangle\,|\tilde{a}\rangle$$

Left-handed spin 3/2
Right-handed spin 1/2

$$b^i_{-1/2}|0\rangle\,|\tilde{\dot{a}}\rangle$$

Left-handed spin 3/2
Right-handed spin 1/2

... similarly for R-NS sector

The light spectrum: R-R sector

$$M^2 = \frac{2}{\alpha'}\left(N_X + \tilde{N}_X + N_\psi + \tilde{N}_\psi\right)$$

$$N_X + N_\psi = \tilde{N}_X + \tilde{N}_\psi$$

$$|a\rangle\,|\tilde{a}\rangle$$

zero-form (scalar)
two-form (antisymmetric tensor)
(self-dual) four-form

$$|a\rangle\,|\tilde{\dot{a}}\rangle$$

one-form (vector)
three-form

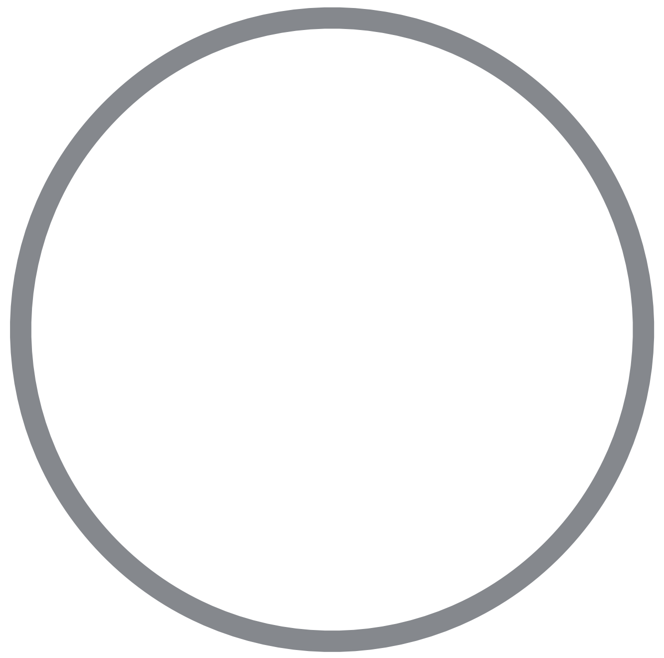
Actually too many states!
The construction is inconsistent!

Need to make a selection of the
massless and massive excitations

See tomorrow ...

A natural book-keeping for the massless and massive excitations is the one-loop partition function.

In field theory one computes the diagram



$$e^{-\Gamma} = \int [D\phi]\, e^{-S_E[\phi]} = \left[\det\left(-\Delta + M^2\right)\right]^{-1/2}$$

Typically, in field theory one in not interested in this quantity. It is a function of the masses of particles of no use (unless the theory is coupled to gravity)

A natural book-keeping for the massless and massive excitations is the one-loop partition function.

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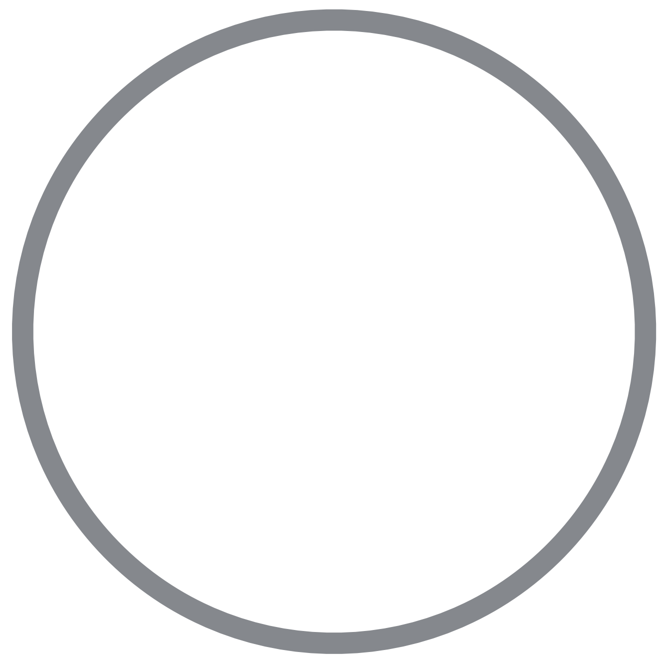

$$\Gamma = -V \int_0^\infty \frac{dt}{t^{1+D/2}} \, \mathrm{Str}\left(e^{-M^2 t}\right)$$

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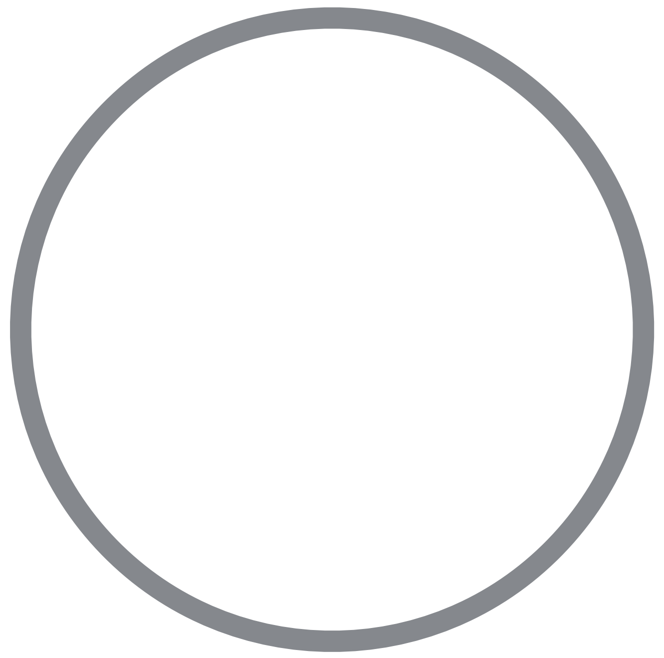
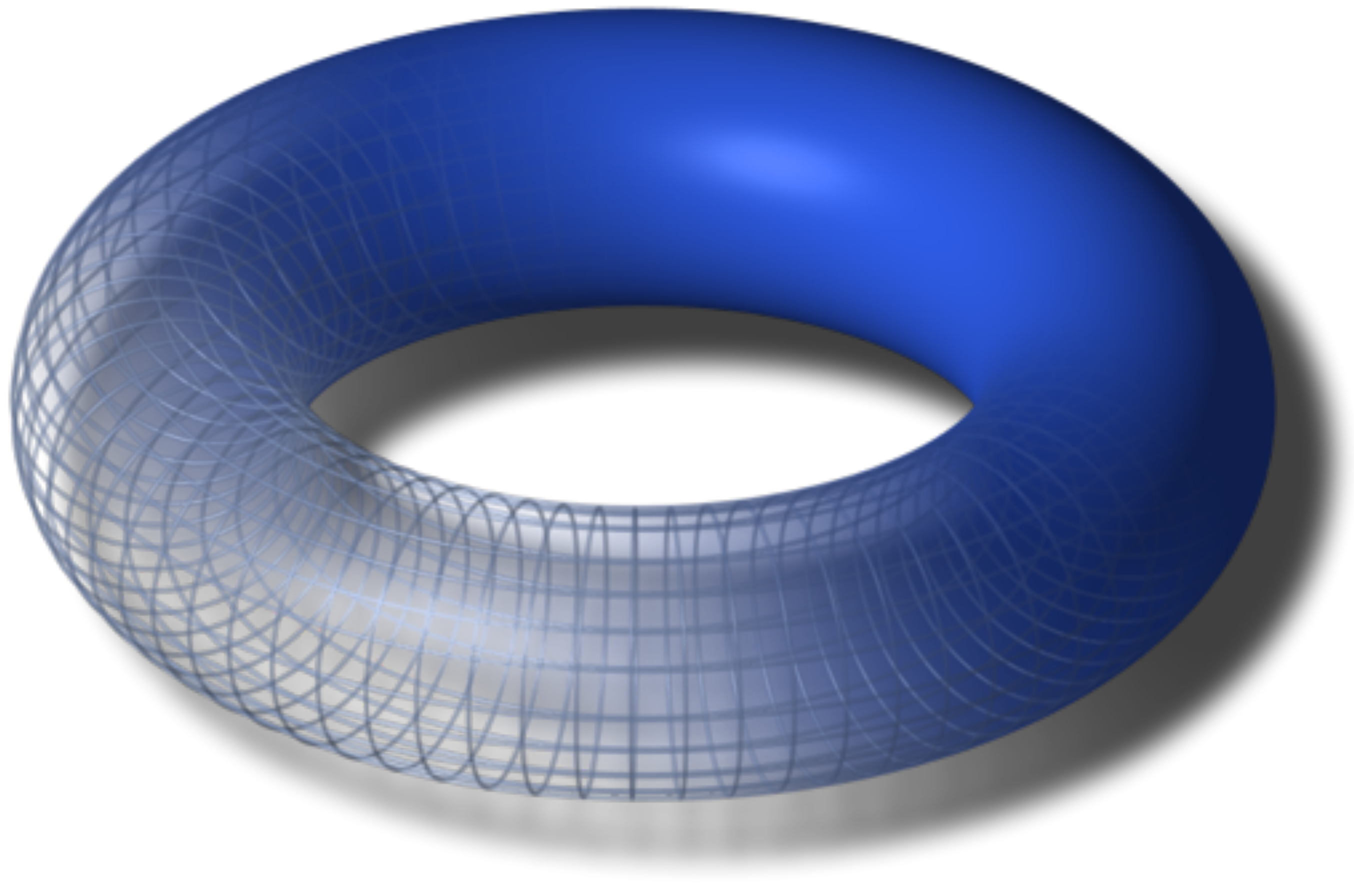
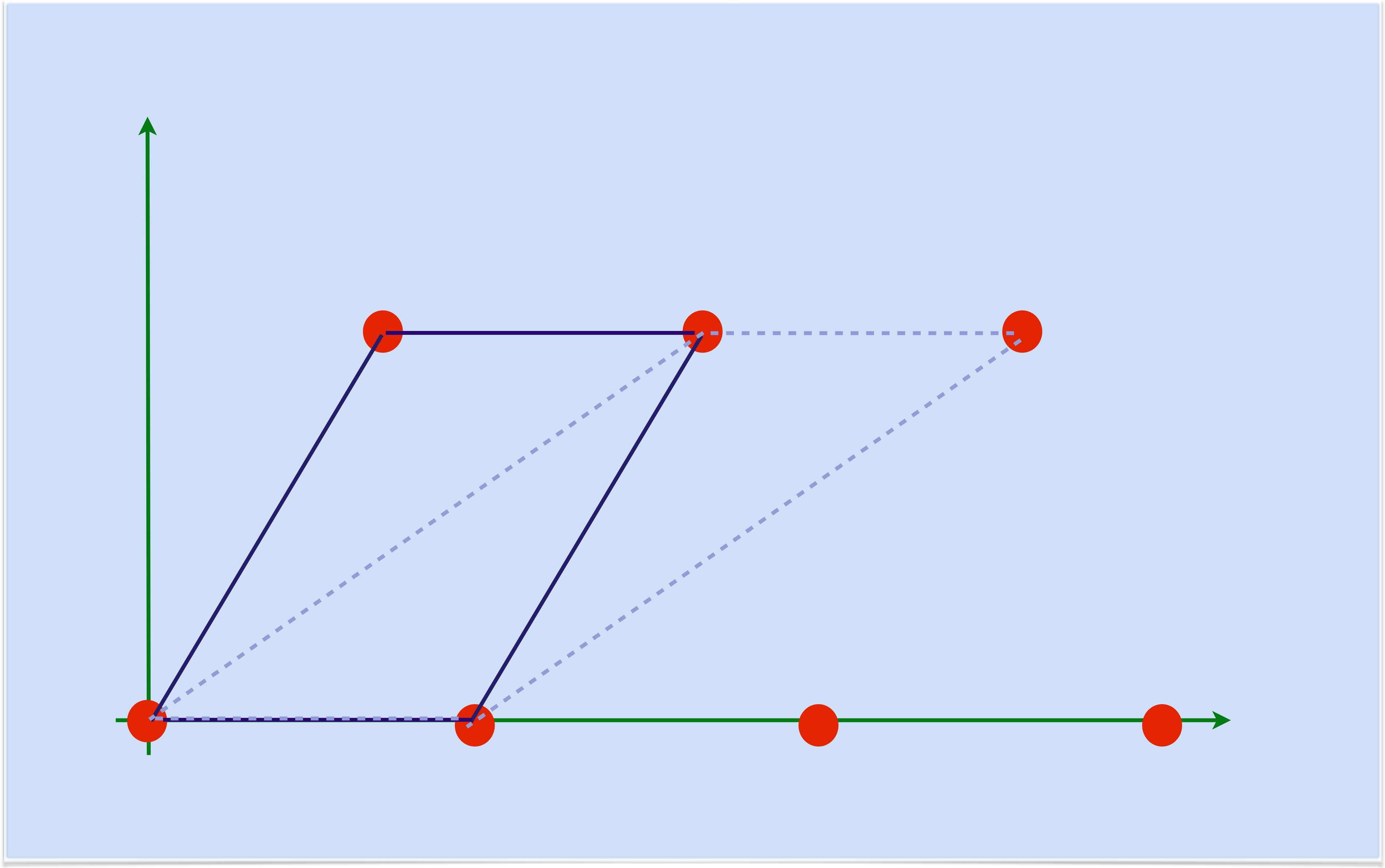


$$\Gamma = -V \int_0^\infty \frac{dt}{t^{1+D/2}} \, \mathrm{Str}\left(e^{-M^2 t}\right)$$

In string theory, however, it teaches us a lot of things since the spectrum contains an infinite tower of massive excitations.

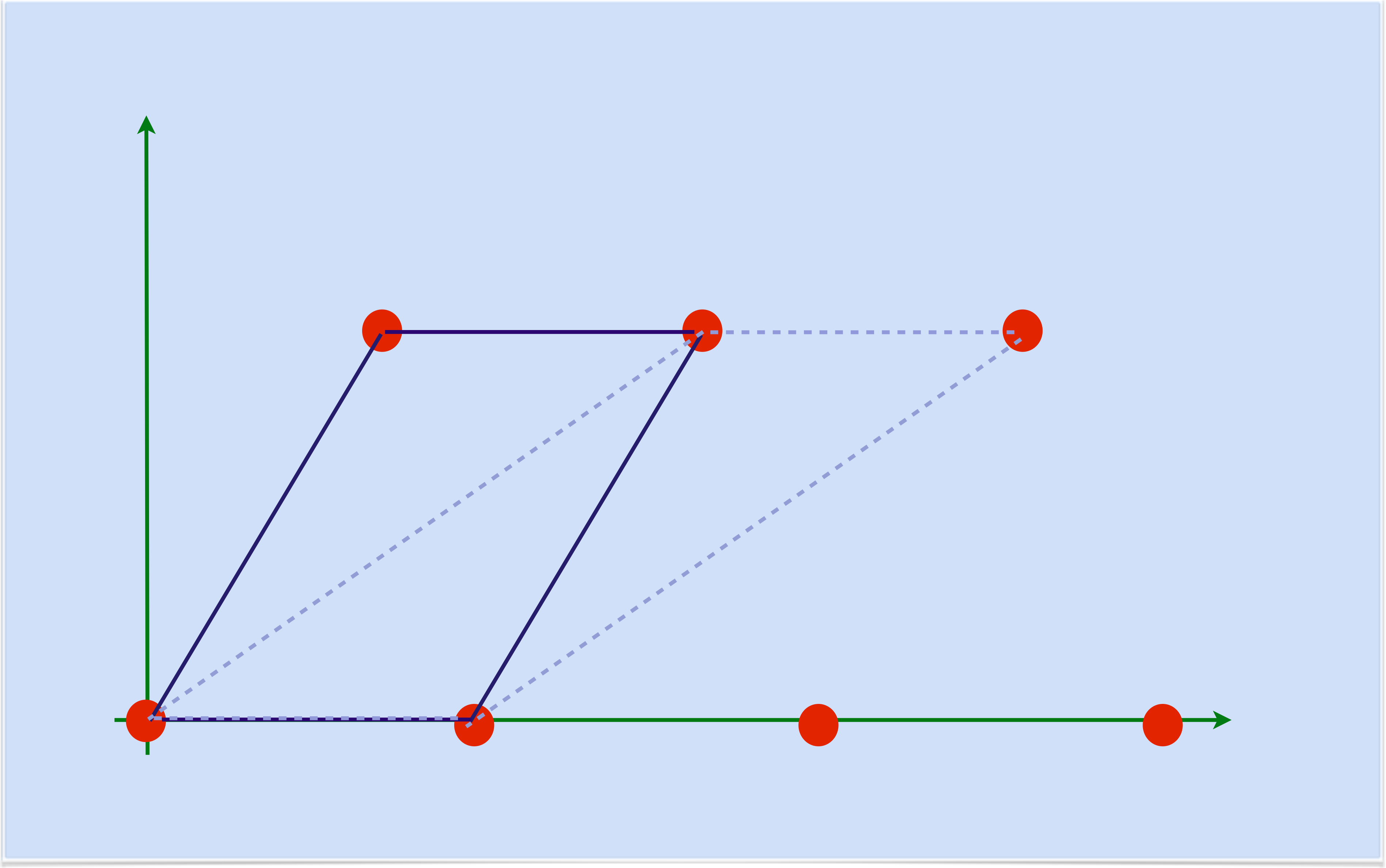
The corresponding diagram in string theory is fatter, and has the topology of a torus





The corresponding diagram in string theory is fatter, and has the topology of a torus

Not all shape of tori are inequivalent. Moreover, we impose that the vacuum energy be the same for equivalent tori.

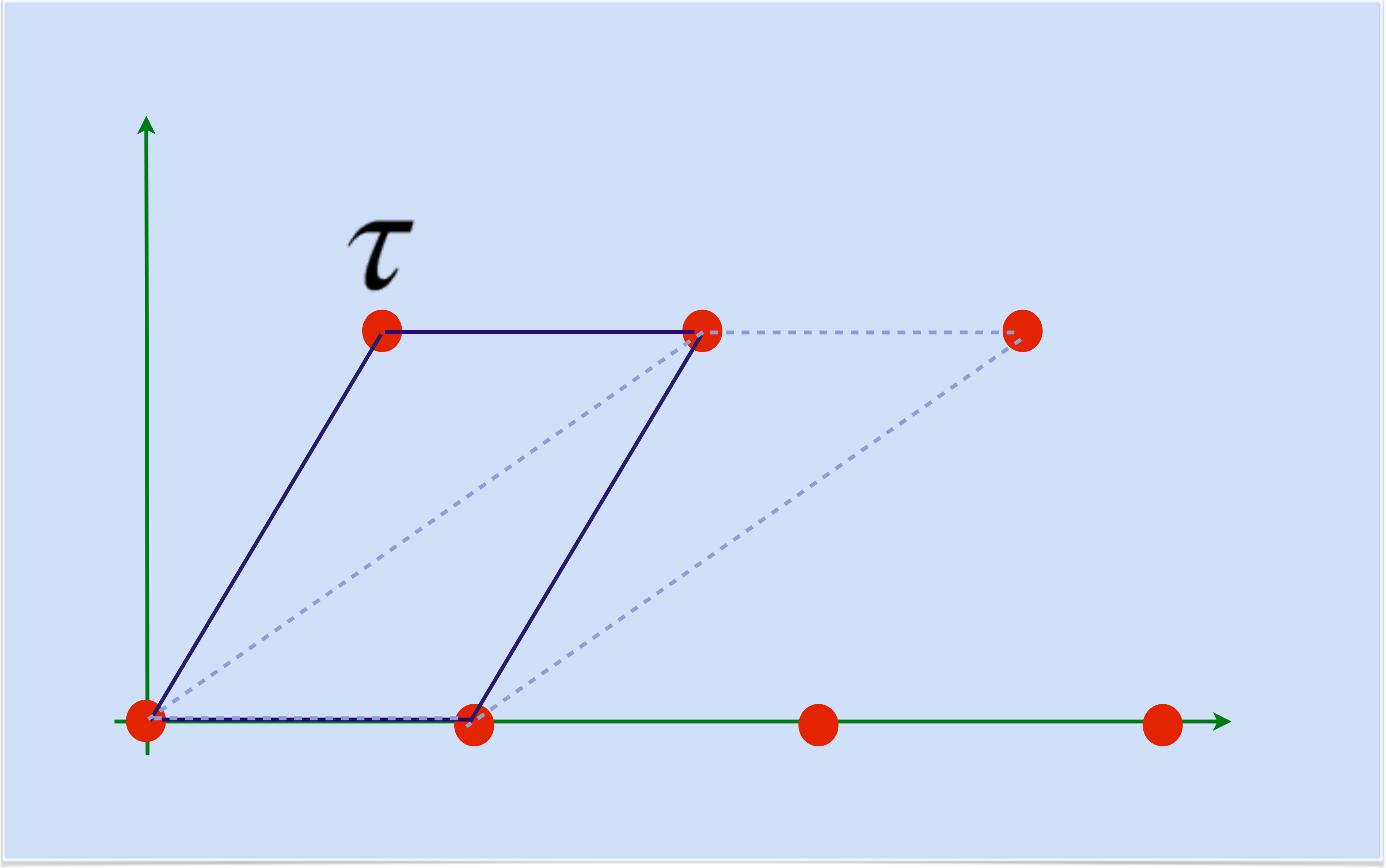


These simple requirements strongly limits our freedom in constructing consistent string vacua, and introduces a natural UV cut-off.

The corresponding diagram in string theory is fatter, and has the topology of a torus

equivalent tori are related by the transformations

$$\tau \to \frac{a\,\tau + b}{c\,\tau + d}$$

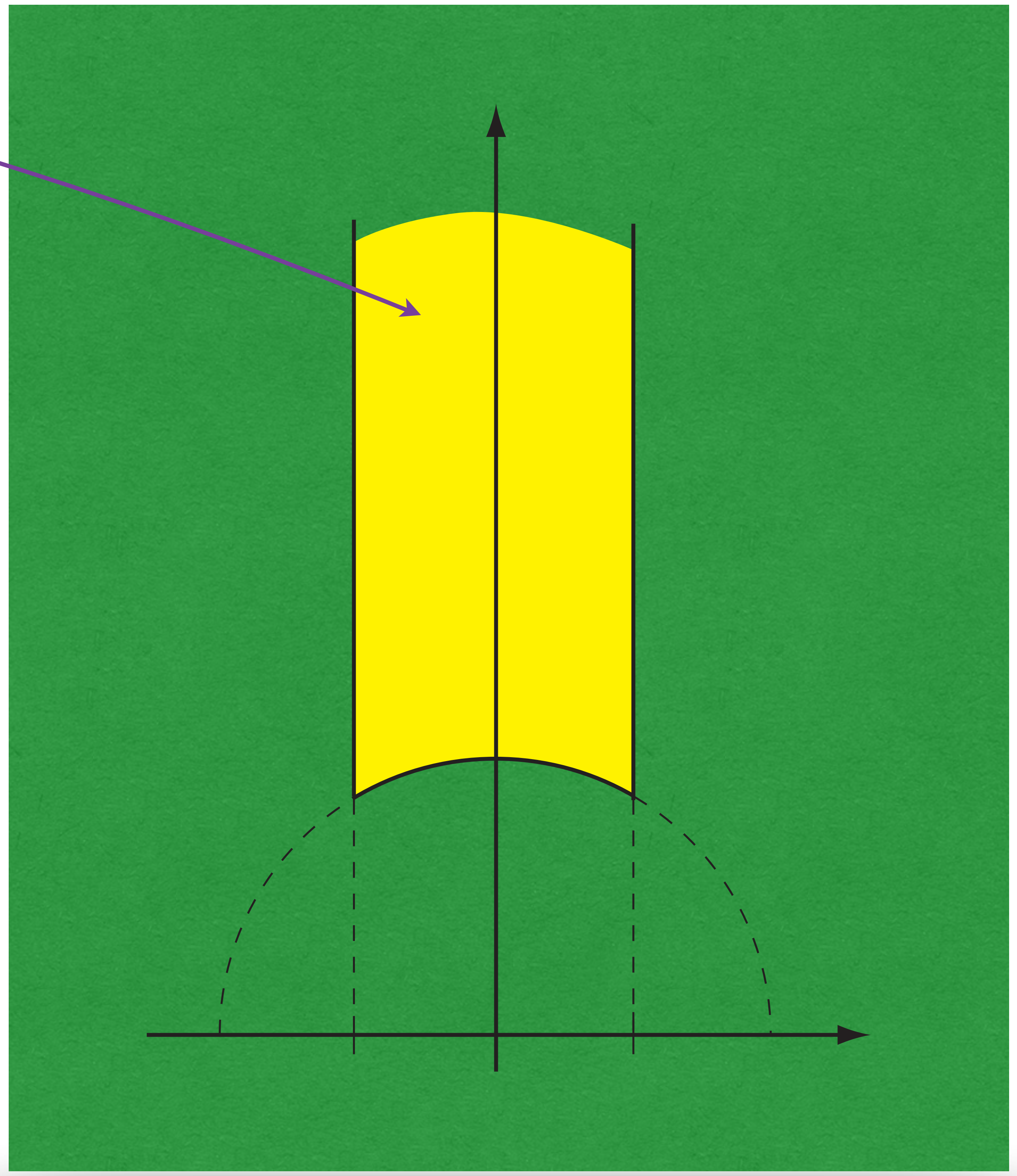


They form the modular group

$$SL(2;\mathbb{Z})$$

The one-loop partition function (free energy) thus reads

$$\Gamma = -\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, \mathcal{Z}(\tau\,,\,\bar{\tau})$$



invariant under the action of the modular group.

Very strong constraint!!!

Modular invariance of the one-loop partition function (free energy) implies that there are only two (plus two!) consistent superstring theories in 10 dimensions.

The type IIA superstring

$$b^i_{-1/2}|0\rangle \, \tilde{b}^j_{-1/2}|\tilde{0}\rangle \qquad b^i_{-1/2}|0\rangle \, |\tilde{\dot{a}}\rangle$$

$$|a\rangle \, |\tilde{\dot{a}}\rangle \qquad |a\rangle \, \tilde{b}^i_{-1/2}|\tilde{0}\rangle$$

Modular invariance of the one-loop partition function (free energy) implies that there are only two (plus two!) consistent superstring theories in 10 dimensions.

The type IIB superstring

$$b^i_{-1/2}|0\rangle\ \tilde{b}^j_{-1/2}|\tilde{0}\rangle \qquad\qquad b^i_{-1/2}|0\rangle\ |\tilde{a}\rangle$$

$$|a\rangle\ |\tilde{a}\rangle \qquad\qquad |a\rangle\ \tilde{b}^i_{-1/2}|\tilde{0}\rangle$$

At low energies these are described by

The type IIA supergravity

$$\left\{ g_{\mu\nu} \, , \, \phi \, , \, B_{\mu\nu} \, ; \, C_{\mu} \, , \, C_{\mu\nu\rho} \, | \, \psi^{a}_{\mu} \, , \, \psi^{\dot{a}}_{\mu} \, , \, \lambda^{a} \, , \, \lambda^{\dot{a}} \right\}$$

The type IIB supergravity

$$\left\{ g_{\mu\nu} \, , \, \phi \, , \, B_{\mu\nu} \, ; \, C \, , \, C_{\mu\nu} \, , \, C^{+}_{\mu\nu\rho\sigma} \, | \, 2 \, \psi^{a}_{\mu} \, , \, 2 \, \lambda^{\dot{a}} \right\}$$

The presence of antisymmetric tensor fields with an increasing number of indexes is typical of higher dimensional (supergravity) theories and indicates the presence of a rich spectrum of **non-perturbative states**!

A *p*-form field naturally couples to electric (*p*-1) branes

In *D* dimensions its magnetic dual is a (*D*-*p*-2)-form field that naturally couples to magnetic (*D*-*p*-3) branes

String theory is not only a theory of strings but includes non-perturbative extended objects

NS 5 brane

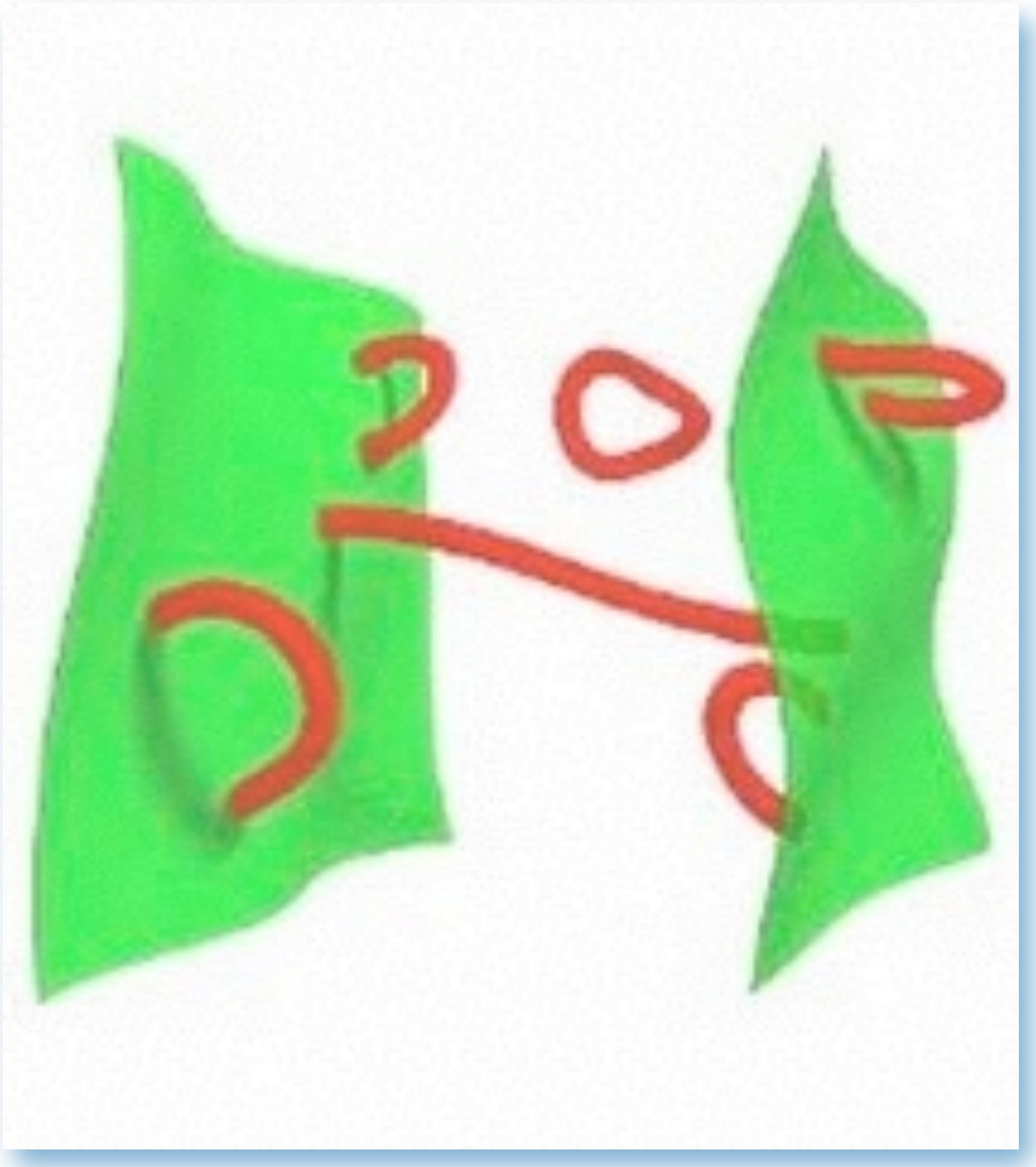
$$M^2 \sim \frac{1}{g_s^2}$$

Dp branes

$$M^2 \sim \frac{1}{g_s}$$

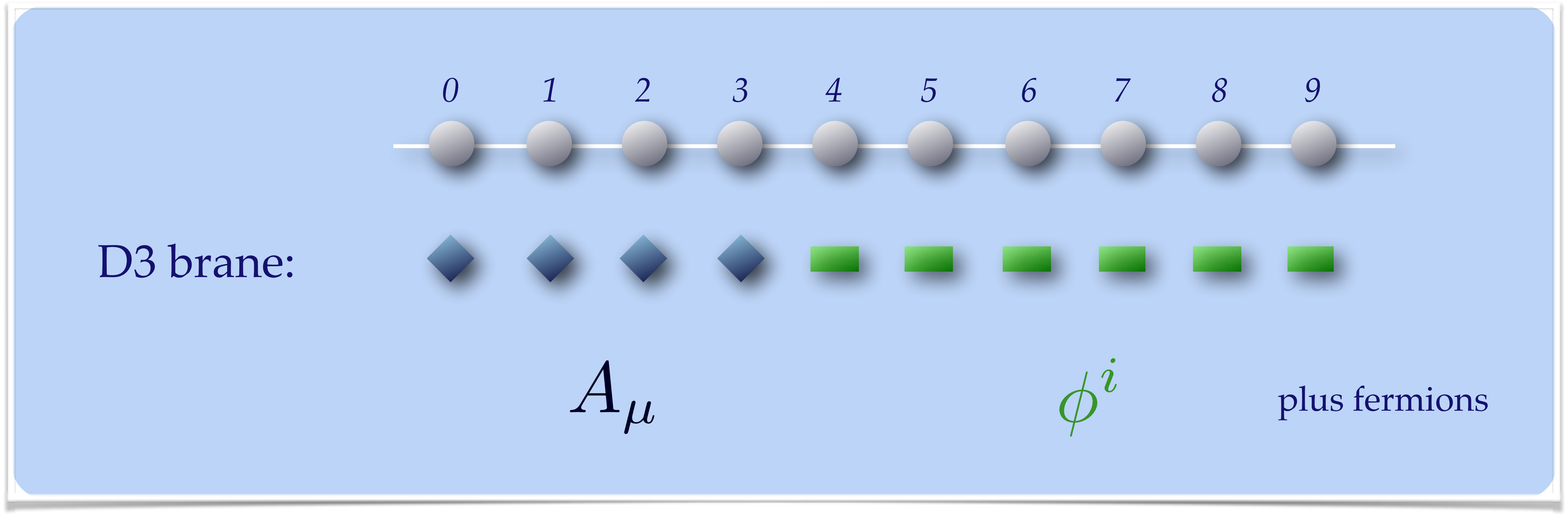
***D-branes** are special since, though non-perturbative in nature their elementary degrees of freedom are **open strings** that allow for a perturbative description of their main properties!*

D-branes identify the hyper-surface where the open string end-points are free to move



***D-branes** are special since, though non-perturbative in nature their elementary degrees of freedom are **open strings** that allow for a perturbative description of their main properties!*

Their excitations are given by the light open-string modes



Piling up *N D*-branes the interactions become non-abelian

$$U(N)\,, \qquad SO(2N)\,, \qquad Sp(2N)$$

D-branes are special since, though non-perturbative in nature their elementary degrees of freedom are open strings that allow for a perturbative description of their main properties!

D-branes are heavy and charged with respect to RR fields.

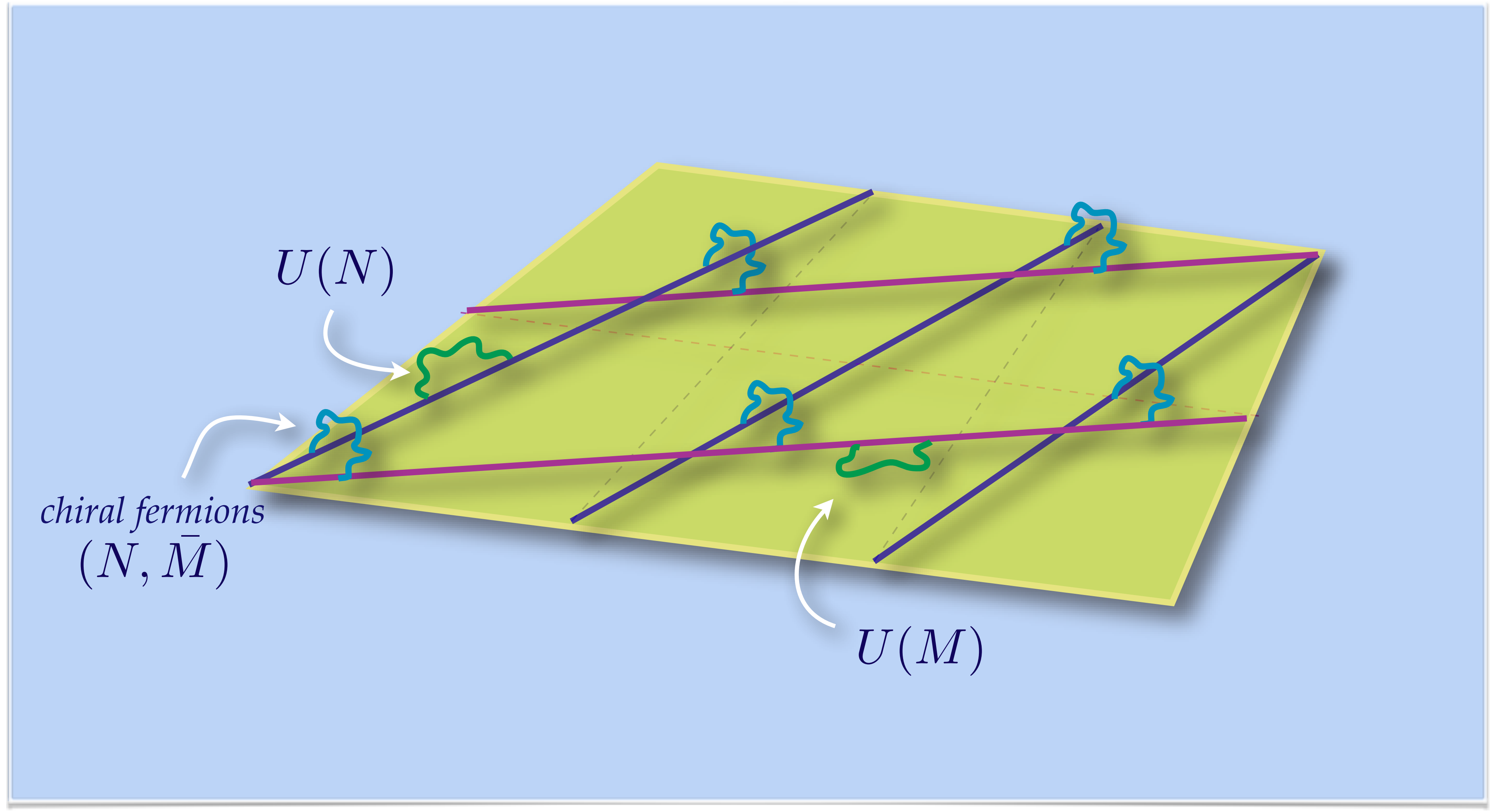
They deform the geometry of space time.

Zooming close to the D-branes it is of *AdSxS* type.

The idea of the AdS/CFT correspondence originates precisely from this dual nature of D-branes

See Papadodimas' lectures

D-branes play a major role also in String Phenomenology



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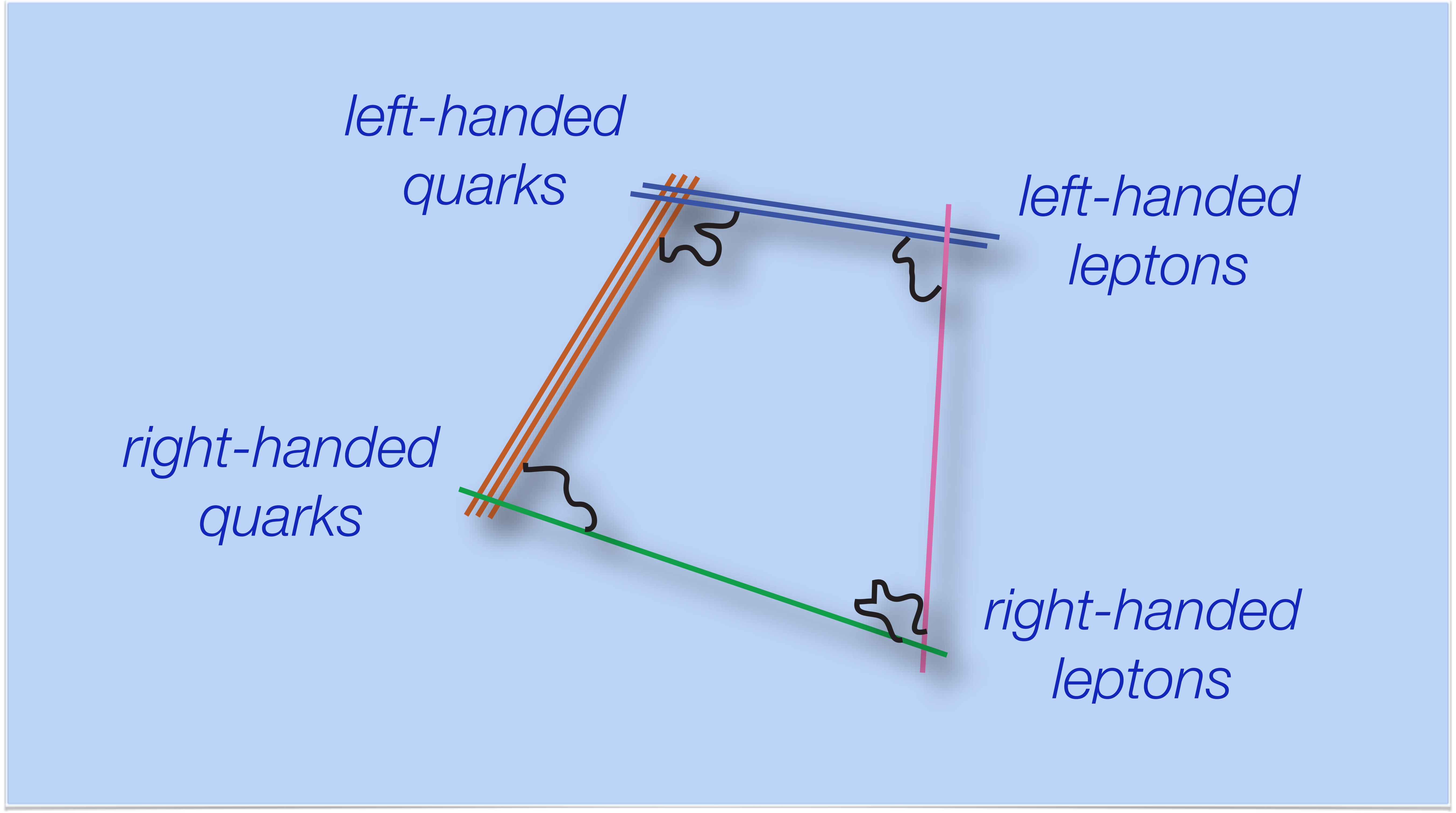
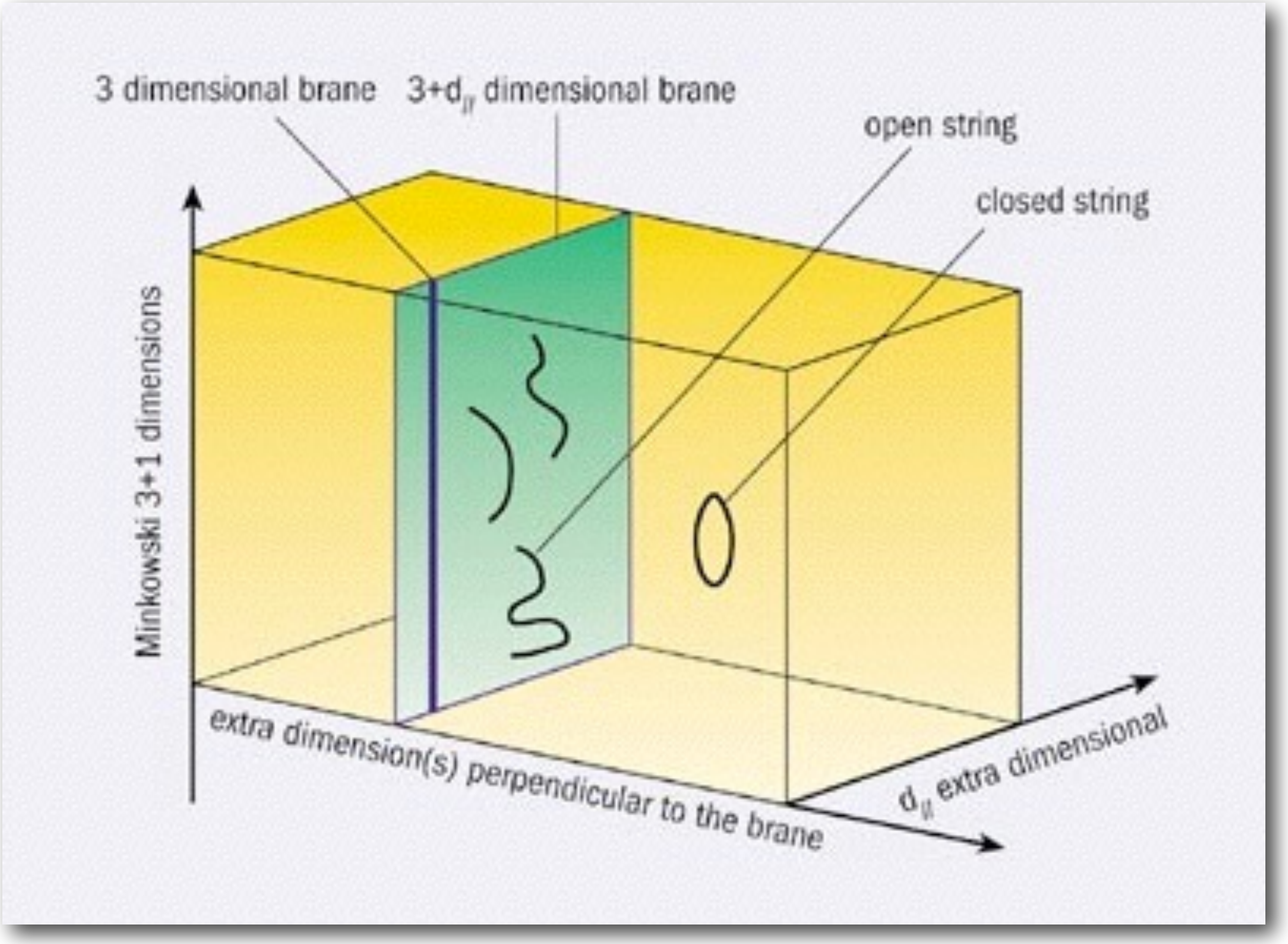


See Anastasopoulos' talk

D-branes play a major role also in String Phenomenology



$$\mathcal{S} = \int d^{10}x \, \frac{\sqrt{G}}{g_s^2 \ell_s^8} R + \int d^{p+1}x \, \frac{\sqrt{g}}{g_s \ell_s^{p-3}} F^2$$

$$\mathcal{S} = \int d^4x \sqrt{g} \left[\frac{V_\parallel V_\perp}{g_s^2 \ell_s^8} R + \frac{V_\parallel}{g_s \ell_s^{p-3}} F^2 \right]$$

D-branes play a major role also in String Phenomenology

$$\ell_{\mathrm{Pl}}^2 \sim \frac{g_{\mathrm{s}}^2\, \ell_{\mathrm{s}}^8}{V_{\|}\, V_{\perp}}\,, \qquad g_{\mathrm{YM}}^2 \sim \frac{g_{\mathrm{s}}\, \ell_{\mathrm{s}}^{p-3}}{V_{\|}}$$

$$\ell_{\mathrm{Pl}}^2 \sim \frac{\ell_{\mathrm{s}}^{11-p}}{V_{\perp}}$$

$$R_{\perp} \sim 1\,\mathrm{mm}$$

$$\ell_{\mathrm{s}}^{-1} \sim 1\,\mathrm{TeV}$$

$$\mathcal{S} = \int d^{10}x\, \frac{\sqrt{G}}{g_{\mathrm{s}}^2 \ell_{\mathrm{s}}^8}\, R + \int d^{p+1}x\, \frac{\sqrt{g}}{g_{\mathrm{s}} \ell_{\mathrm{s}}^{p-3}}\, F^2$$

$$\mathcal{S} = \int d^4x \sqrt{g}\left[\frac{V_{\|} V_{\perp}}{g_{\mathrm{s}}^2 \ell_{\mathrm{s}}^8}\, R + \frac{V_{\|}}{g_{\mathrm{s}} \ell_{\mathrm{s}}^{p-3}}\, F^2\right]$$

Large extra dimensions and low string scale

See Antoniadis' lectures

Despite the lack of a general principle, String Theory has produced a number of astonishing results in Particle Physics and Quantum Gravity

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