The approach to CP conservation in the 2HDM

CORFU
September 2014

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First: generalities about CP violation in the 2HDM

(explicit vs spontaneous CP violation)

Application: CP violating ZZZ vertex

Work with:

Bohdan Grzadkowski, Odd Magne Øgreid

2HDM notation 1

$$
\begin{aligned}
V(\Phi_1,\Phi_2) &= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\} \\
&+ \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&+ \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\} \\
&\equiv Y_{a\bar{b}}\Phi_{\bar{a}}^\dagger\Phi_b + \frac{1}{2}Z_{a\bar{b}c\bar{d}}(\Phi_{\bar{a}}^\dagger\Phi_b)(\Phi_{\bar{c}}^\dagger\Phi_d)
\end{aligned}
$$

No FCNC: $\lambda_6 = 0; \quad \lambda_7 = 0$ (first part)

Allow CPV: $M_{12}^2, \lambda_5, \lambda_6, \lambda_7$ complex

2HDM notation 2

$$\Phi_j = e^{i\xi_j} \left(\begin{array}{c} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{array} \right), \quad j = 1, 2$$

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

$$\eta_3 = -\sin\beta\chi_1 + \cos\beta\chi_2$$

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathcal{M}^2_{\mathrm{diag}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$$

The rotation matrix R

$$R = R_3\, R_2\, R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_3 & \sin\alpha_3 \\ 0 & -\sin\alpha_3 & \cos\alpha_3 \end{pmatrix} \begin{pmatrix} \cos\alpha_2 & 0 & \sin\alpha_2 \\ 0 & 1 & 0 \\ -\sin\alpha_2 & 0 & \cos\alpha_2 \end{pmatrix} \begin{pmatrix} \cos\alpha_1 & \sin\alpha_1 & 0 \\ -\sin\alpha_1 & \cos\alpha_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_1\, c_2 & s_1\, c_2 & s_2 \\ -(c_1\, s_2\, s_3 + s_1\, c_3) & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, s_2\, c_3 + s_1\, s_3 & -(c_1\, s_3 + s_1\, s_2\, c_3) & c_2\, c_3 \end{pmatrix}$$

$$c_i = \cos\alpha_i,\; s_i = \sin\alpha_i$$

3-dimensional parameter space

CP will be conserved on certain manifolds embedded in this space.

In the absence of a Yukawa sector
(which might distinguish the two doublets),
there is a reparametrization invariance

If CP is conserved, or spontaneously violated, then a basis exists in which all the parameters of the potential are real

We follow approach by Gunion & Haber, study invariants:

CP conservation:

Define:

$$
\begin{aligned}
\operatorname{Im} J_1 &= -\frac{2}{v^2}\operatorname{Im}\left[\hat{v}^*_{\bar{a}} Y_{a\bar{b}} Z^{(1)}_{b\bar{d}}\,\hat{v}_d\right] \\
&= -\frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)\operatorname{Im}\lambda_5 \\
\operatorname{Im} J_2 &= \frac{4}{v^4}\operatorname{Im}\left[\hat{v}^*_{\bar{b}}\hat{v}^*_{\bar{c}} Y_{b\bar{e}} Y_{c\bar{f}} Z_{e\bar{a}f\bar{d}}\hat{v}_a\hat{v}_d\right] \\
&= -\frac{v_1^2 v_2^2}{v^8}\left[\left((\lambda_1-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_1^4+2(\lambda_1-\lambda_2)\operatorname{Re}\lambda_5 v_1^2 v_2^2\right. \\
&\qquad\left.-\left((\lambda_2-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_2^4\right]\operatorname{Im}\lambda_5 \\
\operatorname{Im} J_3 &= \operatorname{Im}\left[\hat{v}^*_{\bar{b}}\hat{v}^*_{\bar{c}} Z^{(1)}_{b\bar{e}} Z^{(1)}_{c\bar{f}} Z_{e\bar{a}f\bar{d}}\hat{v}_a\hat{v}_d\right] \\
&= \frac{v_1^2 v_2^2}{v^4}(\lambda_1-\lambda_2)(\lambda_1+\lambda_2+2\lambda_4)\operatorname{Im}\lambda_5
\end{aligned}
$$

CPC: $\operatorname{Im} J_1 = \operatorname{Im} J_2 = \operatorname{Im} J_3 = 0$

CP conservation:

Define:

$$\mathrm{Im}\, J_1 = -\frac{2}{v^2}\mathrm{Im}\left[\hat{v}^*_{\bar{a}} Y_{a\bar{b}} Z^{(1)}_{b\bar{d}} \hat{v}_d\right]$$

$$\longrightarrow \quad = -\frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)\mathrm{Im}\,\lambda_5$$

$$\mathrm{Im}\, J_2 = \frac{4}{v^4}\mathrm{Im}\left[\hat{v}^*_{\bar{b}} \hat{v}^*_{\bar{c}} Y_{b\bar{e}} Y_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right]$$

$$\longrightarrow \quad = -\frac{v_1^2 v_2^2}{v^8}\left[\left((\lambda_1 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2\right) v_1^4 + 2(\lambda_1 - \lambda_2)\mathrm{Re}\,\lambda_5 v_1^2 v_2^2 - \left((\lambda_2 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2\right) v_2^4\right] \mathrm{Im}\,\lambda_5$$

$$\mathrm{Im}\, J_3 = \mathrm{Im}\left[\hat{v}^*_{\bar{b}} \hat{v}^*_{\bar{c}} Z^{(1)}_{b\bar{e}} Z^{(1)}_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right]$$

$$\longrightarrow \quad = \frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)(\lambda_1 + \lambda_2 + 2\lambda_4)\mathrm{Im}\,\lambda_5$$

Second lines valid in “2HDM5”

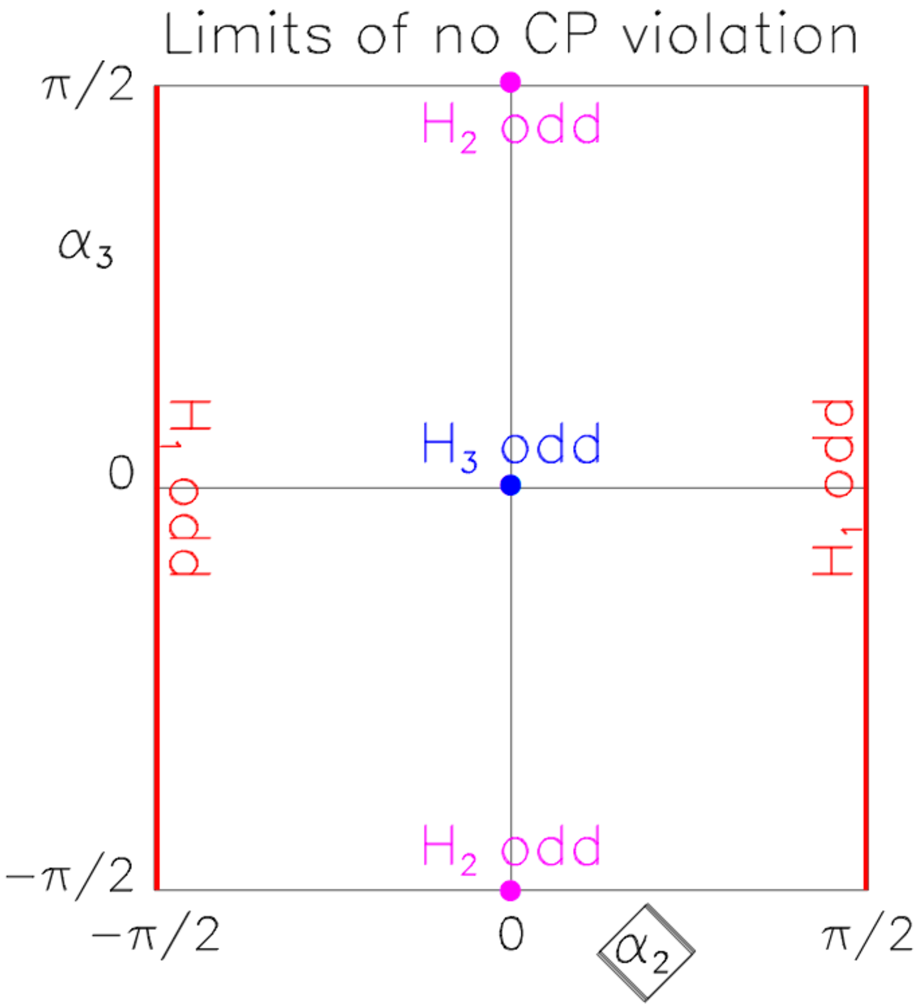
CPC: $\boxed{\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = \mathrm{Im}\, J_3 = 0}$

CP conservation:

(1) Where in the α space do we have CP conservation?

$$\alpha = \{\alpha_1, \alpha_2, \alpha_3\}$$

Answered in part in hep-ph/0702097



Explicit or Spontaneous CP conservation?

more invariants!

$$
\begin{aligned}
I_{Y3Z} &= \mathrm{Im}\left[Z^{(1)}_{a\bar{c}} Z^{(1)}_{e\bar{b}} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}}\right] \\
&= 0 \\
I_{2Y2Z} &= \mathrm{Im}\left[Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z^{(1)}_{f\bar{c}}\right] \\
&= \frac{1}{4}(\lambda_1 - \lambda_2)\mathrm{Im}\ \left[(m_{12}^2)^2 \lambda_5^*\right] \\
&= \frac{v_1^2 v_2^2}{4v^4}(\lambda_1 - \lambda_2)\left[4v^2\mu^2 \mathrm{Re}\,\lambda_5 - 4\mu^4 + v^4(\mathrm{Im}\,\lambda_5)^2\right]\mathrm{Im}\,\lambda_5 \\
I_{3Y3Z} &= \mathrm{Im}\left[Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}}\right] \\
&= -\frac{1}{8}(m_{11}^2 - m_{22}^2)\left[(\lambda_1 - \lambda_3 - \lambda_4)(\lambda_2 - \lambda_3 - \lambda_4) - |\lambda_5|^2\right]\mathrm{Im}\ \left[(m_{12}^2)^2 \lambda_5^*\right] \\
&= -\frac{v_1^2 v_2^2}{8v^6}\left[(\lambda_1 - \lambda_3 - \lambda_4)(\lambda_2 - \lambda_3 - \lambda_4) - |\lambda_5|^2\right] \\
&\times \left[(v_1^2 - v_2^2)(2\mu^2 - v^2(\lambda_3 + \lambda_4 + \mathrm{Re}\,\lambda_5)) + v^2(v_1^2\lambda_1 - v_2^2\lambda_2)\right] \\
&\times \left[4v^2\mu^2 \mathrm{Re}\,\lambda_5 - 4\mu^4 + v^4(\mathrm{Im}\,\lambda_5)^2\right]\mathrm{Im}\,\lambda_5 \\
I_{6Z} &= \mathrm{Im}\left[Z_{a\bar{b}c\bar{d}} Z^{(1)}_{b\bar{f}} Z^{(1)}_{d\bar{h}} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}}\right] \\
&= 0
\end{aligned}
$$

Explicit or Spontaneous CP conservation?

more invariants!

“2HDM5” results simple

$$I_{Y3Z} = \mathrm{Im}\left[Z^{(1)}_{a\bar{c}} Z^{(1)}_{e\bar{b}} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}}\right]$$

$$\longrightarrow \quad = 0$$

$$I_{2Y2Z} = \mathrm{Im}\left[Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z^{(1)}_{f\bar{c}}\right]$$

$$\longrightarrow \quad = \frac{1}{4}(\lambda_1 - \lambda_2)\mathrm{Im}\ \left[(m^2_{12})^2 \lambda^*_5\right]$$

$$\longrightarrow \quad = \frac{v_1^2 v_2^2}{4v^4}(\lambda_1 - \lambda_2)\left[4v^2\mu^2 \mathrm{Re}\,\lambda_5 - 4\mu^4 + v^4(\mathrm{Im}\,\lambda_5)^2\right]\mathrm{Im}\,\lambda_5$$

$$I_{3Y3Z} = \mathrm{Im}\left[Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}}\right]$$

$$\longrightarrow \quad = -\frac{1}{8}(m^2_{11} - m^2_{22})\left[(\lambda_1 - \lambda_3 - \lambda_4)(\lambda_2 - \lambda_3 - \lambda_4) - |\lambda_5|^2\right]\mathrm{Im}\ \left[(m^2_{12})^2 \lambda^*_5\right]$$

$$\longrightarrow \quad = -\frac{v_1^2 v_2^2}{8v^6}\left[(\lambda_1 - \lambda_3 - \lambda_4)(\lambda_2 - \lambda_3 - \lambda_4) - |\lambda_5|^2\right]$$
$$\times\left[(v_1^2 - v_2^2)(2\mu^2 - v^2(\lambda_3 + \lambda_4 + \mathrm{Re}\,\lambda_5)) + v^2(v_1^2\lambda_1 - v_2^2\lambda_2)\right]$$
$$\times\left[4v^2\mu^2\mathrm{Re}\,\lambda_5 - 4\mu^4 + v^4(\mathrm{Im}\,\lambda_5)^2\right]\mathrm{Im}\,\lambda_5$$

$$I_{6Z} = \mathrm{Im}\left[Z_{a\bar{b}c\bar{d}} Z^{(1)}_{b\bar{f}} Z^{(1)}_{d\bar{h}} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}}\right]$$

$$\longrightarrow \quad = 0$$

In general the CP violation is *explicit* if

$$I_{Y3Z} \neq 0 \quad and/or \quad I_{2Y2Z} \neq 0 \quad and/or \quad I_{3Y3Z} \neq 0 \quad and/or \quad I_{6Z} \neq 0$$

(2) Where in the α space is CP violation spontaneous?

Plots for

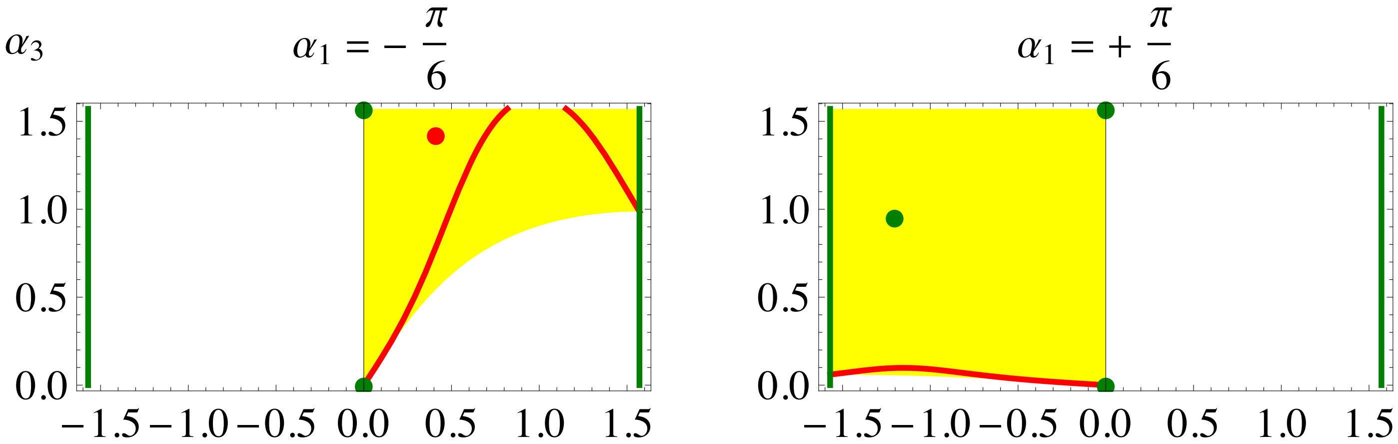
$M_1 = 125\ \text{GeV}, \quad M_2 = 200\ \text{GeV}, \quad M_{H^\pm} = 350\ \text{GeV}, \quad \mu = 250\ \text{GeV}$

$\tan\beta = 2$

Green (lines, dots): CPC

Red (lines, dots): SCPV

Yellow region: ECPV



White regions:

No solution to

$$M_3^2 > 0 \quad \text{and} \quad M_3 > M_2$$

Plots for

$M_1 = 125$ GeV, $\quad M_2 = 200$ GeV, $\quad M_{H^\pm} = 350$ GeV, $\quad \mu = 250$ GeV

$\tan\beta = 2$

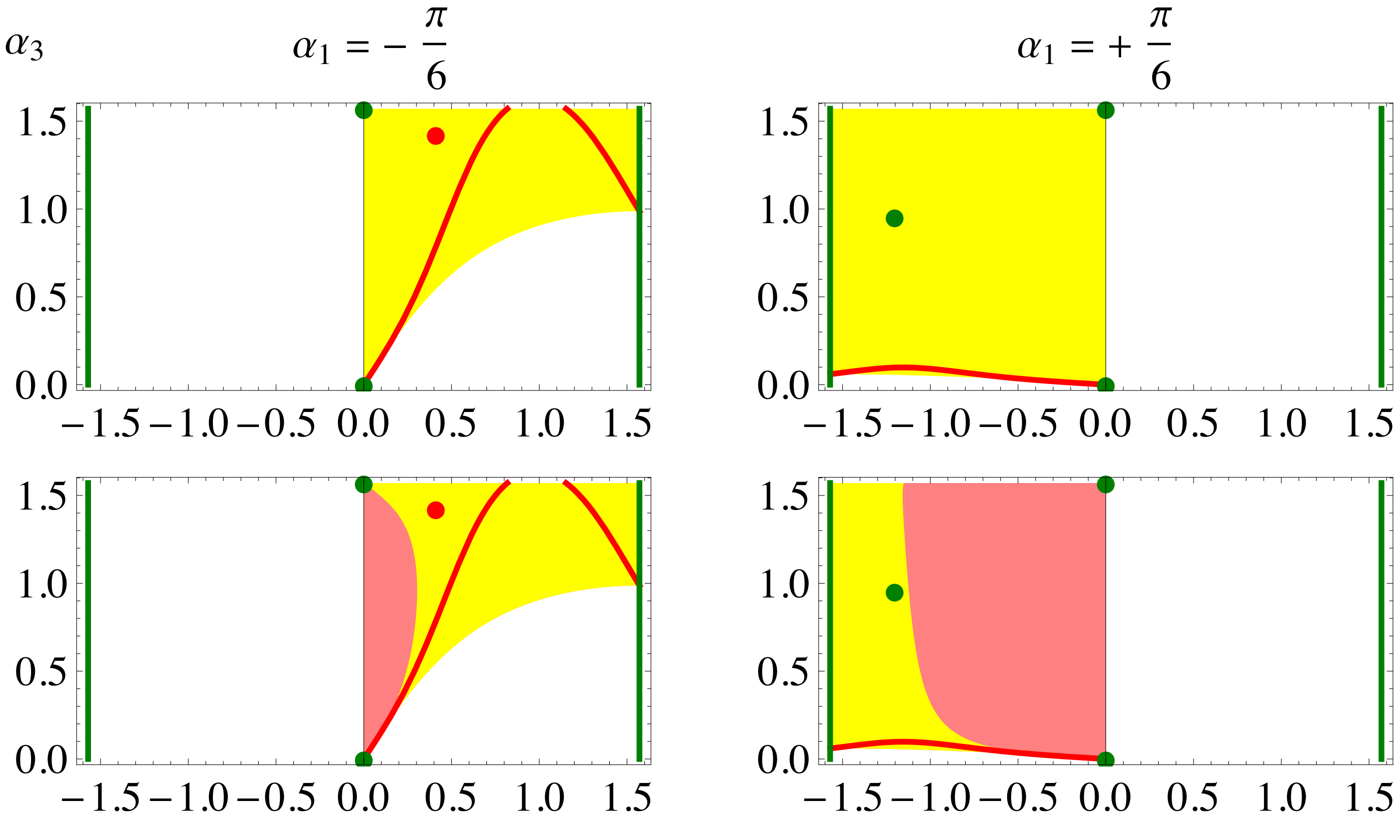
Green (lines, dots): CPC

Red (lines, dots): SCPV

Yellow region: ECPV

Violations:

Pink region: positivity



Plots for

$M_1 = 125$ GeV, $M_2 = 200$ GeV, $M_{H^\pm} = 350$ GeV, $\mu = 250$ GeV

$\tan\beta = 2$

Green (lines, dots): CPC

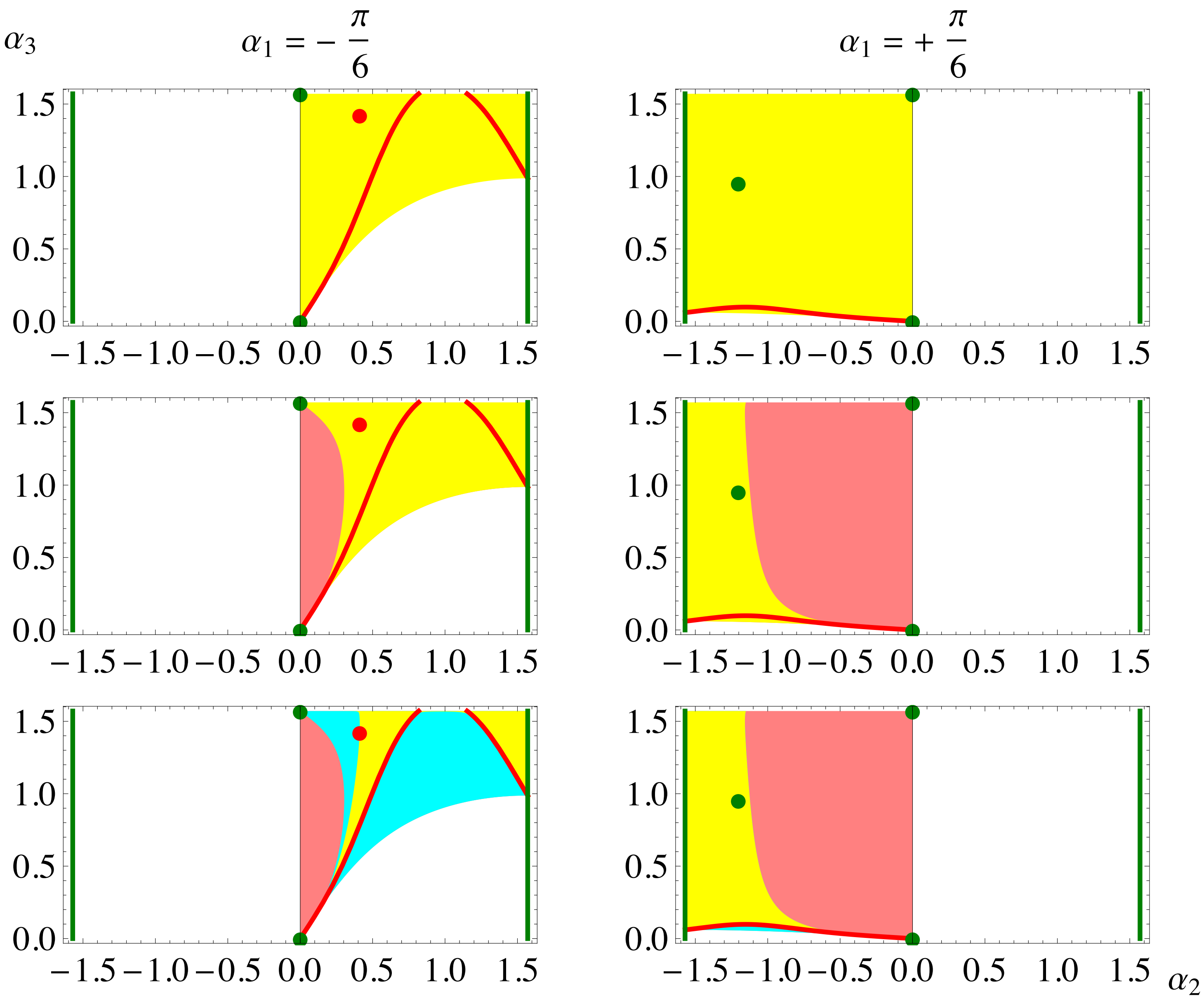
Red (lines, dots): SCPV

Yellow region: ECPV

Excluded:

Pink region: positivity

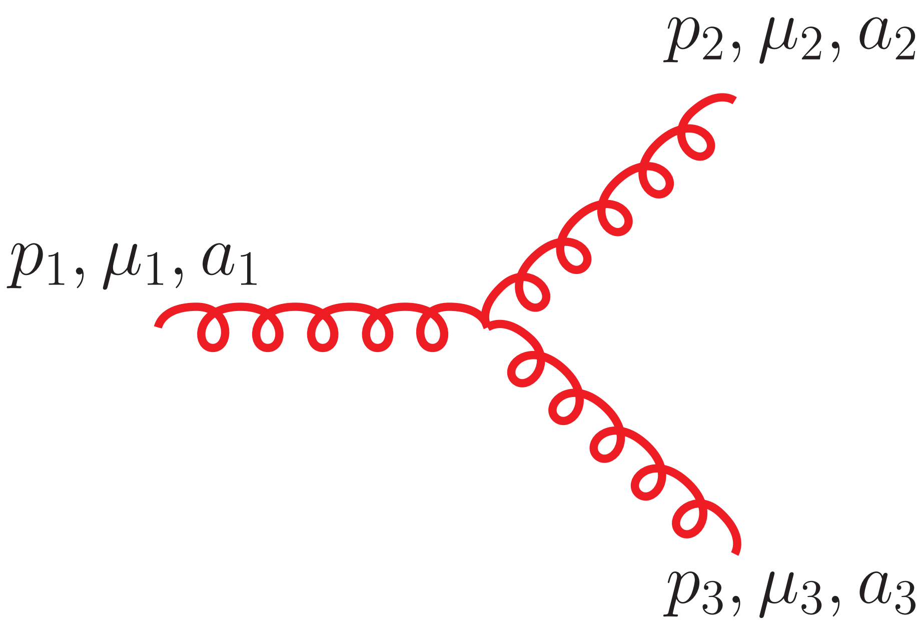
Cyan region: global min



CP violating ZZZ vertex

Preamble

ggg vertex in QCD



SU(3)

Structure:

$$-\mathrm{i}\ g\ f^{a_1 a_2 a_3}\left[g_{\mu_1\mu_2}(p_1 - p_2)_{\mu_3} + g_{\mu_2\mu_3}(p_2 - p_3)_{\mu_1} + g_{\mu_3\mu_1}(p_3 - p_1)_{\mu_2}\right]$$

Origin of ggg coupling, SU(3):

$$\mathcal{L} \ni \mathrm{Tr}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}$$

$$\mathcal{F}^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu - ig[A^\mu, A^\nu]$$

$$F_a^{\mu\nu} = \partial^\nu A_a^\mu - \partial^\mu A_a^\nu + g f_{abc} A_b^\mu A_c^\nu$$

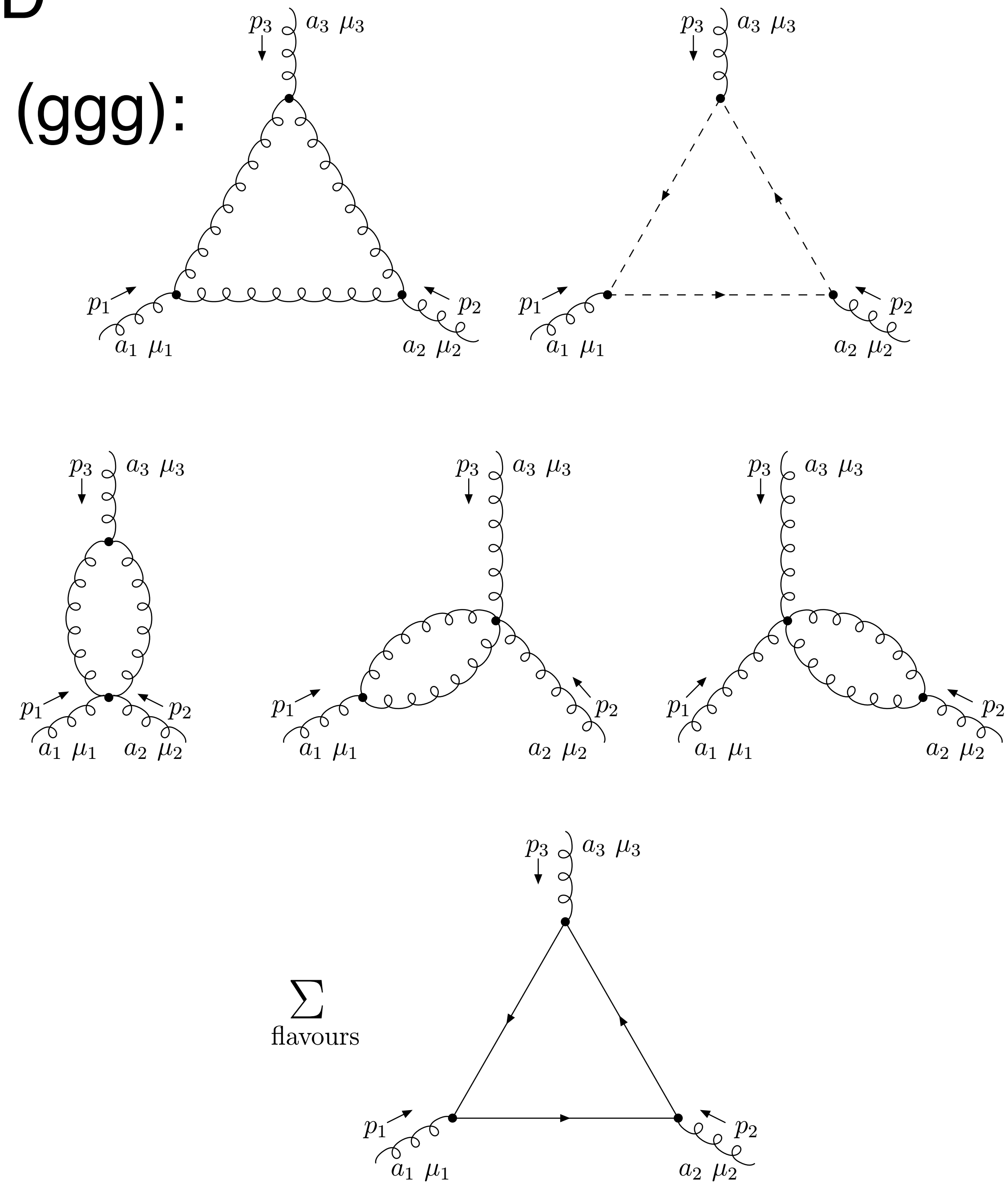
structure constant

SU(2): Completely analogous.

But structure constant is zero unless
all 3 gauge fields are present.
Have W^+W^-Z vertex, but
No ZZZ vertex (at tree level).

Back to QCD

At one loop (ggg):



More Lorentz structures:

tree level structure

$$\Gamma_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) = A(p_1^2,p_2^2;p_3^2)\; g_{\mu_1\mu_2}(p_1-p_2)_{\mu_3} + B(p_1^2,p_2^2;p_3^2)\; g_{\mu_1\mu_2}(p_1+p_2)_{\mu_3}$$
$$-C(p_1^2,p_2^2;p_3^2)\;\Big((p_1p_2)g_{\mu_1\mu_2} - p_{1_{\mu_2}}p_{2_{\mu_1}}\Big)\;(p_1-p_2)_{\mu_3}$$
$$+\tfrac{1}{3}\;S(p_1^2,p_2^2,p_3^2)\;\Big(p_{1_{\mu_3}}p_{2_{\mu_1}}p_{3_{\mu_2}} + p_{1_{\mu_2}}p_{2_{\mu_3}}p_{3_{\mu_1}}\Big)$$
$$+F(p_1^2,p_2^2;p_3^2)\;\Big((p_1p_2)g_{\mu_1\mu_2} - p_{1_{\mu_2}}p_{2_{\mu_1}}\Big)\;\Big(p_{1_{\mu_3}}(p_2p_3) - p_{2_{\mu_3}}(p_1p_3)\Big)$$
$$+H(p_1^2,p_2^2,p_3^2)\Big[-g_{\mu_1\mu_2}\Big(p_{1_{\mu_3}}(p_2p_3)-p_{2_{\mu_3}}(p_1p_3)\Big) + \tfrac{1}{3}\Big(p_{1_{\mu_3}}p_{2_{\mu_1}}p_{3_{\mu_2}}-p_{1_{\mu_2}}p_{2_{\mu_3}}p_{3_{\mu_1}}\Big)\Big]$$
$$+\{\text{ cyclic permutations of } (p_1,\mu_1),(p_2,\mu_2),(p_3,\mu_3)\ \}\,.$$

(in QCD) all conserve CP

ZZZ vertex at one loop

General structure

Gaemers & Gounaris (1979);
Hagiwara, Peccei, Zeppenfeld (1986);
Baur & Rainwater (2000)

Bose symmetry and Lorentz invariance: Coupling vanishes if all are on-shell

For two Zs (Z_2, Z_3) being on-shell:

$$\Gamma_{VVV}^{\mu_1\mu_2\mu_3} = \frac{p_1^2 - M_Z^2}{M_Z^2}\left[i f_4 (p_1^{\mu_2} g^{\mu_1\mu_3} + p_1^{\mu_3} g^{\mu_1\mu_2}) + i f_5 \epsilon^{\mu_1\mu_2\mu_3\rho}(p_2 - p_3)_\rho - f_6^V \epsilon^{\mu_1\mu_2\mu_3\rho} p_{1\rho} \right]$$

Violates CP Preserve CP

plus "scalar" terms $\propto p_1^{\mu_1}$

SM: quark loop contributes to f_5, f_6

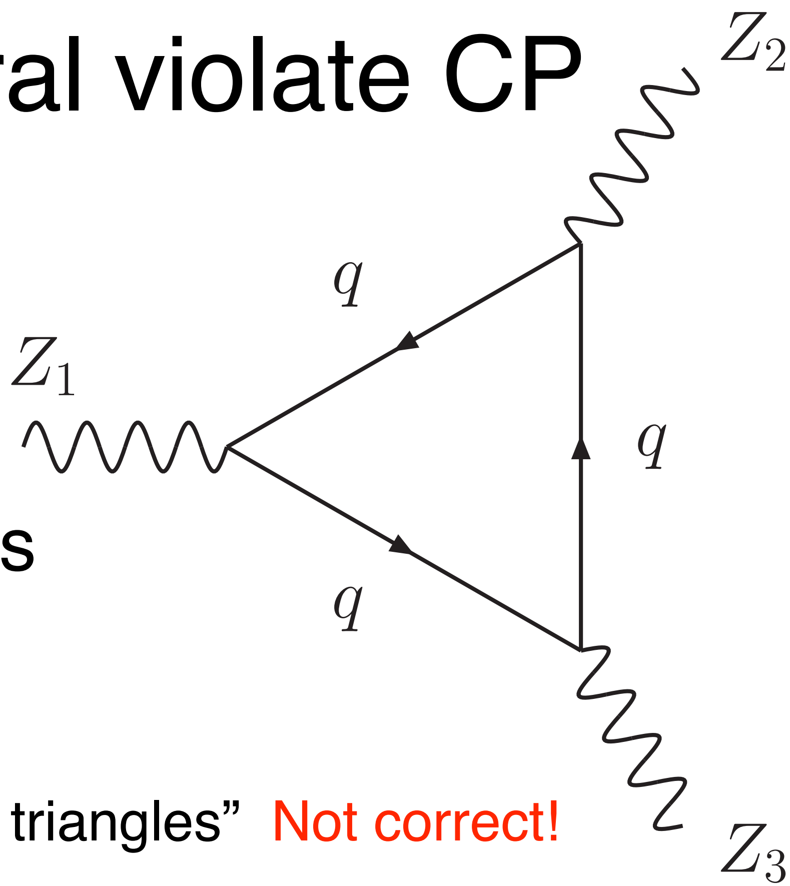
LHC: $|f_4| \lesssim 0.01$

ZZZ vertex at one loop

Z: odd under C

ZZ: even under C

ZZZ vertex will in general violate CP



SM contribution via fermion loops
preserves CP

Gounaris, Layssac, Renard (2000):
“Vertex can only be generated by fermionic triangles” Not correct!

ZZZ vertex at one loop, in 2HDM

non-zero contribution from bosonic sector

Allow for CP violation in Higgs potential

No FCNC: $\lambda_6 = 0; \quad \lambda_7 = 0$ (relax this)

Allow CPV: $M_{12}^2, \lambda_5, \lambda_6, \lambda_7$ complex

CP violation in the 2HDM

- Lavoura and Silva, 1994
- Botella and Silva, 1995
- Gunion and Haber, 2005
- Ivanov, Nishi, Maniatis…, 2006-2007

Gunion and Haber expressed invariants

vev

$$\mathrm{Im}\, J_1 = -\frac{2}{v^2}\mathrm{Im}\left[\hat{v}^*_{\bar{a}} Y_{a\bar{b}} Z^{(1)}_{b\bar{d}} \hat{v}_d\right]$$

$$\mathrm{Im}\, J_2 = \frac{4}{v^4}\mathrm{Im}\left[\hat{v}^*_{\bar{b}} \hat{v}^*_{\bar{c}} Y_{b\bar{e}} Y_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right]$$

$$\mathrm{Im}\, J_3 = \mathrm{Im}\left[\hat{v}^*_{\bar{b}} \hat{v}^*_{\bar{c}} Z^{(1)}_{b\bar{e}} Z^{(1)}_{c\bar{f}} Z_{e\bar{a}f\bar{d}} \hat{v}_a \hat{v}_d\right]$$

CP conserved iff $\boxed{\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = \mathrm{Im}\, J_3 = 0}$

What is the physical content?

Can

$$\operatorname{Im} J_1, \ \operatorname{Im} J_2, \ \operatorname{Im} J_3$$

be rephrased in terms of "physical" quantities?

The physical content

$$\mathrm{Im}\, J_1 = \frac{1}{v^5} \begin{vmatrix} q_1 & q_2 & q_3 \\ e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \end{vmatrix}$$

$$\mathrm{Im}\, J_2 = \frac{2}{v^9} \begin{vmatrix} e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \\ e_1 M_1^4 & e_2 M_2^4 & e_3 M_3^4 \end{vmatrix}$$

$$\mathrm{Im}\, J_{30} = \frac{1}{v^5} \begin{vmatrix} e_1 & e_2 & e_3 \\ q_1 & q_2 & q_3 \\ q_1 M_1^2 & q_2 M_2^2 & q_3 M_3^2 \end{vmatrix}$$

Footnote: $\mathrm{Im}\, J_3 = \mathrm{Im}\, J_{30} + \text{ terms } \propto \mathrm{Im}\, J_1, \mathrm{Im}\, J_2$

Three determinants:

$$\mathrm{Im}\, J_1 = \frac{1}{v^5} \begin{vmatrix} q_1 & q_2 & q_3 \\ e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \end{vmatrix}$$

$$\mathrm{Im}\, J_2 = \frac{2}{v^9} \begin{vmatrix} e_1 & e_2 & e_3 \\ e_1 M_1^2 & e_2 M_2^2 & e_3 M_3^2 \\ e_1 M_1^4 & e_2 M_2^4 & e_3 M_3^4 \end{vmatrix}$$

$$\mathrm{Im}\, J_{30} = \frac{1}{v^5} \begin{vmatrix} e_1 & e_2 & e_3 \\ q_1 & q_2 & q_3 \\ q_1 M_1^2 & q_2 M_2^2 & q_3 M_3^2 \end{vmatrix}$$

Footnote: $\mathrm{Im}\, J_3 = \mathrm{Im}\, J_{30} + \text{ terms } \propto \mathrm{Im}\, J_1, \mathrm{Im}\, J_2$

Couplings:

antisymmetric

$i, j, k : 1, 2, 3$

$$Z^\mu H_i H_j : \quad \frac{g}{2v\cos\theta_{\mathrm{W}}} \epsilon_{ijk} e_k (p_i - p_j)^\mu$$

$$H^+ H^- H_i : \quad - i q_i$$

$$e_i = v_1 R_{i1} + v_2 R_{i2}$$

$q_i = \ldots$ more complicated

Recall CP-conserving 2HDM

Let
$$H_1 = h$$
$$H_2 = H$$
$$H_3 = A$$

Then

(ZHA)	$e_1 \neq 0$	(hH^+H^-)	$q_1 \neq 0$
(ZhA)	$e_2 \neq 0$	(HH^+H^-)	$q_2 \neq 0$
(ZhH)	$e_3 = 0$	(AH^+H^-)	$q_3 = 0$

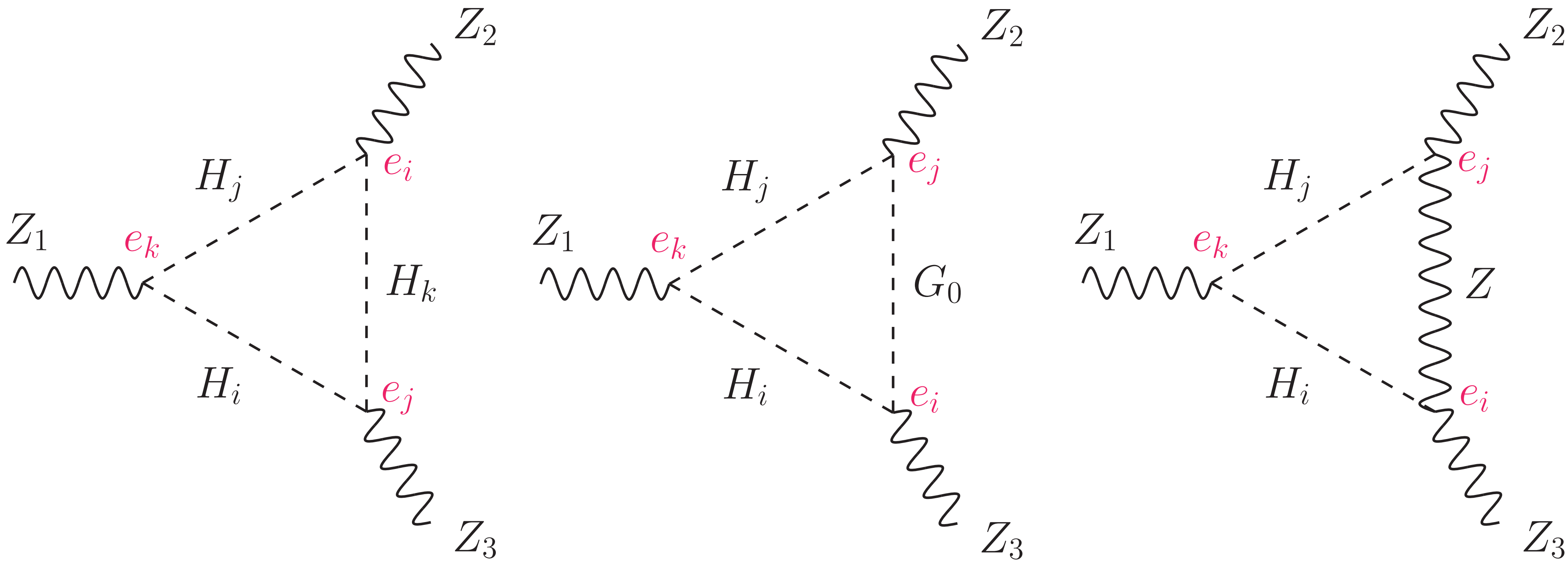
The invariants $\mathrm{Im}\, J_1,\ \mathrm{Im}\, J_2,\ \mathrm{Im}\, J_3$ **vanish**

This result was actually published by Lavoura and Silva in 1994 (see also Botella and Silva, 1995)

$$[\mathrm{Im}\, J_2]_{\text{Higgs Basis}} = -v^6 J_1^{\mathrm{LS}}$$
$$[\mathrm{Im}\, J_1]_{\text{Higgs Basis}} = v^3 J_3^{\mathrm{LS}}$$
$$[\mathrm{Im}\, J_{30}]_{\text{Higgs Basis}} = -v^4 J_2^{\mathrm{LS}}$$

and is reminiscent of the Jarlskog determinant

ZZZ vertex at one loop



Note sum over i, j, k all different.
All three neutral Higgs bosons involved.

ZZZ vertex at one loop

All three neutral Higgs bosons involved.

Proportional to $\mathrm{Im}\, J_2$

$$f_4^Z(p_1^2) = \frac{2\alpha}{\pi \sin^3(2\theta_\mathrm{W})} \frac{M_Z^2}{p_1^2 - M_Z^2} \frac{e_1 e_2 e_3}{v^3}$$
$$\times \sum_{i,j,k} \epsilon_{ijk} \big[C_{001}(p_1^2, M_Z^2, M_Z^2, M_i^2, M_j^2, M_Z^2) + C_{001}(p_1^2, M_Z^2, M_Z^2, M_Z^2, M_j^2, M_k^2)$$
$$+ C_{001}(p_1^2, M_Z^2, M_Z^2, M_i^2, M_Z^2, M_k^2) - C_{001}(p_1^2, M_Z^2, M_Z^2, M_i^2, M_j^2, M_k^2)$$
$$+ M_Z^2 C_1(p_1^2, M_Z^2, M_Z^2, M_i^2, M_Z^2, M_k^2) \big]$$

Z_2, Z_3 assumed on shell

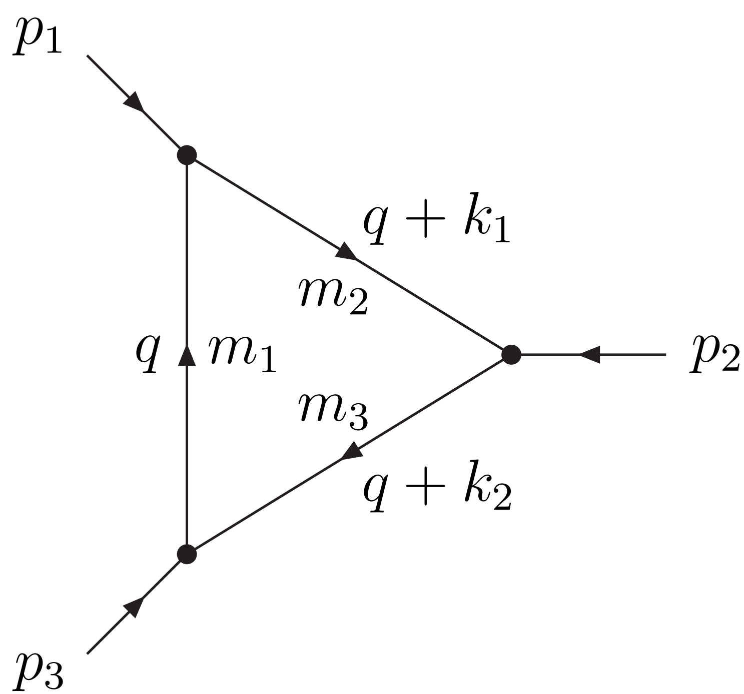
non-symmetric expression

Three-point functions

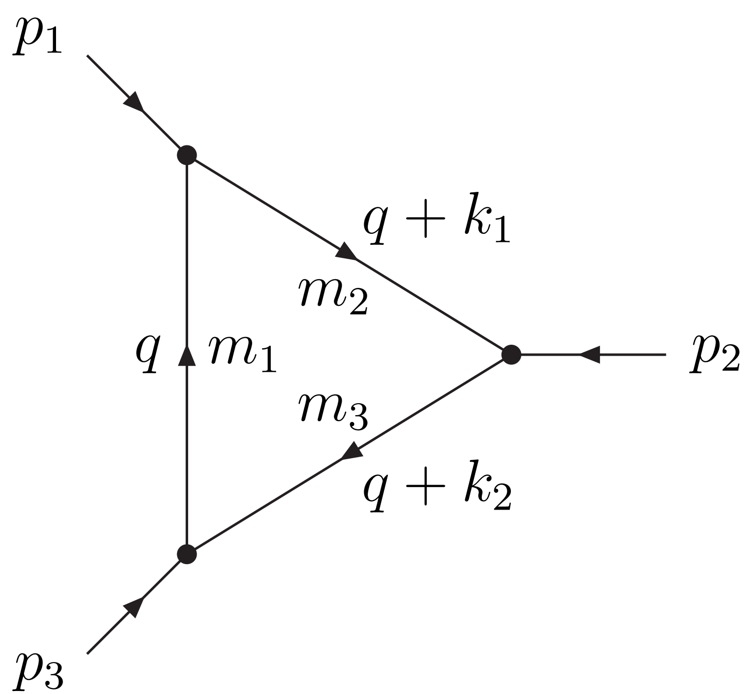
May have up to three Lorentz indices
(from Lorentz vectors in integrand)

Expand in “tensor coefficients”

LoopTools (Thomas Hahn)



Three-point functions



Tensor coefficients:

$$C_\mu = k_{1\mu}C_1 + k_{2\mu}C_2 = \sum_{i=1}^{2} k_{i\mu}C_i \,,$$

$$C_{\mu\nu} = g_{\mu\nu}C_{00} + \sum_{i,j=1}^{2} k_{i\mu}k_{j\nu}C_{ij} \,,$$

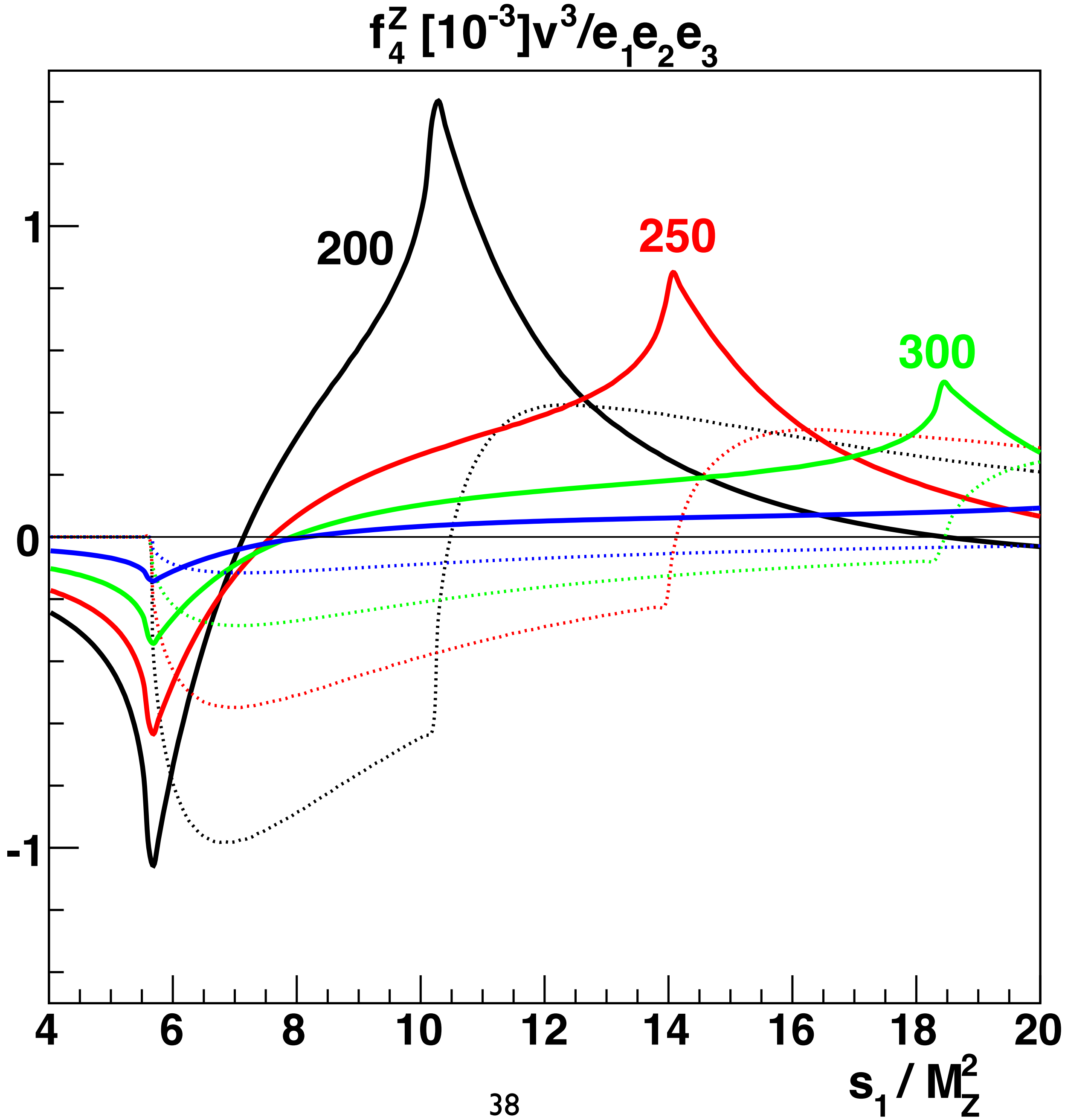
$$C_{\mu\nu\rho} = \sum_{i=1}^{2}\big(g_{\mu\nu}k_{i\rho} + g_{\nu\rho}k_{i\mu} + g_{\mu\rho}k_{i\nu}\big)C_{00i} + \sum_{i,j,\ell=1}^{2} k_{i\mu}k_{j\nu}k_{\ell\rho}C_{ij\ell} \,,$$

Numerical example

$$M_1 = 125 \text{ GeV},$$
$$M_2 = (200, 250, 300, 350) \text{ GeV},$$
$$M_3 = 400 \text{ GeV}$$

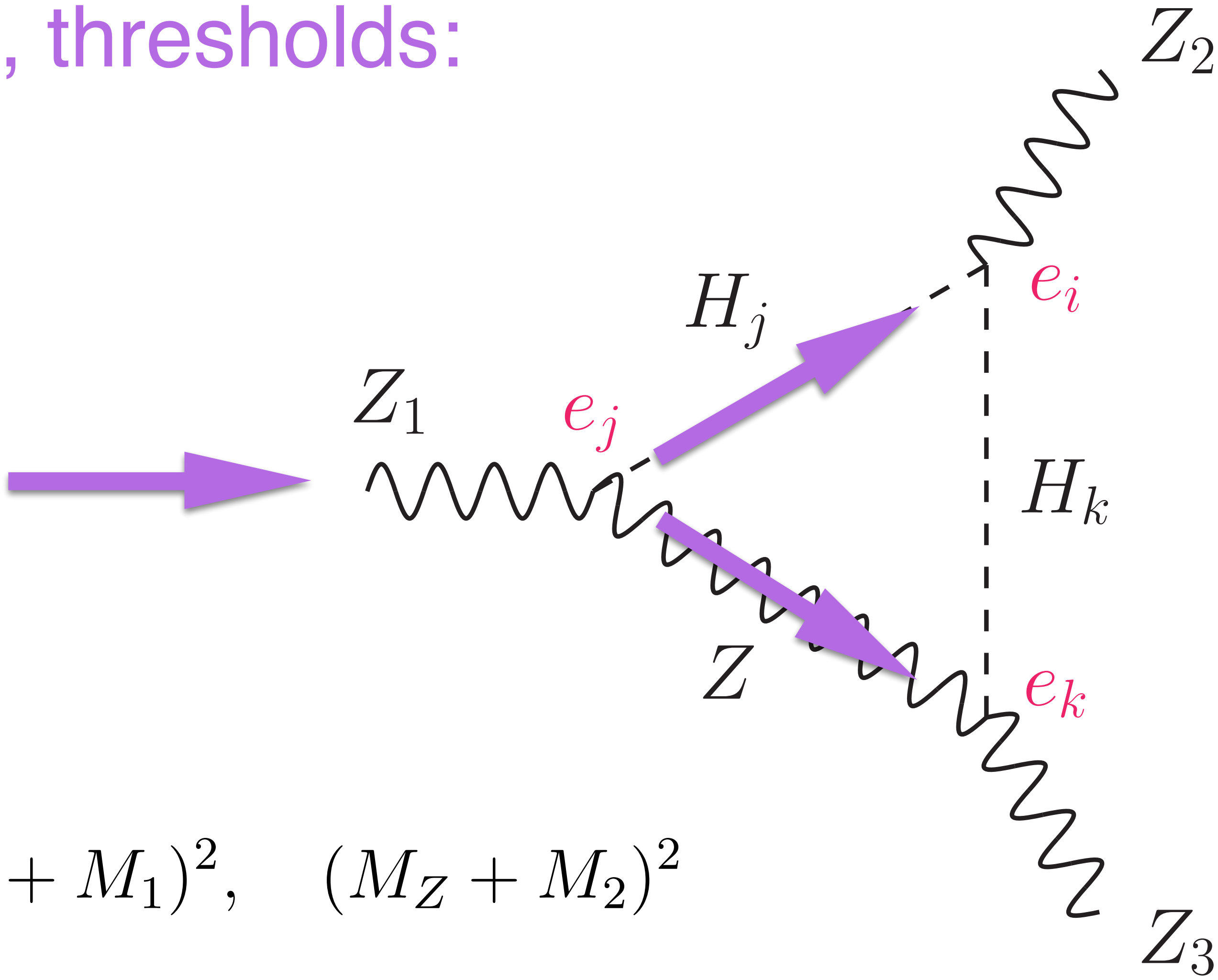
$$f_4^Z(s_1 = p_1^2) \text{ vs } s_1/M_Z^2$$

ZZZ vertex at one loop



Qualitative features

Resonances, thresholds:



at:

$$s_1 = (M_Z + M_1)^2, \quad (M_Z + M_2)^2$$

Product of couplings:

$$e_1^2 + e_2^2 + e_3^2 = v^2$$

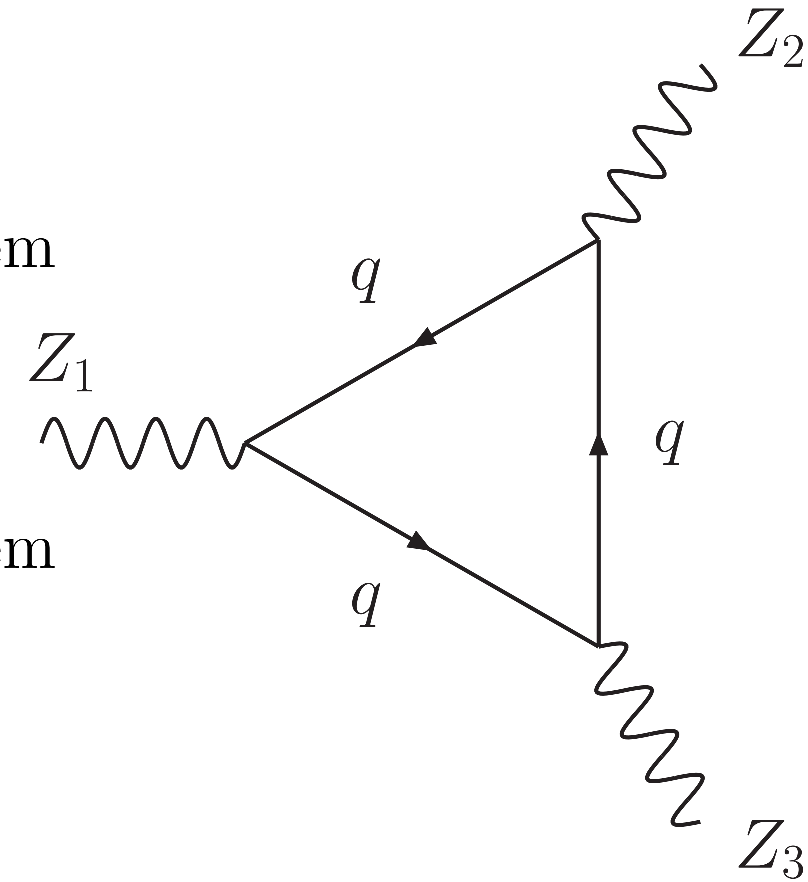
$$\max \frac{e_1 e_2 e_3}{v^3} = \frac{1}{3\sqrt{3}} \simeq 0.1925$$

For benchmarks studied in Basso et al, 1205.6569:

$$\frac{e_1 e_2 e_3}{v^3} = \mathcal{O}(0.01)$$

SM background

- $VVV :\to 0$ Furry's Theorem
- $VVA :\sim \epsilon_{\alpha\beta\gamma\delta}$ Violates P
- $VAA :\to 0$ Furry's Theorem
- $AAA :\sim \epsilon_{\alpha\beta\gamma\delta}$ Violates P



Contributions to f_5

No contribution to f_6 when Z_2, Z_3 are on-shell

The H_1SM limit

The quantity e_i is involved in many couplings

$$H_iH_jZ_\mu : \quad \frac{g}{2v\cos\theta_W}\epsilon_{ijk}e_k(p_i - p_j)_\mu \qquad H_iH_jG_0 : \quad i\frac{M_i^2 - M_j^2}{v^2}\epsilon_{ijk}e_k$$

$$H_iZ_\mu Z_\nu : \quad \frac{ig^2}{2\cos^2\theta_W}e_i\, g_{\mu\nu} \qquad H_iW_\mu^+W_\nu^- : \quad \frac{ig^2}{2}e_i\, g_{\mu\nu}$$

$$H_iG_0G_0 : \quad \frac{-iM_i^2e_i}{v^2} \qquad H_iG^+G^- : \quad \frac{-iM_i^2e_i}{v^2}$$

$$H_iG^+A_\mu W_\nu^- : \quad \frac{ig^2\sin\theta_W}{2v}e_i\, g_{\mu\nu} \qquad H_iG^-A_\mu W_\nu^+ : \quad \frac{ig^2\sin\theta_W}{2v}e_i\, g_{\mu\nu}$$

$$H_iG^+Z_\mu W_\nu^- : \quad -\frac{ig^2}{2v}\frac{\sin^2\theta_W}{\cos\theta_W}e_i\, g_{\mu\nu} \qquad H_iG^-Z_\mu W_\nu^+ : \quad -\frac{ig^2}{2v}\frac{\sin\theta_W^2}{\cos\theta_W}e_i\, g_{\mu\nu}$$

$$H_iG_0Z_\mu : \quad \frac{g}{2v\cos\theta_W}e_i(p_i - p_0)_\mu$$

$$H_iG^+W_\mu^- : \quad i\frac{g}{2v}e_i(p_i - p^+)_\mu \qquad H_iG^-W_\mu^+ : \quad -i\frac{g}{2v}e_i(p_i - p^-)_\mu$$

The H$_1$SM limit

In particular:

$$H_i H_j Z_\mu : \quad \frac{g}{2v\cos\theta_W}\epsilon_{ijk} e_k (p_i - p_j)_\mu \qquad H_i H_j G_0 : \quad i\frac{M_i^2 - M_j^2}{v^2}\epsilon_{ijk} e_k$$

$$H_i Z_\mu Z_\nu : \quad \frac{ig^2}{2\cos^2\theta_W} e_i \, g_{\mu\nu} \qquad H_i W_\mu^+ W_\nu^- : \quad \frac{ig^2}{2} e_i \, g_{\mu\nu}$$

$$H_i G_0 G_0 : \quad \frac{-iM_i^2 e_i}{v^2} \qquad H_i G^+ G^- : \quad \frac{-iM_i^2 e_i}{v^2}$$

$$H_i G^+ A_\mu W_\nu^- : \quad \frac{ig^2 \sin\theta_W}{2v} e_i \, g_{\mu\nu} \qquad H_i G^- A_\mu W_\nu^+ : \quad \frac{ig^2 \sin\theta_W}{2v} e_i \, g_{\mu\nu}$$

$$H_i G^+ Z_\mu W_\nu^- : \quad -\frac{ig^2}{2v}\frac{\sin^2\theta_W}{\cos\theta_W} e_i \, g_{\mu\nu} \qquad H_i G^- Z_\mu W_\nu^+ : \quad -\frac{ig^2}{2v}\frac{\sin\theta_W^2}{\cos\theta_W} e_i \, g_{\mu\nu}$$

$$H_i G_0 Z_\mu : \quad \frac{g}{2v\cos\theta_W} e_i (p_i - p_0)_\mu$$

$$H_i G^+ W_\mu^- : \quad i\frac{g}{2v} e_i (p_i - p^+)_\mu \qquad H_i G^- W_\mu^+ : \quad -i\frac{g}{2v} e_i (p_i - p^-)_\mu$$

The H_1SM limit

In particular:

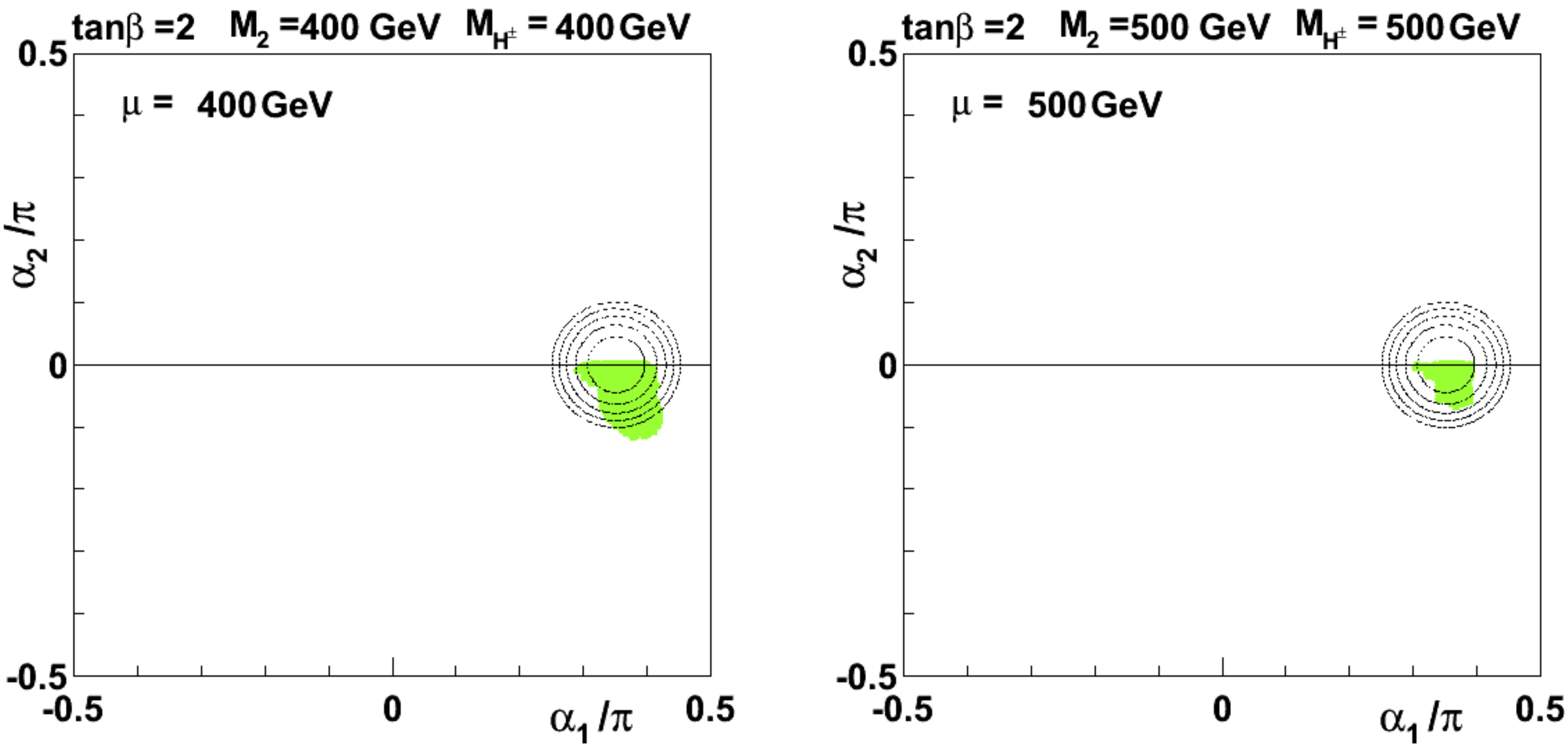
$$H_1 Z_\mu Z_\nu : \qquad \frac{i g^2}{2\cos^2\theta_{\mathrm{W}}} e_1 \, g_{\mu\nu}$$

is very close to SM limit (LHC)

$$e_1 = v \cos\alpha_2 \cos(\beta - \alpha_1)$$

$$\alpha_2 \simeq 0 \quad \alpha_1 \simeq \beta$$

“circles”: 1%, 2%, …, 5% deviation from $e_1=v$ (SM limit)



Green: allowed in 2HDM

Recall

$$\mathrm{Im}\, J_{30} = \frac{1}{v^5} \begin{vmatrix} e_1 & e_2 & e_3 \\ q_1 & q_2 & q_3 \\ q_1 M_1^2 & q_2 M_2^2 & q_3 M_3^2 \end{vmatrix}$$

could still be non-zero, even if

$$e_2 = e_3 = 0$$

Requires $M_3 \neq M_2$

Requires $\lambda_6 \neq 0 \quad \text{and/or} \quad \lambda_7 \neq 0$

Summary

- Invariants Im J_1, Im J_2, Im J_3 can be expressed in terms of physical couplings and masses
- All three neutral Higgses are "involved"
- Couplings lead to a calculable CP-violating ZZZ coupling
- It will be a challenge to measure it
- In H_1SM limit, CP violation possible