On scale- vs conformal invariance

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Plan:

1) History

2) Recent attempts

3) A "conjecture"

Scale- vs. conformal- vs. Weyl invariance

① a scale invariant (effective) action $S = \int d^n x \, L(\varphi, \partial_a \varphi)$

is invariant under $\begin{cases} 'x^a = f^a(x) = \lambda x^a \\ '\varphi(x^a) = \lambda^h \varphi(x^a) \end{cases}$ (h = scaling dim.)

② S is conformal invariant if $f^a(x)$ satisfies the conf. Killing eqn:

$$\partial^a f^b + \partial^b f^a = \frac{2}{n} \eta^{ab} \partial_c f^c$$

③ a Weyl invariant action $S = \int d^n x \sqrt{g} \, L(g_{ab}, \varphi, \partial_a \varphi)$

is invariant under $\begin{cases} 'x^a = x^a \\ 'g_{ab} = e^{\sigma(x)} g_{ab} \\ '\varphi(x^a) = e^{h\sigma(x)} \varphi(x^a) \end{cases}$

Relation between ② and ③

∴ For a diffeomorphism invariant action ② ⊂ ③ with $\partial_a \partial_b \sigma = 0$

$$F = f^{-1}$$

$$\begin{cases} {}''x^a = x^a \\ {}''\varphi(x) = \left(f^c{}_{,c}\right)^h \varphi(x) \\ {}''g_{ab}(x) = e^{\sigma} g_{ab}(x) \end{cases} = \begin{cases} {}''x^a = F^a({}'x) = x^a \\ {}''\varphi(x) = {}'\varphi({}'x) \\ {}''g_{ab}(x) = F_* g_{ab}(x) \end{cases} \circ \begin{cases} {}'x^a = f^a(x) \\ {}'\varphi({}'x) = \left(f^c{}_{,c}\right)^h \varphi(x) \\ {}'g_{ab}({}'x) = g_{ab}(x) \end{cases}$$

weyl　　　　　　　　diff　　　　　　　　conf

$$\sigma(x) = \ln\left(f^c{}_{,c}(x)\right)$$

We shall assume Lorentz- and translation invariance $\Rightarrow$

stress tensor: $T_{ab} = T_{ba}$, $\partial^a T_{ab} = 0$

① is a global symmetry and implies a conserved

Noether current: $j_a = x^b T_{ab} + V_a \leftarrow$ virial current

$\left(Q(V) = \int d^{n-1}x\, V_0 \right.$ generates the scale dimension of φ)

Fact: (Callen, Coleman, Jackiv '70, Coleman, Jackiv '71, Polchinski '88)

If $V_a = \partial^b L_{ab}$, $L_{ab} = L_{ba}$, local then ② is implied.

Γpf: $\exists\, T_{ab}^{\text{imp.}}$, $\eta^{ab} T_{ab}^{\text{imp}} = 0 \Rightarrow j_a = f^b T_{ab}$ is conserved #

... an old question:

$$① \overset{?}{\Longrightarrow} ②$$

is a unitary, scale invariant action automatically conformal?

caution! conformal $\neq$ Weyl!

e.g. the "conformally coupled" scalar

$$S = \int d^4x \sqrt{-g} \left(\partial_a \varphi \partial^a \varphi - \frac{1}{6} R \varphi^2 \right)$$

is Weyl invariant.

relation between ①, ② and ③:

gauge ① to get ③ $L(\varphi, \partial_a\varphi, W_a) = L(\varphi, \partial_a\varphi) + W^a V_a + O(V_a^2)$

virial cur.

gauge pot. for scale

... sometimes W_a can be replaced by a non-minimal coupling to g_{ab} ...

$\longrightarrow$ $\boxed{\text{Ricci gauging}}$

Fact: (Iorio, O'Raifeartaigh, I.S., Wiesendanger '95)

① can be Ricci gauged to get ② iff ① is conformal, ie. ① $\Rightarrow$ ②.

and: if V_a is not well defined e.g. not gauge-invariant, then ① $\not\Rightarrow$ ②

2 Dimensions :

① + unitarity $\Rightarrow$ ② (Zamolodchikov '86)

pf: $\langle 0|T_{ab}(p)\,T_{cd}(-p)|0\rangle = (p_a p_b - \eta_{ab}p^2)(p_c p_c - \eta_{cd}p^2)\,\frac{b}{p^2}$

$+ (b \leftrightarrow c)\,\frac{d}{p^2}$

$\Rightarrow \langle T^a{}_a(p)\,T^c{}_c(-p)\rangle = (b+2d)\,p^2 \Rightarrow$ contact term

$\Rightarrow \langle T(x)\,T(y)\rangle = \Box\,\delta^2(x-y) \Rightarrow T(x) \equiv 0$

$\uparrow$ Reeh-Schlieder thm #

4 Dimensions

+ $L_{ab} = \eta_{ab} L$ (from unitarity)

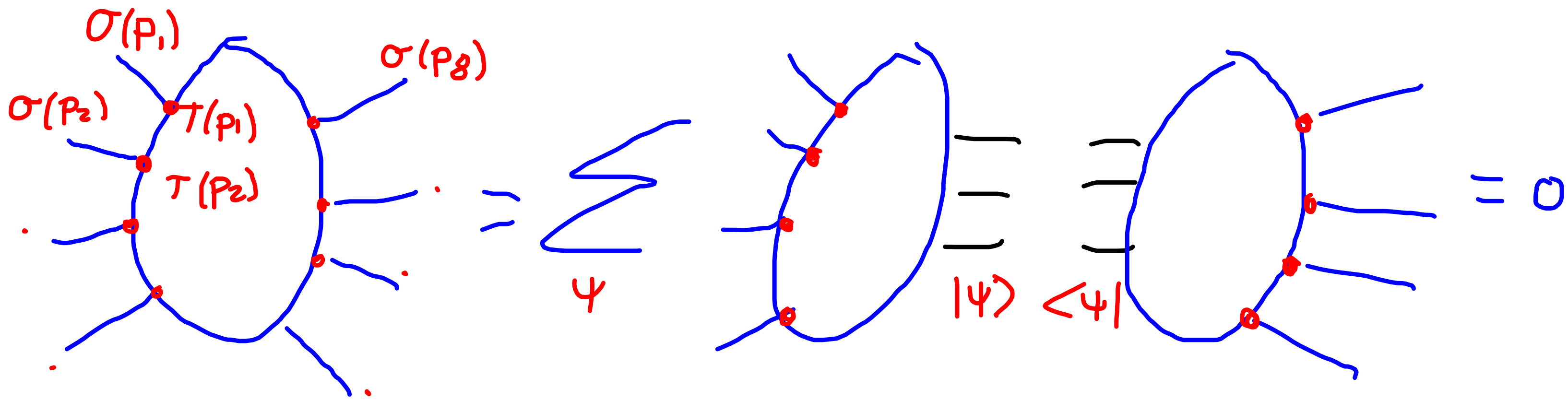
− b and d can depend on $\log(p^2)$ which spoils the pf.

$\longrightarrow$ need another argument ...

- Supersymmetry? a class of unitary, R-symm. fixed points are necessarily superconformal (Antoniadis, Buican '11)

- analyticity of dilaton "scattering"

(Luty, Polchinski, Rattazzi; Dymarski, Komargodski, Schwimmer, Theisen '13)



$$\Rightarrow \langle 0|T(p_1)\cdots T(p_n)|\psi\rangle = 0 \quad \forall |\psi\rangle,\ p_i^2 = 0,\ i = 2,\dots,n$$

$\langle 0|T(p)|\psi\rangle\big|_{p^2=0} = 0$ follows from unitarity (Ponomarev, I.S. '14)

$\overset{?}{\Rightarrow}$ theory decouples from conformal dilaton ($\Box\sigma(x) = 0$)

Does this imply that $T(p) = p^2 L(p)$?

local op.

(Yonekura '14): assume :

$$\langle 0 | \prod_{i/p_i^2=0} T(p_i) | \psi \rangle = 0, \forall \psi \Rightarrow \langle \psi' | \prod_i T(p_i) | \psi'' \rangle = 0, \forall \psi', \psi''$$

then $L(p) \equiv \frac{1}{p^2} T(p)$ satisfies $[L(t,\vec{x}), L(t,\vec{y})] = 0, \vec{x} \neq \vec{y}$

$\overset{?}{\Longrightarrow}$ $L(t,\vec{x})$ local.

"inverse"
wightman

Counter example:

- Maxwell in 3 dimensions: $S = \int d^3x \, F_{ab} F^{ab}$

$T_{ab} = 2 F_{ac} F_b{}^c - \frac{1}{2} g_{ab} F_{de} F^{de}$

$T^a{}_a = \frac{1}{2} F_{cd} F^{cd} \Rightarrow V_a = \frac{1}{2} A_d F_a{}^d$, not gauge invariant!

$\rightarrow$ scale inv., unitary but not conformal.

- rank 2 gauge-potential, B_{ab} in 4 dim.

$V_a = B_{cd} F_a{}^{cd}$

We already know that V_a not gauge-inv. $\Rightarrow$

no Ricci gauging $\Rightarrow$ not conformal. ✓

"Conjecture" (Ponomarev, I.S. '14):

$$\boxed{\begin{array}{l} \mathcal{L}(\varphi, \partial_a\varphi) \text{ scale invariant, unitary, } V_a \text{ exists} \\ \text{then } \mathcal{L}(\varphi, \partial_a\varphi) \text{ is conformal} \end{array}}$$

are there other counter examples ?