Hints of BSM physics in the SM effective potential

Zygmunt Lalak

ITP Warsaw

Corfu Summer Institute 2014

with M. Lewicki and P. Olszewski

arXiv:1402.3826, JHEP

Outline:

- SM effective potential
- Tunneling and Lifetime
- SM phase diagram
- Gauge invariance of tunneling rate
- BSM physics via higher-order operators
- Magnitude of new physics scale
- Derivative interactions
- Summary

SM Effective potential

Standard Model Effective potential

$$V_{SM}(\mu) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum_i \frac{n_i}{64\pi^2} M_i^4 \left[\ln\left(\frac{M_i^2}{\mu^2}\right) - C_i\right]$$

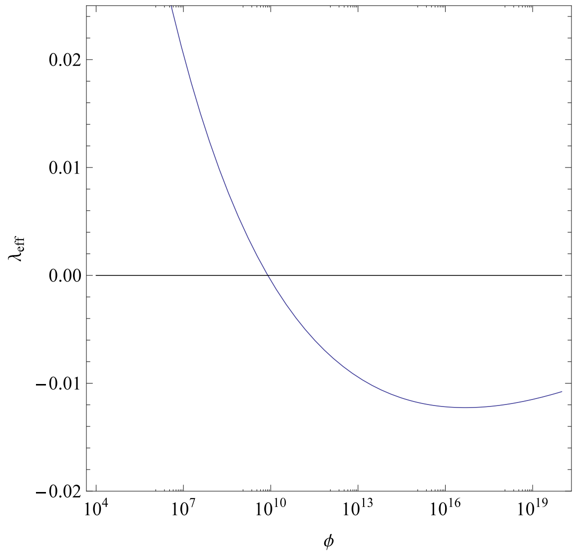
For large field values $m^2 << \phi^2$ and $\mu = \phi$ the potential is very well approximated by

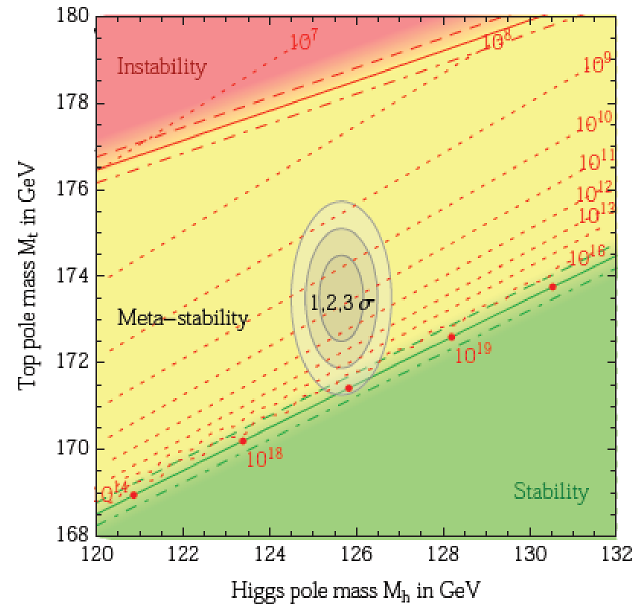
$$V_{SM}(\phi) \approx \phi^4 \left\{ \frac{\lambda}{4} + \frac{1}{64\pi^2} \left[6\left(\frac{g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_2^2}{4}\right) - \frac{5}{6}\right) + 3\left(\frac{g_1^2+g_2^2}{4}\right)^2 \left(\ln\left(\frac{g_1^2+g_2^2}{4}\right) - \frac{5}{6}\right) \right.\right.$$
$$\left.\left. -12\left(\frac{y_t^2}{2}\right)^2 \left(\ln\left(\frac{y_t^2}{2}\right) - \frac{3}{2}\right) + \left(\frac{3\lambda}{2}\right)^2 \left(\ln\left(\frac{3\lambda}{2}\right) - \frac{3}{2}\right) + 3\left(\frac{\lambda}{2}\right)^2 \left(\ln\left(\frac{\lambda}{2}\right) - \frac{3}{2}\right) \right]\right\}$$

$$V_{SM}(\phi) \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4$$

SM Metastability

$$\lambda_{eff} < 0 \Longrightarrow \text{Metastability}$$





D. Buttazzo, et al. [arXiv:1307.3536].
G. Degrassi, et al. [arXiv:1205.6497].

See lectures by G. Degrassi Corfu 2014

Tunneling

Standard semiclassical formalism

S. R. Coleman, Phys. Rev. D **15** (1977) 2929.
C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D **16** (1977) 1762.

$O(4)$ symmetric solution to euclidean equation of motion

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$
$$s = \sqrt{\vec{x}^2 + x_4^2}.$$

with

- $\dot{\phi}(s = 0) = 0$ at the true vacuum
- $\phi(s = \infty) = \phi_{min}$ at the false vacuum

Action of the bounce solution

$$\begin{aligned} S_E &= \int d^4x \left\{ \frac{1}{2} \sum_{\alpha=1}^{4} \left(\frac{\partial \phi(\mathbf{x})}{\partial x^\alpha} \right)^2 + V(\phi(\mathbf{x})) \right\} \\ &= 2\pi^2 \int ds s^3 \left(\frac{1}{2} \dot{\phi}^2(s) + V(\phi(s)) \right), \end{aligned}$$

allows us to calculate decay probability dp of a volume d^3x

$$dp = dt d^3x \frac{S_E^2}{4\pi^2} \left| \frac{det'[-\partial^2 + V''(\phi)]}{det[-\partial^2 + V''(\phi_0)]} \right|^{-1/2} e^{-S_E}.$$

Simplifying

- normalisation factor replaced with width of the barrier $\propto \phi_0$
- size of the universe is $T_U = 10^{10}$yr

we can calculate the lifetime of the false vacuum $(p(\tau) = 1)$

$$\frac{\tau}{T_U} = \frac{1}{\phi_0^4 T_U^4} e^{S_E}.$$

Analytical solution

Analytical solutions for simple potentials

K. M. Lee and E. J. Weinberg, Nucl. Phys. B **267** (1986) 181.

Quartic potential:

$$V(\phi) = \frac{\lambda}{4}\phi^4 \qquad \Longrightarrow \qquad S_E = \frac{8\pi^2}{3|\lambda|}$$

for $\lambda < 0$.

Quartic and linear potential :

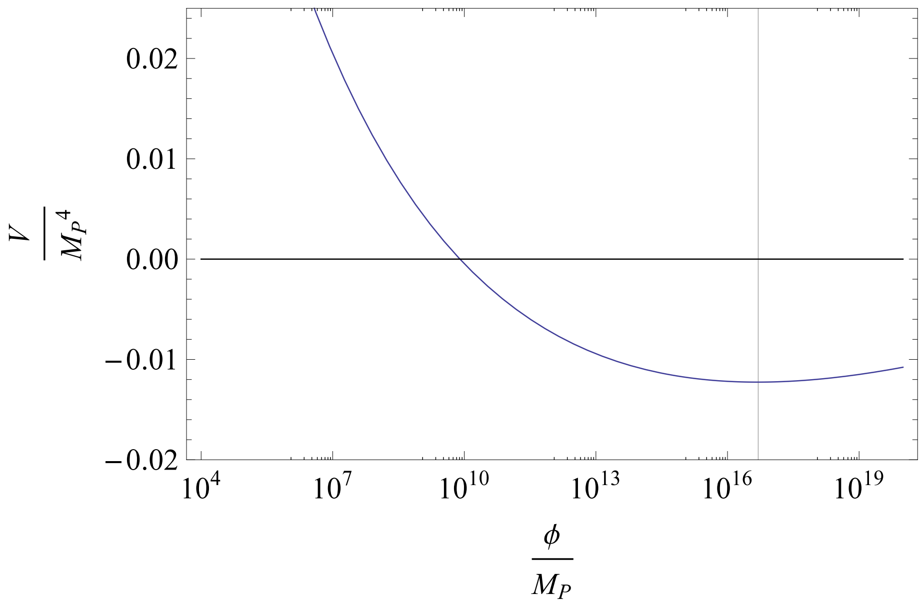
$$V_\eta(\phi) = \begin{cases} \frac{\lambda}{4}\phi^4\,, & \phi \leqslant \eta \\ \frac{\lambda}{4}\eta^4 - K\,(\phi - \eta)\,, & \phi\ > \eta \end{cases}, \qquad \Longrightarrow \qquad \begin{array}{c} S_E = \frac{8\pi^2}{3|\lambda|}(1 - (\gamma + 1)^4) \\ \gamma = \frac{|\lambda|\eta^3}{K} \end{array}$$

for $\lambda < 0$ and $-1 < \gamma < 0$

Approximating by a quartic potential:

$$\frac{\tau}{T_U} = \frac{1}{\phi^4(\lambda_{min})T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}} \approx 10^{540}.$$

lifetime is minimal for ϕ that minimizes $\lambda_{eff}(\phi)$.

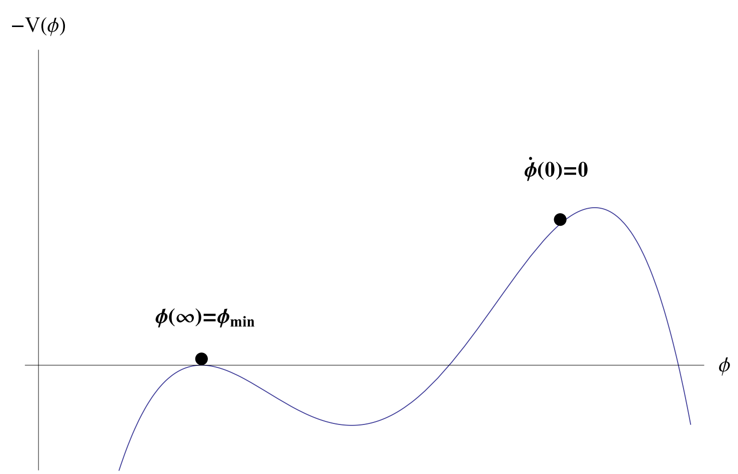


Numerical calculations

Equation we need to solve

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi},$$

is an equation of motion of a particle in potential $-V(\phi)$ with a "time" dependent friction $\frac{3}{s}\dot{\phi}$.



We will use a simple Overshot Undershot algorithm

Gauge dependence of the tunneling rate

It is well known that the effective potential, and in general the effective action, are gauge-dependent objects

However, the statement about the spontaneous breaking of gauge symmetry is gauge invariant (N. K. Nielsen 1975)

The gauge invariant "observables" are the values of the effective potential at the extrema, and the tunneling rate between different minima

When one computes the SM effective potential in a straightforward manner (say naively), nothing looks gauge independent - neither the value of the effective potential at the extrema (see L. Di Luzio and L. Mihaila 2014) nor the tunneling rate (ML,PO,ZL)

This is due to the fact that the new extrema are created radiatively and already one loop effective potential, even in the RGE improved version, contains gauge-dependent terms

$$V^{1,\xi} = -\frac{1}{256\pi^2}\lambda h^4 \left[\xi_B g_1^2 \left(\log \frac{\lambda h^4(\xi_B g_1^2+\xi_W g_2^2)}{4\mu^4} - 3\right) + \xi_W g_2^2 \left(\log \frac{\lambda^3 h^{12}\xi_W^2 g_2^4(\xi_B^2 g_1^2+\xi_W g_2^2)}{64\mu^{12}} - 9\right)\right]$$

As pointed out by A. Andreassen, W. Frost and M. Schwartz 2014, who followed E. Weinberg and D. Metaxas 1996 and S. Coleman and E. Weinberg 1973, the key to save in the calculations the gauge independence of the potantial at the extrema is to realize, that to create extrema radiatively, loop corrections have to cancel between themselves or the tree-level contributions

In CW model $\qquad \lambda \sim \frac{\hbar e^4}{16\pi^2}$

In the SM the equivalent condition is

$$\lambda = \frac{\hbar}{256\pi^2}\left[g_1^4 + 2g_1^2g_2^2 + 3g_2^4 - 48h_t^4 - 3(g_1^2+g_2^2)^2\log\frac{g_1^2+g_2^2}{4} - 6g_2^4\log\frac{g_2^2}{4} + 48y_t^4\log\frac{y_t^2}{2}\right]$$

which holds at the extrema $h = \mu$

Hence λ is formally of the order $\hbar$ and gives higher than one-loop order contribution

It has been shown that that taking this relation into account in counting radiative contributions in the SM makes the value of the potential at the extrema gauge independent at LO and NLO (in powers of $\hbar$)

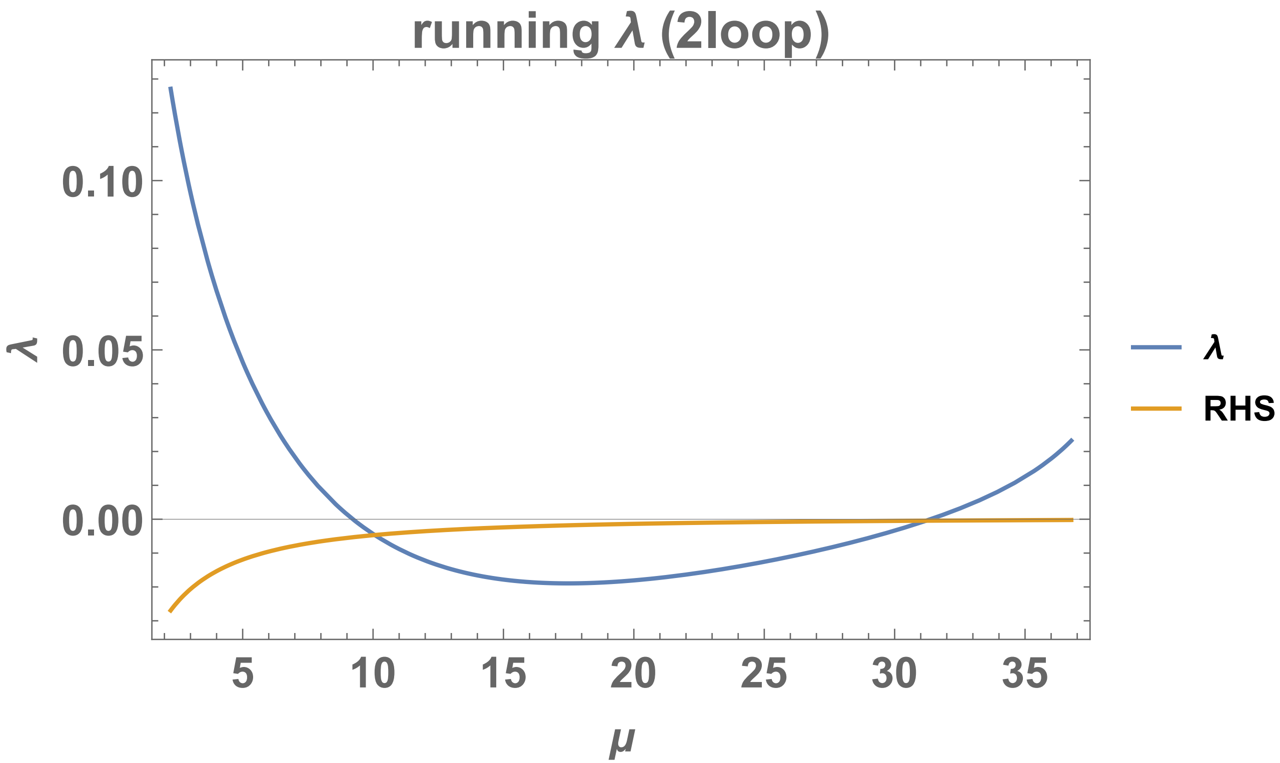
In general, the tunneling rate has the form

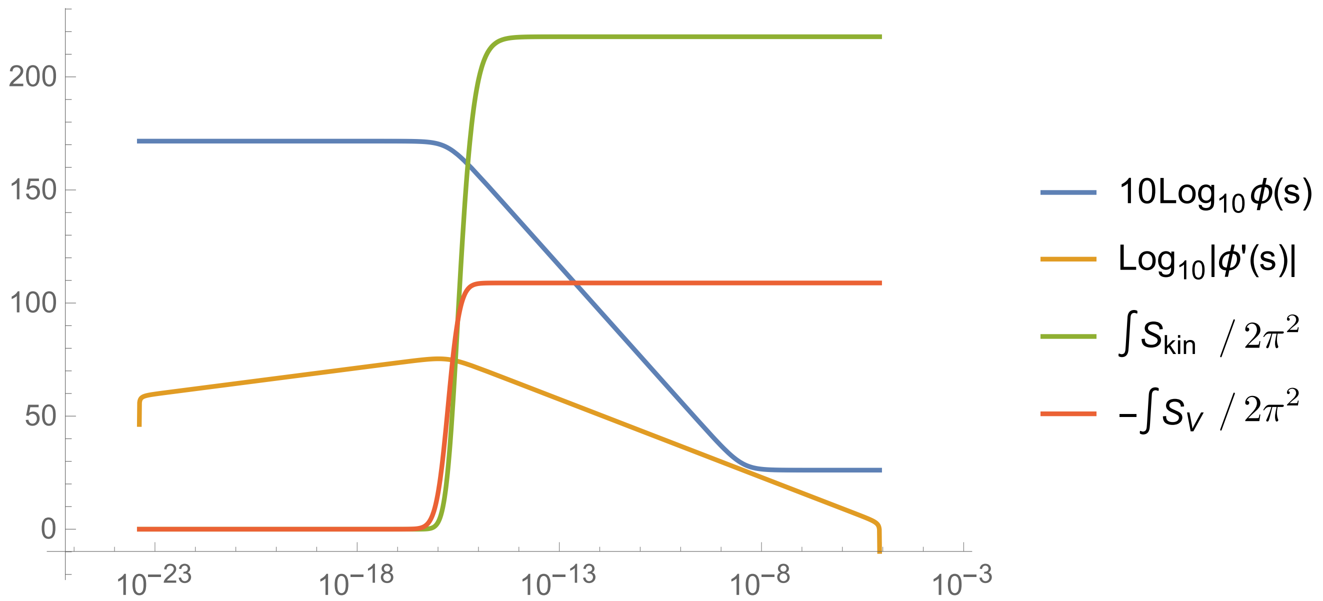
$$\Gamma = Ae^{-B}$$

Weinberg and Mataxas managed to show that if the reordering of the radiative corrections used above holds everywhere, not only near the minima, then indeed the exponent B shall be gauge independent at the NLO.

SM is more complicated. The assumption that the required relation between couplings holds at each scale between the EW minimum and the radiative minimum is not so well justified

$$\lambda = \frac{\hbar}{256\pi^2}\left[g_1^4 + 2g_1^2 g_2^2 + 3g_2^4 - 48h_t^4 - 3(g_1^2+g_2^2)^2 \log\frac{g_1^2+g_2^2}{4} - 6g_2^4 \log\frac{g_2^2}{4} + 48y_t^4 \log\frac{y_t^2}{2}\right]$$





200
150
100
50
0
10^{-23}
10^{-18}
10^{-13}
10^{-8}
10^{-3}
$10\mathrm{Log}_{10}\phi(s)$
$\mathrm{Log}_{10}|\phi'(s)|$
$\int S_{kin} / 2\pi^2$
$-\int S_V / 2\pi^2$

The leading gauge dependence comes from the gauge-dependent anomalous rescaling of the field

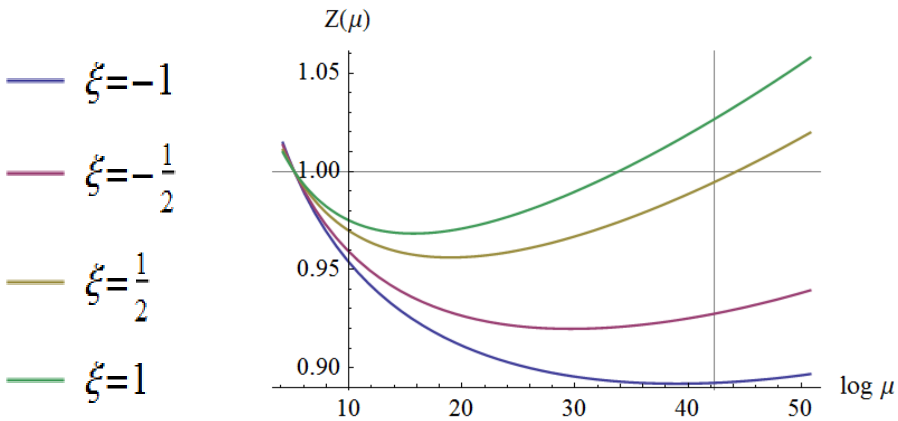
$$\mathcal{L}_{gauge\ fixing} = -\frac{1}{2\xi_W}(\partial^\mu W_\mu^{a\,2}) - \frac{1}{2\xi_B}(\partial^\mu B_\mu{}^2)$$

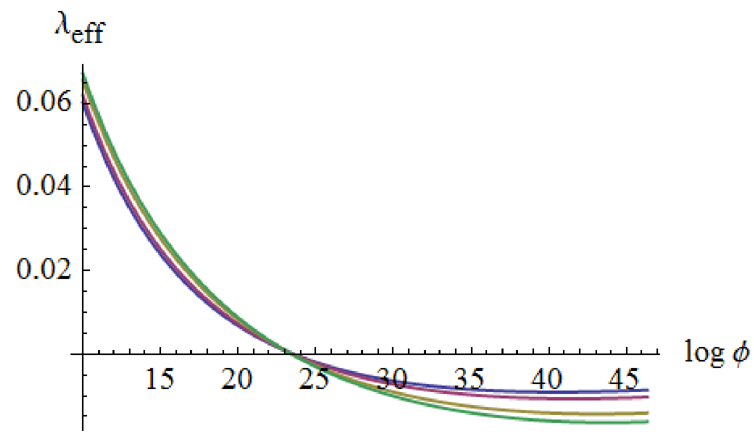
Contributes to:

- 1-loop potential
- γ function of the scalar field

- More important.
- One needs to remember that kinetic contribution to the action is muliplied by Z.

L. Di Luzio, L. Mihaila 1404.7450



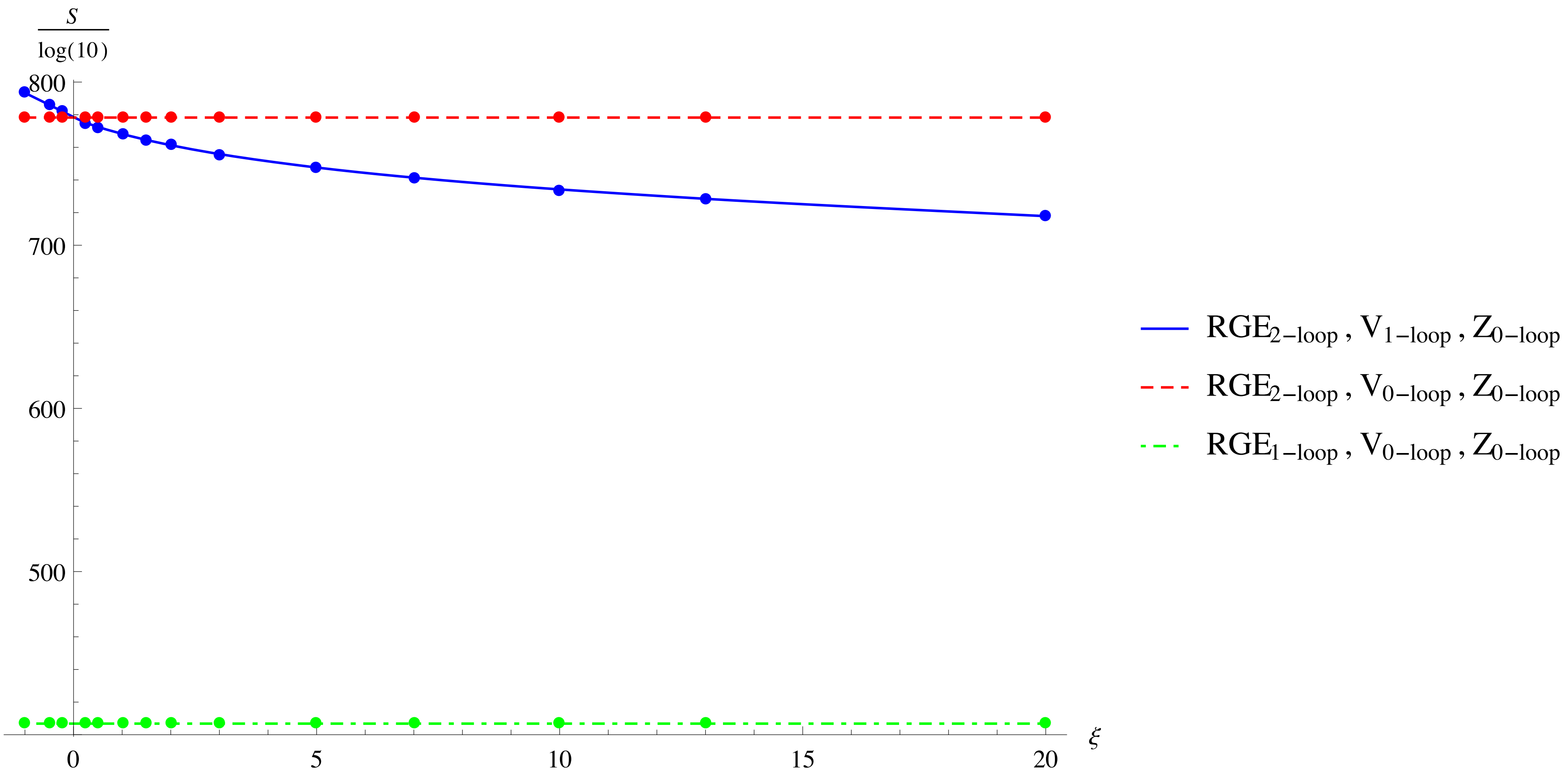


$$\xi = \xi_W(m_t) = \xi_B(m_t)$$

Observation, which allows one to ease the problem, is that once one includes in the euclidean action which is used to compute the bounce the renormalization factor in the 2-derivative term, and treats it consistently as a field dependent quantity, then one can go over to the new field variable $h \to \sqrt{Z(h)}h$ in terms of which the whole action becomes gauge independent at the modified leading order (that is assuming $\lambda \sim \hbar$), and only mildly gauge dependent in the more standard expression, through small logarithmic terms.

Beyond the leading order one needs yet to find proper, possibly non-local, expansion of the action to demonstrate the cancellation of gauge-dependent contributions.

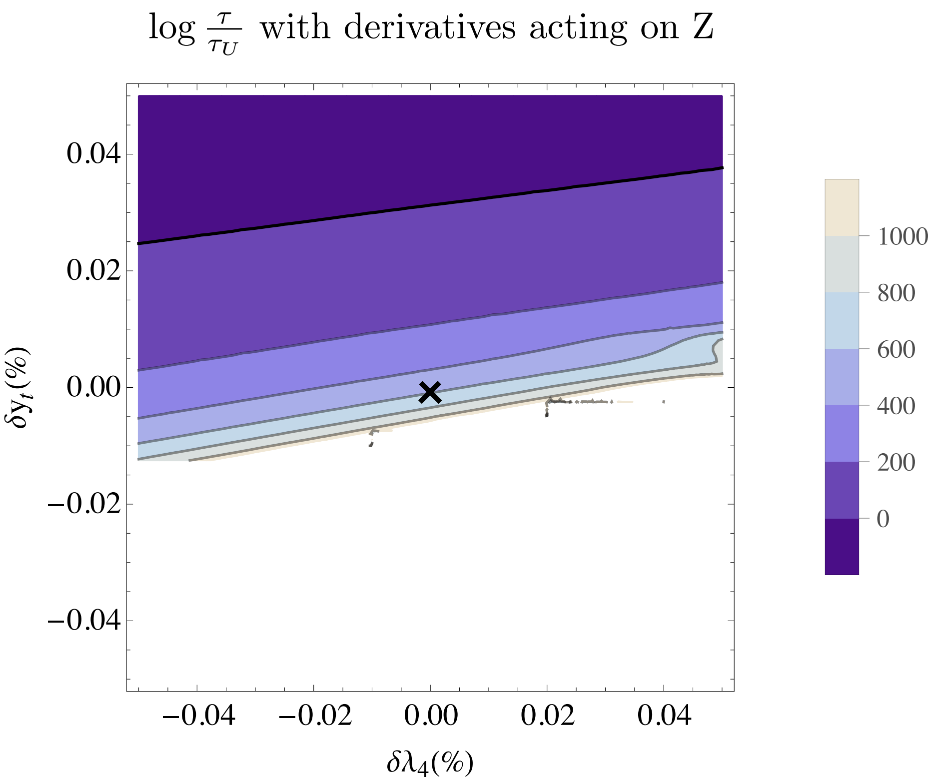
The LO procedure leading to gauge independent estimate the tunneling rate can easily be extended to the analysis of the role of the effective nonrenormalisable operators, and the results shown correspond to such a case.

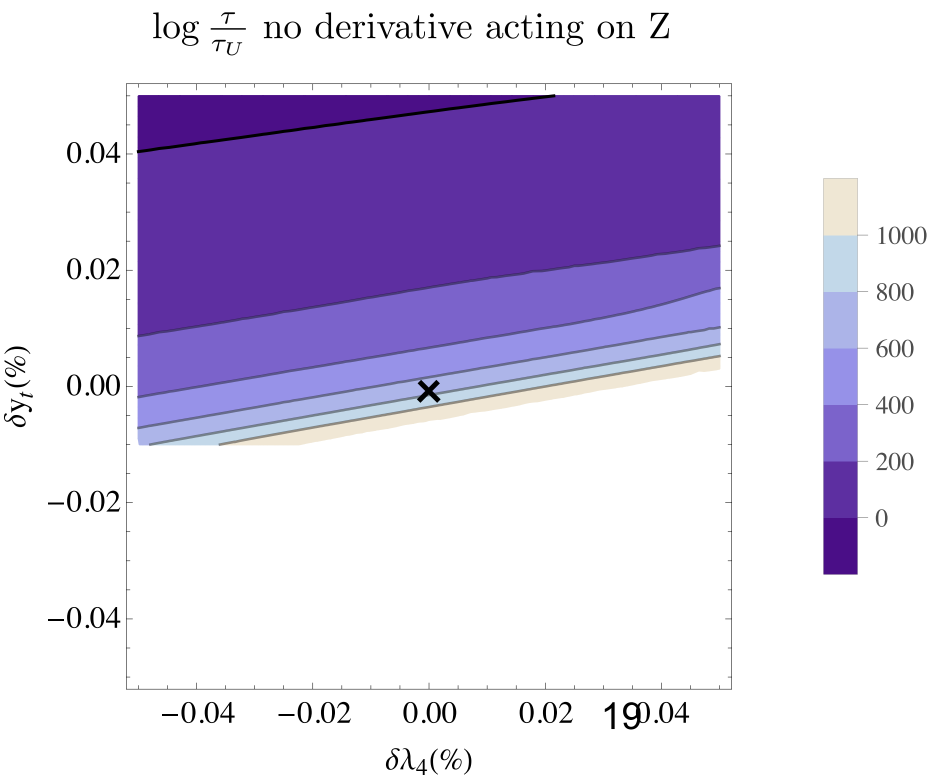


$\frac{S}{\log(10)}$
800
700
600
500
0
5
10
15
20
ξ
$\mathrm{RGE}_{2-\mathrm{loop}}$, $V_{1-\mathrm{loop}}$, $Z_{0-\mathrm{loop}}$
$\mathrm{RGE}_{2-\mathrm{loop}}$, $V_{0-\mathrm{loop}}$, $Z_{0-\mathrm{loop}}$
$\mathrm{RGE}_{1-\mathrm{loop}}$, $V_{0-\mathrm{loop}}$, $Z_{0-\mathrm{loop}}$

Table 1: Lifetime of the EW vacuum for different methods of finding the bounce solution

Potential type	Bounce action S_B	lifetime τ
Tree level, RGE improved	$S_B = 2149$	$\tau = 3,78 * 10^{699}$
$\hbar$-ordered LO	$S_B = 2154$	$\tau = 1.097 * 10^{699}$
RGE improved 1-loop, $Z \neq 1$	$S_B = 1879$	$\tau = 1.09 * 10^{582}$
RGE improved 1-loop, $Z = 1$	$S_B = 2231$	$\tau = 2.55 * 10^{732}$





RGE improved nearly scale invariant SM

Effect of field rescaling becomes real in nearly scale invariant version of the SM (with noninvariant dilaton kinetic term)

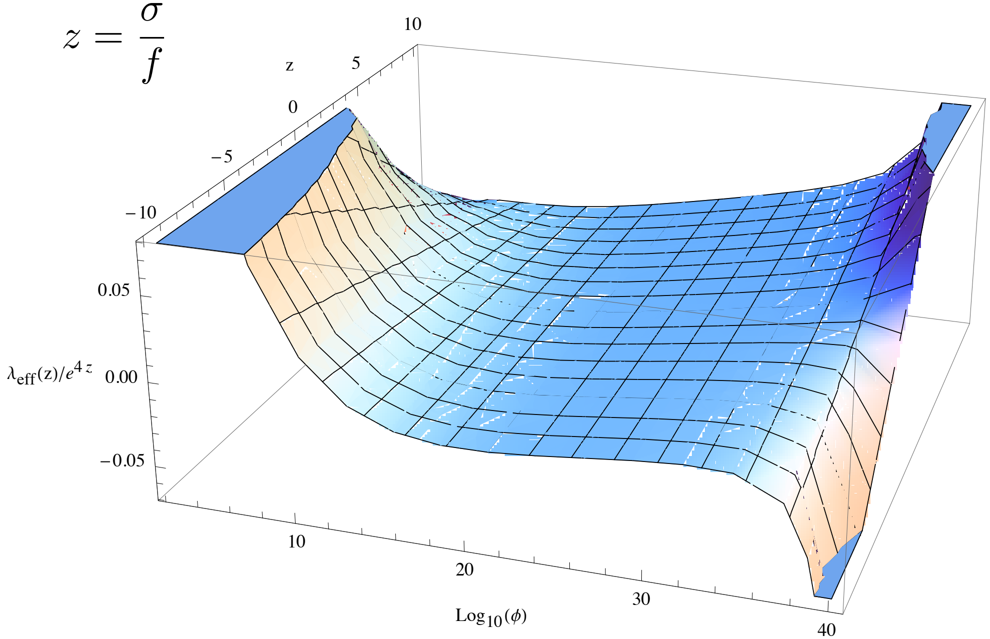
$$V(\Phi, \sigma) = e^{4\sigma/f} V_{SM}(\xi^2(\bar{\mu})\tilde{\Phi}^\dagger\tilde{\Phi};\, \mu(\bar{\mu}), \lambda(\bar{\mu});\, \mu)$$

where $\tilde{\Phi} = e^{-\sigma/f}\Phi$, $\bar{\mu} = e^{\sigma/f}\mu$ and $\xi(\mu) = e^{-\int^{\mu} \frac{d\mu'}{\mu'}\gamma(\mu')}$

The effect comes mostly from the field renormalisation factor $\xi e^{-z} = \xi e^{-\sigma/f}$ which becomes large for negative z

$$V(\Phi,\sigma) = e^{4\sigma/f} V_{SM}(\xi^2(\bar{\mu})\tilde{\Phi}^\dagger\tilde{\Phi};\ \mu(\bar{\mu}), \lambda(\bar{\mu});\ \mu)$$

$$z = \frac{\sigma}{f}$$



More negative values of lambda result in shorter lifetimes for negative z

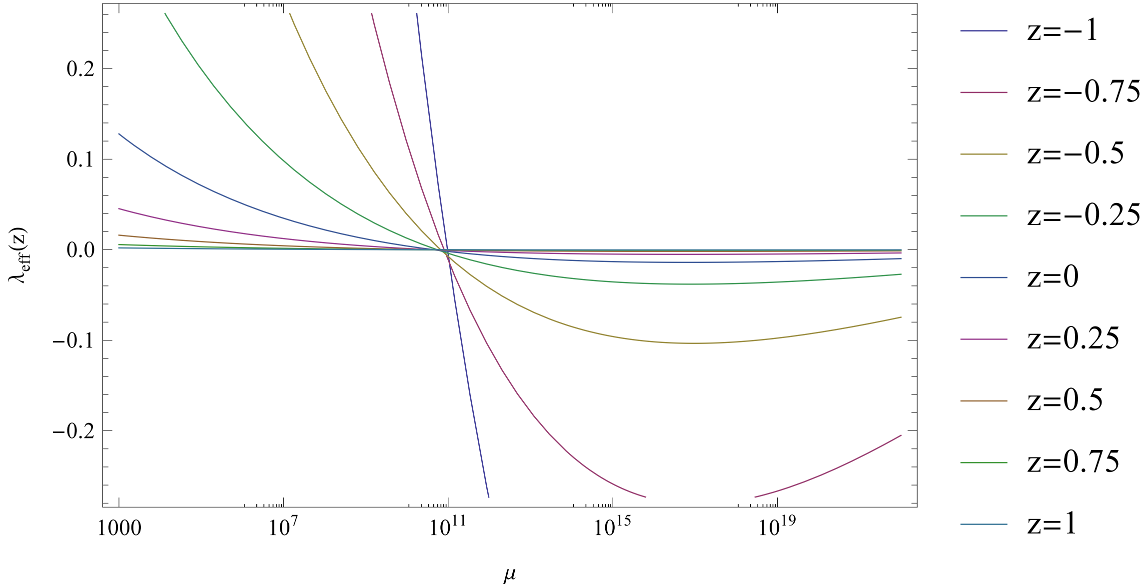


Figure 3: quartic coupling in the effective potential $\lambda_{eff}(z)$.

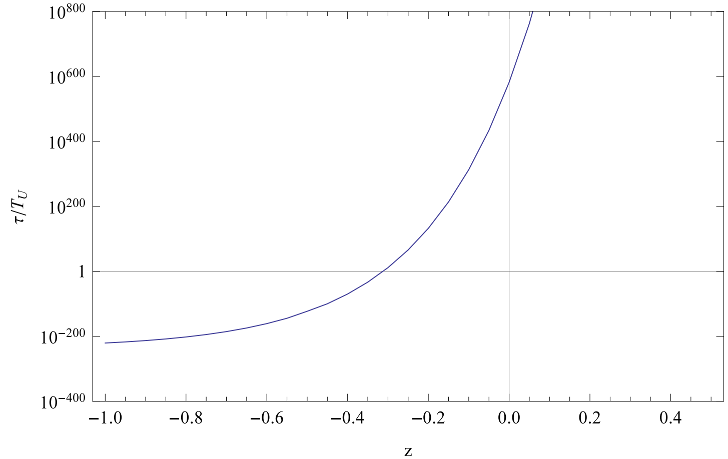


Figure 4: Lifetime of the electroweak vacuum as a function of z.

Effective potential with nonrenormalisable interactions

We add new nonrenormalisable couplings
(similar to V. Branchina and E. Messina, [arXiv:1307.5193].)

$$V \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4 + \frac{\lambda_6}{6!}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8!}\frac{\phi^8}{M_p^4}.$$

New Physics at Planck scale

That modify the potential around the Planck scale:

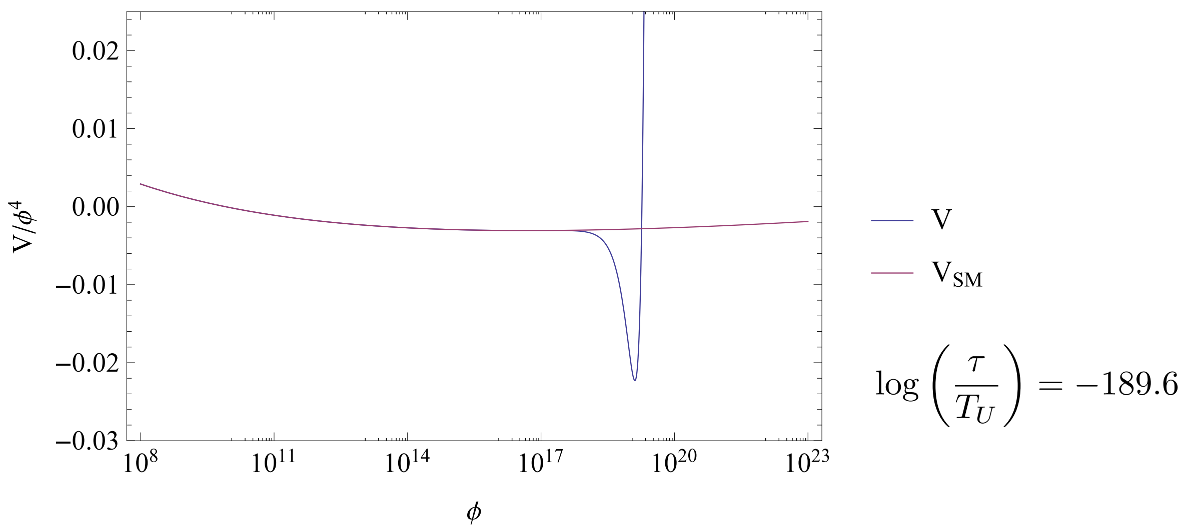


Figure: effective potential with $\lambda_6 = -1$ and $\lambda_8 = 1$.

Numerical vs Analytical

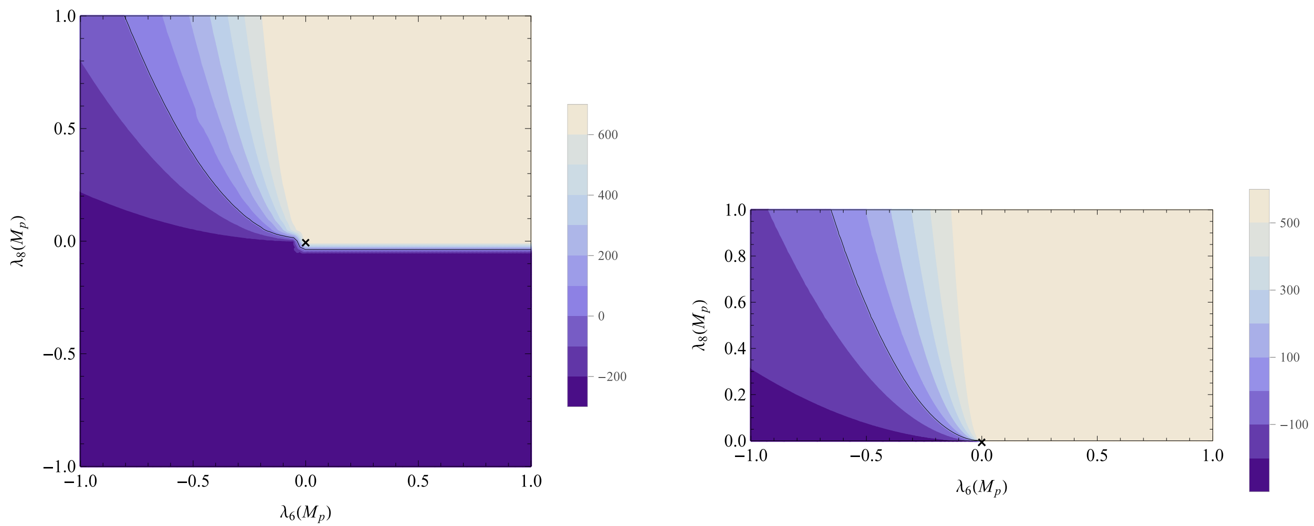


Figure: Decimal logatihm of lifetime of the universe in units of T_U as a function of the nonrenormalisable λ_6 and λ_8 couplings, calculated numerically (left panel) and analytically (right panel).

RG improvement

The correction to the running of the quatric Higgs coupling is of the form

$$\Delta\beta_\lambda = \frac{\lambda_6}{16\pi^2}\frac{m^2}{M_p^2}.$$

One-loop beta functions of new couplings take the form

$$16\pi^2\beta_{\lambda_6} = \lambda_8\frac{m^2}{M_p^2} + 15\lambda_6 6\lambda - 6\lambda_6\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2\right),$$

$$16\pi^2\beta_{\lambda_8} = 35\lambda_6^2 + 28\lambda_8 6\lambda - 8\lambda_8\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2\right).$$

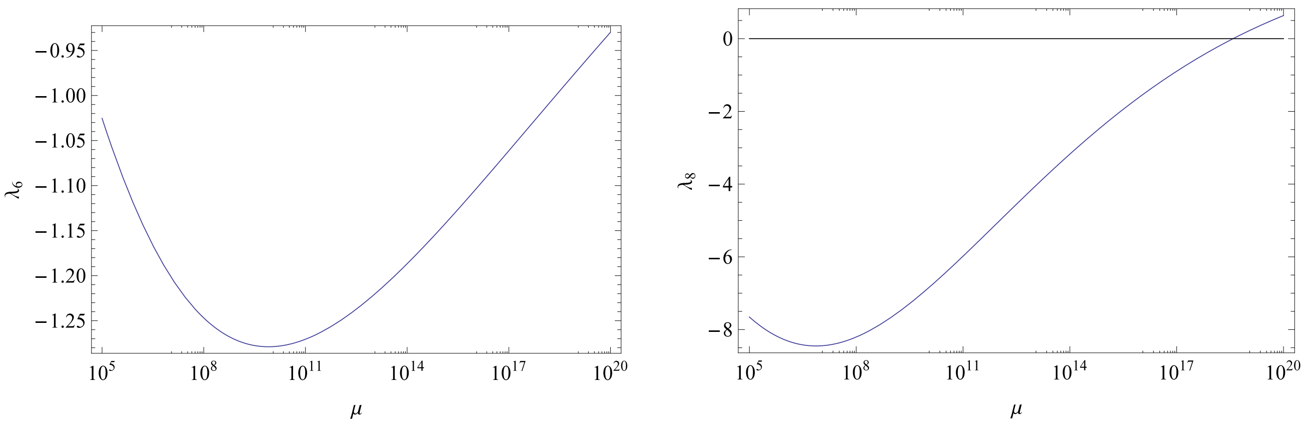


Figure: Example solution with $\lambda_6(M_p) = -1$ and $\lambda_8(M_p) = -0.1$.

Numerical vs Analytical again

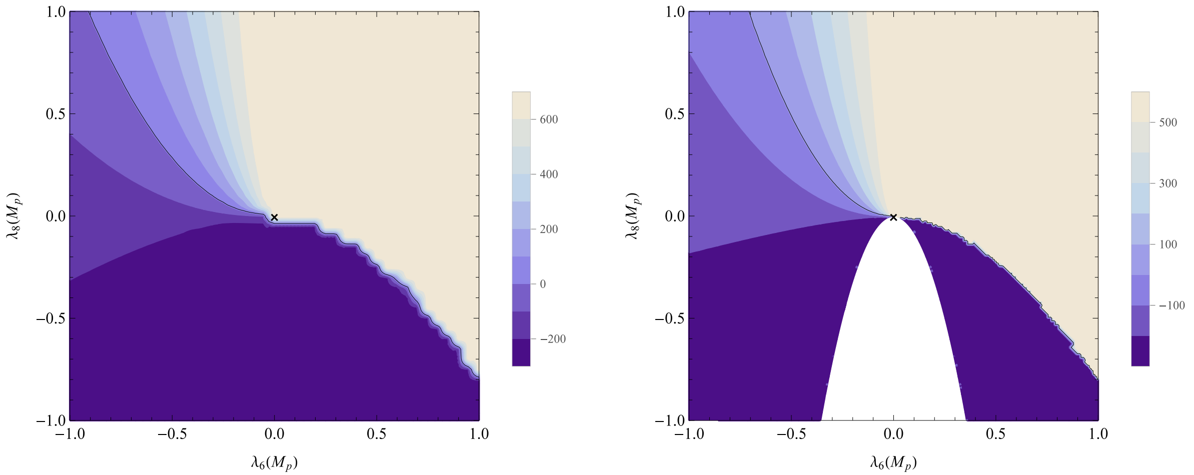


Figure: Decimal logatihm of lifetime of the universe in units of T_U as a function of the nonrenormalisable $\lambda_6(M_p)$ and $\lambda_8(M_p)$ couplings, calculated numerically (left panel) and analytically (right panel).

Comparison

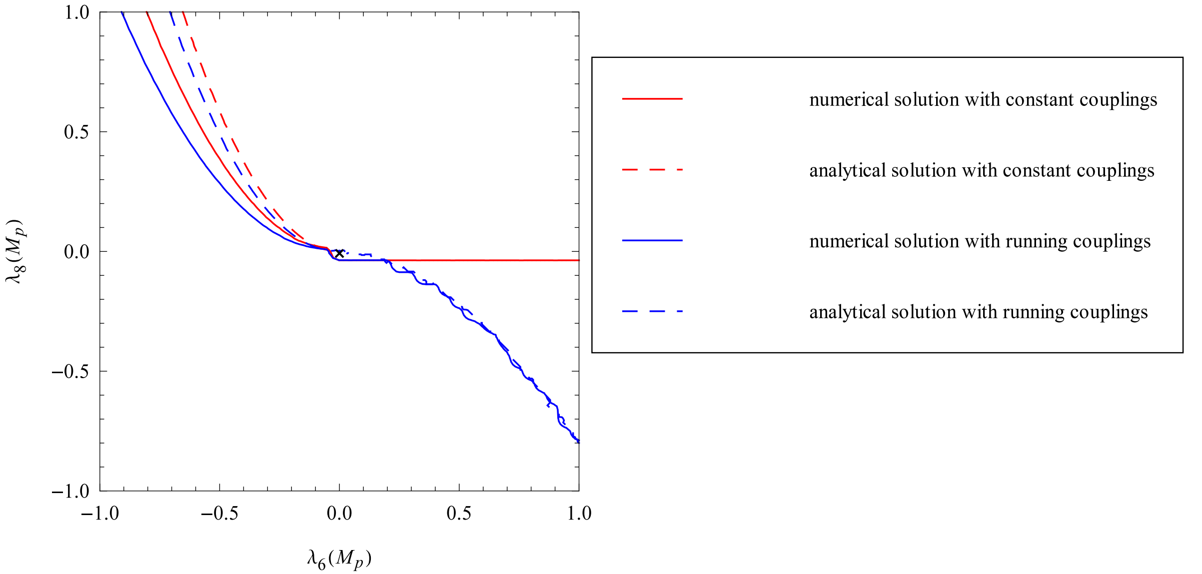
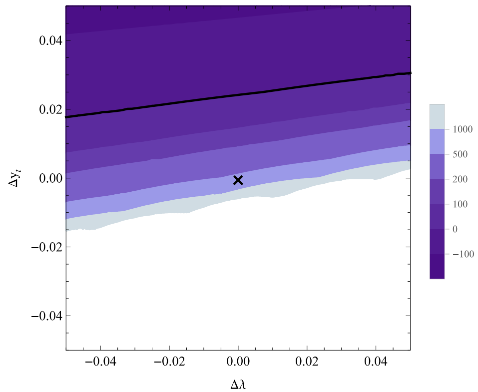


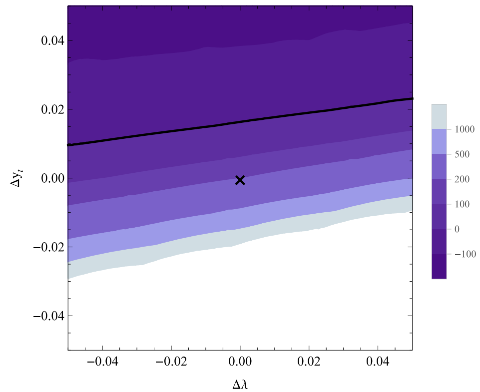
Figure: Contours corresponding to metastability boundary ($\tau = T_u$) obtained using four different methods.

SM phase diagram

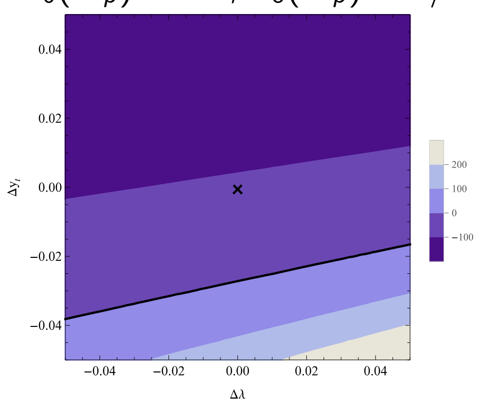
$\lambda_6(M_p) = 0,\ \lambda_8(M_p) = 0$



$\lambda_6(M_p) = -1/2,\ \lambda_8(M_p) = 1$



$\lambda_6(M_p) = -1,\ \lambda_8(M_p) = 1/2$



Magnitude of the suppression scale

Approximate lifetime:

$$\frac{\tau}{T_U} = \frac{1}{\mu^4(\lambda_{min})\, T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}} .$$

Positive λ_6 and λ_8 → stabilizing the potential

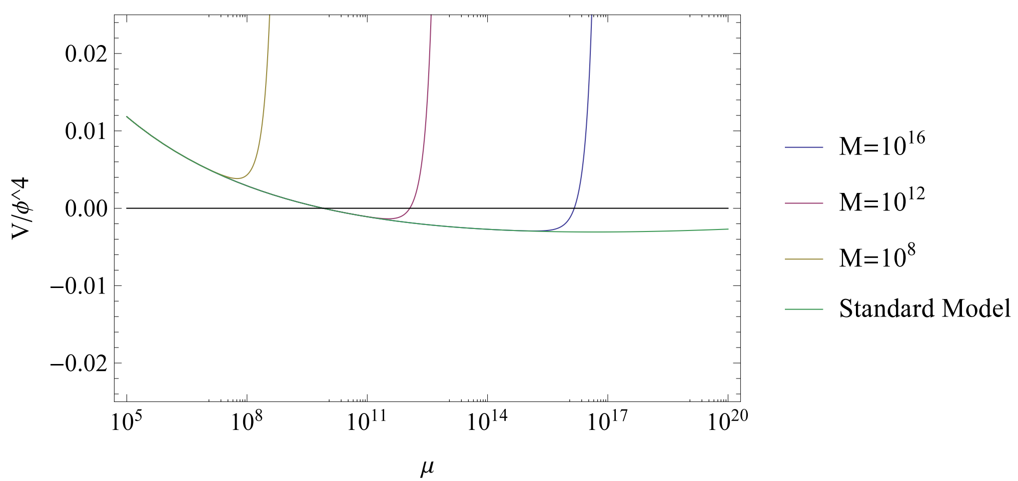


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = \lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$ are, respectively, $\log_{10}(\frac{\tau}{T_U}) = \infty, 1302, 581$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Magnitude of the suppression scale

Positive λ_8 and negative λ_6 $\rightarrow$ New Minimum

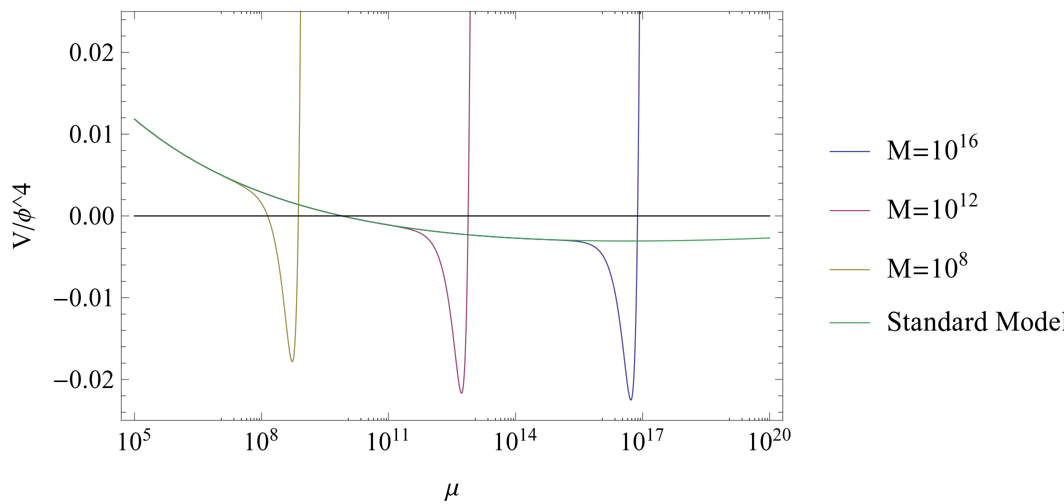
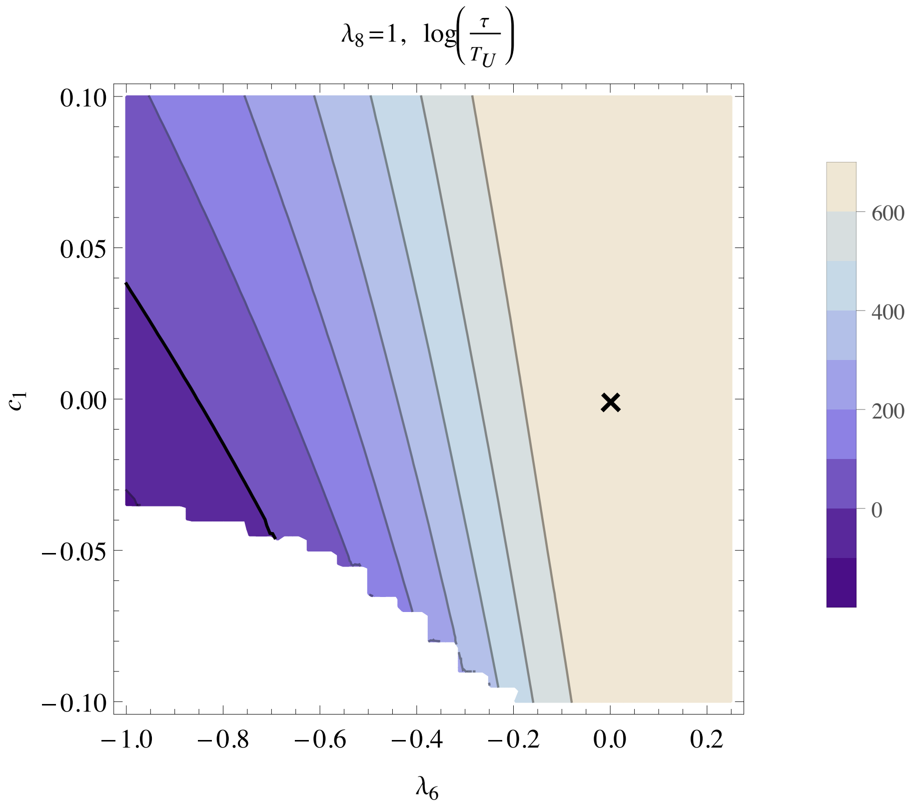


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = -1$ and $\lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$, are, respectively, $\log_{10}(\frac{\tau}{T_U}) = -45, -90, -110$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Derivative terms

$$Z(\phi)\partial_\mu\phi\partial^\mu\phi + \partial_\mu\phi\partial^\mu\phi\left(c_1\frac{\phi^2}{M^2} + c_2\frac{\phi^4}{M^4} + \ldots\right)$$



Lifetime of the EW vacuum doesn't change until negative λ_6 fources the bounce to go up to M

Then increasing $|c_1| = -c_1$ destabilizes the vacuum

$$\partial_s^2\phi + \frac{3}{s}\partial_s\phi = \frac{\partial V}{\partial\phi} \to (1 + c_1\phi^2/M^2)\ (\partial_s^2\phi + \frac{3}{s}\partial_s\phi) \approx \frac{\partial V}{\partial\phi}$$

Summary

- SM vacuum can be stabilized by nonrenormalisable interactions if they appear at suffciently low energy scale $10^{10} - 10^{11}\,\mathrm{GeV}$
- SM vacuum lifetime can be dramatically shortened by nonrenormalisable interactions for any suppression scale
- RG improvement stabilizes significant parts of the parameter space
- Higher-order terms with derivatives can destabilize metastable vacua
- Beyond the leading order one needs yet to find proper expansion of the action to demonstrate perturbatively the cancellation of gauge-dependent contributions.