Noncommutative field theory on $\mathbb{R}^3_\lambda$

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- The easiest one, which is considered here is the one mimicking $\mathfrak{su}(2)$ algebra

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[Hammou,Lagraa,SheikhJabbari PRD 2002] [GraciaBondia, Lizzi, Marmo, Vitale JHEP 2002]

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- It is a subalgebra of the Wick-Voros algebra $\mathbb{R}^4_\theta$, a variation of the Moyal algebra, which exploits the well known realization of three-dimensional Lie algebras as Poisson subalgebras of quadratic-linear functions on $\mathbb{R}^4 \simeq \mathbb{C}^2 \qquad (\mathfrak{isp}(4))$

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- $\mathbb{R}^3_\lambda$ is generated by coordinate functions x^μ

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- it is based on the identification of $\mathbb{R}^3$ with $\mathfrak{g}^*$. Here $\mathfrak{g} = \mathfrak{su}(2)$
- Besides being a Poisson subalgebra, it is also a NC subalgebra wrt the Wick-Voros (and Moyal) star product

$$\phi \star \psi\,(z_a, \bar{z}_a) = \phi(z,\bar{z})\exp(\theta \overleftarrow{\partial}_{z_a}\overrightarrow{\partial}_{\bar{z}_a})\psi(z,\bar{z}), \quad a = 1,2$$

$$[z_a, \bar{z}_b]_\star = \theta\delta_{ab}$$

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$$[x_i \star x_j] = i\lambda\epsilon_{ij}^k x_k$$

x_0 $\star$-commutes with x_i so that we can alternatively define $\mathbb{R}^3_\lambda$ as the $\star$-commutant of x_0; x_0 generates the center of the algebra.

▸ the matrix base

The Wick-Voros product

The Wick-Voros product is introduced through a weighted quantization map which, in two dimensions, associates to functions on the complex plane the operator (Berezin quantization)

$$\hat{\phi} = \hat{\mathcal{W}}_V(\phi) = \frac{1}{(2\pi)^2} \int \mathrm{d}^2 z \, \hat{\Omega}(z, \bar{z}) \phi(z, \bar{z})$$

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where

$$\hat{\Omega}(z, \bar{z}) = \int \mathrm{d}^2 \eta \, e^{-(\eta \bar{z} - \bar{\eta} z)} e^{\theta \eta a^\dagger} e^{-\theta \bar{\eta} a}$$

$a, a^\dagger$ are the usual (configuration space) creation and annihilation operators, with commutation relations

$$[a, a^\dagger] = \theta.$$

The inverse map which is the analogue of the Wigner map is represented by:

$$\phi(z,\bar{z}) = \mathcal{W}_V^{-1}(\hat{\phi}) = \langle z|\hat{\phi}|z\rangle$$

with $|z\rangle$ the *coherent states* defined by $a|z\rangle = z|z\rangle$.

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The Wick-Voros product is then defined as

$$\phi \star \psi := \mathcal{W}_V^{-1}\left(\hat{\mathcal{W}}_V(\phi)\hat{\mathcal{W}}_V(\psi)\right) = \langle z|\hat{\phi}\,\hat{\psi}|z\rangle$$

Unlike the Moyal product

$$\int \phi \star \psi = \int \psi \star \phi \neq \int \phi \cdot \psi$$

the algebra $\mathbb{R}^3_\lambda$

The Wick-Voros matrix base for $\mathbb{R}^4_\theta$

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$$\phi(\bar{z}, z) = \sum_{pq} \tilde{\phi}_{pq} \bar{z}^p z^q \,, \quad p, q \in \mathbb{N} \quad \tilde{\phi}_{pq} \in \mathbb{C}$$

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$$\begin{aligned} a_1|n_1,n_2\rangle &= \sqrt{\theta}\sqrt{n_1}|n_1-1,n_2\rangle, \ a_1^\dagger|n\rangle = \sqrt{\theta}\sqrt{n_1+1}|n_1+1,n_2\rangle, \\ a_2|n_1,n_2\rangle &= \sqrt{\theta}\sqrt{n_2}|n_1,n_2-1\rangle, \ a_2^\dagger|n\rangle = \sqrt{\theta}\sqrt{n_2+1}|n_1,n_2+1\rangle \end{aligned}$$

Wick-Voros product

we get

$$\hat{\phi} = \sum_{P,Q\in\mathbb{N}^2} \phi_{PQ} |P\rangle\langle Q| \quad \phi_{PQ} \in \mathbb{C} \quad |P\rangle := |p_1, p_2\rangle$$

$$|P\rangle = \frac{a_1^{\dagger p_1} a_2^{\dagger p_2}}{[P!\theta^{|P|}]^{1/2}} |0\rangle, \quad \forall P = (p_1, p_2) \in \mathbb{N}^2,$$

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$$f_{PQ}(z, \bar{z}) = \langle z_1, z_2 | \hat{f}_{PQ} | z_1, z_2\rangle = \frac{e^{-\frac{\bar{z}_1 z_1 + \bar{z}_2 z_2}{\theta}}}{\sqrt{P!Q!\theta^{|P+Q|}}} \bar{z}_1^{p_1} \bar{z}_2^{p_2} z_1^{q_1} z_2^{q_2}$$

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with $\hat{f}_{PQ} := |P\rangle\langle Q|$ and usual nice properties

$$\begin{aligned} f_{MN} \star f_{PQ}(z,\bar{z}) &= \delta_{NP} f_{MQ}(z,\bar{z}) \\ \int d^2 z_1 d^2 z_2 \, f_{PQ}(z,\bar{z}) &= (\pi\theta)^2 \delta_{PQ} \end{aligned}$$

The star product becomes a matrix product

$$\phi \star \psi(z, \bar{z}) = \sum \phi_{MN} \psi_{PQ} f_{MN} \star f_{PQ} = \sum \phi_{MP} \psi_{PQ} f_{MQ}$$

and the integral becomes a trace

$$\int \phi \star \psi \star \ldots = (\pi\theta)^2 \,\mathrm{Tr}\, \Phi\Psi \ldots$$

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Consider the number operators $\hat{N}_1 = a_1^\dagger a_1, \ \hat{N}_2 = a_2^\dagger a_2$ with eigenvalues n_1, n_2.

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with $j(j+1)$ and m eigenvalues of $\hat{X}_i \hat{X}_i$ and $\hat{X}_3$ resp.

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$$\hat{f}_{NP} = |n_1, n_2 >< p_1, p_2| \longrightarrow |j+m, j-m >< \tilde{\jmath} + \tilde{m}, \tilde{\jmath} - \tilde{m}| \equiv \hat{v}^{j\tilde{\jmath}}_{m\tilde{m}}$$

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$$f_{NP}(\bar{z}, z) \longrightarrow v^{j\tilde{\jmath}}_{m\tilde{m}}(\bar{z}, z)$$

For this to be a base in $\mathbb{R}^3_\lambda$ we impose it to $\star$-commute with x_0

$$x_0 \star v^{j\tilde{j}}_{m\tilde{m}}(z,\bar{z}) - v^{j\tilde{j}}_{m\tilde{m}} \star x_0(z,\bar{z}) = \lambda(j-\tilde{j})v^{j\tilde{j}}_{m\tilde{m}}$$

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This fixes $j = \tilde{j}$. We have then

$$\phi(x_i, x_0) = \sum_j \sum_{m,\tilde{m}=-j}^{j} \phi^j_{m\tilde{m}} v^j_{m\tilde{m}}$$

with

$$v^j_{m\tilde{m}} := v^{jj}_{m\tilde{m}} = e^{-\frac{\bar{z}_a z_a}{\theta}} \frac{\bar{z}_1^{j+m} z_1^{j+\tilde{m}} \bar{z}_2^{j-m} z_2^{j-\tilde{m}}}{\sqrt{(j+m)!(j-m)!(j+\tilde{m})!(j-\tilde{m})!\theta^{4j}}}$$

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The star product acquires the simple form

$$v^j_{m\tilde{m}} \star v^{\tilde{j}}_{n\tilde{n}} = \delta^{j\tilde{j}} \delta_{\tilde{m}n} v^j_{m\tilde{n}}$$

$$\int v^j_{m\tilde{m}} \star v^{\tilde{j}}_{n\tilde{n}} = \pi^2\theta^2 \delta^{j\tilde{j}} \delta_{\tilde{m}n} \delta_{m\tilde{n}}.$$

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the infinite matrix Φ gets rearranged into a block-diagonal form, each block being the $(2j+1) \times (2j+1)$ matrix $\Phi^j = \{\phi^j_{mn}\}, \ -j \leq m, n \leq j$.

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the infinite matrix Φ gets rearranged into a block-diagonal form, each block being the $(2j+1) \times (2j+1)$ matrix
$\Phi^j = \{\phi^j_{mn}\}, \ -j \le m, n \le j$.
The integral is defined through the pullback to $\mathbb{R}^4_\theta$

$$\int_{\mathbb{R}^3_\lambda} \phi := \frac{\kappa^3}{\pi^2 \theta^2} \int_{\mathbb{R}^4_\theta} \pi^\star(\phi) = \kappa^3 \sum_j \mathrm{Tr}_j \Phi^j$$

with Tr_j the trace in the $(2j+1) \times (2j+1)$ subspace.

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- The integral as a trace: $\int \phi \star \psi \star \ldots \star \xi = \kappa^3 \sum_j \mathrm{Tr}_j \Phi^j \Psi^j \ldots \Xi^j$

- All derivations of $\mathbb{R}^3_\lambda$ are inner $D_\mu \to [x_\mu, \cdot]_\star$ (D_0 is trivial because $[x_0, f]_\star = 0$ *for* $f \in \mathbb{R}^3_\lambda$)
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- Indeed, the natural Laplacian operator constructed with inner derivations $\sum_\mu [x_\mu, [x_\mu, \phi]_\star]_\star$, reduces to the usual Laplacian on the fuzzy sphere
- we propose

$$\Delta\phi = \alpha \sum_i D_i^2 \phi + \frac{\beta}{\kappa^4} x_0 \star x_0 \star \phi$$

$D_i = \kappa^{-2}[x_i, \; \cdot \;]_\star, \; i = 1, .., 3 \qquad \alpha, \beta$ real parameters and

$$x_0 \star \phi = x_0 \phi + \frac{\lambda}{2} x_i \partial_i \phi$$

contains the dilation operator in the radial direction.

With a slight modification the highest derivative term of the Laplacian can be made proportional to the ordinary Laplacian on $\mathbb{R}^3$, for the parameters α and β appropriately chosen.

$$\begin{aligned}
\sum_i [x_i, [x_i, \phi]_\star]_\star &= \lambda^2 \left[x^i \partial_i (x^j \partial_j \phi + x^i \partial_i \phi)\right] - \lambda^2 x_0^2 \partial^2 \phi \\
x_0 \star x_0 \star \phi + \frac{\lambda}{2} x_0 \star \phi &= \frac{\lambda^2}{4} \left[x^i \partial_i (x^j \partial_j \phi + x^i \partial_i \phi)\right] \\
&+ \lambda x_0 (x^i \partial_i \phi + \phi) + x_0^2 \phi
\end{aligned}$$

With this choice, and $\alpha/\beta = -1/4$, we obtain a term proportional to the ordinary Laplacian, multiplied by x_0^2, plus lower derivatives.

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$$V^{j_1 j_2 j_3 j_4}_{p_1\tilde{p}_1;p_2\tilde{p}_2;p_3\tilde{p}_3;p_4\tilde{p}_4} = \frac{g}{4!}\delta^{j_1 j_2}\delta^{j_2 j_3}\delta^{j_3 j_4}\delta_{\tilde{p}_1 p_2}\delta_{\tilde{p}_2 p_3}\delta_{\tilde{p}_3 p_4}\delta_{\tilde{p}_4 p_1}$$

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x_+ &= \frac{\lambda}{\theta}\bar{z}_1 z_2 = \lambda \sum_{j,m} \sqrt{(j+m)(j-m+1)}\, v^j_{m\,m-1} \\
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x_3 &= \frac{\lambda}{2\theta}(\bar{z}_1 z_1 - \bar{z}_2 z_2) = \lambda \sum_{j,m} m v^j_{m\,m} \\
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and compute

$$
\begin{aligned}
S_k[\phi] &= \kappa^3 \sum \phi^{j_1}_{m_1 \tilde{m}_1} \left(\Delta(\alpha,\beta) + \mu^2 \mathbf{1}\right)^{j_1 j_2}_{m_1 \tilde{m}_1; m_2 \tilde{m}_2} \phi^{j_2}_{m_2 \tilde{m}_2} \\
&= \kappa^3 \, \mathrm{Tr}\left(\Phi(\Delta(\alpha,\beta) + \mu^2 \mathbf{1})\Phi\right)
\end{aligned}
$$

with

$$(\Delta + \mu^2 \mathbf{1})^{j_1 j_2}_{m_1 \tilde{m}_1; m_2 \tilde{m}_2} = \frac{1}{\pi^2 \theta^2} \int v^{j_1}_{m_1 \tilde{m}_1} \star (\Delta(\alpha, \beta) + \mu^2 \mathbf{1}) v^{j_2}_{m_2 \tilde{m}_2}$$

$$= \frac{\lambda^2}{\kappa^4} \delta^{j_1 j_2} \{ \delta_{\tilde{m}_1 m_2} \delta_{m_1 \tilde{m}_2} D^{j_2}_{m_2 \tilde{m}_2} - \delta_{\tilde{m}_1, m_2+1} \delta_{m_1, \tilde{m}_2+1} B^{j_2}_{m_2, \tilde{m}_2}$$

$$- \delta_{\tilde{m}_1, m_2-1} \delta_{m_1, \tilde{m}_2-1} H^{j_2}_{m_2, \tilde{m}_2} \}$$

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- In the matrix base the interaction term is diagonal, the kinetic term is not (cfr. Grosse-Wulkenhaar)
- The action factorizes into an infinite sum of contributions $S[\Phi] = \sum_{j \in \frac{\mathbb{N}}{2}} S^{(j)}[\Phi]$

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$$(P(\alpha,\beta))^{j_1 j_2}_{p_1,\tilde{p}_1;p_2\tilde{p}_2} = \sum_{l=0}^{2j_1} \sum_{k=-l}^{l} \frac{\delta^{j_1 j_2}}{(2j_1+1)(\frac{\lambda^2}{\kappa^4}\gamma + \mu^2)} ({Y^{j_1}_{lk}}^{\dagger})_{p_1\tilde{p}_1} (Y^{j_2}_{lk})_{p_2\tilde{p}_2}$$

with

$$\gamma = \left(\alpha l(l+1) + \beta j^2\right)$$

$$(Y^j_{lk})_{m\tilde m} =< \hat v^j_{m\tilde m} | \hat Y^j_{lk} >= \sqrt{2j+1}(-1)^{j-\tilde m} \left(\begin{array}{cc|c} j & j & l \\ m & -\tilde m & k \end{array} \right)$$

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The scalar action
The propagator

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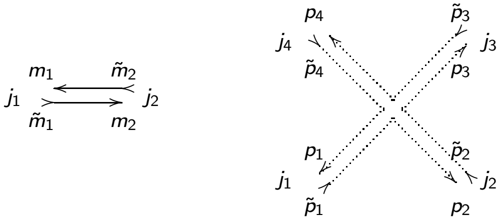
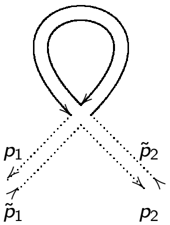
Once we have the propagator and the vertex we can compute correlation functions



(1)

One-loop calculations



Planar diagram contributing to the 2-point correlation function

$$\mathcal{A}^{j_1 j_2\,P}_{p_1\tilde{p}_1;p_2\tilde{p}_2} = \frac{\kappa^4}{\lambda^2}\delta^{j_1 j_2}\delta_{\tilde{p}_1 p_2}\delta_{p_1\tilde{p}_2}\sum_{l=0}^{2j_1}(-1)^{2j_1}\frac{2l+1}{(2j_1+1)(\gamma(j_1,l;\alpha\beta)+\frac{\kappa^4}{\lambda^2}\mu^2)}$$

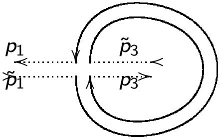
which is finite for all j

In the propagating (fuzzy harmonics) base

$$\tilde{\mathcal{A}}^{j_1 j_2\, P}_{l_1 k_1; l_2 k_2} = \frac{\kappa^4}{\lambda^2}\delta^{j_1 j_2}\sum_{l=0}^{2j_1}\frac{2l+1}{\alpha l(l+1)+\beta j_1^2+\frac{\kappa^4}{\lambda^2}\mu^2}(-1)^{k_2}\delta_{-k_1 k_2}\delta_{l_1 l_2}.$$

When fixing $j_1 = j_2 = j$ and $\beta = 0$ we retrieve the result for the fuzzy sphere

S. Vaidya, Phys. Lett. B **512**, 403 (2001); C. -S. Chu, J. Madore, H. Steinacker, JHEP **0108**, 038 (2001)



Nonplanar diagram contributing to the two-point function

$$\mathcal{A}^{j_1 j_3 \, NP}_{p_1 \tilde{p}_1; p_3 \tilde{p}_3} = \frac{\kappa^4}{\lambda^2} \delta^{j_1 j_3} \sum_{l=0}^{2j_1} \frac{1}{\left(\gamma(j_1, l, \alpha, \beta) + \frac{\kappa^4}{\lambda^2}\mu^2\right)} \times$$

$$\sum_k (-1)^{p_1 + \tilde{p}_1} \left(\begin{array}{cc|c} j_1 & j_1 & l \\ \tilde{p}_3 & -p_1 & k \end{array} \right) \left(\begin{array}{cc|c} j_1 & j_1 & l \\ p_3 & -\tilde{p}_1 & k \end{array} \right)$$

can be seen to be finite for all values of the indices

In the propagating base

$$\tilde{\mathcal{A}}^{j_1 j_2\, NP}_{l_1 k_1; l_2 k_2} = \frac{\kappa^4}{\lambda^2}\delta^{j_1 j_2}\sum_{l=0}^{2j_1}\frac{(2j_1+1)(2l+1)}{\left(\alpha l(l+1)+\beta j_1^2+\frac{\kappa^4}{\lambda^2}\mu^2\right)}\times$$

$$(-1)^{l_1+l+2j_1-k_1}\delta_{l_1 l_2}\delta_{k_1,-k_2}\left\{\begin{array}{ccc} j_1 & j_1 & l_1 \\ j_1 & j_1 & l \end{array}\right\}$$

In agreement with

S. Vaidya, Phys. Lett. B **512**, 403 (2001); C. -S. Chu, J. Madore, H. Steinacker, JHEP **0108**, 038 (2001)

for $j_1 = j_2, \beta = 0$

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- gauge models (in preparation with Antoine Géré and J.-C. Wallet)