Prospects for 2HDM CP violation in light of the LHC Higgs signal

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- CP violation in 2HDM
- The H1SM limit
- Implications of the LHC Higgs signal - *Does any CP-violation remain?*
- Numerical strategy and illustrations of the H1SM limit
- Prospect for measuring CP violation
- Summary

◇ B.G., O. M. Ogreid and P. Osland, "Diagnosing CP properties of the 2HDM", JHEP **1401**, 105 (2014),

◇ B.G., O. M. Ogreid and P. Osland, in progress

CP violation in 2HDM

Motivations for 2HDM:

- Baryon asymmetry and Sakharov conditions for baryogenesis
  - Baryon number non-conservation,
  - C- and CP-violation,
  - Thermal inequilibrium,

  Extra sources of the CP-violation are required

- Possibility of large (tree-level generated) FCNC, e.g. $t \to cH$ decays, interesting non-standard flavour physics

- 2HDM provide a framework for light new physics that is easily tolerated by the Higgs boson discovery.
  see e.g.
  B. Dumont, J. F. Gunion, Y. Jiang and S. Kraml, "Constraints on and future prospects for Two-Higgs-Doublet Models in light of the LHC Higgs signal", Phys. Rev. D **90**, 035021 (2014) [arXiv:1405.3584 [hep-ph]].

The 2HDM potential:

$$
\begin{aligned}
V(\phi_1,\phi_2) \;=\; & -\frac{1}{2}\left\{m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + \left[m_{12}^2\phi_1^\dagger\phi_2 + \text{H.c.}\right]\right\} + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 \\
& + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right] + \left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right]\left[(\phi_1^\dagger\phi_2) + \text{H.c.}\right]
\end{aligned}
$$

Yukawa couplings:

$$
\mathcal{L}_Y^{(q)} = \bar{Q}_L\left(\tilde{\Gamma}_1\tilde{\Phi}_1 + \tilde{\Gamma}_2\tilde{\Phi}_2\right)u_R + \bar{Q}_L\left(\Gamma_1\Phi_1 + \Gamma_2\Phi_2\right)d_R + \text{H.c.}
$$

then

$$
M_u = -\tilde{\Gamma}_1\langle\tilde{\Phi}_1\rangle - \tilde{\Gamma}_2\langle\tilde{\Phi}_2\rangle \qquad M_d = -\Gamma_1\langle\Phi_1\rangle - \Gamma_2\langle\Phi_2\rangle
$$

The type II model:
$\mathbb{Z}_2$ softly broken (by $m_{12}^2 \neq 0$): $\Phi_1 \to -\Phi_1$ and $d_R \to -d_R \Rightarrow \lambda_6 = \lambda_7 = 0$, $\tilde{\Gamma}_1 = \Gamma_2 = 0$

In an arbitrary basis, the vevs may be complex, and the Higgs-doublets can be written

$$\Phi_j = e^{i\xi_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2.$$

Here v_j are real numbers, so that $v_1^2 + v_2^2 = v^2$. The fields η_j and χ_j are real. The phase difference between the two vevs is defined as

$$\xi \equiv \xi_2 - \xi_1.$$

Next, let's define the Goldston bosons G_0 and $G^\pm$ by an orthogonal rotation

$$\begin{pmatrix} G_0 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \qquad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix}$$

where $s_\beta \equiv \sin\beta$ and $c_\beta \equiv \cos\beta$ for $\tan\beta \equiv v_2/v_1$. Then G_0 and $G^\pm$ become the massless Goldstone fields. $H^\pm$ are the charged scalars.

The model contains three neutral scalar mass-eigenstates, which are linear compositions of the η_i,

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix},$$

with the 3×3 orthogonal rotation matrix R satisfying

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathcal{M}^2_{\mathrm{diag}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2),$$

and with $M_1 \leq M_2 \leq M_3$. A convenient parametrization of the rotation matrix R is

$$R = R_3 R_2 R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_1\, c_2 & s_1\, c_2 & s_2 \\ -(c_1\, s_2\, s_3 + s_1\, c_3) & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, s_2\, c_3 + s_1\, s_3 & -(c_1\, s_3 + s_1\, s_2\, c_3) & c_2\, c_3 \end{pmatrix}$$

Couplings:

$$H_i Z_\mu Z_\nu : \quad \frac{ig^2}{2\cos^2\theta_W} e_i\, g_{\mu\nu}, \qquad H_i W_\mu^+ W_\nu^- : \quad \frac{ig^2}{2} e_i\, g_{\mu\nu}$$

where

$$e_i \equiv v_1 R_{i1} + v_2 R_{i2}$$

In terms of the mixing angles

$$\begin{aligned} e_1 &= v\cos\alpha_2\cos(\beta-\alpha_1) \\ e_2 &= v[\cos\alpha_3\sin(\beta-\alpha_1) - \sin\alpha_2\sin\alpha_3\cos(\beta-\alpha_1)] \\ e_3 &= -v[\sin\alpha_3\sin(\beta-\alpha_1) + \sin\alpha_2\cos\alpha_3\cos(\beta-\alpha_1)] \end{aligned}$$

Note that

$$e_1^2 + e_2^2 + e_3^2 = v^2.$$

Couplings:

$$(Z^\mu H_i H_j): \quad \frac{g}{2v\cos\theta_W}\epsilon_{ijk}e_k(p_i-p_j)^\mu,$$

$$H_i H^- H^+: \quad -iq_i$$

where

$$\begin{aligned} q_i \;=\; & \frac{2e_i}{v^2}M^2_{H^\pm} - \frac{R_{i2}v_1 + R_{i1}v_2 - R_{i3}vt_\xi}{v_1 v_2 c_\xi}\mu^2 + \frac{g_i - R_{i3}v^3 t_\xi}{v^2 v_1 v_2}M_i^2 + \frac{R_{i3}v^3}{2v_1 v_2 c_\xi^2}\mathrm{Im}\,\lambda_5 \\ & -\frac{v^2\left(R_{i3}vt_\xi - R_{i2}v_1 + R_{i1}v_2\right)}{2v_2^2 c_\xi}\mathrm{Re}\,\lambda_6 - \frac{v^2\left(R_{i3}vt_\xi + R_{i2}v_1 - R_{i1}v_2\right)}{2v_1^2 c_\xi}\mathrm{Re}\,\lambda_7 \end{aligned}$$

and $g_i \equiv v_1^3 R_{i2} + v_2^3 R_{i1}$.

CP conservation:

CP is conserved if and only if:

$$\begin{aligned}
\mathrm{Im}\, J_1 &= \frac{1}{v^5} \sum_{i,j,k} \epsilon_{ijk} M_i^2 e_i e_j q_k = 0 \\
\mathrm{Im}\, J_2 &= \frac{e_1 e_2 e_3}{v^9} \sum_{i,j,k} \epsilon_{ijk} M_i^4 M_k^2 = 0 \\
\mathrm{Im}\, J_{30} &= \frac{1}{v^5} \sum_{i,j,k} \epsilon_{ijk} M_i^2 q_i e_j q_k = 0.
\end{aligned}$$

where J_i are the weak basis invariants found by Lavoura, Silva and Botella (1994, 1995) and discussed by Davidson, Gunion and Haber (2005), Ivanov (2006, 2007), Nishi (2006) and M. Maniatis, A. von Manteuffel and O. Nachtmann (2008).

CP conservation:

The conditions for CP conservation could be rewritten as:

$$\begin{aligned}
\mathrm{Im}\, J_1 &= \frac{1}{v^5}\left[M_1^2 e_1(e_2 q_3 - e_3 q_2) + M_2^2 e_2(e_3 q_1 - e_1 q_3) + M_3^2 e_3(e_1 q_2 - e_2 q_1)\right] = 0 \\
\mathrm{Im}\, J_2 &= \frac{e_1 e_2 e_3}{v^9}(M_2^2 - M_1^2)(M_3^2 - M_2^2)(M_1^2 - M_3^2) = 0 \\
\mathrm{Im}\, J_{30} &= \frac{1}{v^5}\left[M_1^2 q_1(e_2 q_3 - e_3 q_2) + M_2^2 q_2(e_3 q_1 - e_1 q_3) + M_3^2 q_3(e_1 q_2 - e_2 q_1)\right] = 0
\end{aligned}$$

The H1SM limit

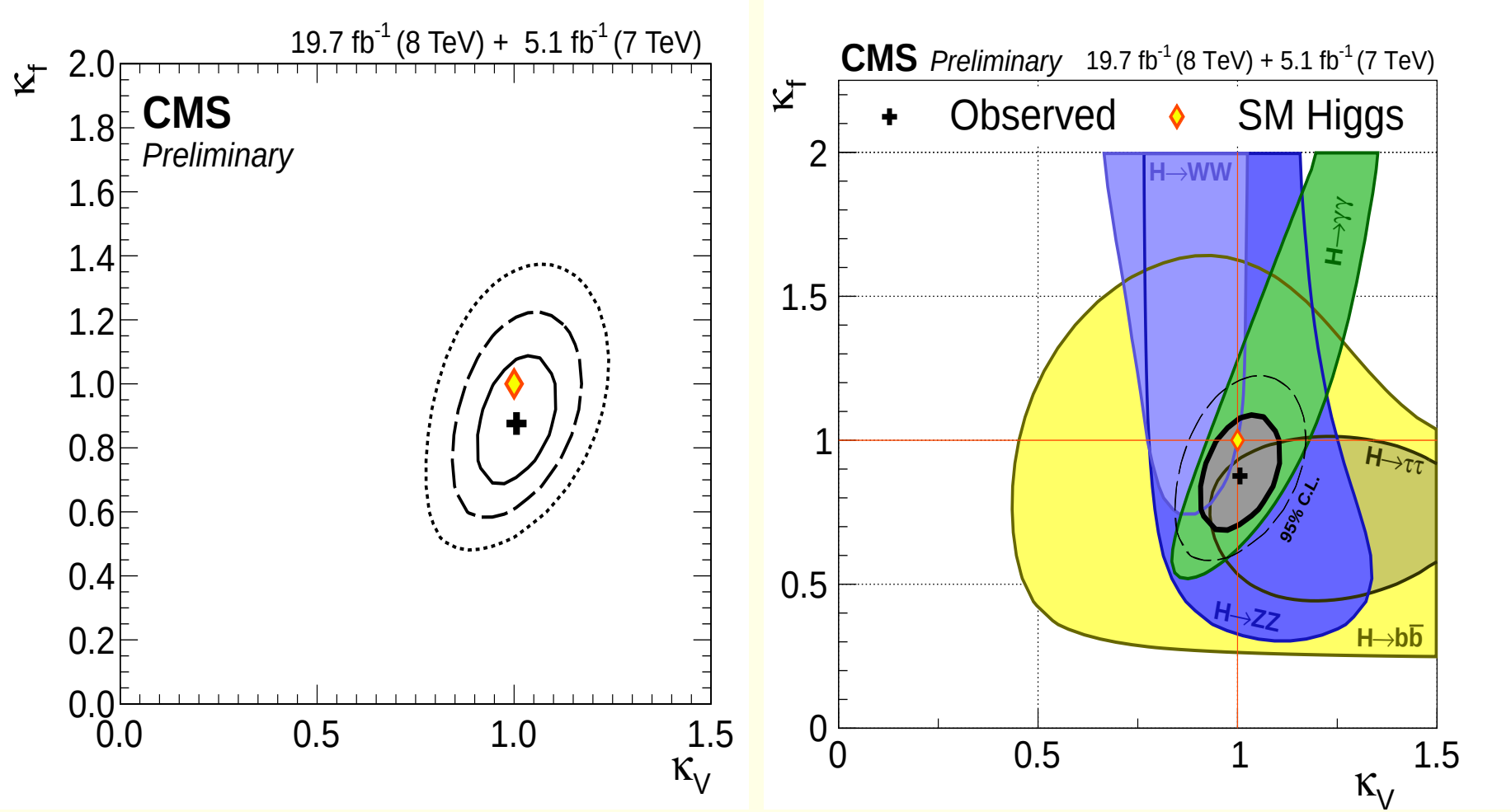


Figure 1: CMS PAS HIG-14-009: Results of 2D likelihood scans for the (κ_V, κ_f). Left: the solid, dashed, and dotted contours show the 68%, 95%, and 99.7% CL regions, respectively. Right: the 68% CL contours for individual channels and for the overall combination (thick curve), the dashed contour bounds the 95% CL region.

The LHC Higgs data suggest that HZZ and HW^+W^- couplings are close to the SM prediction.

$$\Downarrow$$

$$H_i Z_\mu Z_\nu : \quad \frac{ig^2}{2\cos^2\theta_{\rm W}} e_i\, g_{\mu\nu}, \qquad H_i W^+_\mu W^-_\nu : \quad \frac{ig^2}{2} e_i\, g_{\mu\nu}$$

We define (within 2HDM) the H1SM limit as $e_1 = v$

Then

$$e_1^2 + e_2^2 + e_3^2 = v^2 \qquad \Rightarrow \qquad e_2 = e_3 = 0$$

Note that no assumption has been made concerning the mass scale of beyond the SM physics: M_2, M_3 and μ^2 defined as

$$\mathrm{Re}\, m_{12}^2 = \frac{2v_1 v_2}{v^2}\mu^2.$$

The coupling of H_1 to a pair of vector bosons, e_1, could be written as follows:

$$e_1 = v \cos(\alpha_2) \cos(\alpha_1 - \beta)$$

The most general solution of the H1SM limit (known also as the alignment condition) $e_1 = v$, $e_2 = 0$, $e_3 = 0$:

$$\alpha_2 = 0 \qquad \alpha_1 = \beta$$

The rotation matrix in this case becomes

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1\, c_3 & c_1\, c_3 & s_3 \\ s_1\, s_3 & -c_1\, s_3 & c_3 \end{pmatrix}$$

Note that the mixing matrix could be written in this case as

$$R = R_3 R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Does the H1SM limit allow for CP-violation?

Couplings:

In the H1SM limit Z couples only to H_2H_3:

$$(Z^\mu H_i H_j): \quad \frac{g}{2v\cos\theta_W}\epsilon_{ijk}e_k(p_i-p_j)^\mu \neq 0 \ \text{ only if } \ \mathrm{i}=2 \ \text{ and } \ \mathrm{j}=3$$

Couplings between H_i and H^+H^- are given in the H1SM limit (for $\xi=0$) by:

$$\begin{aligned}
q_1 &= \frac{1}{v}\left(2M_{H^\pm}^2 - 2\mu^2 + M_1^2\right)\\
q_2 &= +c_3\left[\frac{(c_\beta^2-s_\beta^2)}{vc_\beta s_\beta}(M_2^2-\mu^2) + \frac{v}{2s_\beta^2}\mathrm{Re}\,\lambda_6 - \frac{v}{2c_\beta^2}\mathrm{Re}\,\lambda_7\right] + s_3\frac{v}{2c_\beta s_\beta}\mathrm{Im}\,\lambda_5,\\
q_3 &= -s_3\left[\frac{(c_\beta^2-s_\beta^2)}{vc_\beta s_\beta}(M_3^2-\mu^2) + \frac{v}{2s_\beta^2}\mathrm{Re}\,\lambda_6 - \frac{v}{2c_\beta^2}\mathrm{Re}\,\lambda_7\right] + c_3\frac{v}{2c_\beta s_\beta}\mathrm{Im}\,\lambda_5
\end{aligned}$$

In the H1SM limit the expressions for the CPV invariants become

$$\begin{aligned}
\mathrm{Im}\, J_1 &= \frac{1}{v^5}\left[M_1^2 e_1(e_2 q_3 - e_3 q_2) + M_2^2 e_2(e_3 q_1 - e_1 q_3) + M_3^2 e_3(e_1 q_2 - e_2 q_1)\right] \to 0 \\
\mathrm{Im}\, J_2 &= \frac{e_1 e_2 e_3}{v^9}(M_2^2 - M_1^2)(M_3^2 - M_2^2)(M_1^2 - M_3^2) \to 0 \\
\mathrm{Im}\, J_{30} &= \frac{1}{v^5}\left[M_1^2 q_1(e_2 q_3 - e_3 q_2) + M_2^2 q_2(e_3 q_1 - e_1 q_3) + M_3^2 q_3(e_1 q_2 - e_2 q_1)\right] \\
&\to \frac{e_1 q_2 q_3}{v^3}(M_3^2 - M_2^2)
\end{aligned}$$

- Note that $e_1 = v$ implies no CP violation in $H_i VV$ couplings ($\mathrm{Im}\, J_2 = 0$), the only possible CP violation may appear in cubic scalar couplings $H_2 H^+ H^-$ and $H_3 H^+ H^-$, proportional to q_2 and q_3, respectively.

- The necessary condition for CP violation is that both $H_2 H^+ H^-$ *and* $H_3 H^+ H^-$ must exist *together* with non-zero $Z H_2 H_3$ vertex. The latter implies that for CP invariance either H_2 or H_3 would have to be odd under CP, on the other hand if *both* of them couple to $H^+ H^-$ (that is CP even), then there is no way to preserve CP.

In the case $\lambda_6 = \lambda_7 = 0$ the $(\mathcal{M}^2)_{13}$ and $(\mathcal{M}^2)_{23}$ are related as follows

$$(\mathcal{M}^2)_{13} = \tan\beta (\mathcal{M}^2)_{23}$$

As a consequence of the above relation there is a constraint that relates mass eigenvalues, mixing angles and $\tan\beta$ (Khater and Osland, 2003):

$$M_1^2 R_{13}(R_{12}\tan\beta - R_{11}) + M_2^2 R_{23}(R_{22}\tan\beta - R_{21}) + M_3^2 R_{33}(R_{32}\tan\beta - R_{31}) = 0$$

$$\Downarrow$$

H1SM limit $(\alpha_2 = 0, \alpha_1 = \beta) \Rightarrow$ $\boxed{(M_2^2 - M_3^2) s_3 c_3 s_\beta = 0}$

$$\Downarrow$$

- $M_2 \neq M_3$, but $\alpha_3 = 0, \pi/2$, then $q_3 = 0, q_2 = 0$, respectively, so no CP violation, or
- $M_2 = M_3$, therefore $\mathrm{Im}\, J_3 = 0$, so again no CP violation.

The form of R in the H1SM limit

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1\, c_3 & c_1\, c_3 & s_3 \\ s_1\, s_3 & -c_1\, s_3 & c_3 \end{pmatrix}$$

suggests that H_2 and/or H_3 might have CP-violating interactions. In order to resolve this puzzle it is useful to note that the limit implies that the (23) bloc of the scalar mass matrix squared (after diagonalization) is proportional to the unit matrix. Therefore without changing kinetic or mass terms, the following orthogonal transformation allows to rotate away α_3:

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} \to R_3 \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, \tag{1}$$

As a consequence the resulting mixing matrix is just $R = R_1$, implying no CPV consistently with $\mathrm{Im}\, J_3 = 0$.

Numerical strategy and illustrations

Parameters:

- for 2HDM5 ($\mathbb{Z}_2$ imposed, so $\lambda_6 = \lambda_7 = 0$):

$$\mathcal{P}_5 \equiv \{M^2_{H^\pm}, \mu^2, M^2_1, M^2_2, v_1, v_2, \xi = 0, \alpha_1, \alpha_2, \alpha_3\}$$

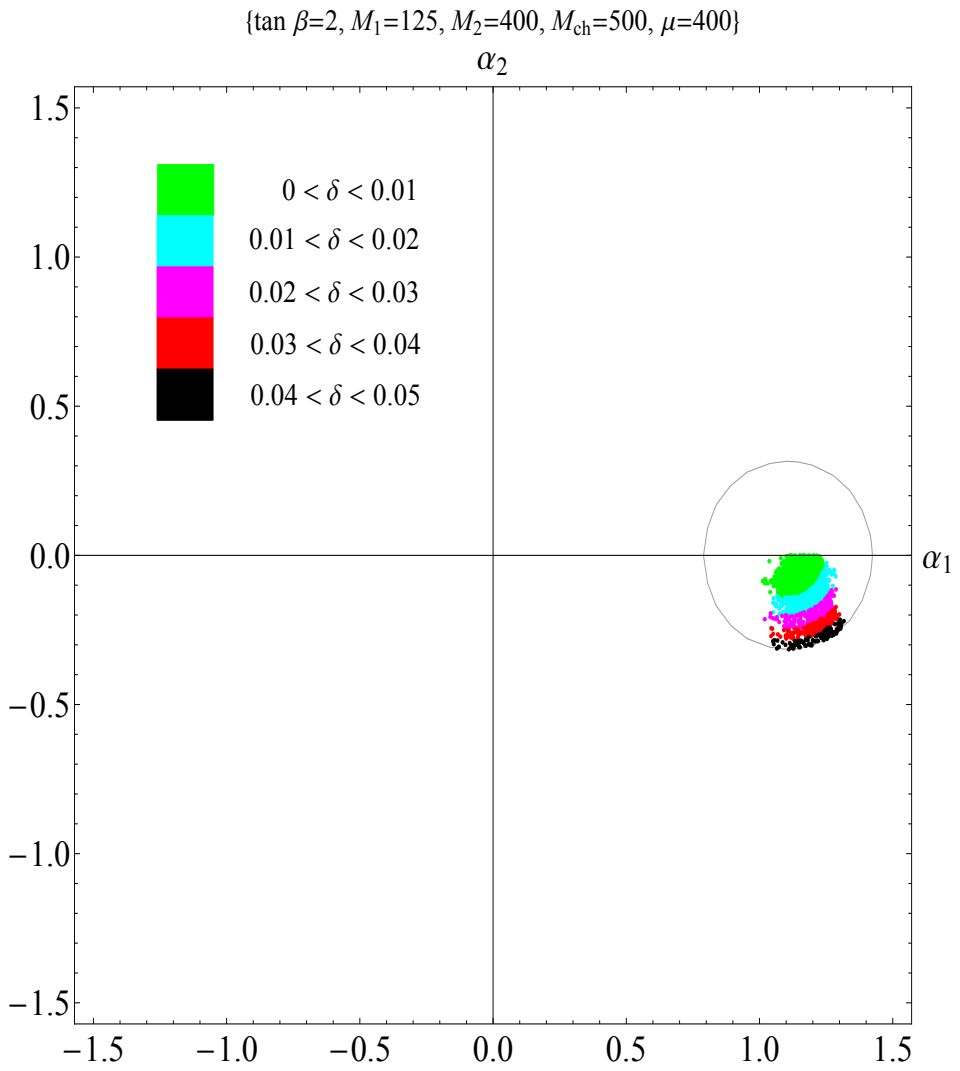
- for 2HDM67 ($\mathbb{Z}_2$ not imposed, so $\lambda_6 \neq 0, \lambda_7 \neq 0$):

$$\mathcal{P}_{67} \equiv \{M^2_{H^\pm}, \mu^2, M^2_1, M^2_2, M^2_3, \mathrm{Im}\lambda_5, \mathrm{Re}\lambda_6, \mathrm{Re}\lambda_7, v_1, v_2, \xi = 0, \alpha_1, \alpha_2, \alpha_3\}$$

Plots shown in next slides have been obtained adopting the following strategy:

- 2HDM5 ($\mathbb{Z}_2$ imposed, so $\lambda_6 = \lambda_7 = 0$):
  - $M^2_{H^\pm}$, μ^2, M_2, $\tan\beta$ are fixed parameters
  - scan over $\alpha_1, \alpha_2, \alpha_3$ for chosen maximal deviation $\delta \equiv |e_1/v - 1|$ and imposing $M_1 < M_2 < M_3$, vacuum stability and unitarity.

- for 2HDM67 ($\mathbb{Z}_2$ not imposed, so $\lambda_6 \neq 0, \lambda_7 \neq 0$):
  - $M^2_{H^\pm}$, μ^2, M_2, M_3, and $\tan\beta$ are fixed parameters
  - scan over $\alpha_1, \alpha_2, \alpha_3$, $\mathrm{Im}\lambda_5, \mathrm{Re}\lambda_6, \mathrm{Re}\lambda_7$, for chosen maximal deviation $\delta \equiv |e_1/v - 1|$ and imposing $M_1 < M_2 < M_3$, vacuum stability and unitarity.



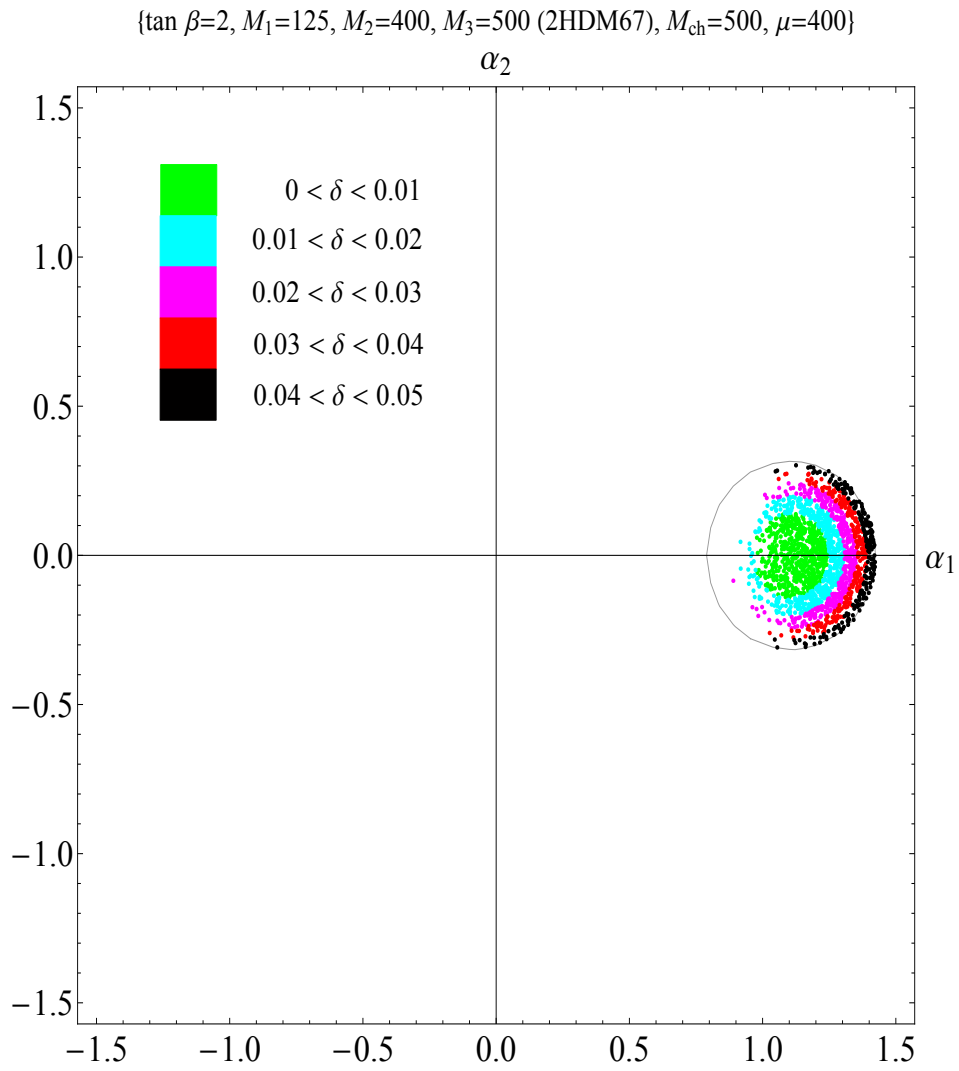


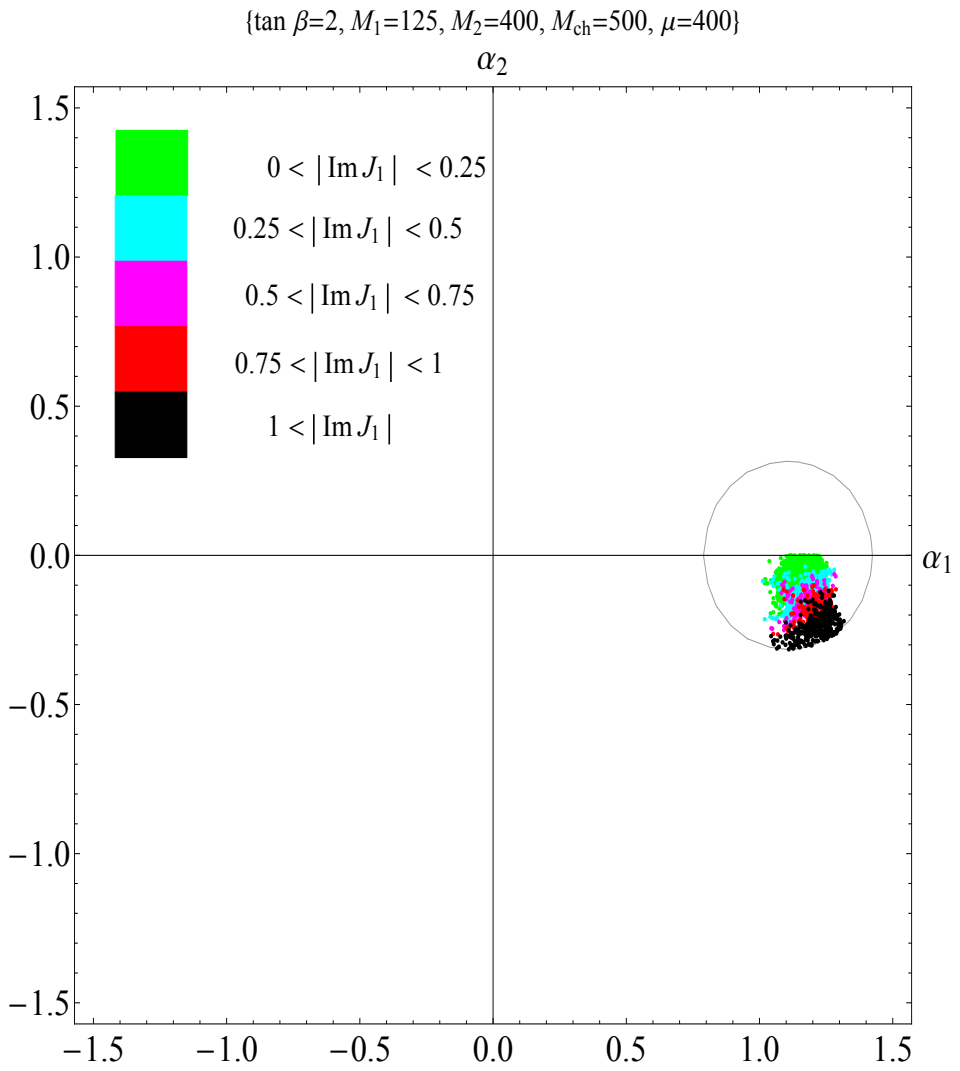
Figure 2: Allowed regions in the (α_1, α_2) space for $\tan\beta = 2$ corresponding to maximal deviation $\delta \equiv |e_1/v - 1| = 0.05$ within 2HDM5 ($\mathbb{Z}_2$ imposed) and 2HDM67, are shown in the left and right panels, respectively. Coloring corresponds to ranges of δ shown in the legend. Vacuum stability and unitarity constraints are satisfied. Parameters adopted are shown in the plot.

The goal is to see

How much CP violation remain
for a given maximal deviation $\delta \equiv |e_1/v - 1|$ from the H1SM limit?

$\Downarrow$

For points inside "circles" we calculate $\mathrm{Im}\, J_1$, $\mathrm{Im}\, J_2$ and $\mathrm{Im}\, J_{30}$



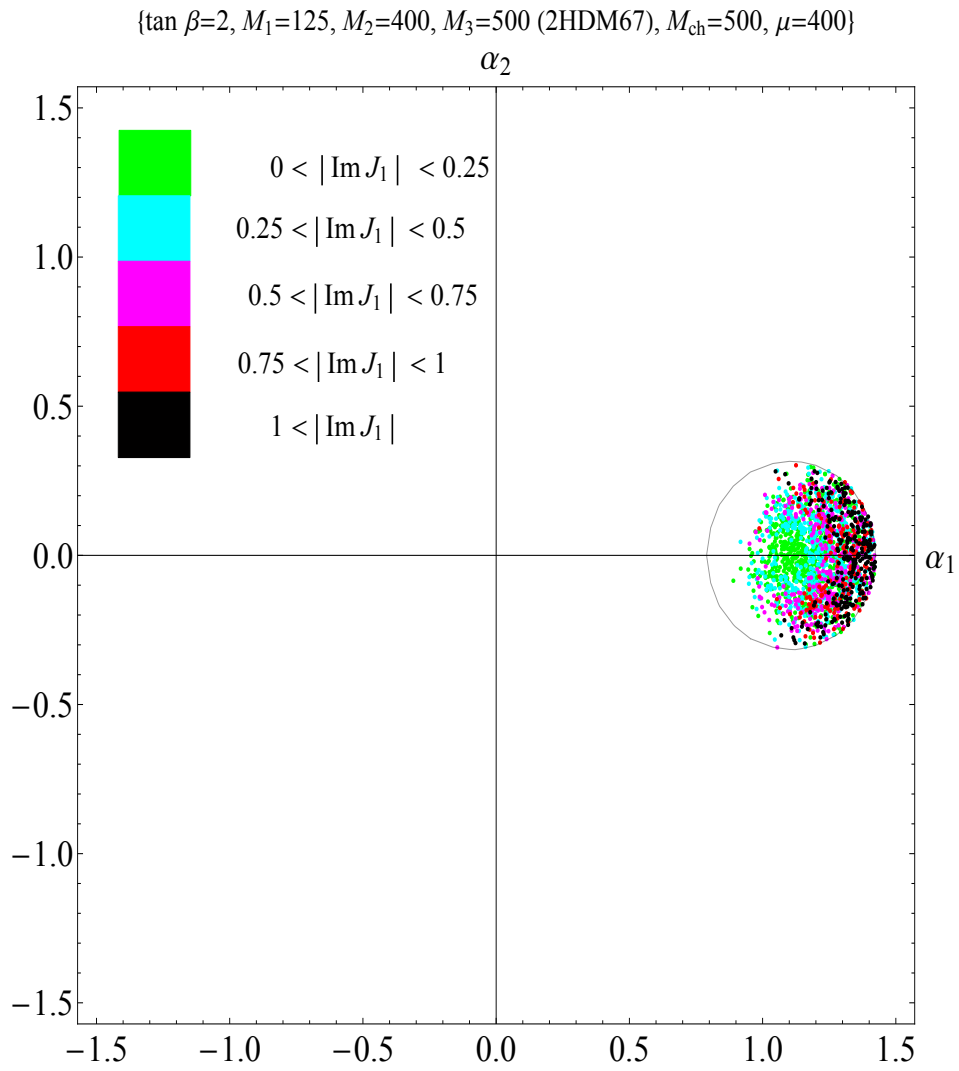
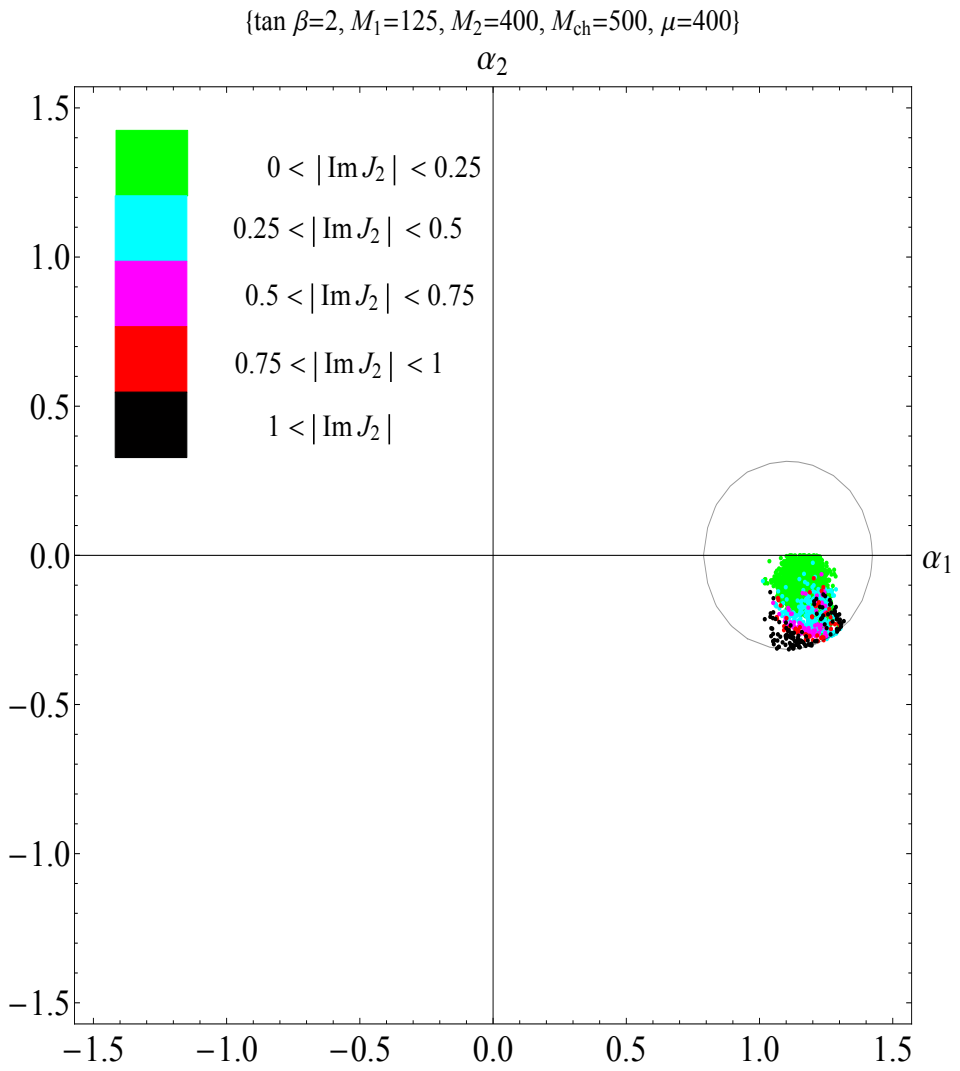


Figure 3: Allowed regions in the (α_1, α_2) space for $\tan\beta = 2$ corresponding to maximal deviation $\delta \equiv |e_1/v - 1| = 0.05$ within 2HDM5 ($\mathbb{Z}_2$ imposed) and 2HDM67, are shown in the left and right panels, respectively. Coloring corresponds to ranges of $|\mathrm{Im}\, J_1|$ shown in the legend. Vacuum stability and unitarity constraints are satisfied. Parameters adopted are shown in the plot.



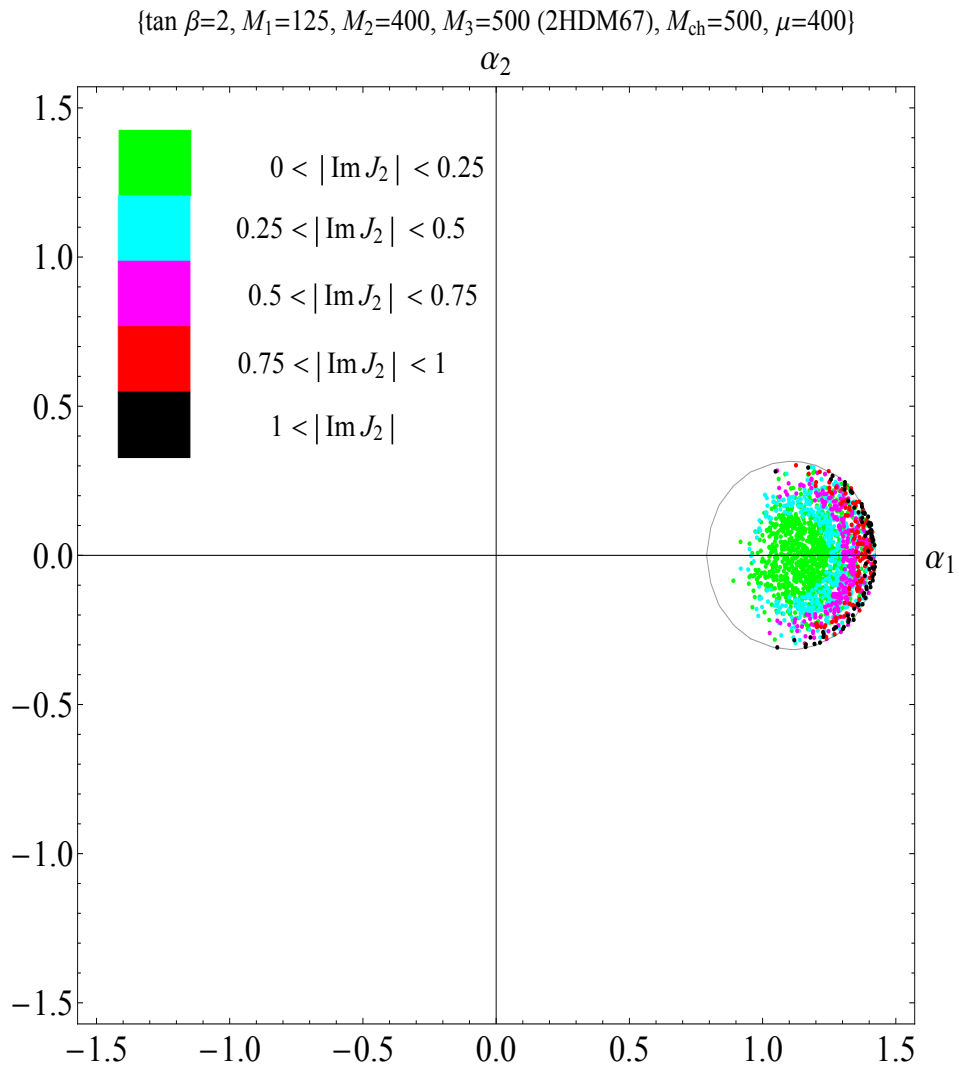
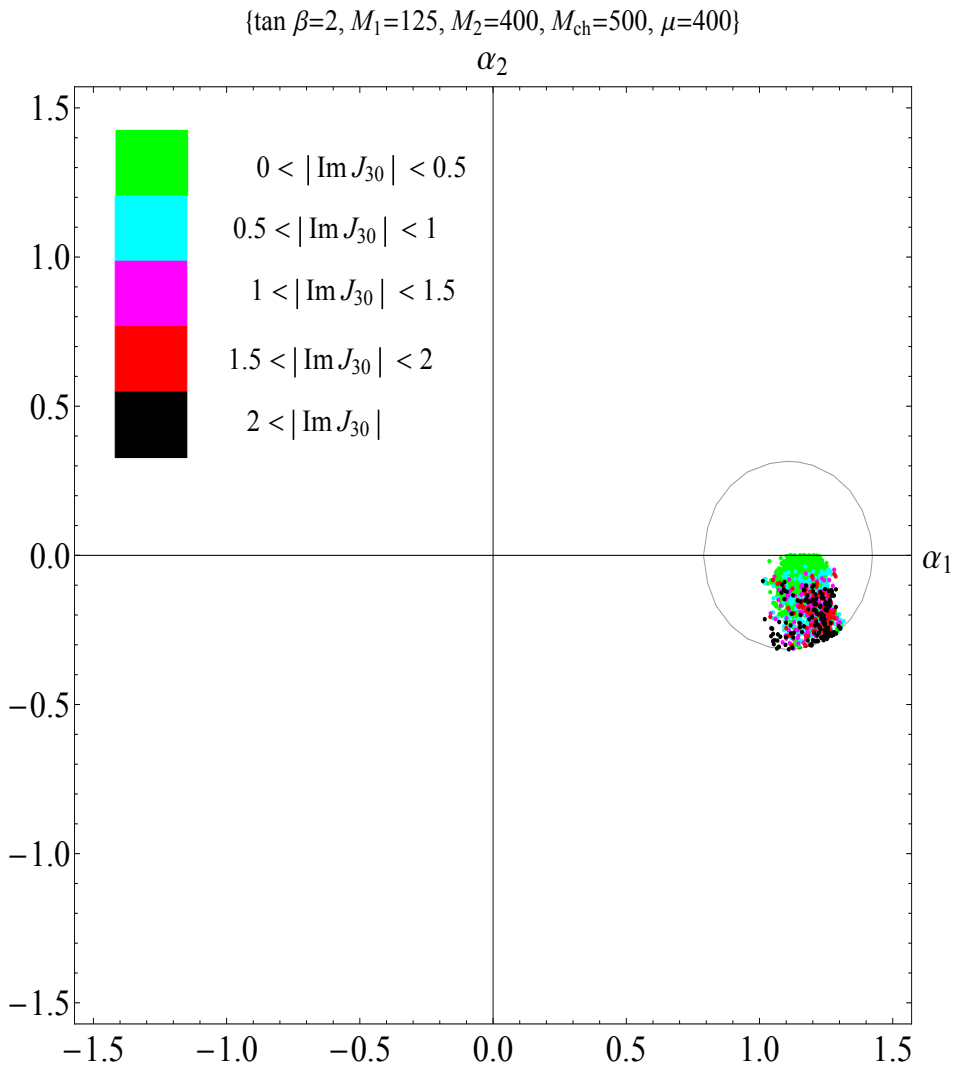


Figure 4: Allowed regions in the (α_1, α_2) space for $\tan\beta = 2$ corresponding to maximal deviation $\delta \equiv |e_1/v - 1| = 0.05$ within 2HDM5 ($\mathbb{Z}_2$ imposed) and 2HDM67, are shown in the left and right panels, respectively. Coloring corresponds to ranges of $|\mathrm{Im}\, J_2|$ shown in the legend. Vacuum stability and unitarity constraints are satisfied. Parameters adopted are shown in the plot.



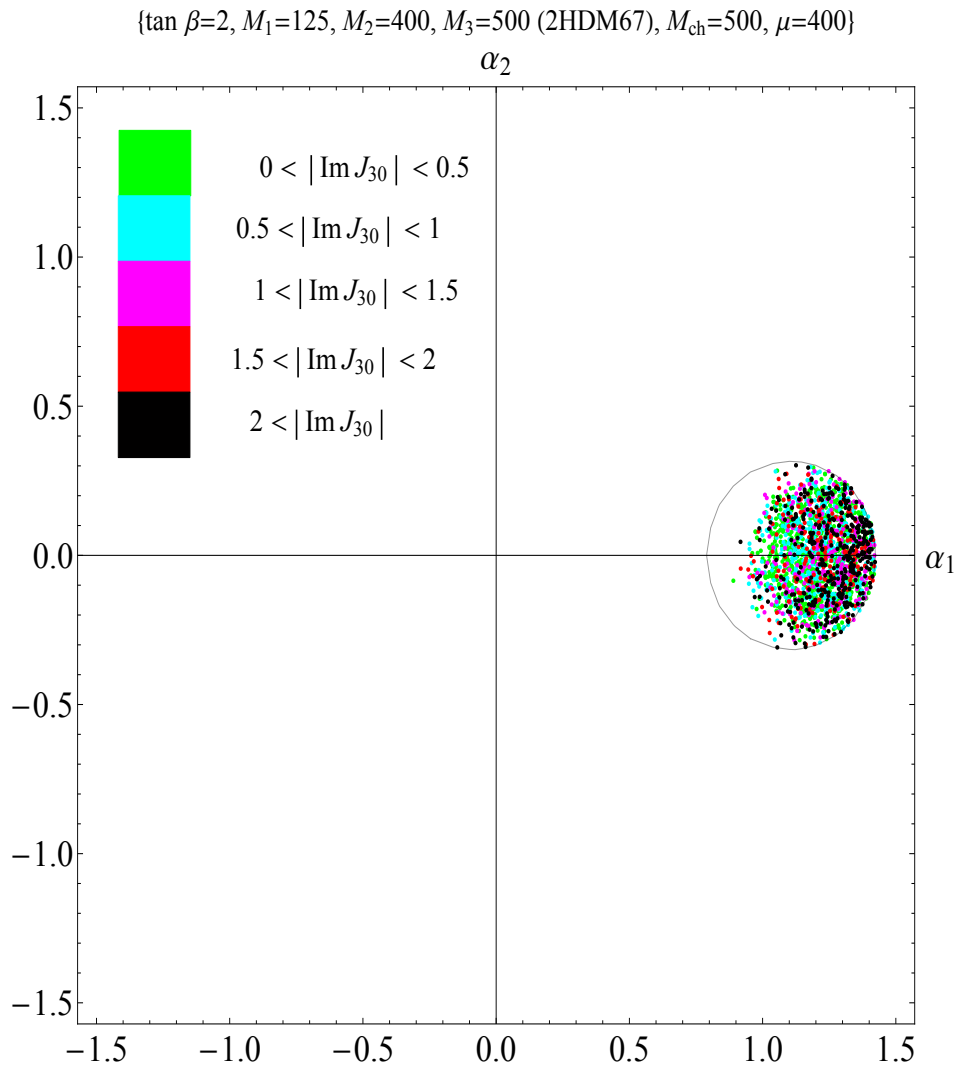


Figure 5: Allowed regions in the (α_1, α_2) space for $\tan\beta = 2$ corresponding to maximal deviation $\delta \equiv |e_1/v - 1| = 0.05$ within 2HDM5 ($\mathbb{Z}_2$ imposed) and 2HDM67, are shown in the left and right panels, respectively. Coloring corresponds to ranges of $|\mathrm{Im}\, J_3|$ shown in the legend. Vacuum stability and unitarity constraints are satisfied. Parameters adopted are shown in the plot.

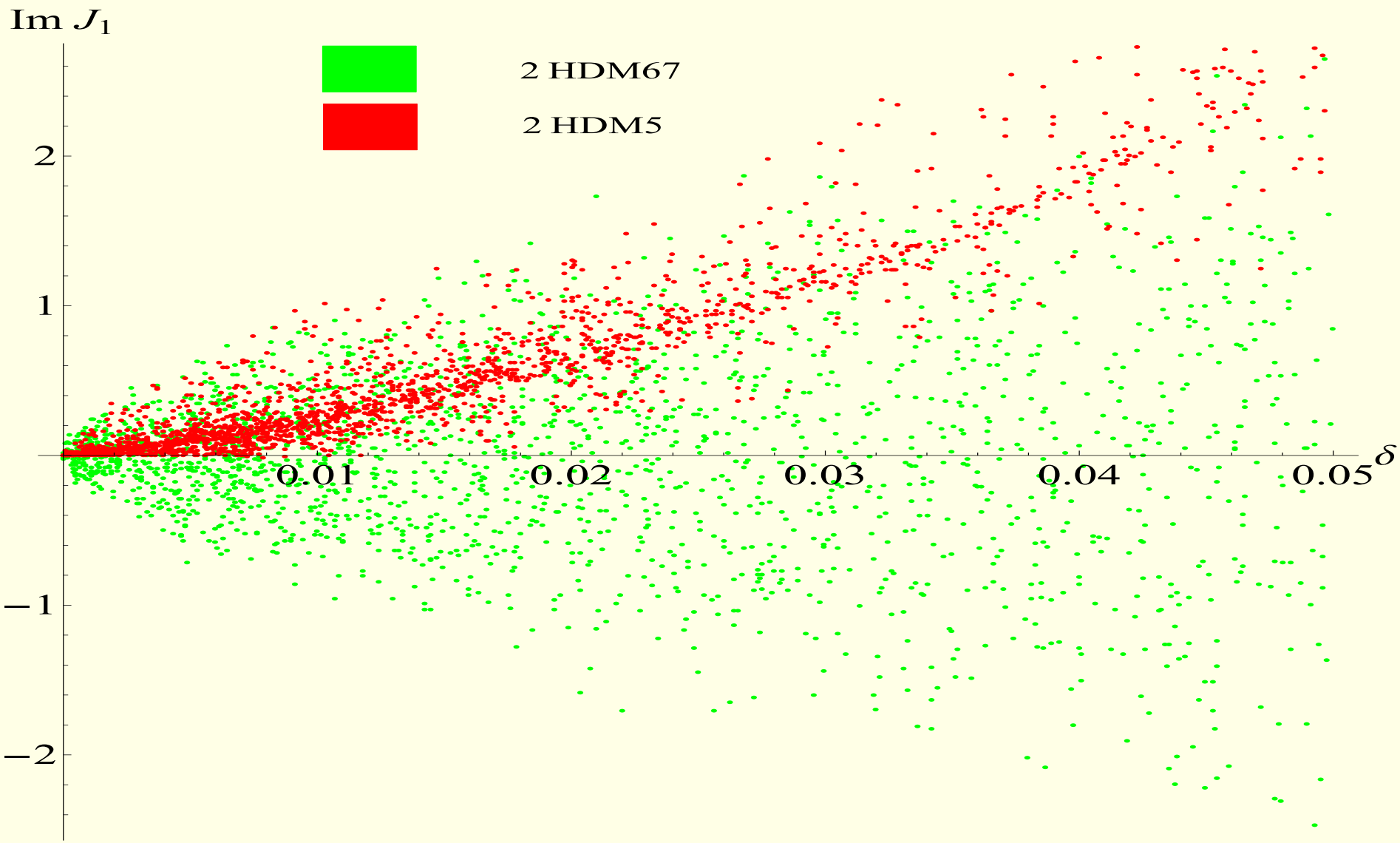


Figure 6: Correletion between $\mathrm{Im}\, J_1$ and the maximal deviation $\delta \equiv |e_1/v - 1| = 0.05$. Green and red dots correspond to 2HDM67 and 2HDM5, respectively.

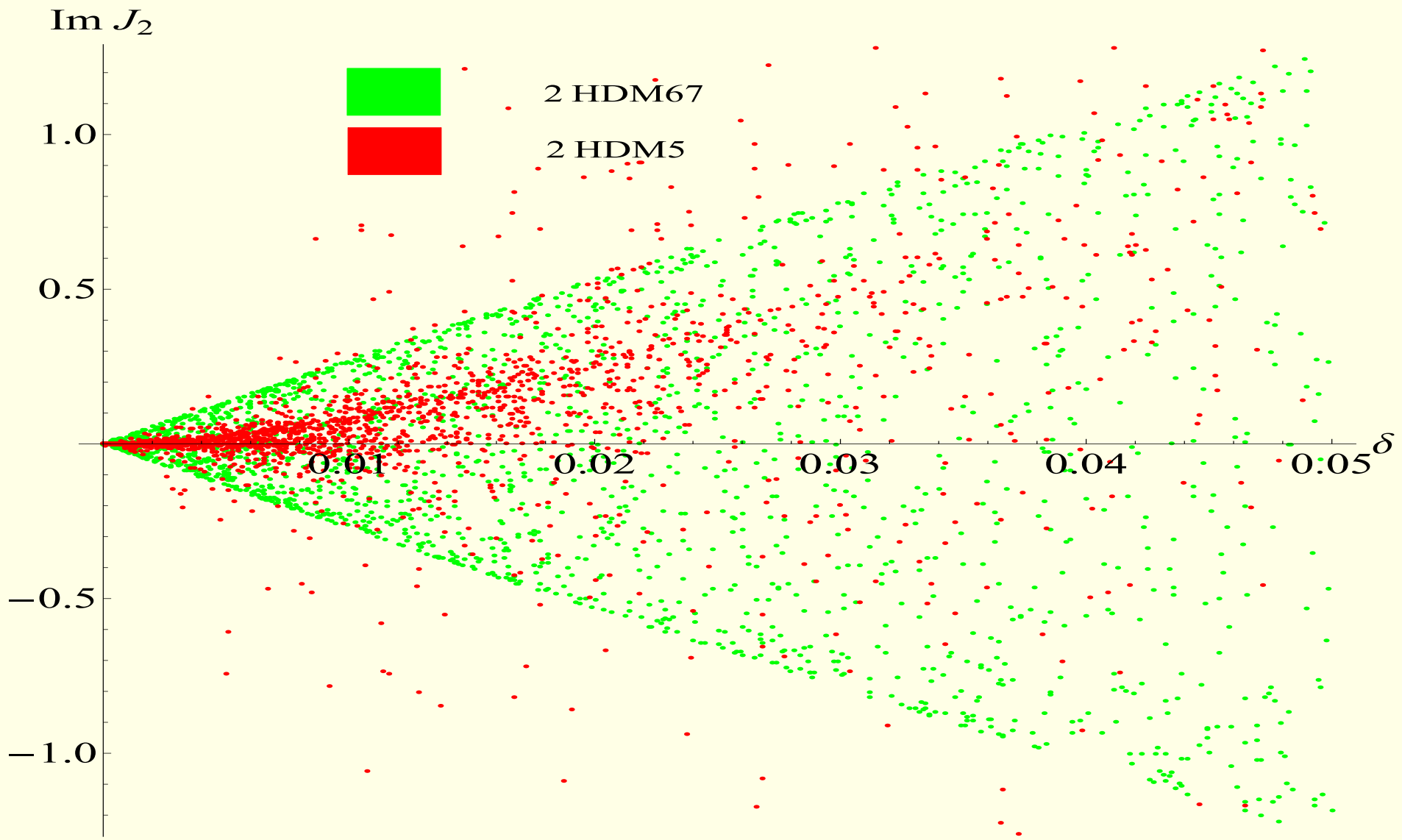


Figure 7: Correletion between $\mathrm{Im}\, J_2$ and the maximal deviation $\delta \equiv |e_1/v - 1| = 0.05$. Green and red dots correspond to 2HDM67 and 2HDM5, respectively.

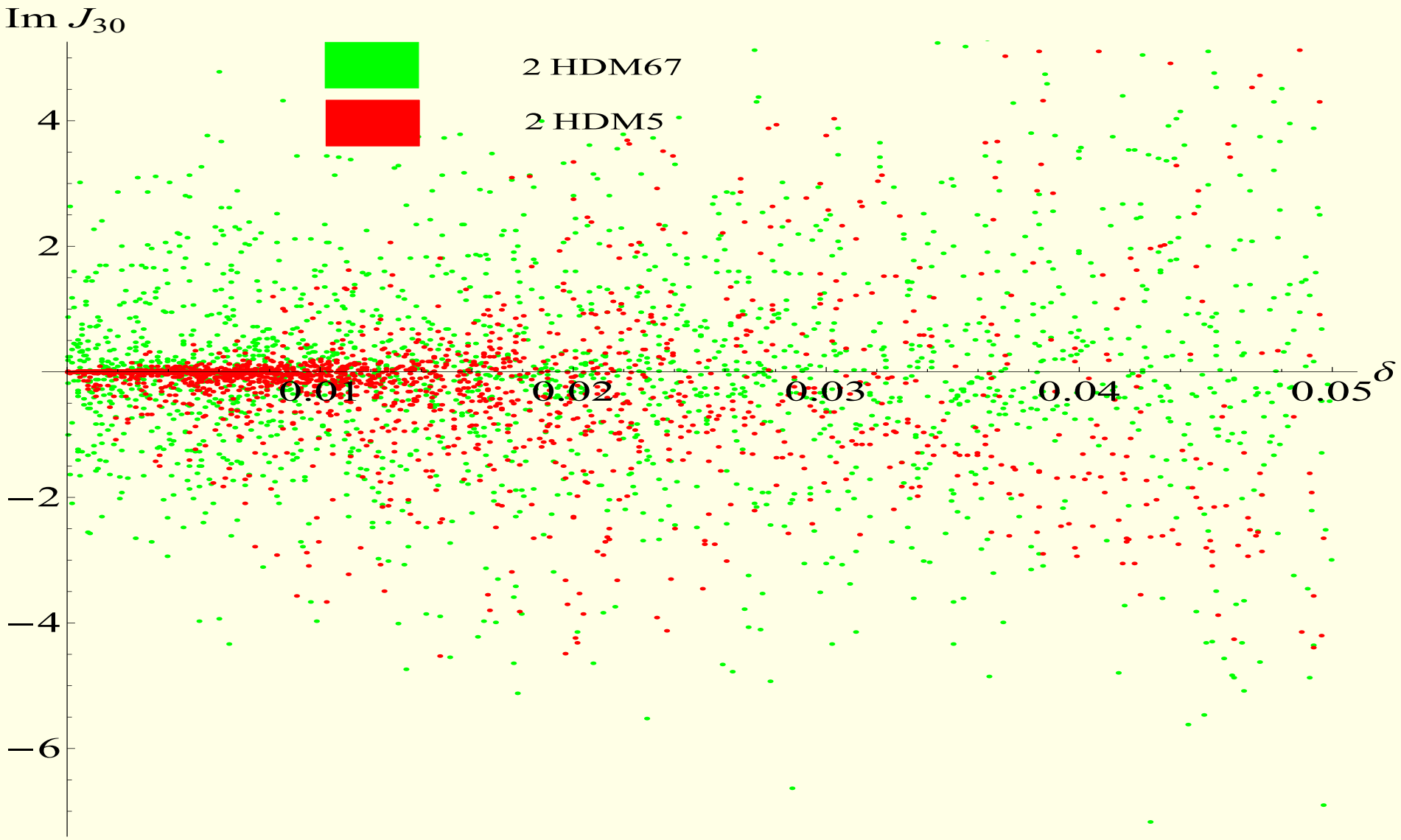


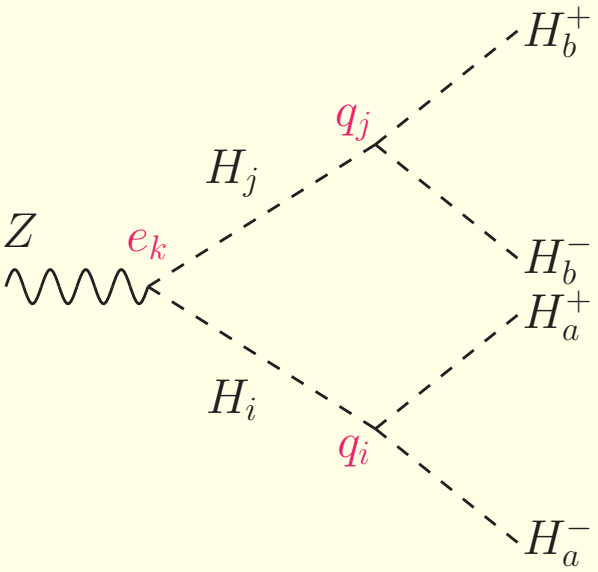
Figure 8: Correletion between $\mathrm{Im}\, J_{30}$ and the maximal deviation $\delta \equiv |e_1/v - 1| = 0.05$. Green and red dots correspond to 2HDM67 and 2HDM5, respectively.

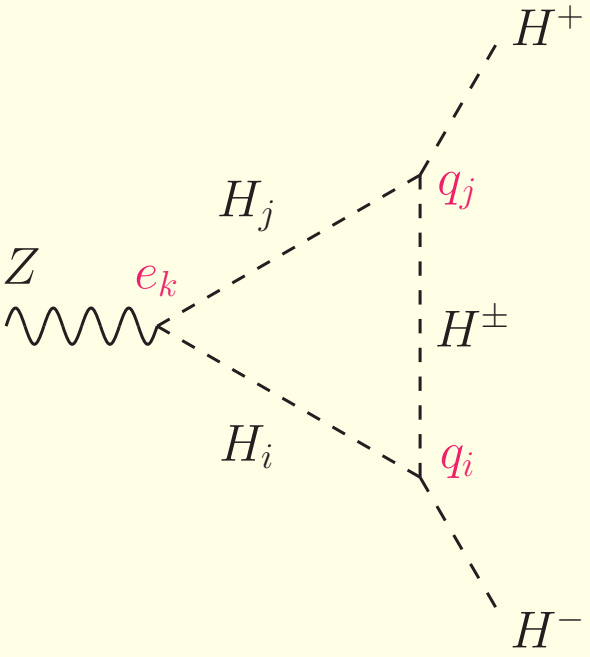
Prospect for measuring CP violation

In the H1SM limit ($e_1 = v, e_2 = 0, e_3 = 0$):

$$\mathrm{Im}\, J_{30} \to \frac{e_1 q_2 q_3}{v^3}(M_3^2 - M_2^2)$$

Couplings needed: e_1, q_2 and q_3





$$\mathcal{M} \propto \mathrm{Im}\, J_{30}$$

Summary

- 2HDM allows for extra sources of CP-violation that might be useful to explain baryon asymmetry.
- We have defined the H1SM limit as $e_1 = v$, so that H_1 couples to VV as in the SM.
- In the H1SM limit there is no CP-violation if $\lambda_6 = \lambda_7 = 0$ ($\mathbb{Z}_2$ imposed).
- The requirement of extra sources of CP-violation in the presence of light extra scalars favours the most general 2HDM with $\lambda_6 \neq 0$ and $\lambda_7 \neq 0$ (no $\mathbb{Z}_2$ symmetry).
- The requirement of extra sources of CP-violation in the presence of light extra scalars implies an interesting possibility of large FCNC that couple to Higgs bosons (in progress).