Generalized Geometry and D-brane

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History

- D-brane and its effective action

It is known that the D-brane effective action is given by DBI action. The symmetry of the DBI action is diffeomorphism on the world volume and gauge transformation of the gauge field on it. Recently it was argued that the DBI action is invariant under a bigger symmetry [Gliozzi 2011], the non-linearly realized Poincare symmetry of the target space. The way they found this result is not systematic, "try and error method", but rather motivated by the **naive expectations**:

Poincare symmetry is spontaneously broken and broken symmetry can be realized nonlinearly, with NG boson.

Broken symmetry $\quad Poincare\ symmetry \quad \epsilon^i + \omega^{ia}$

NG boson $\quad Scalar\ \Phi^i$

The form of Effective action is restricted.

Contents

Our expectation:

- The NG bosons are the scalar bosons $\Phi^i(\sigma^a)\ (a = 0 \cdots p,\ i = p+1 \cdots D-1)$
- T-duality exchanges scalar bosons with gauge bosons $A_a(\sigma^a)$, so these should be also some kind of NG bosons, and there should be the related broken symmetries and then the non-linearly realized symmetries.

Let’s find those symmetries!

Use of Geometry

- Gliozzi's pioneering work was clarified by Casalbuoni et al., using a geometrical approach.

Gliozzi's mysterious transformation of $A_a(\sigma^a)$ under broken Lorentz symmetry is a "compensating diffeomorphism"

They could derive the same transformation rules systematically and concluded.

- The gauge field is a covariant field of the non-linear realization of the Poincare group. So not a NG mode.

We need a proper geometrical setting to realize our expectation.

- For this purpose, we need Geometry with proper T-duality action: Generalized Geometry works well!

Questions

- Can Gauge boson be understood as NG boson?
- What is the broken symmetry?
- Is broken symmetry nonlinearly realized?
- How far is that bigger symmetry characterize the effective action of the D-brane?

- What is the good geometric picture of the D-brane?

2. Generalized Geometry

Short review

- The generalized tangent bundle over the target space M : conbination of tangent and cotangent bundle

$$\mathbb{T}M = TM \oplus T^*M \ \rightarrow \ M$$

$$v + \xi \in \Gamma(\mathbb{T}M), \ v \in \Gamma(TM), \ \xi \in \Gamma(T^*M)$$

anchor map $\pi : \Gamma(\mathbb{T}M) \to \Gamma(TM), \ \pi(v+\xi) = v$

Inner product : $\langle u+\xi, v+\eta \rangle = \frac{1}{2}(\imath_u \eta + \imath_v \xi)$ $\quad O(D,D)$

Lie bracket $\rightarrow$ Dorfman bracket

$$[u+\xi, v+\eta] = [u,v] + \mathcal{L}_u \eta - \imath_v d\xi,$$

- Generalization of

Lie algebroid $\rightarrow$ Courant algebroid

$\Gamma(TM)$ $\qquad$ $\Gamma(\mathbb{T}M)$

Not antisymmetric
no Jacobi
but Leibniz like

Symmetry of courant algebroid $\mathbb{T}M$

- Diff(M): For a diffeo. $f : M \to M$,

$$u + \xi \mapsto f_*(u) + f^{*-1}(\xi),$$

- B-transformation: For $B \in \Omega^2_{\text{closed}}(M)$,

$$e^B(u + \xi) \mapsto u + \xi + \imath_u B,$$

Shift of 1-form

Note: for general B, H-twisted Courant algebroid

Summary

	Differential geometry	Generalized Geometry
	TM	$TM \oplus T^*M$
	$v = v^M \partial_M$	$v + \xi = v^M \partial_M + \xi_M dx^M$
	Lie bracket $[\cdot, \cdot]$	Dorfman bracket $[\cdot, \cdot]$
	Lie algebroid	Courant algebroid
symmetry	Diff.	$\mathrm{Diff}(M) \ltimes \Omega^2_{\mathrm{closed}}(M)$
generaotr	$\mathcal{L}_v$	$\mathcal{L}_{v+\xi}$

3. D-brane as Dirac structure

- Reasons why GOOD Geometric characterizaion of D-brane is Dirac structure

Standard description of D-brane:

Hyper surface with line bundle

- Embedding $\varphi \ : \ \Sigma \ni \sigma^a \ \hookrightarrow \ x^M(\sigma) \in M$

Fluctuations:

- Scalar fields Φ^i (in static gauge): transverse displacements

- Gauge fields A_a:

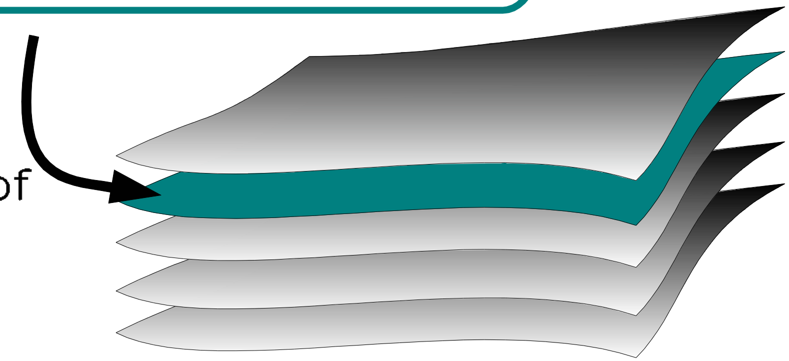
In static gauge: they are function of $x^a = \sigma^a$

Geometrical image of D-brane

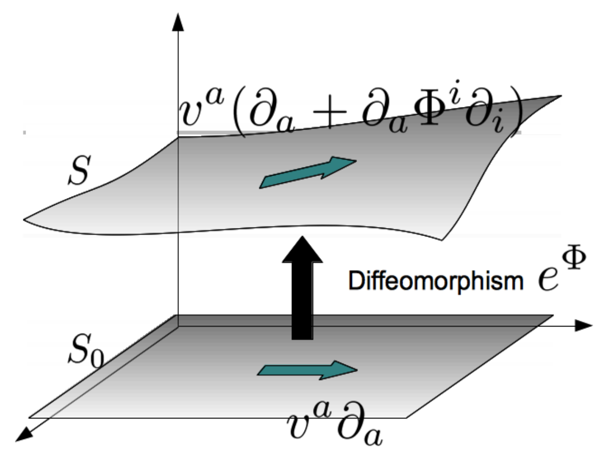
Static gauge: $\Phi^i(x^a)$ x^i independent

a D-brane = a leaf

Fit with the foliation picture



Mathematically, a Lie algebroid (Lie bracket of $v \in \Gamma(TS)$) defines a foliation.



Scalar fields Φ^i define a vector field defined on the whole target space M, and generates a diffeomorphism $-\mathcal{L}_\Phi$.

Dirac structure

A Dirac structure is a subbundle $L \subset \mathbb{T}M$ of rank D such that

- Isotropic: L is self orthogonal $L = L^{\perp}$ $\langle a, b \rangle = 0$ for $\forall a, b \in \Gamma(L)$
- Involutive: L is closed under the Dorfman bracket

Example:

Span$\{\partial_0, ..., \partial_p, dx^1 \cdots dx^{D-1}\}$ ($\sim$ flat D-p brane)

On the Dirac subbundle, Dorfman [,] becomes antisymmetric and Lie algebroid.

Special subbundle in G.G.

 (Lie (anti) + anchor+Jacobi)

D-brane in Generalized Geometry

D-brane = sub-bundle → Dirac structure

Subbundle $L = \text{span}\{\partial_a, dx^i\} \subset \mathbb{T}M$: A section has the form

$$V_L = v^a(x)\partial_a + \xi_i(x)dx^i \quad \in L.$$

Fluctuations are unified as Generalized Lie derivative $\mathcal{L}_{\Phi+A}$. Fluctuated brane is

$$L_{\mathcal{F}} = e^{-\mathcal{L}_{\Phi+A}} L \subset \mathbb{T}M,$$

The sections of $L_{\mathcal{F}}$ can also be represented as a graph:

$$V + \mathcal{F}(V) = v^a(x)(\partial_a + \partial_a\Phi^i\partial_i + F_{ab}dx^b) + \xi_i(x)(dx^i - \partial_a\Phi^i dx^a), \quad V \in \Gamma(L)$$

$$\mathcal{F} = F_{ab}dx^a \wedge dx^b + \partial_a\Phi^i dx^a \wedge \partial_i \quad \in \Gamma(\wedge^2 L^*), \quad \text{F=dA}$$

Symmetry of the Dirac structure

Generalized diffeo. $\mathcal{L}_{\epsilon+\lambda}$, Diffeo. $\ltimes$ B-field gauge transformation:

$$\epsilon = \epsilon_{||} + \epsilon_{\perp} = \epsilon^M \partial_M = \epsilon^a \partial_a + \epsilon^i \partial_i$$
$$\Lambda = \Lambda_{\perp} + \Lambda_{||} = \Lambda_M dx^M = \Lambda_i dx^i + \Lambda_a dx^a.$$

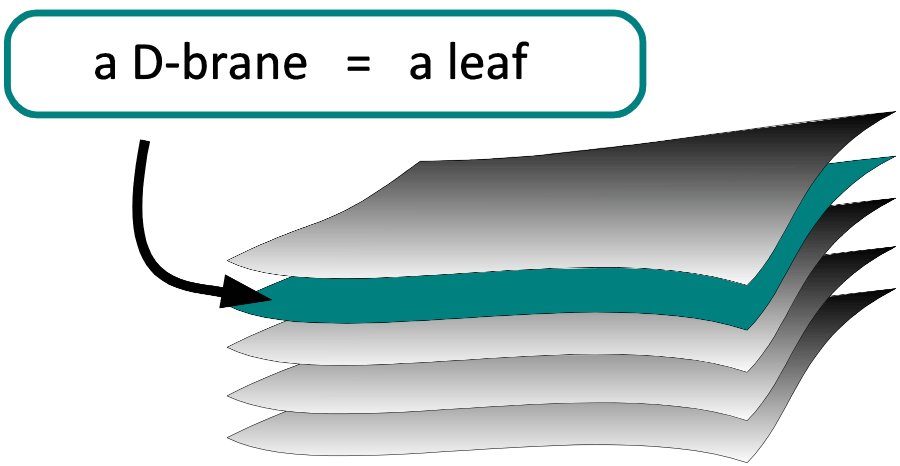
Note that $\epsilon_{||} + \Lambda_{\perp} \in L$ and $\epsilon_{\perp} + \Lambda_{||} \in L^*$.

Symmetry:

1. Generalized Diffeo which preserve foliation:

2. Generalized Diffoe which preserve leaf:

$$L\text{-}Diff \subset F\text{-}Diff$$



Non-linear transformation

F-Diff is the full symmetry of D-brane

Putting the D-brane as a Leaf, the symmetry is broken to

F-Diff → L-Diff

then NG boson, nonlinear realization follow.

We sprit the generalized diffeo as

$$-\mathcal{L}_{\epsilon+\Lambda}(V+\mathcal{F}(V)) = -\mathcal{L}_{\epsilon+\Lambda}V - \mathcal{L}_{\epsilon+\Lambda}(\mathcal{F}(V))$$

$$\stackrel{\text{def.}}{=} \delta V^{(L)} + \mathcal{F}(\delta V^{(L)}) + (\delta\mathcal{F})(V).$$

$$\begin{aligned} \delta A_a &= \Lambda_a - \epsilon^c F_{ca} + \Lambda_k \partial_a \Phi^k, \\ \delta \Phi^i &= \epsilon^i - \epsilon^c \partial_c \Phi^i. \end{aligned}$$

Unified picture of scalar boson and gauge boson

Symmetry of the DBI action

We show that the DBI action has a symmetry under the transformation given of full $\mathrm{Diff}(M) \ltimes \Omega^2_{\mathrm{closed}}(M)$ transformation.

We need to see the metric structure in Generalized Geometry.

Metric can be defined by specifying subbundle like Dirac structure.

See this subbundle from different Dirac structure, we see the T-duality of g+B and find various metric on D-brane

Various metrics

As a map $TM \rightarrow T^*M$

$$E = g + B \qquad G = \begin{pmatrix} -Bg^{-1} & g^{-1} \\ g - Bg^{-1}B & g^{-1}B \end{pmatrix}$$

As a map $L \rightarrow L^*$

$$t = s + a \qquad G = \begin{pmatrix} -s^{-1}a & s^{-1} \\ s - as^{-1}a & as^{-1} \end{pmatrix},$$

With fluctuation

$$t_{\mathcal{F}} = s + a + \mathcal{F} \quad G = \begin{pmatrix} -s^{-1}(a+\mathcal{F}) & s^{-1} \\ s - (a+\mathcal{F})s^{-1}(a+\mathcal{F}) & (a+\mathcal{F})s^{-1} \end{pmatrix}$$

Note that $s_{\mathcal{F}} = s - (a+\mathcal{F})s^{-1}(a+\mathcal{F})$

Symmetry of the DBI action

Using the product representation of the DBI action and the transformation rule of the determinants we can prove the invariance of the DBI action under Non-linear symmetry.

$$\det^{\frac{1}{4}} g \det^{\frac{1}{4}} s_{\mathcal{F}} = \sqrt{\det(\varphi^*_\Phi(g+B) - F)_{ab}} = \mathcal{L}_{DBI}.$$

$$S_{\mathrm{DBI}} = \int_M \mathcal{L}_{\mathrm{DBI}}\ \delta^{(D-p-1)}(x^i - \Phi^i(x^a))\ dx^0 \wedge \cdots \wedge dx^{D-1},$$

$$\delta\mathcal{L}_{\mathrm{DBI}} = -\epsilon^M \partial_M \mathcal{L}_{\mathrm{DBI}} - (\partial_c \epsilon^c + \partial_c \Phi^k \partial_k \epsilon^c)\mathcal{L}_{\mathrm{DBI}}.$$

Discussion

- D-brane as Dirac structre
- Broken symmetry in Target space
- NG boson Φ^i and A_a
- Non-linear transformation
- Invariance of DBI action

- Understand the multiple D-branes
- Unified description of the D-brane from various Dirac structure. This relates to Seiberg-Witten map. [In preparation.]

See also [Schupp et al.]