

# Theory of Electroweak Interactions

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# Electroweak Standard Model

= Quantum Field Theory

gauge invariant

with spontaneous symmetry breaking

# gauge invariance

- powerful symmetry principle
- determines structure of interactions
- guarantees renormalizability → precision tests

## gauge invariance

- powerful symmetry principle
- determines structure of interactions
- guarantees renormalizability → precision tests

## symmetry breaking

- massive leptons and quarks
- massive vector bosons of the weak interaction

## Electroweak interactions – a successful (hi)story

- the symmetry group  $SU(2) \times U(1)$ 
  - discovery of neutral currents 1973
- the W and Z bosons
  - discovery 1983 at SPS (CERN)
- the coupling structure from local gauge invariance
  - precise measurements at LEP/SLC 1989 - 2000
- the Higgs mechanism and Yukawa interactions
  - top discovery at Tevatron 1995
  - Higgs discovery at LHC 2012

# Outline

1. Gauge Theories
2. Higgs mechanism
3. Electroweak interaction and Standard Model
4. Phenomenology of  $W$  and  $Z$  bosons, precision tests
5. Higgs bosons

# **1. Gauge theories**

# Constructing QED – main steps

- start with  $\mathcal{L}_0 = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi$  for free fermion field  $\psi$   
symmetric under global gauge transformations  
 $\psi' = e^{i\alpha} \psi$ ,  $\alpha$  real
- perform minimal substitution  $\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu$   
 $\Rightarrow$  invariance under local gauge transformations  
 $\psi' = e^{i\alpha(x)} \psi$ ,  $A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \alpha(x)$ 
  - involves additional vector field  $A_\mu$
  - induces interaction between  $A_\mu$  and  $\psi$   
 $e (\bar{\psi} \gamma^\mu \psi) A_\mu \equiv e j^\mu A_\mu$
- make  $A_\mu$  a dynamical field by adding

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

# Non-Abelian gauge theories

Generalization: “phase” transformations that do not commute

$$\psi \rightarrow \psi' = U\psi \quad \text{with} \quad U_1 U_2 \neq U_2 U_1$$

requires **matrices**, i.e.  $\psi$  is a multiplet

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{pmatrix}, \quad U = n \times n \text{-matrix}$$

each  $\psi_k = \psi_k(x)$  is a Dirac spinor

## (i) global symmetry

starting point:  $\mathcal{L}_0 = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi$

where  $\bar{\psi} = (\bar{\psi}_1, \dots, \bar{\psi}_n)$

consider unitary matrices:  $U^\dagger = U^{-1}$

$$\psi' = U\psi \quad \Rightarrow \quad \bar{\psi}' = \bar{\psi} U^\dagger = \bar{\psi} U^{-1}$$

$$\Rightarrow \quad \bar{\psi}'\psi' = \bar{\psi}\psi, \quad \bar{\psi}'\gamma^\mu \partial_\mu \psi' = \bar{\psi}\gamma^\mu \partial_\mu \psi$$

*if  $U$  does not depend on  $x$*

$\Rightarrow \mathcal{L}_0$  is invariant under  $\psi \rightarrow U\psi$

$U$ : global gauge transformation

similar for

scalar fields:

$$\phi \rightarrow \phi' = U\phi, \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{pmatrix}$$

each  $\psi_k = \phi_k(x)$  is a scalar field,  $\phi^\dagger = (\phi_1^\dagger, \dots, \phi_n^\dagger)$

terms  $\phi^\dagger \phi, (\partial_\mu \phi)^\dagger (\partial^\mu \phi)$  are invariant

$\Rightarrow \mathcal{L}_0 = (\partial_\mu \phi)^\dagger (\partial^\mu \phi)$  is invariant

relevant in physics:

*the special unitary  $n \times n$ -matrices with  $\det=1$*

group  $SU(n)$

examples:

$$SU(2) : \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad e.g. \quad \psi = \begin{pmatrix} \psi_\nu \\ \psi_e \end{pmatrix} \quad \text{isospin}$$

$$SU(3) : \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \quad e.g. \quad \psi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix} \quad \text{colour}$$

$SU(n)$

matrices  $U$  can be written as exponentials

$$U(\theta_1, \dots, \theta_N) = e^{i\theta_a T_a} \quad \text{sum over } a = 1, \dots, N$$

$\theta_1, \dots, \theta_N$  : real parameters

$T_1, \dots, T_N$  :  $n \times n$ -matrices, generators,  $T_a^\dagger = T_a$

infinitesimal  $\theta$  :  $U = 1 + i\theta_a T_a \quad (+O(\theta^2))$

N-dimensional Lie Group

det=1 and unitarity

$$\Rightarrow \boxed{N = n^2 - 1}$$

$n = 2$  :  $N = 3$ ,  $n = 3$  :  $N = 8$

commutators     $[T_a, T_b] \neq 0$     non-Abelian

$$[T_a, T_b] = i f_{abc} T_c$$

*Lie Algebra*

$f_{abc}$  : real numbers, **structure constants**

$f_{abc} = -f_{bac} = \dots$  antisymmetric

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*Lie Algebra*

$f_{abc}$  : real numbers, **structure constants**

$f_{abc} = -f_{bac} = \dots$  antisymmetric

$$SU(2) \quad f_{abc} = \epsilon_{abc} \quad (\text{like angular momentum})$$

$$T_a = \frac{1}{2} \sigma_a, \quad \sigma_a : \text{Pauli matrices } (a=1,2,3)$$

commutators     $[T_a, T_b] \neq 0$     non-Abelian

$$[T_a, T_b] = f_{abc} T_c$$

*Lie Algebra*

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$$SU(3) \quad T_a = \frac{1}{2} \lambda_a, \quad \lambda_a : \text{Gell-Mann matrices } (a=1,\dots,8)$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \dots$$

## (ii) local symmetry

now:  $\theta_a = \theta_a(x)$  for  $a = 1, \dots, N$

covariant derivative  $\partial_\mu \rightarrow D_\mu = \partial_\mu - ig \mathbf{W}_\mu$

vector field  $\mathbf{W}_\mu$  is  $n \times n$  matrix:  $\mathbf{W}_\mu(x) = T_a W_\mu^a(x)$

induces interaction term  $\mathcal{L}_0 \rightarrow \mathcal{L}_0 + \mathcal{L}_{\text{int}}$

with  $\mathcal{L}_{\text{int}} = g \bar{\Psi} \gamma^\mu \mathbf{W}_\mu \Psi = g (\bar{\Psi} \gamma^\mu T_a \Psi) W_\mu^a \equiv j_a^\mu W_\mu^a$

For a multiplet of scalar fields  $\Phi$ :

$$\mathcal{L}_0 = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) \rightarrow \mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi)$$

$\mathcal{L}$  is invariant under local gauge transformations

$$\Psi \rightarrow \Psi' = U \Psi ,$$

$$\mathbf{W}_\mu \rightarrow \mathbf{W}'_\mu = U \mathbf{W}_\mu U^{-1} - \frac{i}{g} (\partial_\mu U) U^{-1}$$

for infinitesimal transformations:

$$W_\mu^a \rightarrow W'_\mu{}^a = W_\mu^a + \frac{1}{g} \partial_\mu \theta^a + f_{abc} W_\mu^b \theta^c$$

crucial property of covariant derivative

$$D'_\mu U = U D_\mu$$

(iii) dynamics of  $W_\mu^a$  fields

need: additional term  $\mathcal{L}_W \Rightarrow$  e.o.m., propagators

naive:  $\sum_a (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)^2$  *not gauge invariant*

instead:

$$\begin{aligned} \mathbf{F}_{\mu\nu} &= D_\mu \mathbf{W}_\nu - D_\nu \mathbf{W}_\mu \equiv F_{\mu\nu}^a T_a \\ &= \partial_\mu \mathbf{W}_\nu^a - \partial_\nu \mathbf{W}_\mu^a - ig [\mathbf{W}_\mu, \mathbf{W}_\nu] \\ &= \frac{i}{g} [D_\mu, D_\nu] \end{aligned}$$

gauge transformation:  $\mathbf{W}_\mu \rightarrow \mathbf{W}'_\mu, D_\mu \rightarrow D'_\mu$   
 $\Rightarrow \mathbf{F}_{\mu\nu} \rightarrow \mathbf{F}'_{\mu\nu} = U \mathbf{F}_{\mu\nu} U^{-1}$

$$\Rightarrow \text{Tr} (\mathbf{F}'_{\mu\nu} \mathbf{F}'^{\mu\nu}) = \text{Tr} (U \mathbf{F}_{\mu\nu} U^{-1} U \mathbf{F}^{\mu\nu} U^{-1}) = \text{Tr} (\mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu})$$

*gauge invariant*

# Lagrangian:

$$\mathcal{L}_W = -\frac{1}{2} \text{Tr} (\mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu}) = -\frac{1}{4} \sum_a F_{\mu\nu}^a F^{a,\mu\nu}$$

components of  $\mathbf{F}_{\mu\nu}$       [using normalization  $\text{Tr}(T_a T_b) = \frac{1}{2} \delta_{ab}$  ]

$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g f_{abc} W_\mu^b W_\nu^c$$

$F_{\mu\nu}^a$  = Abelian + non-Abelian

$$\mathcal{L}_W = \underbrace{\text{quadratic}}_{\text{free part}} + \underbrace{\text{cubic} + \text{quartic}}_{\text{tri- and quadri-linear interactions}}$$

$$\mathcal{L}_W = -\frac{1}{4} (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)^2 \quad \Rightarrow \text{ propagator}$$

$$= -\frac{1}{2} g f_{abc} (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) W^{b,\mu} W^{c,\nu}$$

$$= -\frac{1}{4} g^2 f_{abc} f_{ade} W_\mu^b W_\nu^c W^{d,\mu} W^{e,\nu}$$

new type of couplings:

- self-couplings of vector fields (gauge couplings)
- universal coupling constant  $g$  for fermions and vector fields

## propagator $D_{\rho\nu}$ for massless spin-1 particles

$$[\square g^{\mu\rho} - \partial^\mu \partial^\rho] D_{\rho\nu}(x-y) = g^\mu{}_\nu \delta^4(x-y)$$

in momentum space

$$(-k^2 g^{\mu\rho} + k^\mu k^\rho) D_{\rho\nu}(k) = g^\mu{}_\nu$$

has no solution ( $\det = 0$ )

way out: add gauge fixing term  $\mathcal{L}_{\text{fix}} = -\frac{1}{2\xi} (\partial_\mu W^\mu)^2$

$$\left[ -k^2 g^{\mu\rho} + \left(1 - \frac{1}{\xi}\right) k^\mu k^\rho \right] D_{\rho\nu}(k) = g^\mu{}_\nu$$

which now has a solution:

$$D_{\rho\nu}(k) = \frac{1}{k^2 + i\epsilon} \left[ -g_{\nu\rho} + (1 - \xi) \frac{k_\nu k_\rho}{k^2} \right]$$

## Faddeev-Popov ghosts, BRS symmetry

[important for quantization and renormalization]

gauge group  $G$ , generators  $T_a$ , structure constants  $f_{abc}$

for quantization:  $\mathcal{L} = \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{ghost}}$

$$\mathcal{L}_{\text{fix}} = \frac{1}{2} \sum F_a^2, \quad F_a = \partial_\mu W^{a,\mu}$$

requires ghost fields  $c_a$  and anti-ghosts  $\bar{c}_a$

$$\mathcal{L}_{\text{ghost}} = (\partial^\mu \bar{c}_a) (D_\mu^{\text{adj}})_{ab} c_b, \quad D_\mu^{\text{adj}} = \partial_\mu - ig W_\mu^r T_r^{\text{adj}}$$

- $\mathcal{L}$  is symmetric under BRS transformations

$$\delta W_\mu^a = (D_\mu^{\text{adj}})_{ab} c_b \quad [W_\mu^a \rightarrow W_\mu^a + \delta W_\mu^a \quad \text{etc.}]$$

$$\delta \bar{c}_a = -\partial^\nu W_\nu^a, \quad \delta c_a = -\frac{1}{2} g f_{abc} c_b c_c$$

BRS [*Becchi, Rouet, Stora*] symmetry guarantees

- renormalizability
- gauge invariant and unitary  $S$  matrix

important: ST identities = symmetry relations between  
Green functions, valid to all orders

basic quantity: effective action  $\Gamma(\mathcal{L})$   
generating functional of vertex functions

$$\frac{\delta \Gamma}{\delta \varphi_i \delta \varphi_j \dots} \Big|_{\varphi=0} = \Gamma_{\varphi_i \varphi_j \dots}$$

classical action:  $\Gamma_{\text{cl}}(\mathcal{L}) = \int d^4x \mathcal{L}$   
 $\Rightarrow$  tree level vertices

general: vertex functions with loop contributions,  
building blocks for renormalization

**BRS symmetry:** invariance of  $\Gamma$  under BRS transformations,

$$S(\Gamma) = \int d^4x \left[ \frac{\delta\Gamma}{\delta\varphi_i} \delta\varphi_i + \dots \right] = 0 \quad S: \text{ST-operator}$$

$$\Rightarrow \frac{\delta S(\Gamma)}{\delta\varphi_j \dots} \Big|_{\varphi=0} = 0 \quad \text{relations between vertex functions}$$

### Slavnov-Taylor (ST) identities

$\Rightarrow$  determines the structure of the counter terms  
for renormalization

all UV divergences in vertex functions can be removed  
by (multiplicative) renormalization of parameters and  
fields in the classical Lagrangian/action

## **2. Higgs mechanism**

## Vector field for massive spin-1 particles

generic vector field  $A_\mu(x)$ ,  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

free Lagrangian  $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2} A_\mu A^\mu$

e.o.m.  $[(\square + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu] A_\nu = 0$

solutions  $\epsilon_\nu^{(\lambda)} e^{\pm ikx}$  with  $k^2 = m^2$

3 orthogonal polarization vectors  $\epsilon_\nu^{(\lambda)}$  with polarization sum

$$\sum_{\lambda=1}^3 \epsilon_\mu^{(\lambda)*} \epsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$$

longitudinal polarization  $\epsilon_\nu \simeq k_\nu/m$  for high momentum

propagator     $D_{\rho\nu}(x - y)$

solution of     $\left[ (\square + m^2) g^{\mu\rho} - \partial^\mu \partial^\rho \right] D_{\rho\nu}(x - y) = g^\mu_\nu \delta^4(x - y)$

in momentum space

$$\left[ (-k^2 + m^2) g^{\mu\rho} + k^\mu k^\rho \right] D_{\rho\nu}(k) = g^\mu_\nu$$

solution

$$D_{\rho\nu}(k) = \frac{1}{k^2 - m^2 + i\epsilon} \left( -g_{\nu\rho} + \frac{k_\nu k_\rho}{m^2} \right)$$

problem: weak interaction, gauge bosons are massive

mass terms  $\sim M^2 W_\mu^a W^{a,\mu}$  spoil local gauge invariance

- bad high energy behaviour of amplitudes and cross sections, conflict with unitarity  
reason: longitudinal polarization  $\epsilon^\mu \simeq \frac{k^\mu}{M} \sim k^\mu$
- bad divergence of higher orders with loop diagrams  
reason: propagators contain  $-g_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}$ 
  - $\Rightarrow$  additional powers of momenta in loop integration
  - $\Rightarrow$  spoil renormalizability

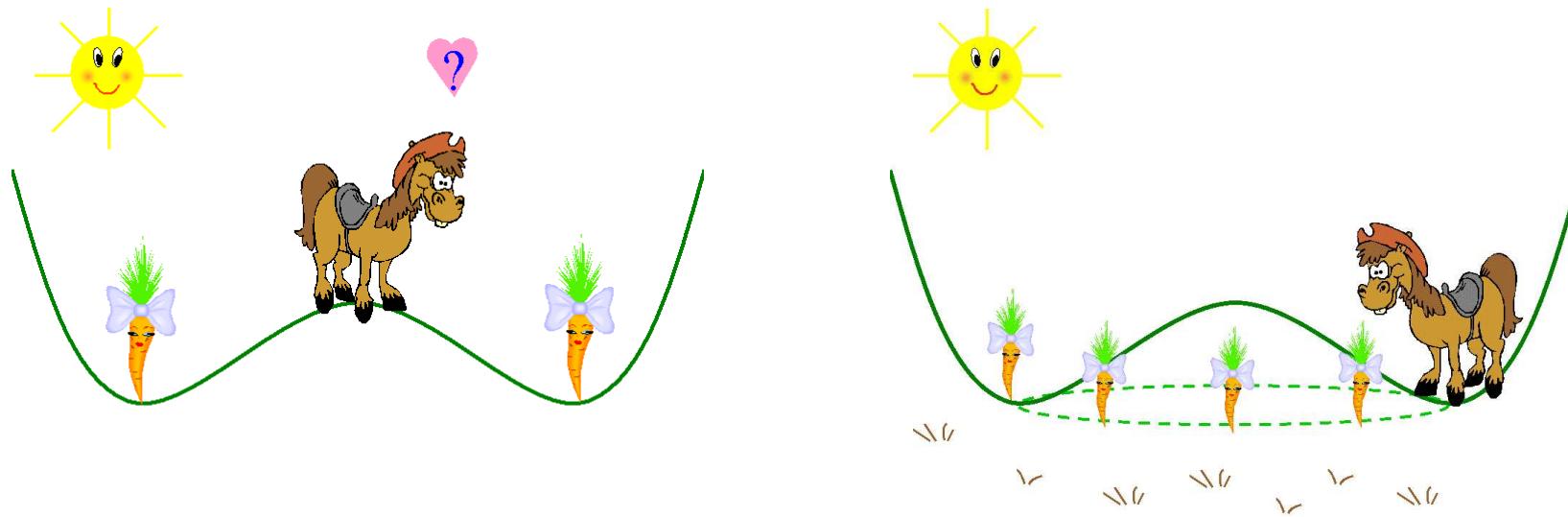
**renormalizable theories:** divergences can be removed by a finite number of counter terms

**gauge invariant theories:** counter terms for parameters (and fields)

# Spontaneous symmetry breaking (SSB)

physical system: has a symmetry

ground state: not symmetric

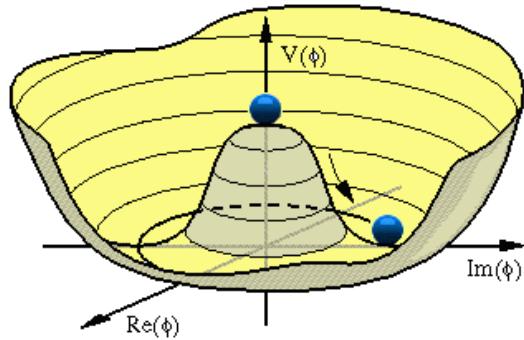


[A. Pich]

example: complex scalar field  $\phi \neq \phi^\dagger$

Lagrangian with interaction  $V$  (potential), minimum at  $\phi_0 = v$

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi)$$



$V = V(|\phi|)$  :  *$\mathcal{L}$  symmetric under  $\phi \rightarrow e^{i\alpha} \phi$ ,  $U(1)$*

$v \neq 0$  :  $\phi'_0 = e^{i\alpha} v \neq \phi_0$  *not symmetric*

$V = V(|\phi'_0|) = V(|\phi_0|)$  : *vacuum is degenerate*

write  $\phi(x) = \eta(x)e^{i\theta(x)}$ ,  $\eta$  and  $\theta$  real

$V(|\phi|) = V(\eta)$ , minimum at  $\eta = v$ :  $V'(v) = 0$ ,  $V''(v) > 0$

expand around minimum:  $\eta(x) = v + \frac{1}{\sqrt{2}} H(x)$

$$V(\eta) = V(v) + \frac{1}{2} V''(v) \cdot \frac{1}{2} H^2 + \dots$$

$$\mathcal{L} = \frac{1}{2} |\partial_\mu H|^2 - \underbrace{\frac{1}{2} V''(v) \cdot \frac{1}{2} H^2}_{m_H^2 > 0} + v^2 |\partial_\mu \theta|^2 + \dots$$

- $H$  field is massive
- $\theta$  field is massless, no  $\theta^2$  term: **Goldstone field**
- special case of **Goldstone theorem**:  
for each broken generator  $T_a$  with  $T_a \phi_0 \neq 0$   
there is a massless Goldstone field  $\theta(x)$

## SSB in gauge theories

$$\mathcal{L} = (D_\mu \phi)^+ (D^\mu \phi) - V(|\phi|) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad D_\mu = \partial_\mu - ieA_\mu$$

invariant under local U(1) transformations:

$$\phi'(x) = e^{i\alpha(x)} \phi(x) = e^{i\alpha(x)} e^{i\theta(x)} \eta(x)$$

$$A'_\mu(x) = A_\mu(x) + \frac{1}{e} \partial_\mu \alpha(x)$$

choose  $\alpha(x) = -\theta(x)$ :  $\phi'(x) = \eta(x)$

$$\mathcal{L} = |(\partial - ieA'_\mu)\eta|^2 - V(\eta) - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu}$$

massless  $\theta$  field removed (unphysical)

$$\begin{aligned}\mathcal{L} &= |(\partial_\mu - ieA'_\mu)(\textcolor{red}{v} + \frac{1}{\sqrt{2}}H)|^2 - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - V \\ &= \underbrace{-\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \textcolor{red}{v}^2 e^2 A'_\mu A'^\mu}_{\text{massive } A\text{-field, } m_A \sim ev} + \underbrace{\frac{1}{2}[(\partial_\mu H)^2 - m_H^2 H^2]}_{\text{neutral scalar, } m_H \neq 0} + \dots\end{aligned}$$

in this special gauge: no Goldstone field unitary gauge

$A_\mu$ -field propagator:

$$\frac{i}{k^2 - m_A^2 + i\varepsilon} \underbrace{\left( -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2} \right)}_{\text{polarization sum of 3 pol. states}}$$

massive vector field without spoiling gauge symmetry of  $\mathcal{L}$

## two different gauges

properties	$\phi$ field	$A_\mu$ field
symmetry manifest	$H, \theta$	2 polarizations (transverse)
physics manifest	$H$	3 polarizations (2 transverse + 1 longitudinal)

$\theta \rightarrow$  *longitudinal polarization of  $A_\mu$*

### **3. Electroweak interaction and Standard Model**

## preliminaries

Dirac matrices:  $\gamma^\mu$  ( $\mu = 0, 1, 2, 3$ ),  $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}$

$\bar{\Gamma} = \gamma^0 (\Gamma)^\dagger \gamma^0$ ,  $\Gamma$  any Dirac matrix oder product of matrices

further Dirac matrix:  $\gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\gamma_5 \gamma^\mu + \gamma^\mu \gamma_5 = 0, \quad \bar{\gamma}_5 = -\gamma_5, \quad \gamma_5^2 = 1$$

chiral fermions:

$\psi^L = \frac{1-\gamma_5}{2} \psi$  left-handed spinor, L-chiral spinor

$\psi^R = \frac{1+\gamma_5}{2} \psi$  right-handed spinor, R-chiral spinor

projectors on right/left chirality:  $\omega_\pm = \frac{1 \pm \gamma_5}{2}$ ,  $(\omega_\pm)^2 = \omega_\pm$

## chiral currents:

$$\overline{\psi^L} \gamma^\mu \psi^L = \overline{\psi} \gamma^\mu \frac{1-\gamma_5}{2} \psi \equiv J_L^\mu \quad \text{left-handed current}$$

$$\overline{\psi^R} \gamma^\mu \psi^R = \overline{\psi} \gamma^\mu \frac{1+\gamma_5}{2} \psi \equiv J_R^\mu \quad \text{right-handed current}$$

$$J_V^\mu = \overline{\psi} \gamma^\mu \psi = J_L^\mu + J_R^\mu \quad \text{vector current}$$

$$J_A^\mu = \overline{\psi} \gamma^\mu \gamma_5 \psi = -J_L^\mu + J_R^\mu \quad \text{axialvector current}$$

## mass term:

$$m \overline{\psi} \psi = m (\overline{\psi^L} \psi^R + \overline{\psi^R} \psi^L)$$

connects  $L$  and  $R$  !

symmetry group:  $SU(2)_I \times U(1)_Y$

$SU(2)_I$  : weak isospin, generators  $T_I^a = \frac{1}{2} \sigma^a$  for L, = 0 for R

$U(1)_Y$  : weak hypercharge, generator  $Y$

$$T_I^3 + Y/2 = Q$$

Fermion content of the SM:

(ignoring possible right-handed neutrinos)

			$T_I^3$	$Q$
leptons:	$\Psi_L^L = \begin{pmatrix} \nu_e^L \\ e^L \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu^L \\ \mu^L \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau^L \\ \tau^L \end{pmatrix},$	$+ \frac{1}{2}$ $- \frac{1}{2}$	0 -1	
	$\psi_l^R = e^R, \quad \mu^R, \quad \tau^R,$	0	-1	
quarks:	$\Psi_Q^L = \begin{pmatrix} u^L \\ d^L \end{pmatrix}, \quad \begin{pmatrix} c^L \\ s^L \end{pmatrix}, \quad \begin{pmatrix} t^L \\ b^L \end{pmatrix},$	$+ \frac{1}{2}$ $- \frac{1}{2}$	$+\frac{2}{3}$ $-\frac{1}{3}$	
(Each quark exists in 3 colours!)	$\psi_u^R = u^R, \quad c^R, \quad t^R,$	0	$+\frac{2}{3}$	
	$\psi_d^R = d^R, \quad s^R, \quad b^R,$	0	$-\frac{1}{3}$	

# gauge boson content

$$\begin{aligned}
 SU(2)_I : & \quad \text{generators} \quad T_I^1, T_I^2, T_I^3 \\
 & \quad \text{gauge fields} \quad W_\mu^1, W_\mu^2, W_\mu^3 \\
 & \quad \text{also:} \quad W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2), \quad W_\mu^3
 \end{aligned}$$

$U(1)_Y :$	generator	$Y$
	gauge field	$B_\mu$

## gauge boson content

- |             |                     |   |
|-------------|---------------------|---|
| $SU(2)_I :$ | <b>generators</b>   | $T_I^1, T_I^2, T_I^3$   |
|             | <b>gauge fields</b> | $W_\mu^1, W_\mu^2, W_\mu^3$                                       |
|             | <i>also:</i>        | $W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2), W_\mu^3$ |
| $U(1)_Y :$  | <b>generator</b>    | $Y$   |
|             | <b>gauge field</b>  | $B_\mu$   |
- notation:  $\not{\partial} = \gamma^\mu \partial_\mu, \quad \not{a} = \gamma^\mu a_\mu$

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\text{ferm}} = i\overline{\psi_f}\not{\partial}\psi_f = i\overline{\Psi_L^L}\not{\partial}\Psi_L^L + i\overline{\Psi_Q^L}\not{\partial}\Psi_Q^L + i\overline{\psi_l^R}\not{\partial}\psi_l^R + i\overline{\psi_u^R}\not{\partial}\psi_u^R + i\overline{\psi_d^R}\not{\partial}\psi_d^R$$

Minimal substitution:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ig_2 T_I^a W_\mu^a + ig_1 \frac{1}{2} Y B_\mu = D_\mu^L \omega_- + D_\mu^R \omega_+,$$

$$D_\mu^L = \partial_\mu - \frac{ig_2}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{i}{2} \begin{pmatrix} g_2 W_\mu^3 - g_1 Y^L B_\mu & 0 \\ 0 & -g_2 W_\mu^3 - g_1 Y^L B_\mu \end{pmatrix},$$

$$D_\mu^R = \partial_\mu + ig_1 \frac{1}{2} Y^R B_\mu$$

Photon identification:

“rotation”:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad c_W = \cos \theta_W, s_W = \sin \theta_W, \quad \theta_W = \text{mixing angle}$$

$$D_\mu^L \Big|_{A_\mu} = -\frac{i}{2} A_\mu \begin{pmatrix} -g_2 s_W - g_1 c_W Y^L & 0 \\ 0 & g_2 s_W - g_1 c_W Y^L \end{pmatrix} \stackrel{!}{=} ie A_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

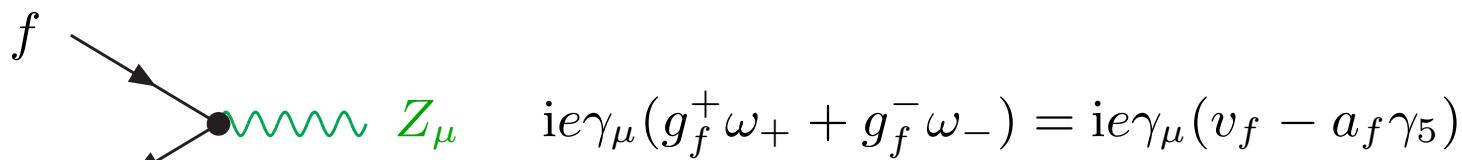
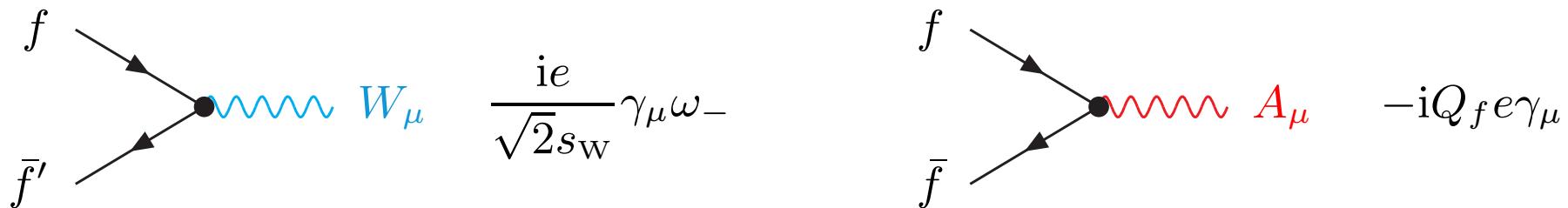
- charged difference in doublet  $Q_1 - Q_2 = 1 \rightarrow g_2 = \frac{e}{s_W}$
- normalize  $Y^{\text{L/R}}$  such that  $g_1 = \frac{e}{c_W}$

↪  $Y$  fixed by “Gell-Mann–Nishijima relation”:  $Q = T_I^3 + \frac{Y}{2}$

## Fermion–gauge-boson interaction:

$$\begin{aligned} \mathcal{L}_{\text{ferm, YM}} = & \frac{e}{\sqrt{2}s_W} \overline{\Psi}_F^L \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix} \Psi_F^L + \frac{e}{2c_W s_W} \overline{\Psi}_F^L \sigma^3 Z \Psi_F^L \\ & - e \frac{s_W}{c_W} Q_f \overline{\psi}_f Z \psi_f - e Q_f \overline{\psi}_f A \psi_f \quad (f=\text{all fermions}, F=\text{all doublets}) \end{aligned}$$

## Feynman rules:



with  $g_f^+ = -\frac{s_W}{c_W} Q_f, \quad g_f^- = -\frac{s_W}{c_W} Q_f + \frac{T_{I,f}^3}{c_W s_W},$

$$v_f = -\frac{s_W}{c_W} Q_f + \frac{T_{I,f}^3}{2 c_W s_W}, \quad a_f = \frac{T_{I,f}^3}{2 c_W s_W}$$

## gauge field Lagrangian (Yang-Mills Lagrangian)

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

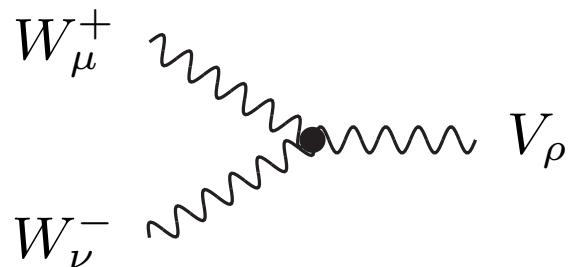
$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of “physical” fields:

$$\begin{aligned} \mathcal{L}_{\text{YM}} = & -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\ & - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ & + (\text{trilinear interaction terms involving } AW^+W^-, ZW^+W^-) \\ & + (\text{quadrilinear interaction terms involving } \\ & \quad AAW^+W^-, AZW^+W^-, ZZW^+W^-, W^+W^-W^+W^-) \end{aligned}$$

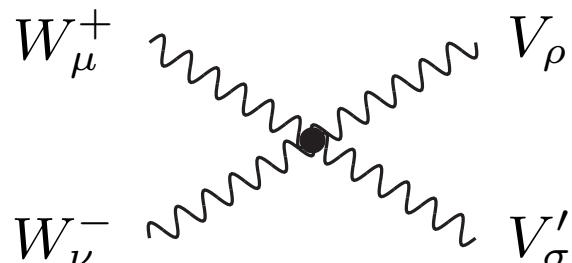
## Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$ieC_{WWV} \left[ g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu \right]$$

$$\text{with } C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_W}{s_W}$$



$$ie^2 C_{WWVV'} \left[ 2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho} \right]$$

$$\text{with } C_{WW\gamma\gamma} = -1, \quad C_{WW\gamma Z} = \frac{c_W}{s_W},$$

$$C_{WWZZ} = -\frac{c_W^2}{s_W^2}, \quad C_{WWWW} = \frac{1}{s_W^2}$$

# Higgs mechanism $\Rightarrow$ masses of W and Z bosons

spontaneous breaking  $SU(2)_I \times U(1)_Y \rightarrow U(1)_Q$   
 unbroken em. gauge symmetry, massless photon

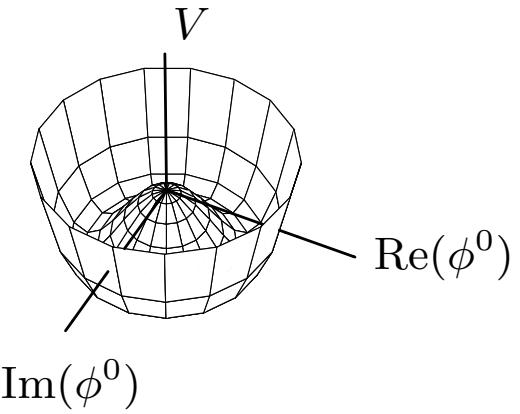
Minimal scalar sector with complex scalar doublet  $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2, \quad \mu^2, \lambda > 0,$$

$= SU(2)_I \times U(1)_Y$  symmetric

$$V(\Phi) = \text{minimal for } |\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$$



ground state  $\Phi_0$  (=vacuum expectation value of  $\Phi$ ) not unique

specific choice  $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$  not gauge invariant  $\Rightarrow$  spontaneous symmetry breaking

elmg. gauge invariance unbroken, since  $Q\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0 = 0$

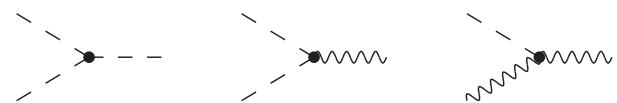
Field excitations in  $\Phi$ :

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + H(x) + i\chi(x)) \end{pmatrix}$$

Gauge-invariant Lagrangian of Higgs sector:  $(\phi^- = (\phi^+)^{\dagger})$

$$\begin{aligned} \mathcal{L}_H &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad \text{with } D_\mu = \partial_\mu - ig_2 \frac{\sigma^a}{2} W_\mu^a + i \frac{g_1}{2} B_\mu \\ &= (\partial_\mu \phi^+) (\partial^\mu \phi^-) - \frac{iev}{2s_W} (W_\mu^+ \partial^\mu \phi^- - W_\mu^- \partial^\mu \phi^+) + \frac{e^2 v^2}{4s_W^2} W_\mu^+ W^{-,\mu} \\ &\quad + \frac{1}{2} (\partial \chi)^2 + \frac{ev}{2c_W s_W} Z_\mu \partial^\mu \chi + \frac{e^2 v^2}{4c_W^2 s_W^2} Z^2 + \frac{1}{2} (\partial H)^2 - \mu^2 H^2 \end{aligned}$$

+ (trilinear  $SSS$ ,  $SSV$ ,  $SVV$  interactions)



+ (quadrilinear  $SSSS$ ,  $SSVV$  interactions)



Implications:

- gauge-boson masses:  $M_W = \frac{ev}{2s_W}$ ,  $M_Z = \frac{ev}{2c_W s_W} = \frac{M_W}{c_W}$ ,  $M_\gamma = 0$
- physical Higgs boson  $H$ :  $M_H = \sqrt{2\mu^2}$  = free parameter
- would-be Goldstone bosons  $\phi^\pm, \chi$ : unphysical degrees of freedom

general gauge: Goldstone fields  $\phi^\pm, \chi$  are present

required: gauge fixing term  $\mathcal{L}_{fix}$

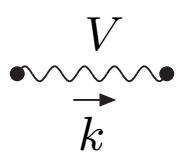
$R_\xi$  gauge:

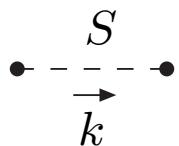
$$\mathcal{L}_{fix} = -\frac{1}{2\xi_\gamma} (F^\gamma)^2 - \frac{1}{2\xi_Z} (F^Z)^2 - \frac{1}{2\xi_W} (F^\pm)^2$$

with the gauge-fixing functionals  $F^a$ : ( $\xi_V$  = arbitrary gauge-fixing parameters)

$$F^\pm = \partial W^\pm \mp i\xi_W M_W \phi^\pm, \quad F^Z = \partial Z - \xi_Z M_Z \chi, \quad F^\gamma = \partial A$$

- elimination of mixing terms ( $W_\mu^\pm \partial^\mu \phi^\mp$ ), ( $Z_\mu \partial^\mu \chi$ ) in Lagrangian  
 $\hookrightarrow$  decoupling of gauge and would-be Goldstone fields (no mix propagators)
- boson propagators:

  $D_{\mu\nu}^{VV}(k) = -i \left[ \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - M_V^2} + \frac{k_\mu k_\nu}{k^2} \frac{\xi_V}{k^2 - \xi_V M_V^2} \right], \quad V = W, Z, \gamma$

  $D^{SS}(k) = \frac{i}{k^2 - \xi_V M_V^2}, \quad S = \phi, \chi$

- important special cases:

- ◊  $\xi_V = 1$ : ‘t Hooft–Feynman gauge  
 $\hookrightarrow$  convenient gauge-boson propagators  $\frac{-ig_{\mu\nu}}{k^2 - M_V^2}$
- ◊  $\xi_W, \xi_Z \rightarrow \infty$ : “unitary gauge”  
 $\hookrightarrow$  elimination of would-be Goldstone bosons

## Fermion masses

fermions in chiral representations of gauge symmetry

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad e_R \quad \Rightarrow \text{mass term } m_e (\bar{e}_L e_R + \bar{e}_R e_L) = m_e \bar{e} e$$

not gauge invariant

solution of the SM: introduce Yukawa interaction

= new interaction of fermions with the Higgs field

gauge invariant interaction,  $g_e$  = Yukawa coupling constant

$$\mathcal{L}_{\text{Yuk}} = g_e [\overline{\psi^L} \Phi e_R + \overline{e_R} \Phi^\dagger \psi^L]$$

most transparent in unitary gauge

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

apply to the first lepton generation

$$\psi^L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R :$$

$$\frac{g_e}{\sqrt{2}} \left[ (\overline{\nu}_L, \overline{e}_L) \begin{pmatrix} 0 \\ v + H \end{pmatrix} e_R + \overline{e}_R (0, v + H) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right]$$

$$= \underbrace{\frac{g_e}{\sqrt{2}} v}_{m_e} [\overline{e}_L e_R + \overline{e}_R e_L] + \underbrace{\frac{g_e}{\sqrt{2}} H}_{m_e/v} [\overline{e}_L e_R + \overline{e}_R e_L]$$

$$= m_e \overline{e} e + \frac{m_e}{v} H \overline{e} e$$

## 3 generations of leptons and quarks

Lagrangian for Yukawa couplings:

$$\mathcal{L}_{\text{Yuk}} = -\overline{\Psi_L^L} G_l \psi_l^R \Phi - \overline{\Psi_Q^L} G_u \psi_u^R \tilde{\Phi} - \overline{\Psi_Q^L} G_d \psi_d^R \Phi + \text{h.c.}$$

- $G_l, G_u, G_d = 3 \times 3$  matrices in 3-dim. space of generations ( $\nu$  masses ignored)
- $\tilde{\Phi} = i\sigma^2 \Phi^* = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$  = charge conjugate Higgs doublet,  $Y_{\tilde{\Phi}} = -1$

Fermion mass terms:

mass terms = bilinear terms in  $\mathcal{L}_{\text{Yuk}}$ , obtained by setting  $\Phi \rightarrow \Phi_0$ :

$$\mathcal{L}_{m_f} = -\frac{v}{\sqrt{2}} \overline{\psi_l^L} G_l \psi_l^R - \frac{v}{\sqrt{2}} \overline{\psi_u^L} G_u \psi_u^R - \frac{v}{\sqrt{2}} \overline{\psi_d^L} G_d \psi_d^R + \text{h.c.}$$

↪ diagonalization by unitary field transformations  $(f = l, u, d)$

$$\hat{\psi}_f^{\text{L/R}} \equiv U_f^{\text{L/R}} \psi_f^{\text{L/R}} \quad \text{such that} \quad \frac{v}{\sqrt{2}} U_f^{\text{L}} G_f (U_f^{\text{R}})^\dagger = \text{diag}(m_f)$$

$$\Rightarrow \text{standard form: } \mathcal{L}_{m_f} = -m_f \overline{\hat{\psi}_f^{\text{L}}} \hat{\psi}_f^{\text{R}} + \text{h.c.} = -m_f \overline{\hat{\psi}_f} \hat{\psi}_f$$

## Quark mixing:

- $\psi_f$  correspond to eigenstates of the gauge interaction
- $\hat{\psi}_f$  correspond to mass eigenstates,  
for **massless neutrinos** define  $\hat{\psi}_\nu^L \equiv U_l^L \psi_\nu^L \rightarrow$  no lepton-flavour changing

Yukawa and gauge interactions in terms of mass eigenstates:

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = & -\frac{\sqrt{2}m_l}{v} \left( \phi^+ \overline{\hat{\psi}_\nu^L} \hat{\psi}_l^R + \phi^- \overline{\hat{\psi}_l^R} \hat{\psi}_\nu^L \right) + \frac{\sqrt{2}m_u}{v} \left( \phi^+ \overline{\hat{\psi}_u^R} V \hat{\psi}_d^L + \phi^- \overline{\hat{\psi}_d^L} V^\dagger \hat{\psi}_u^R \right) \\ & - \frac{\sqrt{2}m_d}{v} \left( \phi^+ \overline{\hat{\psi}_u^L} V \hat{\psi}_d^R + \phi^- \overline{\hat{\psi}_d^R} V^\dagger \hat{\psi}_u^L \right) - \frac{m_f}{v} i \operatorname{sgn}(T_{I,f}^3) \chi \overline{\hat{\psi}_f} \gamma_5 \hat{\psi}_f \\ & - \frac{m_f}{v} (v + H) \overline{\hat{\psi}_f} \hat{\psi}_f, \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{ferm, YM}} = & \frac{e}{\sqrt{2}s_W} \overline{\hat{\Psi}_L^L} \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix} \hat{\psi}_L^L + \frac{e}{\sqrt{2}s_W} \overline{\hat{\Psi}_Q^L} \begin{pmatrix} 0 & VW^+ \\ V^\dagger W^- & 0 \end{pmatrix} \hat{\psi}_Q^L \\ & + \frac{e}{2c_W s_W} \overline{\hat{\Psi}_F^L} \sigma^3 \not{Z} \hat{\Psi}_F^L - e \frac{s_W}{c_W} Q_f \overline{\hat{\psi}_f} \not{Z} \hat{\psi}_f - e Q_f \overline{\hat{\psi}_f} \not{A} \hat{\psi}_f \end{aligned}$$

- only charged-current coupling of quarks modified by  $V = U_u^L (U_d^L)^\dagger = \text{unitary}$   
(Cabibbo–Kobayashi–Maskawa (CKM) matrix)
- Higgs–fermion coupling strength =  $\frac{m_f}{v}$

## Features of the CKM mixing:

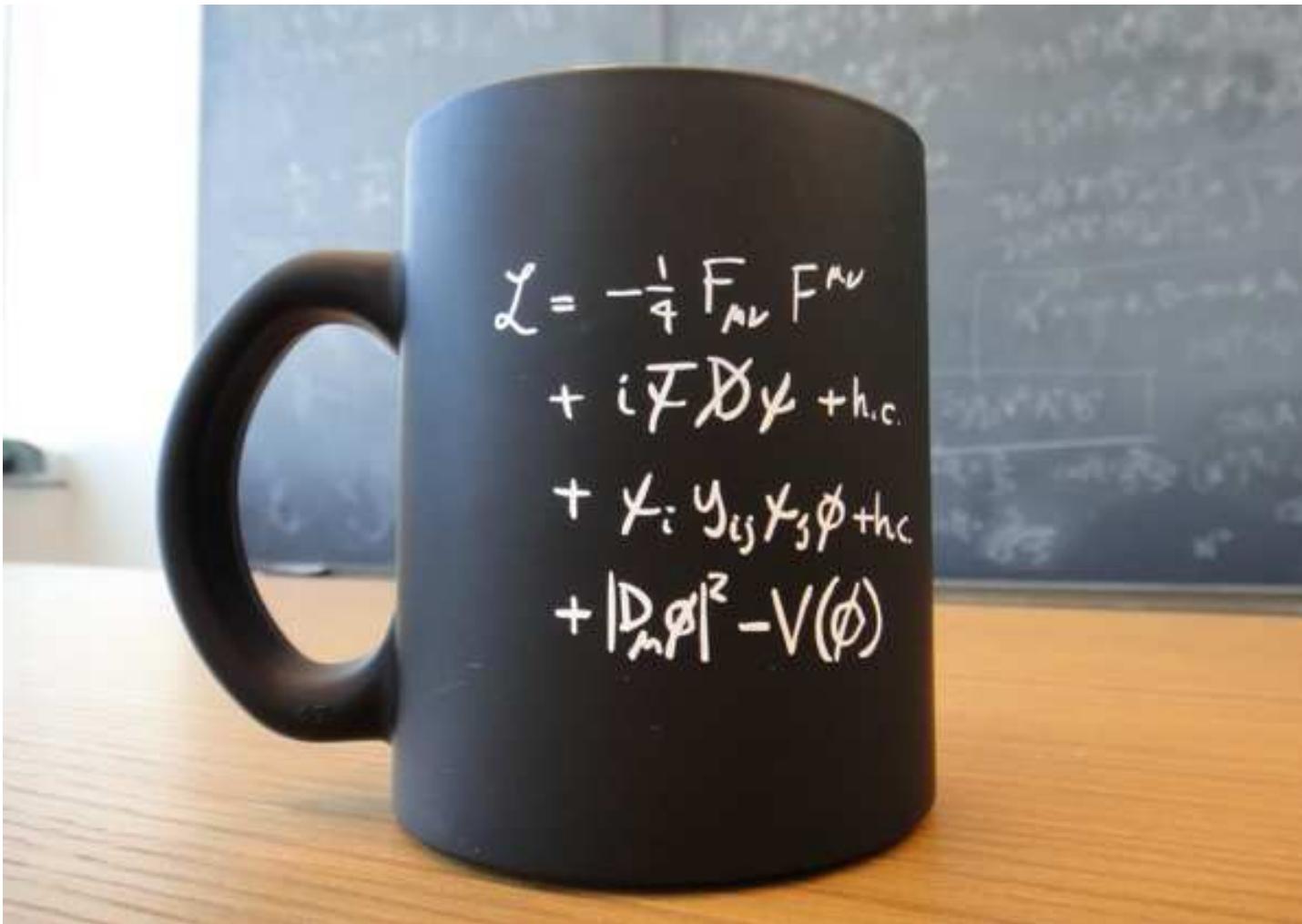
- $V$  = 3-dim. generalization of Cabibbo matrix  $U_C$
- $V$  is parametrized by 4 free parameters: 3 real angles, 1 complex phase  
→ **complex phase is the only source of CP violation in SM**

counting:

$$\begin{aligned} & \left( \begin{array}{c} \text{\#real d.o.f.} \\ \text{in } V \end{array} \right) - \left( \begin{array}{c} \text{\#unitarity} \\ \text{relations} \end{array} \right) - \left( \begin{array}{c} \text{\#phase diffs. of} \\ u\text{-type quarks} \end{array} \right) - \left( \begin{array}{c} \text{\#phase diffs. of} \\ d\text{-type quarks} \end{array} \right) - \left( \begin{array}{c} \text{\#phase diff. between} \\ u\text{- and } d\text{-type quarks} \end{array} \right) \\ & = 18 - 9 - 2 - 2 - 1 = 4 \end{aligned}$$

- **no flavour-changing neutral currents in lowest order,**  
flavour-changing suppressed by factors  $G_\mu(m_{q_1}^2 - m_{q_2}^2)$  in higher orders  
("Glashow–Iliopoulos–Maiani mechanism")

# The Standard Model Lagrangian



- renormalizable  $\Rightarrow$  precision calculations
- quantum effects in precision observables detectable
- involve Higgs mass dependence

## **4. Phenomenology of W and Z bosons and precision tests**

## features of the ew Standard Model

- Higgs boson probably found, all other particles confirmed
- consistent quantum field theory
  - in accordance with unitarity
  - renormalizable  $\Rightarrow$  predictions at higher orders
- formal parameters:  $g_2, g_1, v, \lambda, g_f, V_{\text{CKM}}$   
physical parameters:  $\alpha, M_W, M_Z, M_H, m_f, V_{\text{CKM}}$

# Basic parameters and relations

ew mixing angle:  $s_W \equiv \sin \theta_W, \quad c_W \equiv \cos \theta_W$

gauge coupling constants:  $g_2 = \frac{e}{s_W}, \quad g_1 = \frac{e}{c_W}$

vector boson masses:  $M_W = \frac{1}{2}g_2 v = \frac{ev}{2s_W}$

$$M_Z = \frac{ev}{2s_W c_W} = \frac{M_W}{c_W}$$

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}$$

neutral current (NC) couplings:  $a_f = \frac{g_2}{2c_W} T_3^f$

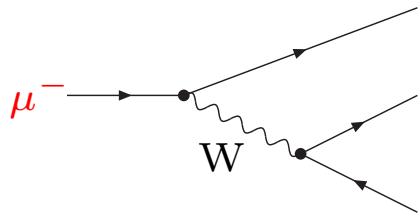
$$v_f = \frac{g_2}{2c_W} (T_3^f - 2Q_f s_W)$$

# observables and experiments

- Muon decay:

$$\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$$

determination of the Fermi constant



$$G_\mu = \frac{\pi \alpha M_Z^2}{\sqrt{2} M_W^2 (M_Z^2 - M_W^2)} + \dots$$

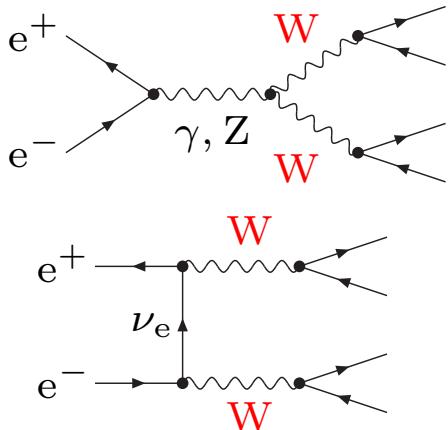
- Z production (LEP1/SLC):

$$e^+ e^- \rightarrow Z \rightarrow f \bar{f}$$

various precision measurements at the  
Z resonance:  $M_Z$ ,  $\Gamma_Z$ ,  $\sigma_{\text{had}}$ ,  $A_{\text{FB}}$ ,  $A_{\text{LR}}$ , etc.  
 $\Rightarrow$  good knowledge of the  $Z f \bar{f}$  sector

- W-pair production (LEP2/ILC):

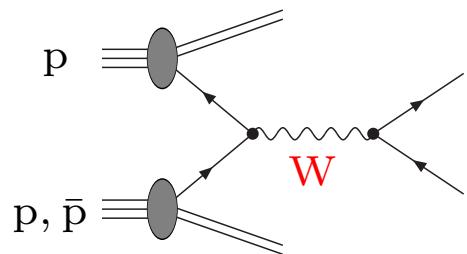
$$e^+ e^- \rightarrow WW \rightarrow 4f (+\gamma)$$



- measurement of  $M_W$
- $\gamma WW/ZWW$  couplings
- quartic couplings:  $\gamma\gamma WW$ ,  $\gamma ZWW$

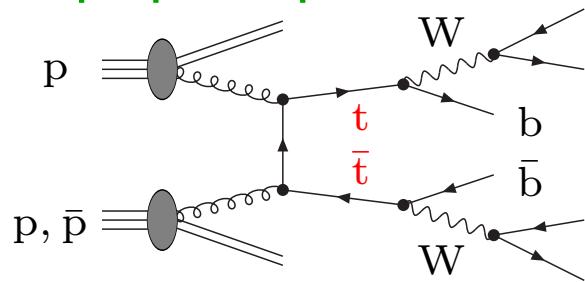
# experiments at hadron colliders

- **W production** (Tevatron/LHC):  $pp, p\bar{p} \rightarrow W \rightarrow l\nu_l (+\gamma)$



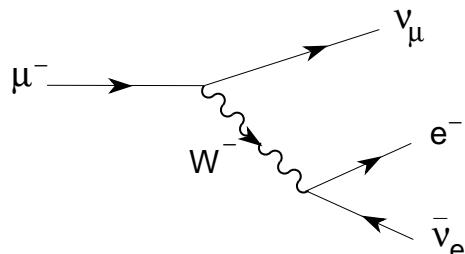
- measurement of  $M_W$
- bounds on  $\gamma WW$  coupling

- **top-quark production** (Tevatron/LHC):  $pp, p\bar{p} \rightarrow t\bar{t} \rightarrow 6f$



- measurement of  $m_t$

## $\mu$ decay



$$\mathcal{M} = \left( \frac{ig_2}{2\sqrt{2}} \right)^2 J_\rho^{(\mu)} \frac{-ig^{\rho\sigma}}{q^2 - M_W^2} J_\sigma^{(e)}$$

$$|q|^2 \simeq m_\mu^2 \ll M_W^2 : \quad \mathcal{M} = -\frac{g_2^2}{8M_W^2} J_\rho^{(\mu)} J^\rho(e)$$

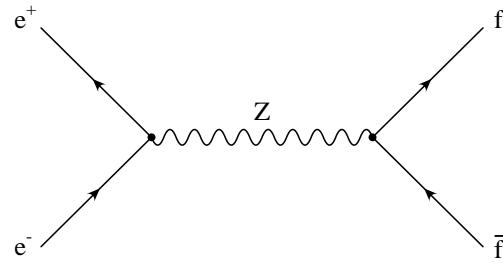
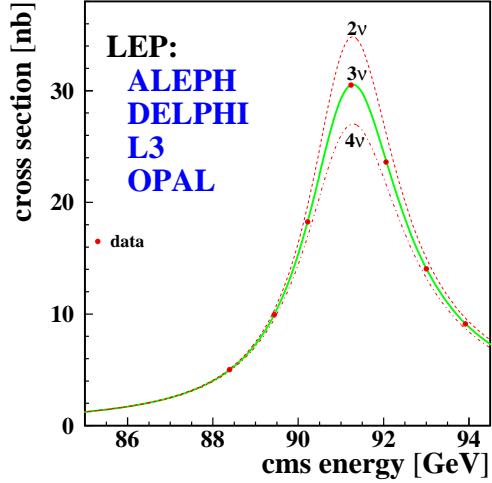
**Fermi model**    *with point-like 4-fermion interaction:*

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} J_\rho^{(\mu)} J^\rho(e) \quad \text{low-energy limit of SM}$$

$$\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{e^2}{8s_W^2 c_W^2 M_Z^2} = \frac{\pi\alpha}{2s_W^2 c_W^2 M_Z^2} = \frac{\pi\alpha}{2(1-M_W^2/M_Z^2)M_W^2}$$

$$G_F = 1.16637(1) \cdot 10^{-5} \text{ GeV}^{-2}$$

# Z resonance



$$\mathcal{M} = J_\mu^{(e)} \frac{-ig^{\mu\nu}}{s - M_Z^2 + iM_Z\Gamma_Z} J_\nu^{(f)}$$

*propagator with finite width  $\Gamma_Z$  (unstable particle)*

$$\Gamma_Z = \sum_f \Gamma(Z \rightarrow f\bar{f}), \quad \Gamma(Z \rightarrow f\bar{f}) = \frac{M_Z}{12\pi} (v_f^2 + a_f^2)$$

*differential cross section at  $s = M_Z^2$ :*

$$\frac{d\sigma}{d\Omega} \sim (v_e^2 + a_e^2)(v_f^2 + a_f^2) (1 + \cos^2 \theta) + (2v_e a_e)(2v_f a_f) \cdot 2 \cos \theta$$

$$\Rightarrow \text{forward-backward asymmetry} \quad A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_e A_f$$

$$A_f = \frac{2v_f a_f}{v_f^2 + a_f^2}$$

*polarized cross section for  $e_{L,R}^-$ :*

$$\Rightarrow \text{left-right asymmetry} \quad A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e$$

asymmetries determine  $\sin^2 \theta_W$

# input from experiments

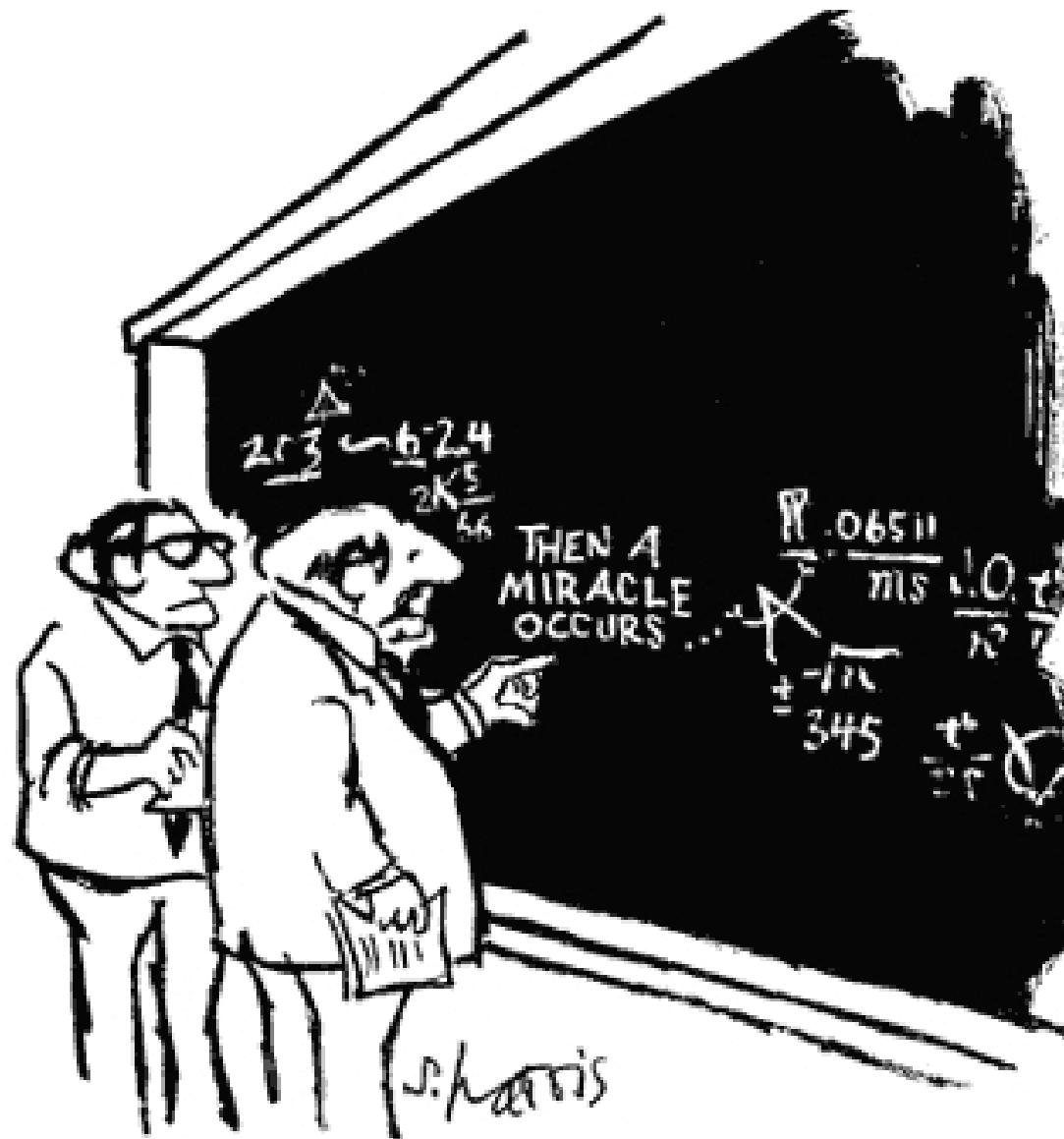
- LEP1/SLC:  $e^+e^- \rightarrow Z \rightarrow f\bar{f}$   
LEP1:  $\sim 4 \times 10^6$  events/experiment  
4 experiments (1989 – 1995)
- LEP2:  $e^+e^- \rightarrow W^+W^-$   
 $\mathcal{O}(10^4)$  W pairs (1996 – 2000)
- Tevatron:  $q\bar{q}' \rightarrow W \rightarrow l\nu, q\bar{q}'$   
( $p\bar{p}$ )  $q\bar{q}' \rightarrow t\bar{t}, t \rightarrow W^+b \rightarrow \dots$
- low-energy experiments ( $\mu$  decay,  $\nu N$  scattering,  $\nu e$  scattering, atomic parity violation, ... )

# experimental results (selection)

$M_Z$ [GeV]	$= 91.1875 \pm 0.0021$	0.002%
$\Gamma_Z$ [GeV]	$= 2.4952 \pm 0.0023$	0.09%
$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$= 0.23148 \pm 0.00017$	0.07%
$M_W$ [GeV]	$= 80.385 \pm 0.015$	0.02%
$m_t$ [GeV]	$= 173.2 \pm 0.9$	0.52%
$G_F$ [GeV $^{-2}$ ]	$= 1.16637(1)10^{-5}$	0.001%

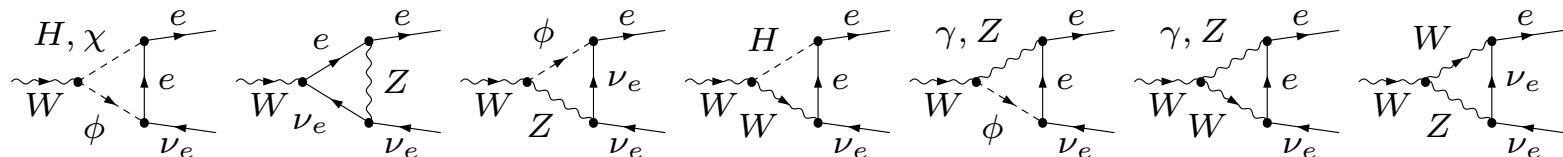
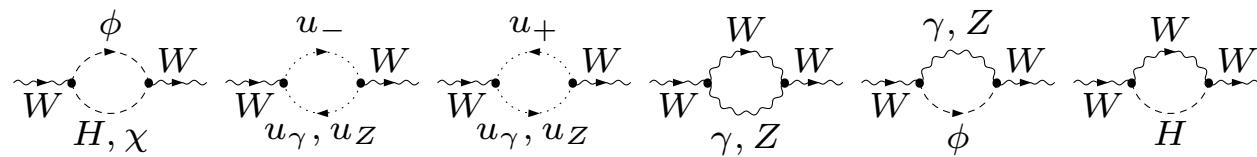
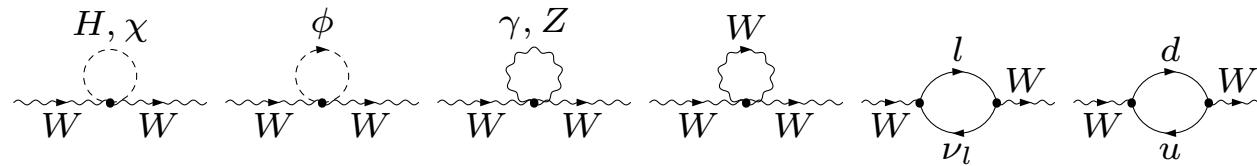
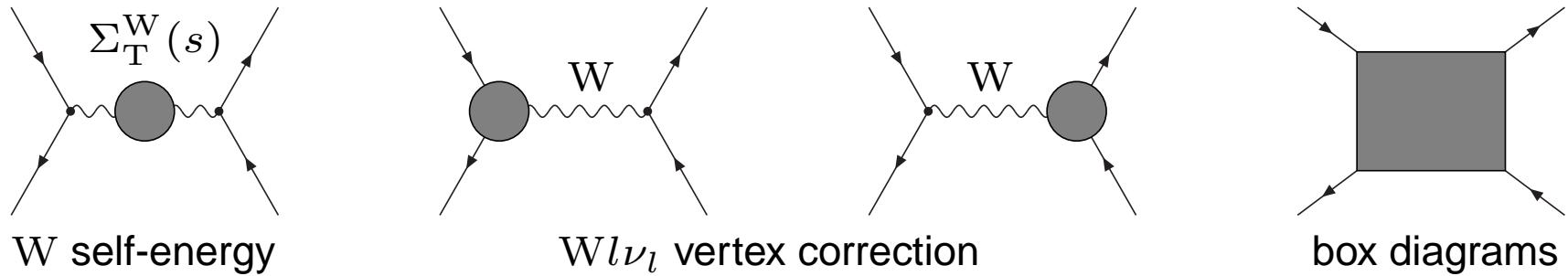
loop effects are at least one order of magnitude larger than experimental uncertainties

# precise experiments need precise calculations

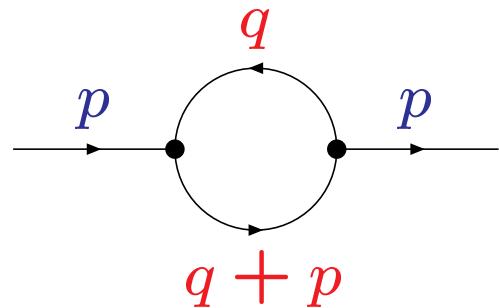


"I think you should be more explicit here in step two."

# example: 1-loop diagrams for $\mu$ decay amplitude



Example of loop integral:


$$\sim \int d^4q \frac{1}{(q^2 - m_1^2) [(q + p)^2 - m_2^2]}$$

$$q \rightarrow \infty : \sim \int^{\infty} \frac{q^3 dq}{q^4} = \int^{\infty} \frac{dq}{q} \rightarrow \infty$$

⇒ integral diverges for large  $q$

⇒ theory in this form not physically meaningful

needs (i) regularization

(ii) renormalization

## Regularization:

theory modified such that expressions become mathematically meaningful

⇒ “regulator” introduced, removed at the end

e.g. cut-off in loop integral

$$\int_0^\infty d^4 q \rightarrow \int_0^\Lambda d^4 q; \quad \Lambda \rightarrow \infty \text{ at the end}$$

technically more convenient: dimensional regularization

$$\int d^4 q \rightarrow \int d^D q, \quad D = 4 - \varepsilon; \quad D \rightarrow 4 \text{ at the end}$$

## Renormalization:

- absorption of divergencies
- determination of physical meaning of parameters  
order by order in perturbation theory

add counterterms that absorb divergent parts

- parameters in  $\mathcal{L}$  are formal, “bare parameters”  
 $g_0 = g + \delta g$  for a coupling,     $m_0 = m + \delta m$  for a mass
- $g, m$  are “physical”, i.e. measurable

mass renormalization,  $m_0^2 = m^2 + \delta m^2$

Physical mass: pole of propagator

inverse propagator up to 1-loop order:

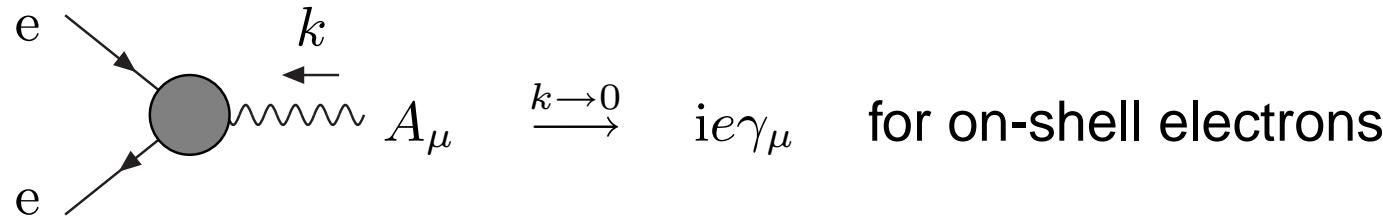
$$\text{---} + \text{---} \circ \text{---} + \text{---} \times \text{---} + \dots$$

$p^2 - m^2$        $\Sigma(p^2)$        $- \delta m^2$

**on-shell renormalization:**  $\delta m^2 = \text{Re } \Sigma(m^2)$

charge renormalization:  $e_0 = e + \delta e$

$\delta e$  cancels loop contributions to  $ee\gamma$  vertex in the Thomson limit



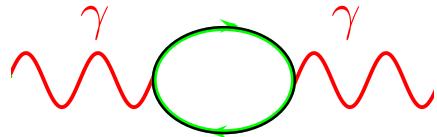
$\Rightarrow e = \text{elementary charge of classical electrodynamics}$

fine-structure constant  $\alpha(0) = \frac{e^2}{4\pi} = 1/137.03599976$

$\delta e$  contains photon vacuum polarization  $\Pi^\gamma(k^2 = 0)$ :

$$\Pi^\gamma(0) = \underbrace{\Pi^\gamma(0) - \Pi^\gamma(M_Z^2)}_{\text{non-perturbative}} + \underbrace{\Pi^\gamma(M_Z^2)}_{\text{perturbative}}$$

## photon vacuum polarization



$$\Pi^\gamma(M_Z^2) - \Pi^\gamma(0) \equiv \Delta\alpha \quad \rightarrow \quad \alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha} \simeq \frac{1}{129}$$

$$\Delta\alpha = \Delta\alpha_{\text{lept}} + \Delta\alpha_{\text{had}},$$

$$\Delta\alpha_{\text{lept}} = 0.031498 \quad (\text{3-loop})$$

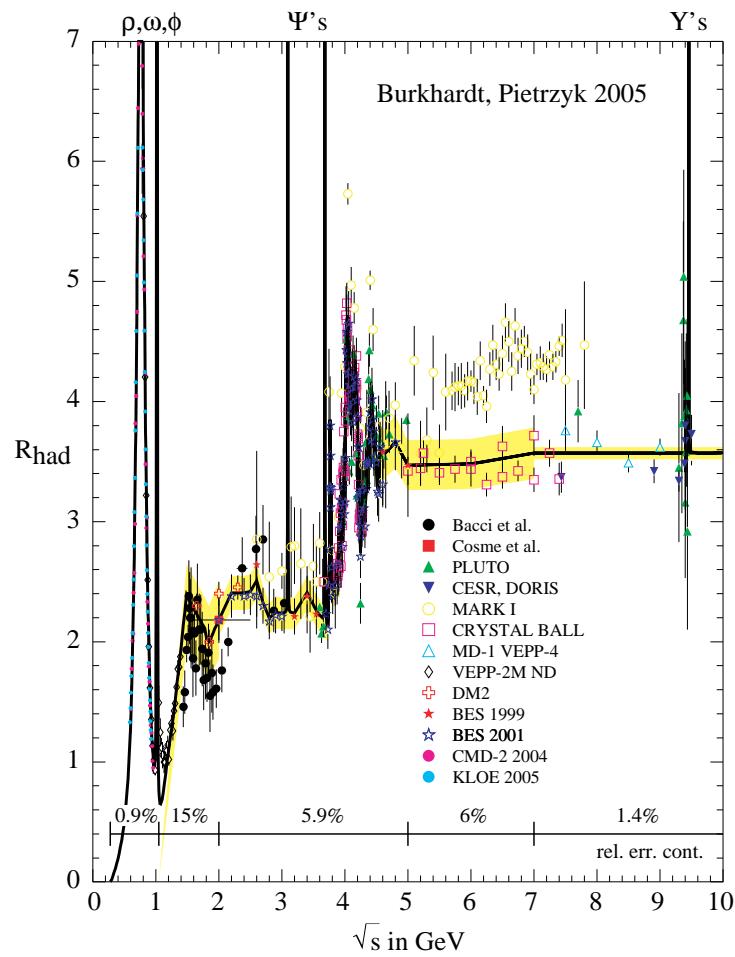
$$\begin{aligned}\Delta\alpha_{\text{had}} &= 0.02750 \pm 0.00033 \\ &= 0.02757 \pm 0.00010\end{aligned}$$

*arXiv:1010.4180*

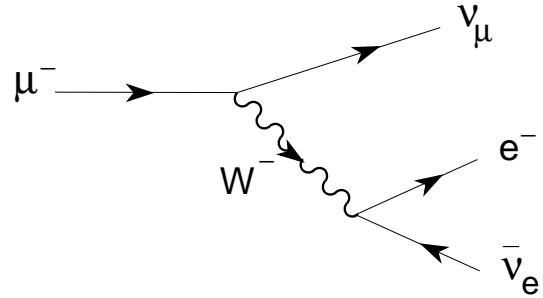
$$\Delta\alpha_{\text{had}} = -\frac{\alpha}{3\pi} M_Z^2 \operatorname{Re} \int_{4m_\pi^2}^\infty ds' \frac{R_{\text{had}}(s')}{s'(s' - M_Z^2 - i\epsilon)}$$

$$R_{\text{had}} =$$

$$\frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)}$$

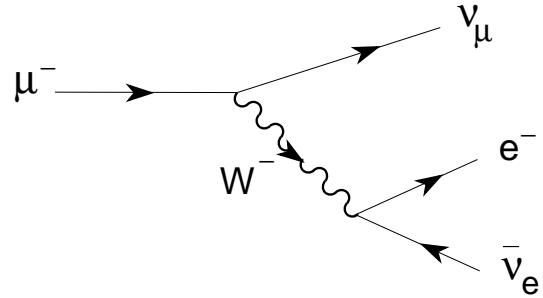


# $M_W - M_Z$ correlation



$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 (1 - M_W^2/M_Z^2)}$$

# $M_W - M_Z$ correlation



$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 (1 - M_W^2/M_Z^2)}$$

$M_W = 80.939 \pm 0.002 \text{ GeV}$     *from*     $G_F, \alpha, M_Z$

$M_W = 79.965 \pm 0.005 \text{ GeV}$     *with*     $\alpha \rightarrow \alpha(M_Z)$

$M_W = 80.385 \pm 0.015 \text{ GeV}$     *exp.*     $37\sigma / 28\sigma$

## with loop contributions

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 (1 - M_W^2/M_Z^2)} \cdot (1 + \Delta r)$$

$\Delta r$  : quantum correction

$$\Delta r = \Delta r(m_t, M_H)$$

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2} \Delta\rho + \dots$$

$$\Delta\rho \sim \frac{m_t^2}{M_W^2}$$

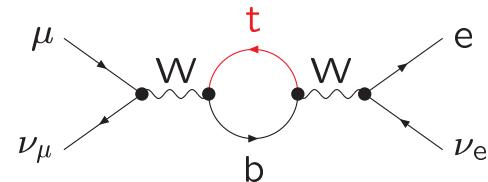
determines W mass

$$M_W = M_W(\alpha, G_F, M_Z, m_t, M_H)$$

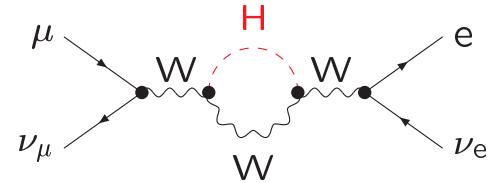
complete at 2-loop order

## 1-loop examples

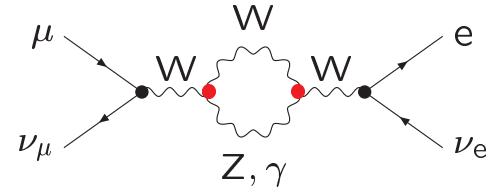
- top quark



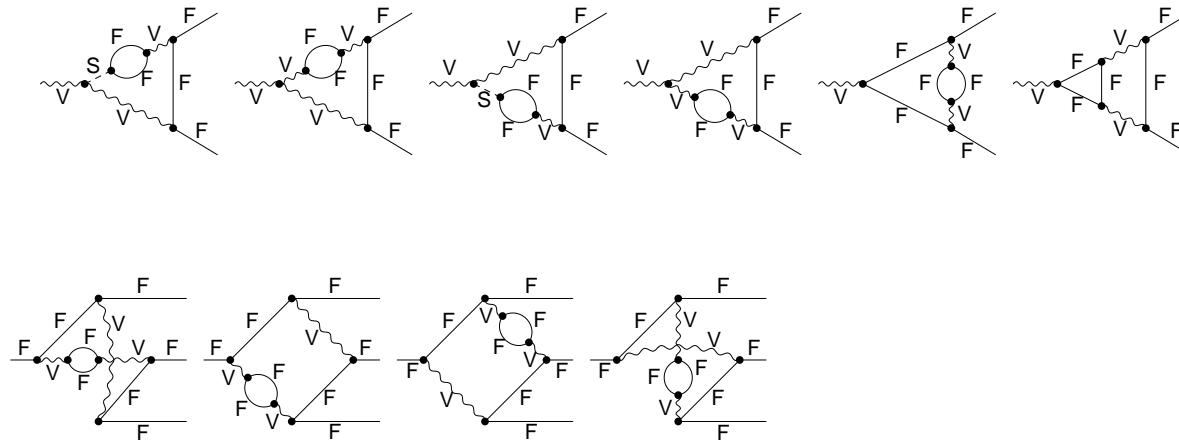
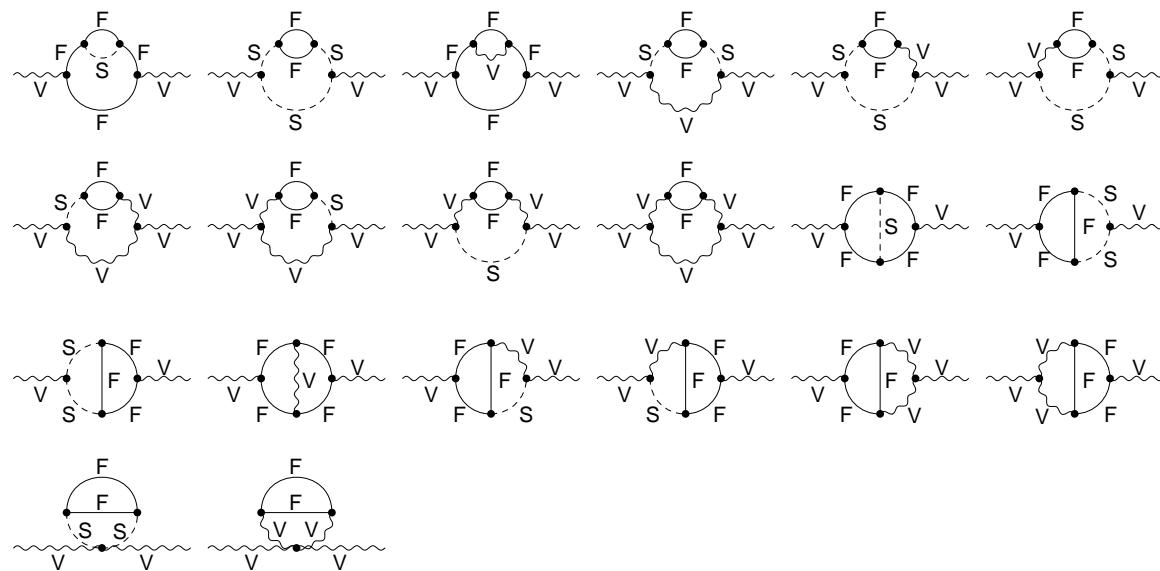
- Higgs boson



- gauge-boson self-couplings

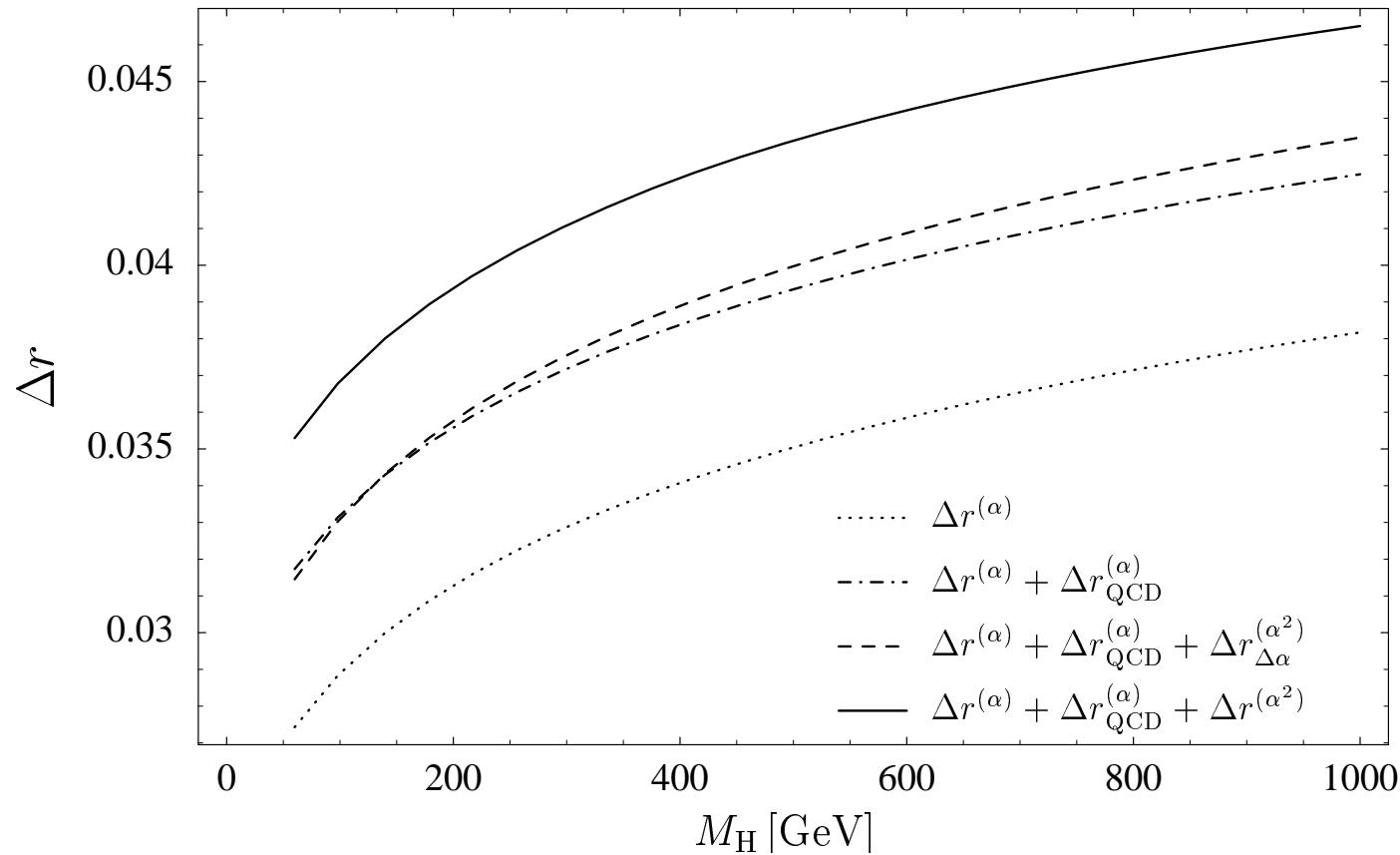


full structure of SM



*2-loop examples*

# effects of higher-order terms on $\Delta r$

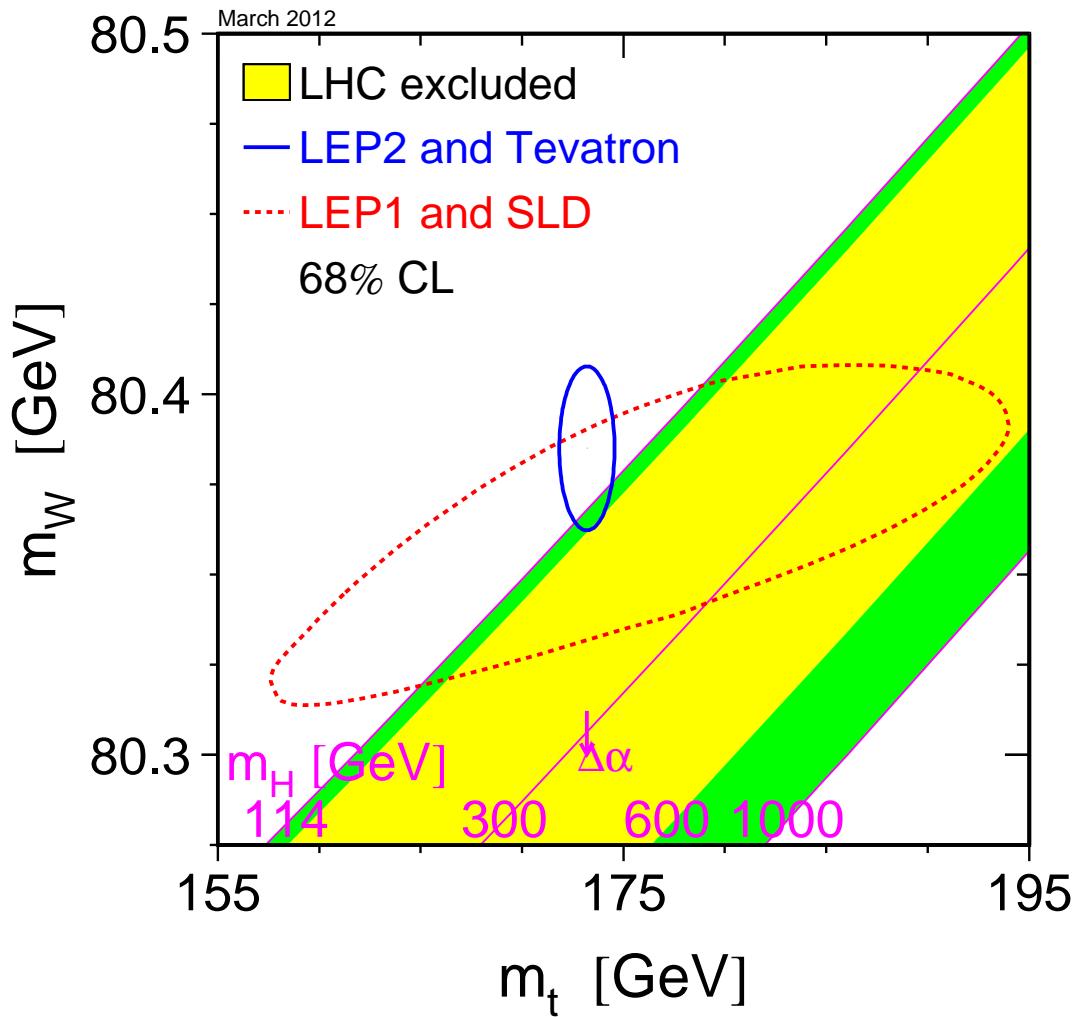


variation of  $\Delta r$  by 0.001  $\Rightarrow \delta M_W = 18 \text{ MeV}$

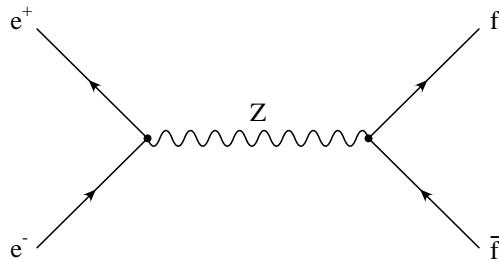
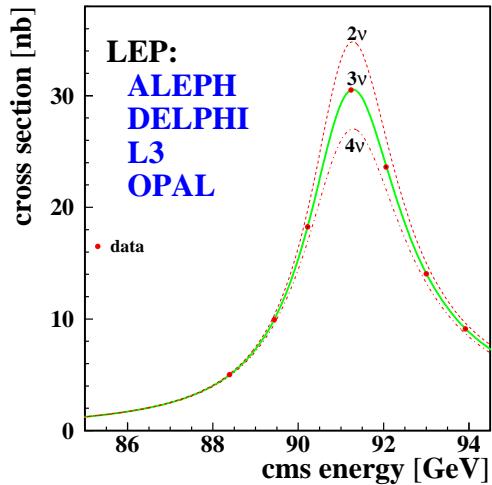
3-loop ( $\Delta\rho$ )  $\Rightarrow \delta M_W = 12 \text{ MeV}$

present exp. error:  $\Delta M_W = 15 \text{ MeV}$  / theo: 4 MeV

# LEP Electroweak Working Group



# Z resonance

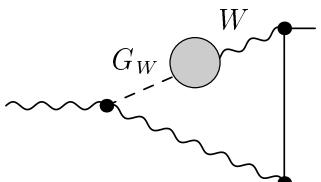


- effective  $Z$  boson couplings with higher-order  $\Delta g_{V,A}$

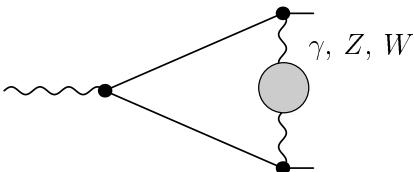
$$v_f \rightarrow g_V^f = v_f + \Delta g_V^f, \quad a_f \rightarrow g_A^f = a_f + \Delta g_A^f$$

- effective ew mixing angle (for  $f = e$ ):

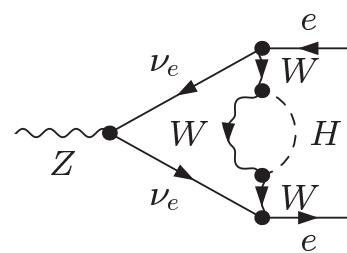
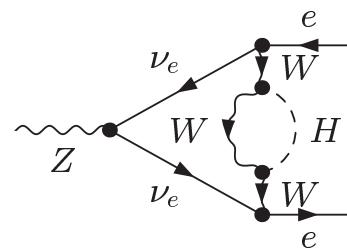
$$\sin^2 \theta_{\text{eff}} = \frac{1}{4} \left( 1 - \text{Re} \frac{g_V^e}{g_A^e} \right) = \kappa \cdot \left( 1 - \frac{M_W^2}{M_Z^2} \right)$$



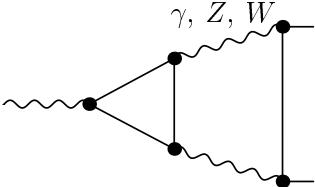
a)



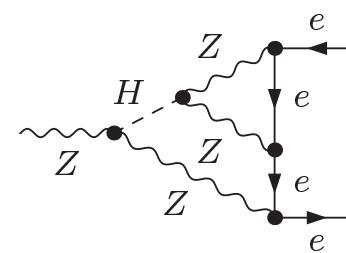
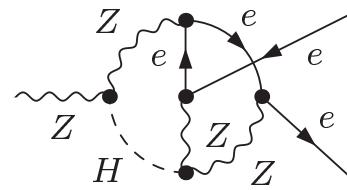
b)



c)



d)



## *2-loop examples for $Z$ couplings*

complete 2-loop calculation available for  $\sin^2 \theta_{\text{eff}}$

## EW 2-loop calculations for $\Delta r$

*Freitas, Hollik, Walter, Weiglein*

*Awramik, Czakon*

*Onishchenko, Veretin*

## EW 2-loop calculations for $\sin^2 \theta_{\text{eff}}$

*Awramik, Czakon, Freitas, Weiglein*

*Awramik, Czakon, Freitas*

*Hollik, Meier, Uccirati*

## universal terms at 3- and 4-loops (EW and QCD)

*van der Bij, Chetyrkin, Faisst, Jikia, Seidensticker*

*Faisst, Kühn Seidensticker, Veretin*

*Boughezal, Tausk, van der Bij*

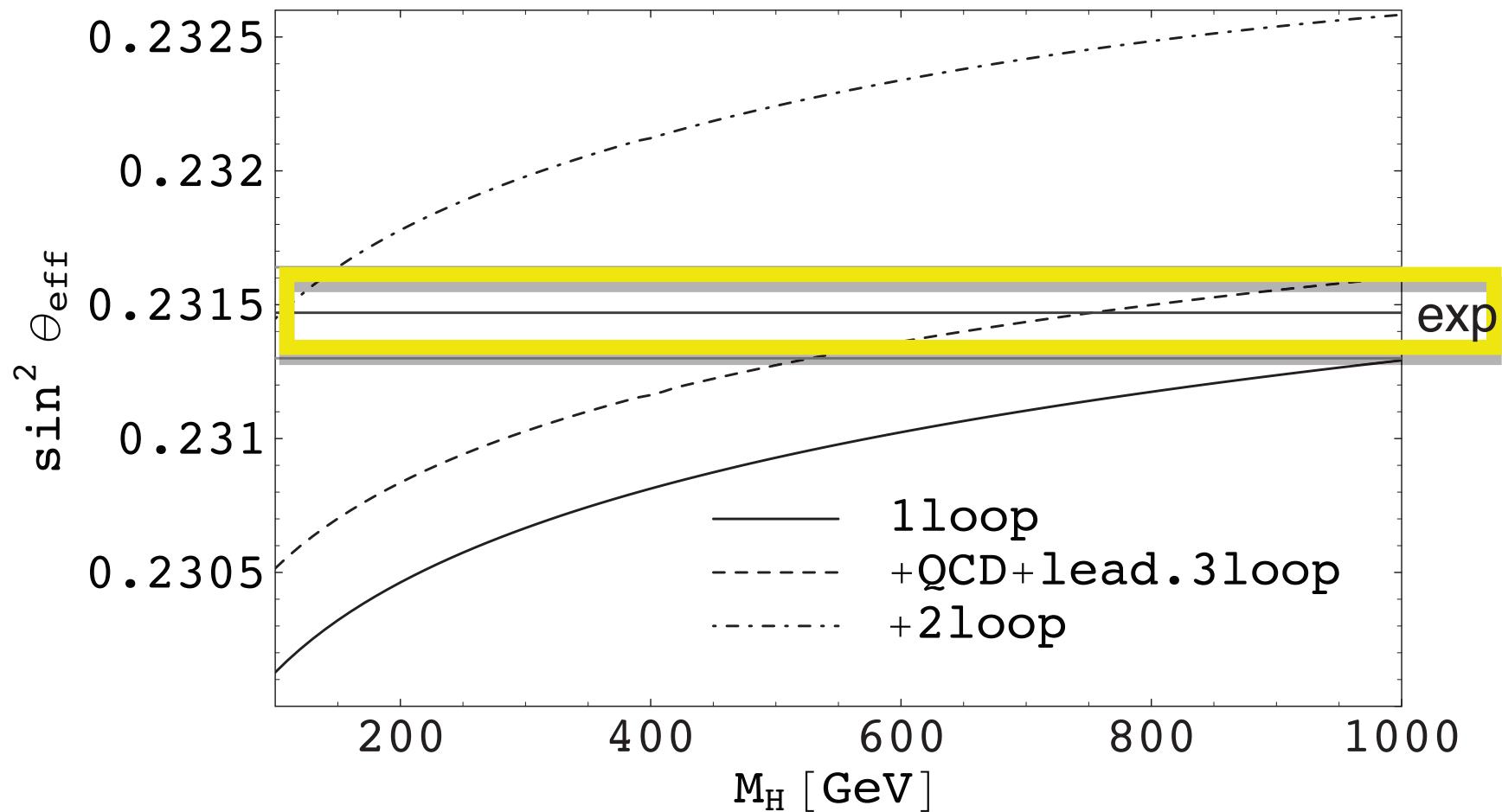
*Schröder, Steinhauser*

*Chetyrkin, Faisst, Kühn*

*Chetyrkin, Faisst, Kühn, Maierhofer, Sturm*

*Boughezal, Czakon*

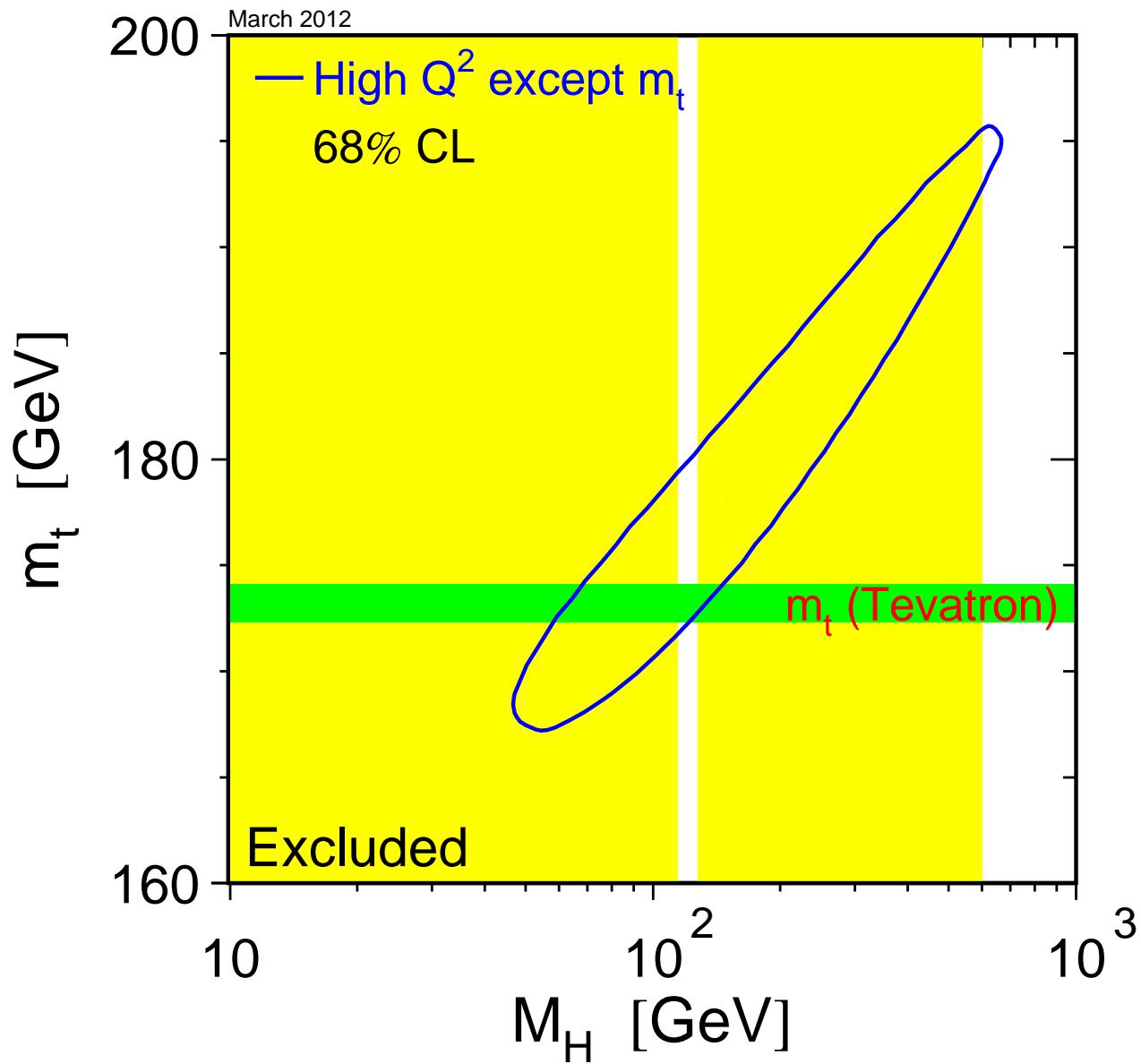
## importance of two-loop calculations



lowest order:  $\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.22290 \pm 0.00029$

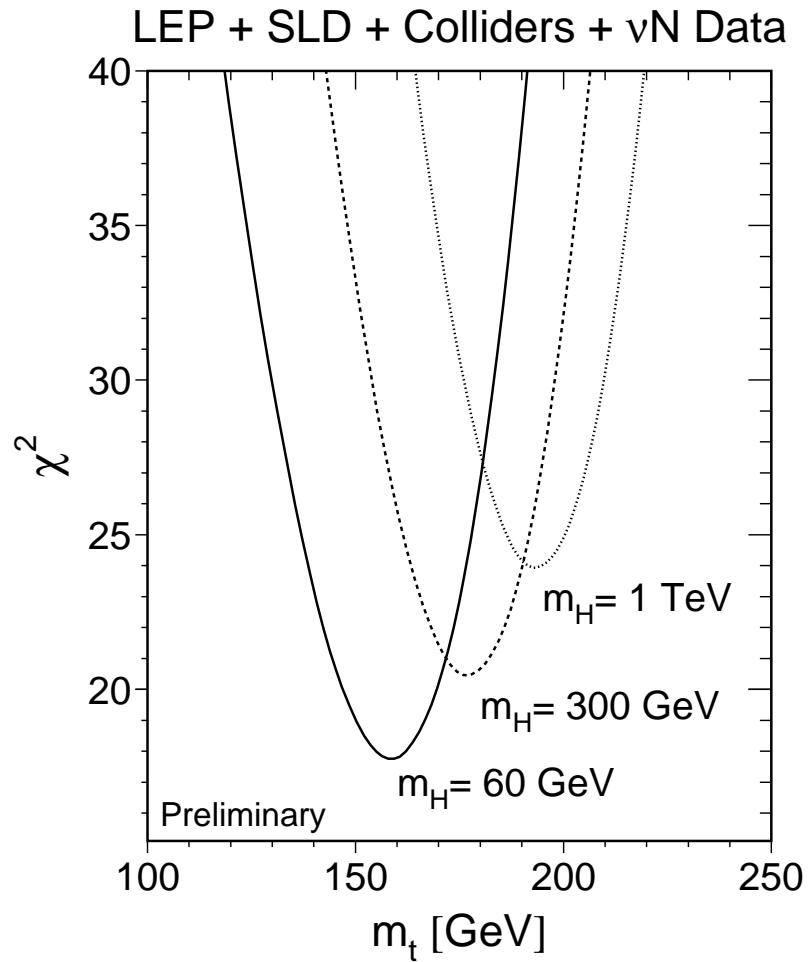
exp. value:  $\sin^2 \theta_{\text{eff}} = 0.23153 \pm 0.00016$

## Global analysis within the SM



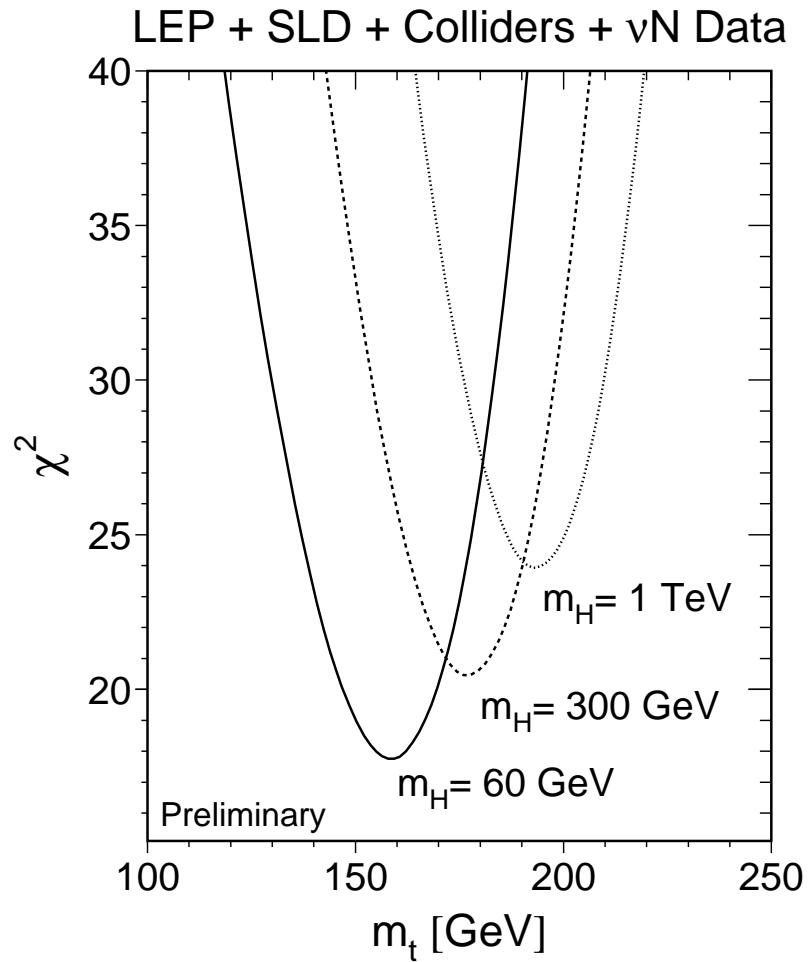
before the top quark was discovered (< 1995):

indirect mass determination  $\Rightarrow m_t = 178 \pm 8^{+17}_{-20} \text{ GeV}$



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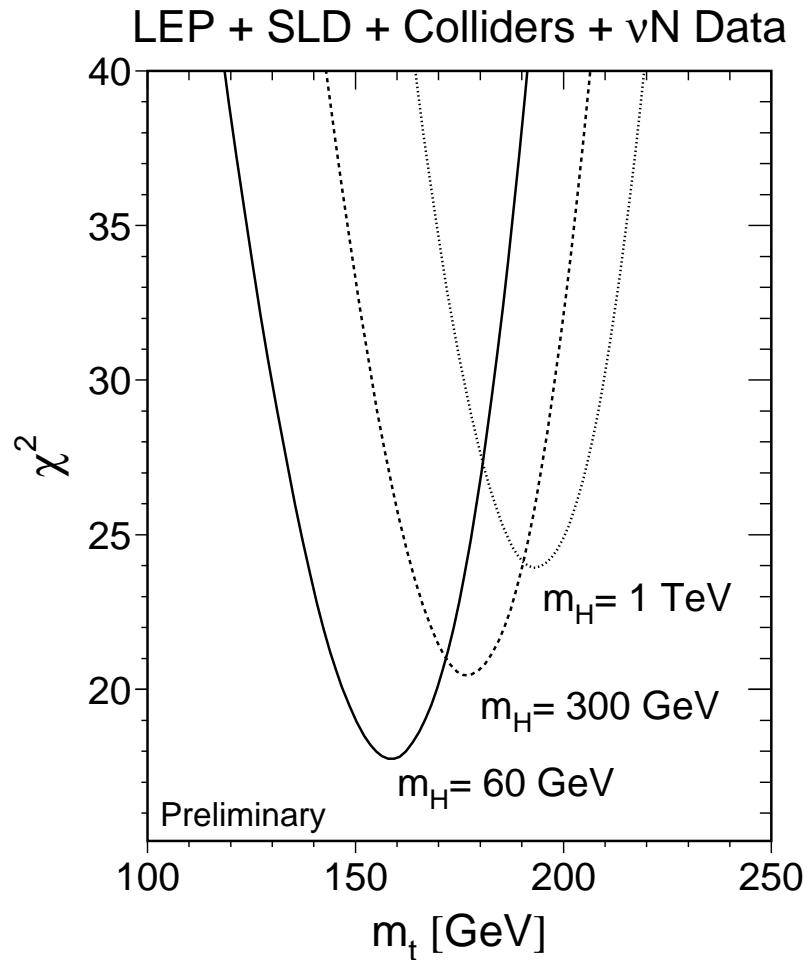


top discovery: *Tevatron 1995*

$m_t = 180 \pm 12 \text{ GeV}$

before the top quark was discovered (< 1995):

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top discovery: *Tevatron 1995*

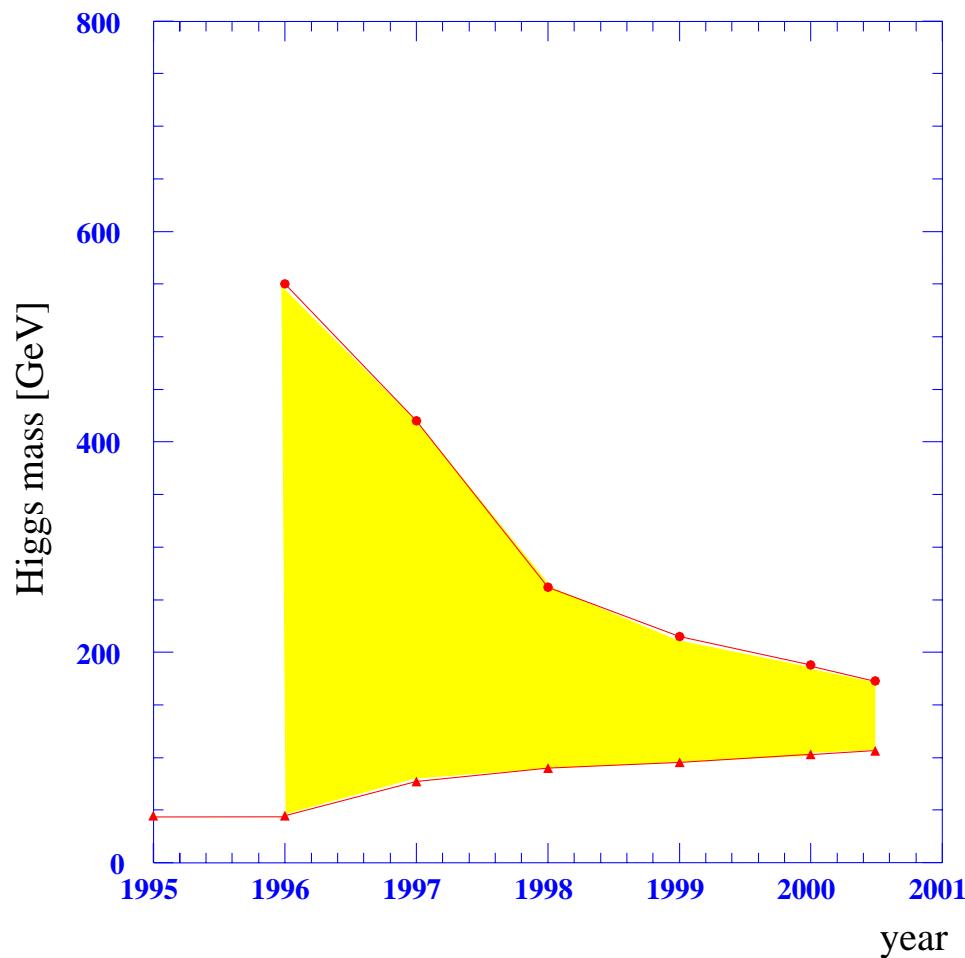
$m_t = 180 \pm 12 \text{ GeV}$

today:

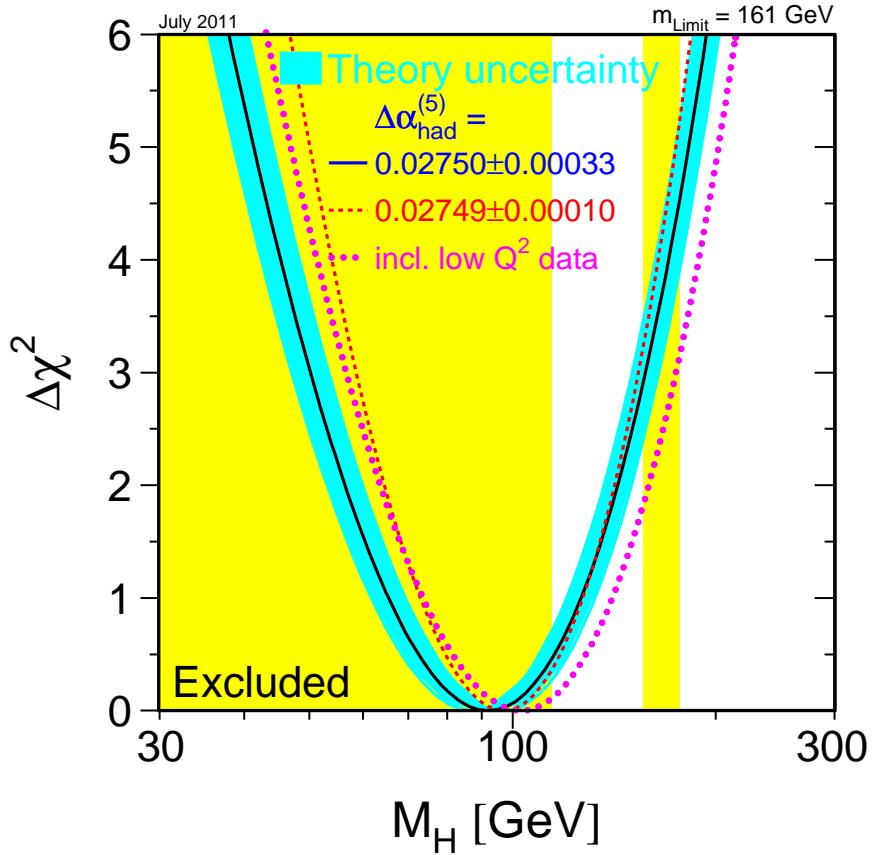
$m_t = 173.2 \pm 0.9 \text{ GeV}$

# The way to the Higgs boson

development of bounds from direct and indirect searches



# Global fit to the Higgs boson mass

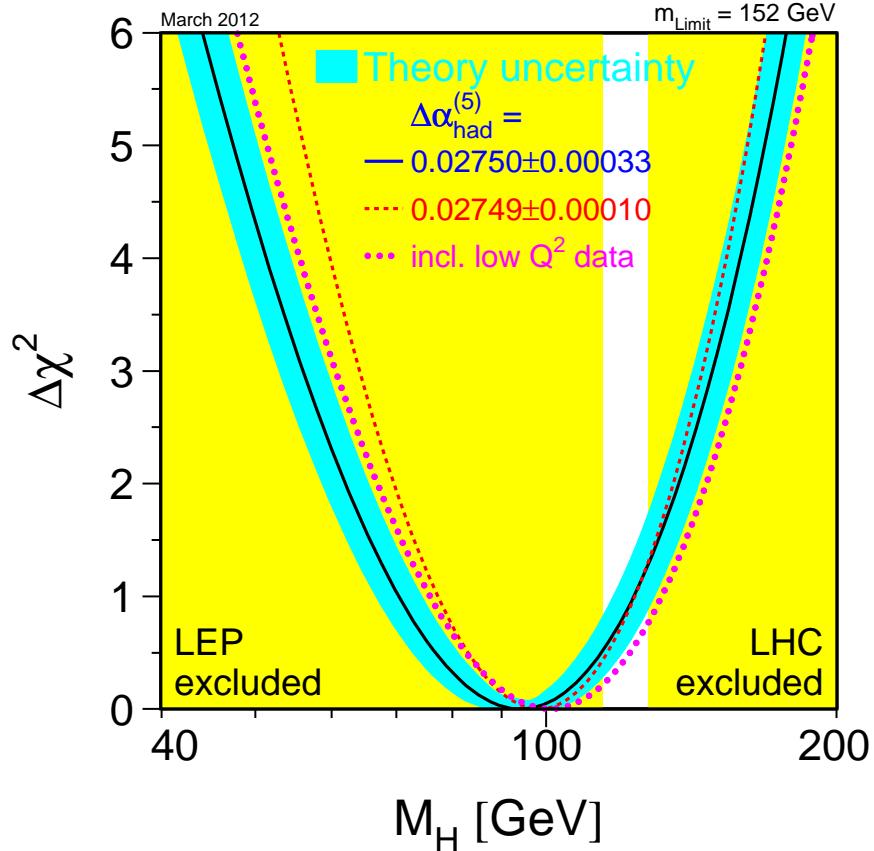


blueband: Theory uncertainty

“Precision Calculations  
at the  $Z$  Resonance”  
CERN 95-03

[Bardin, WH, Passarino (eds.)]

$M_H < 161 \text{ GeV}$  (at 95% C.L.)



after the 2011 results  
from the LHC  
on the Higgs boson mass

$M_H < 152$  GeV (95% C.L.)

$$M_H = 94^{+29}_{-24} \text{ GeV}$$

## **5. Higgs bosons**

Higgs potential: 
$$V = -\mu^2 (\Phi^\dagger \Phi)^2 + \frac{\lambda}{4} (\Phi^\dagger \Phi)^4$$

Higgs field in unitary gauge:  $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$

$H(x)$ : *real scalar field, describes neutral spin-0 bosons*

minimum of  $V$ :  $v = \frac{2\mu}{\sqrt{\lambda}}, \quad M_H = \mu\sqrt{2}$

$$\Rightarrow \lambda = \frac{4\mu^2}{v^2} = \frac{2M_H^2}{v^2}$$

$$V = \frac{M_H^2}{2} H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4$$

$M_H$  is the only free parameter

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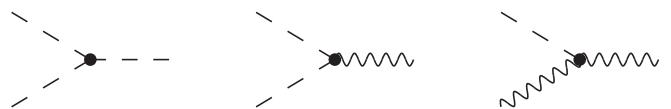
$M_H$  is the only free parameter

general gauge:  $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}} [v + \textcolor{blue}{H}(x) + i\chi(x)] \end{pmatrix}$

## gauge invariant Lagrangian of the Higgs sector

$$\begin{aligned}
 \mathcal{L}_H &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad \text{with } D_\mu = \partial_\mu - i g_2 \frac{\sigma^a}{2} W_\mu^a + i \frac{g_1}{2} B_\mu \\
 &= (\partial_\mu \phi^+)(\partial^\mu \phi^-) - \frac{iev}{2s_W} (W_\mu^+ \partial^\mu \phi^- - W_\mu^- \partial^\mu \phi^+) + \frac{e^2 v^2}{4s_W^2} W_\mu^+ W^{-,\mu} \\
 &\quad + \frac{1}{2} (\partial \chi)^2 + \frac{ev}{2c_W s_W} Z_\mu \partial^\mu \chi + \frac{e^2 v^2}{4c_W^2 s_W^2} Z^2 + \frac{1}{2} (\partial H)^2 - \mu^2 H^2
 \end{aligned}$$

+ (trilinear  $SSS$ ,  $SSV$ ,  $SVV$  interactions)



+ (quadrilinear  $SSSS$ ,  $SSVV$  interactions)

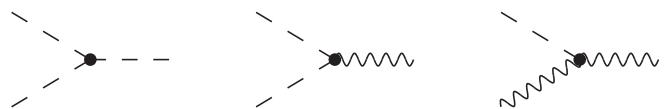


$\Rightarrow$  **H-V-V gauge interactions,  $V=W$  and  $Z$**

## gauge invariant Lagrangian of the Higgs sector

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 \end{aligned}$$

+ (trilinear  $SSS$ ,  $SSV$ ,  $SVV$  interactions)



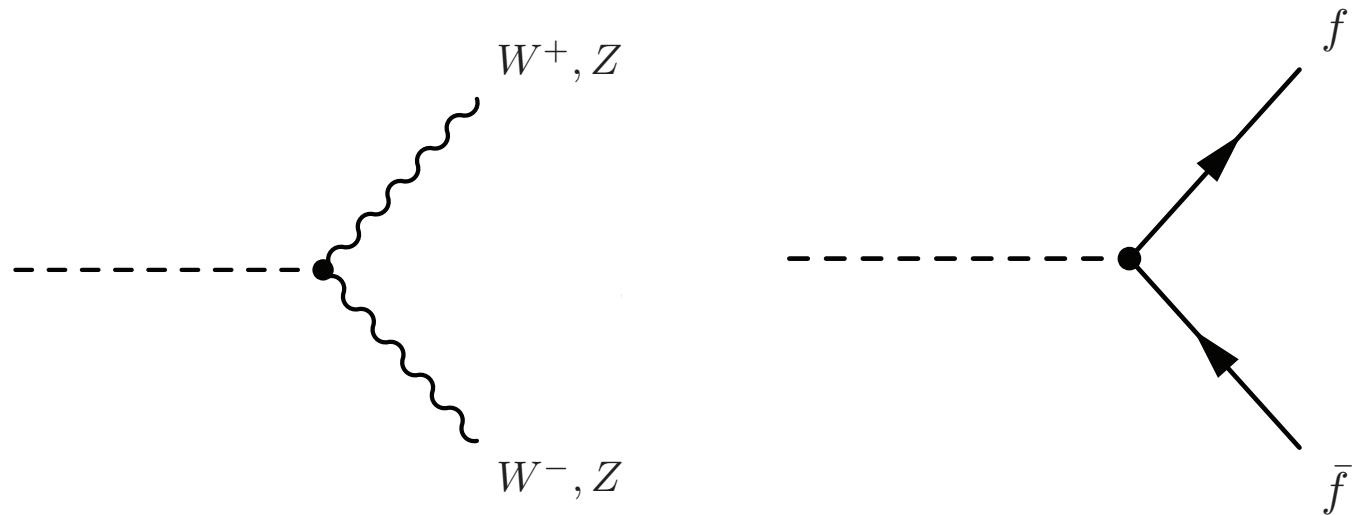
+ (quadrilinear  $SSSS$ ,  $SSVV$  interactions)



$\Rightarrow$  **H-V-V gauge interactions,  $V=W$  and  $Z$**

$$\mathcal{L}_{\text{Yuk}} = - \sum_f (m_f + \frac{m_f}{v} H) \bar{\psi}_f \psi_f + \dots (ff \chi, \phi^\pm)$$

$\Rightarrow$  **H-f-f Yukawa interactions**



$$g_2 M_W, \quad g_2 \frac{M_Z}{c_W}$$

$$\frac{m_f}{v} = \frac{g_2 m_f}{2 M_W}$$

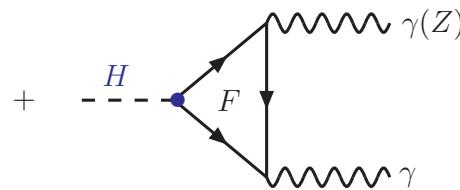
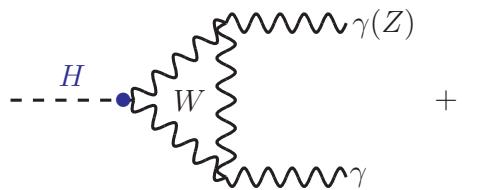
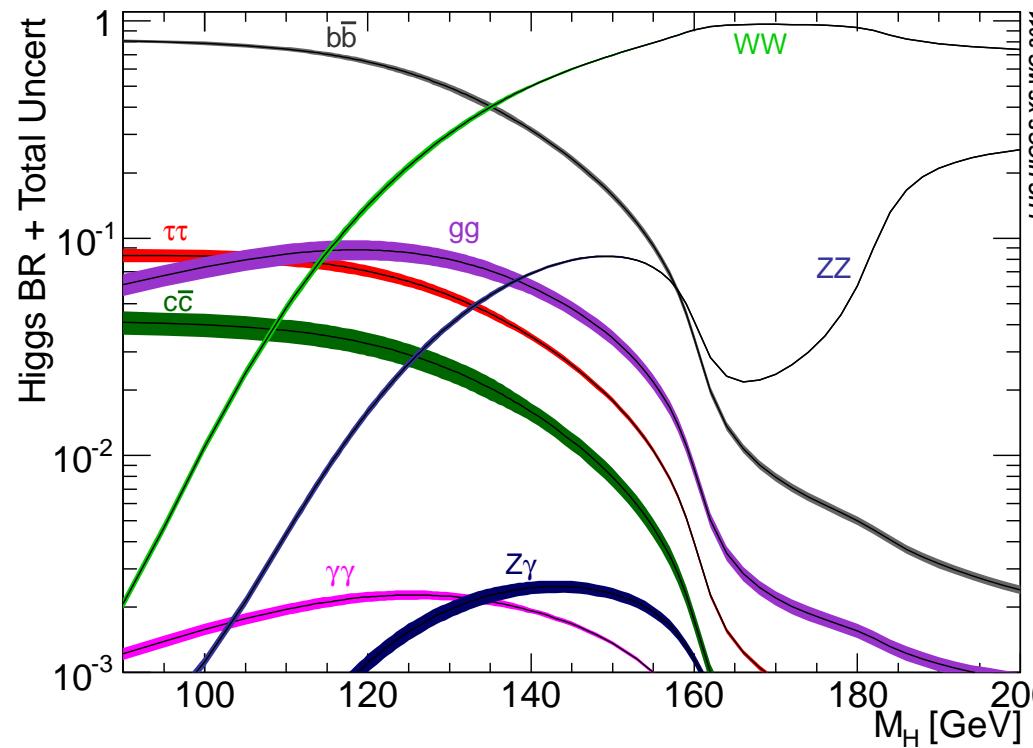
$$\Gamma(H \rightarrow f\bar{f}) = N_c \frac{G_F M_H}{4\pi\sqrt{2}} m_f^2, \quad N_C = 3 \text{ (1) for quarks (leptons)}$$

$$\Gamma(H \rightarrow VV) = \frac{G_F M_H^3}{8\pi\sqrt{2}} F(r) \left(\frac{1}{2}\right)_Z, \quad r = \frac{M_V}{M_H}$$

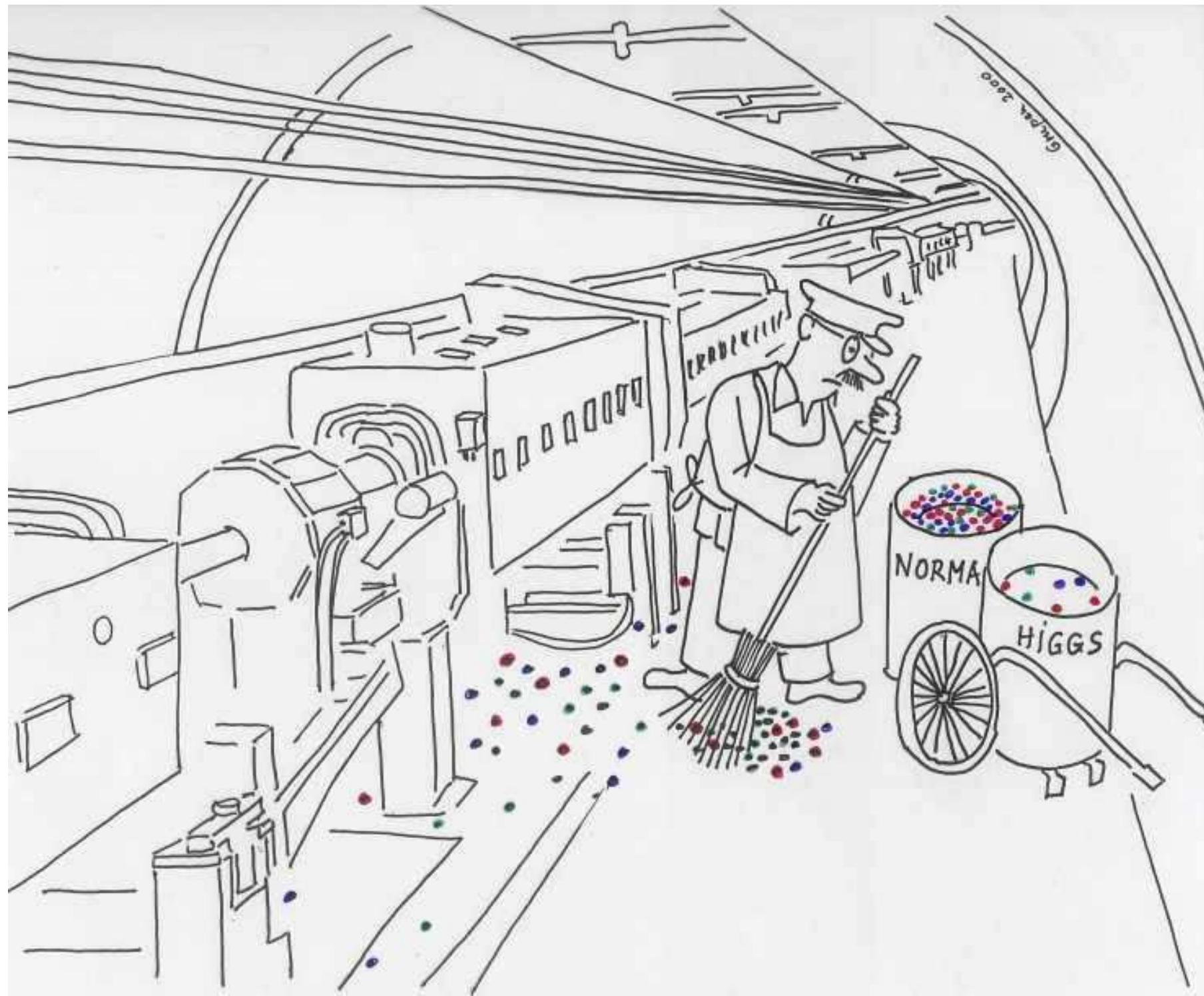
# Higgs boson decay channels

branching ratios

$$BR(H \rightarrow X) = \frac{\Gamma(H \rightarrow X)}{\Gamma(H \rightarrow \text{all})}$$

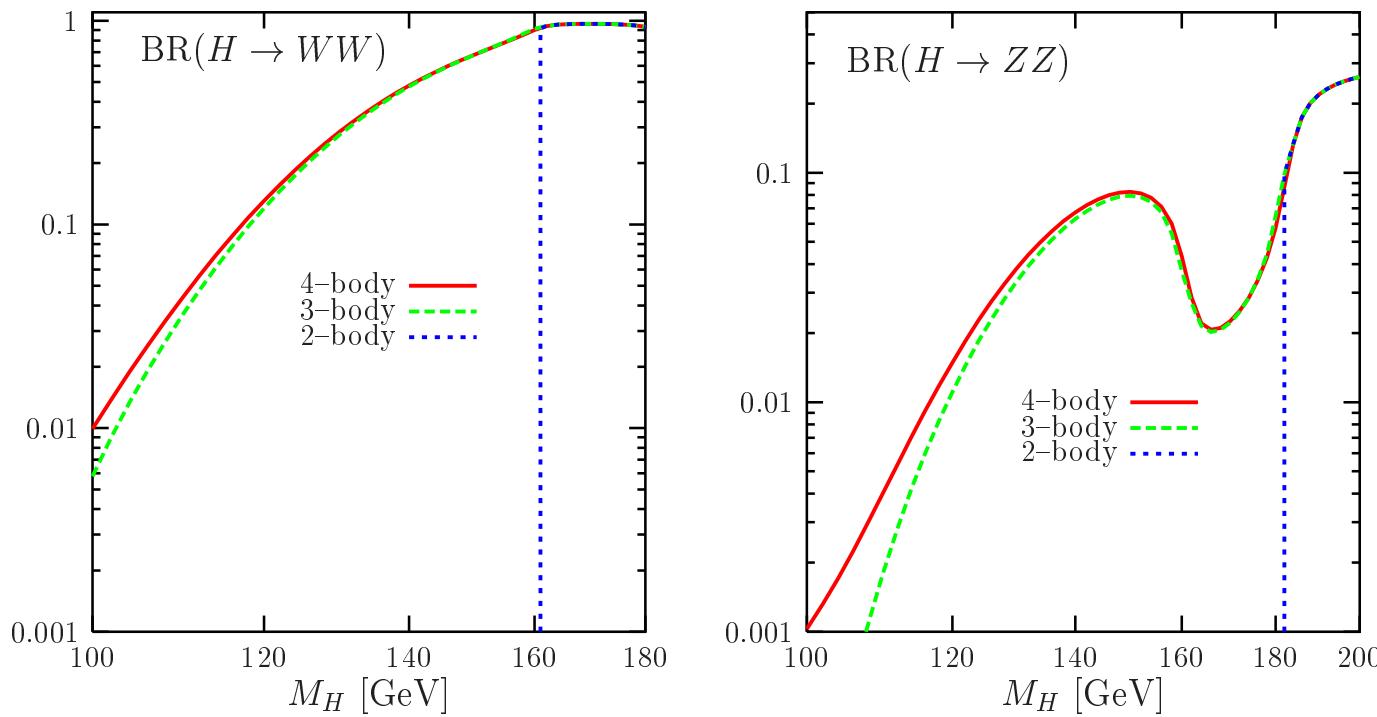
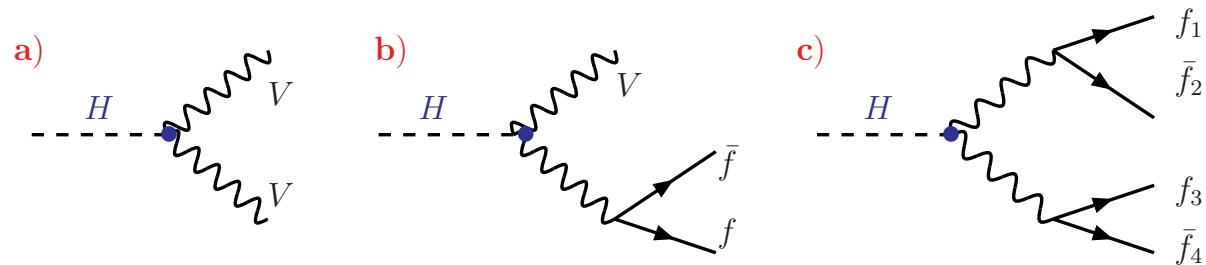


loop-induced (rare) decays



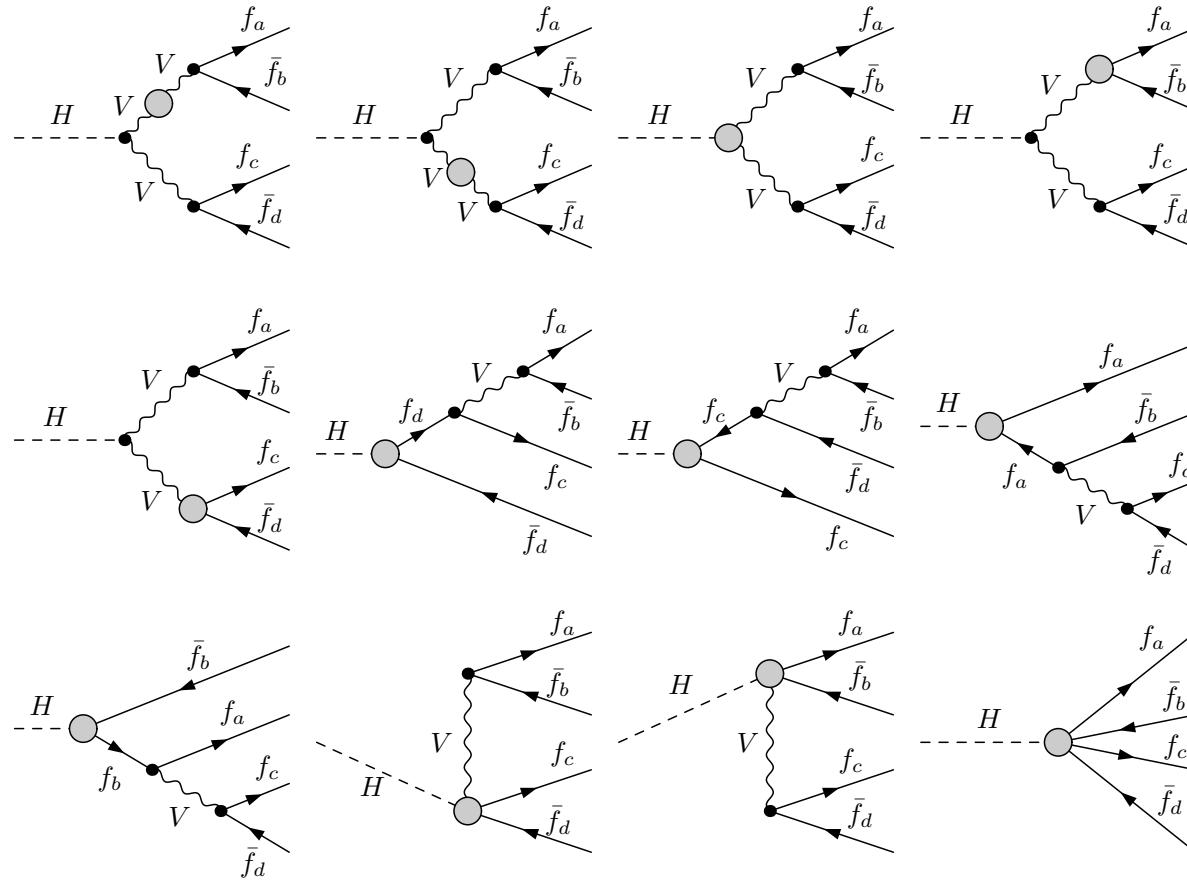
# Higgs decays into 4 fermions

also below  $VV$  threshold with one or two  $V$  off-shell

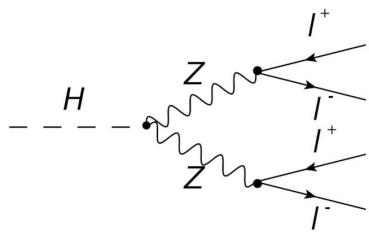
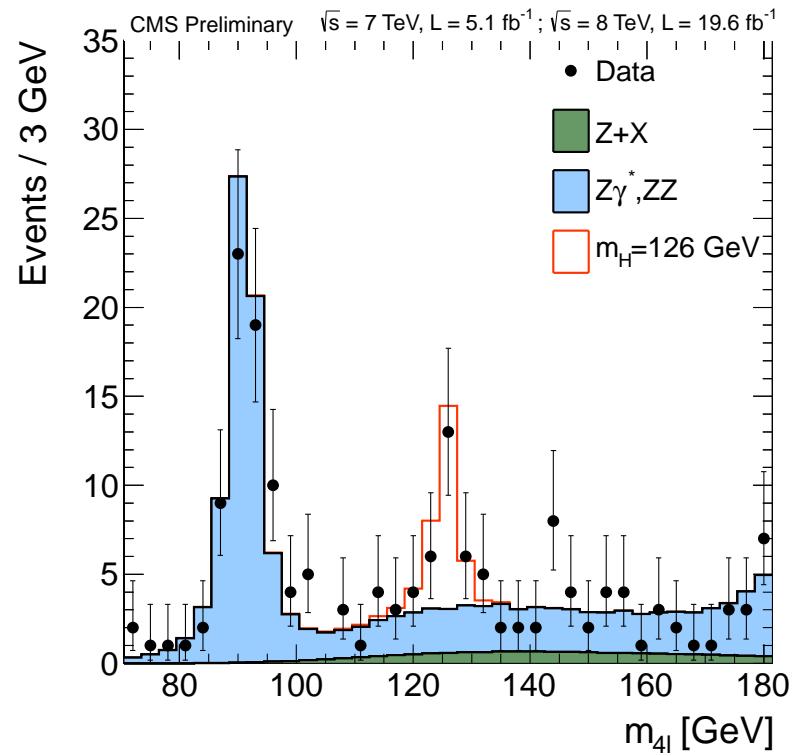
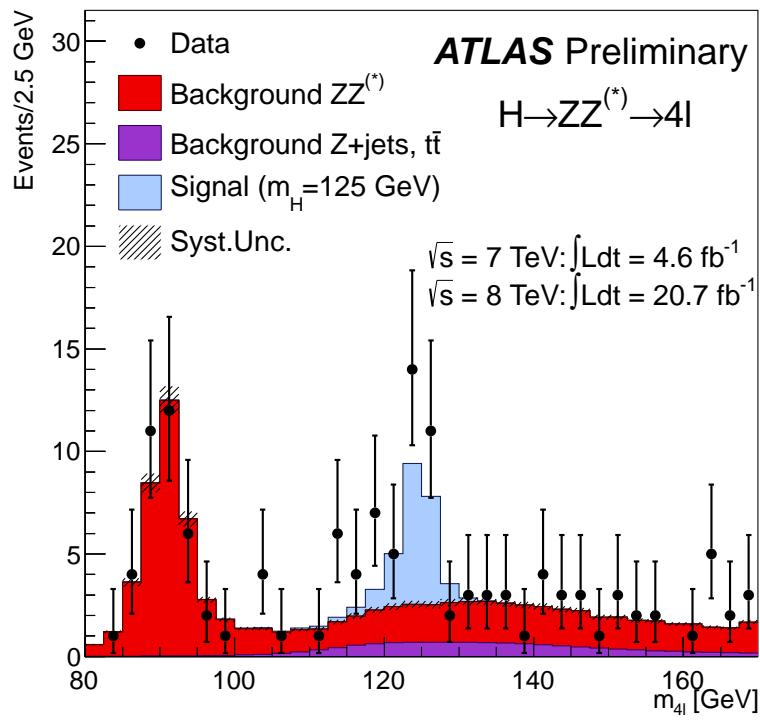


$$H \rightarrow VV \rightarrow 4f$$

needs also background processes + h.o.



$H \rightarrow ZZ \rightarrow l^+l^- l^+l^-$



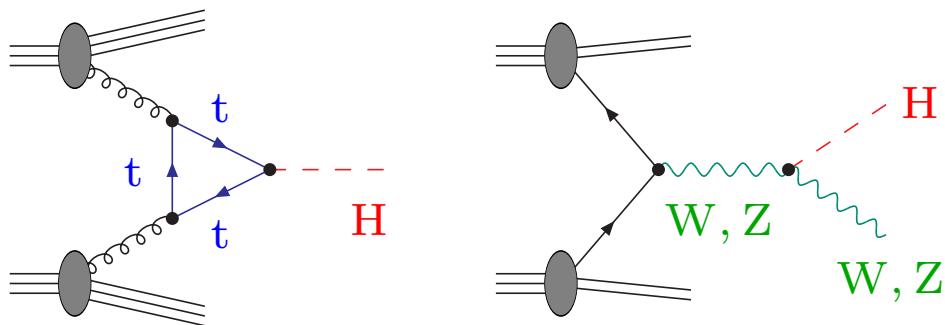
signal + background

## The direct search for the Higgs boson

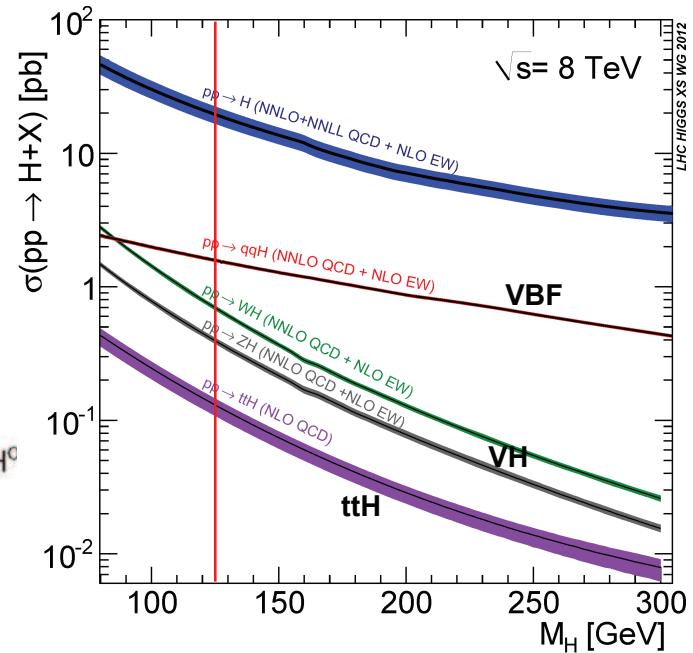
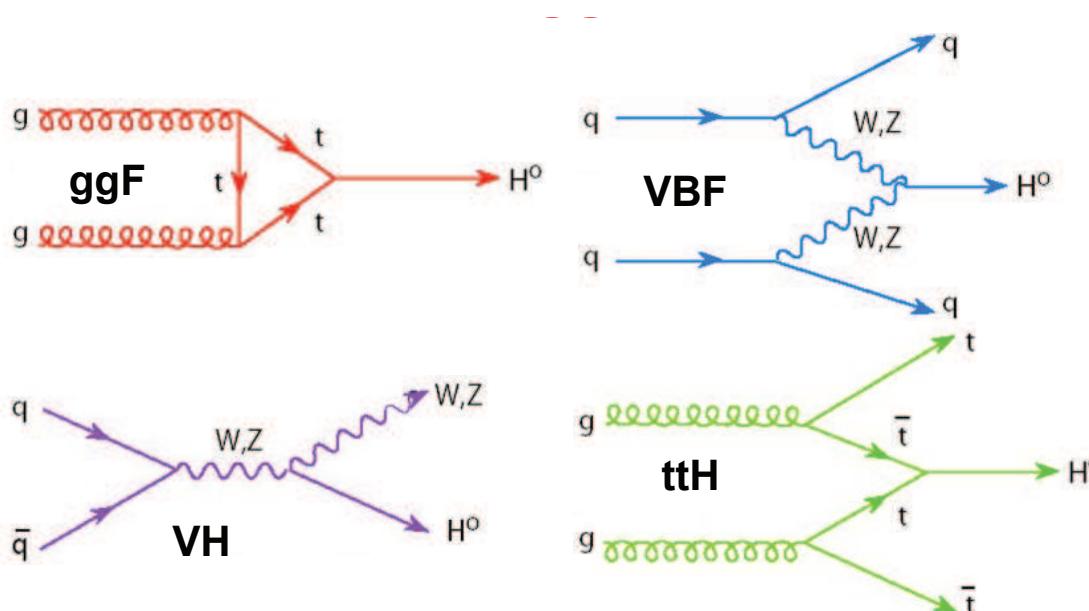
Higgs production at LEP:



Higgs production at the Tevatron:



# Higgs production at the LHC



*Handbook of Higgs Cross sections,  
arXiv:1101.0593, arXiv:1201.3084*



“ I think we have it . . . ”

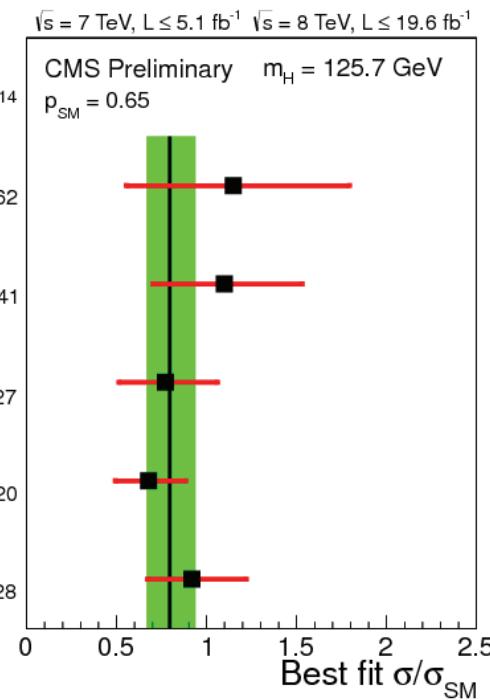
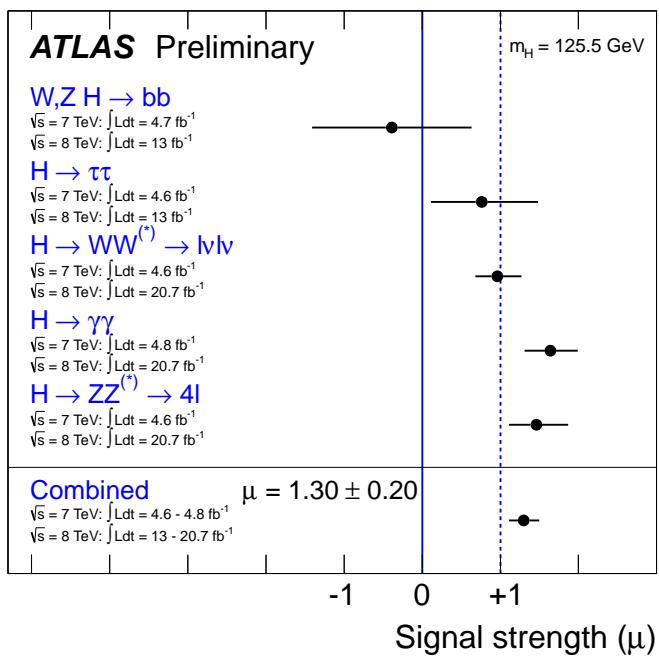


“ I think we have it . . . ”



. . . which one?

# A Standard Model Higgs boson at the LHC?



**H mass ATLAS (GeV)**

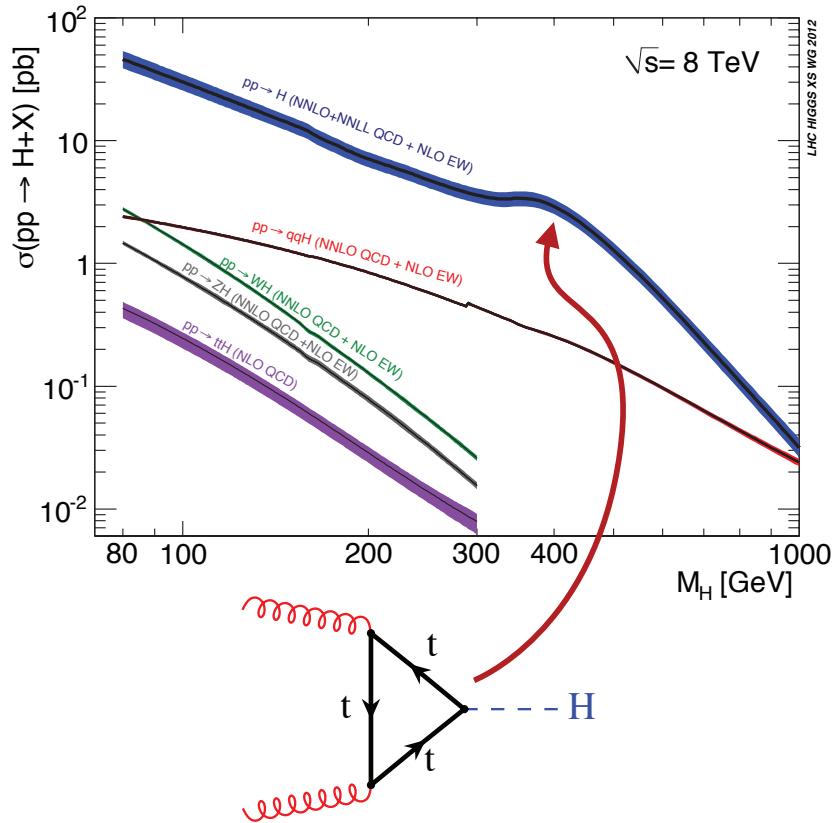
$$125.5 \pm 0.2 \pm 0.6$$

**H mass CMS (GeV)**

$$125.7 \pm 0.3 \pm 0.3$$

Theory:  $\sigma(pp \rightarrow H) \cdot BR(H \rightarrow X)$

# cross section for Higgs-boson production – theory



NLO: Spira, Djouadi, Graudenz, Zerwas '91, '93  
Dawson '91 **~80%**

NNLO: RH, Kilgore '02  
Anastasiou, Melnikov '02  
Ravindran, Smith, v. Neerven '03 **~30%**

Resummation:

Catani, de Florian, Grazzini, Nason '02  
Ahrens, Becher, Neubert, Zhang '08 **~10%**

Electroweak:

Actis, Passarino, Sturm, Uccirati '08  
Aglietti, Bonciani, Degrassi, Vicini '04  
Degrassi, Maltoni '04  
Djouadi, Gambino '94 **~5%**

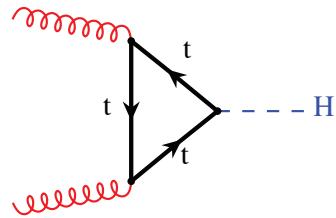
Mixed EW/QCD:

Anastasiou, Boughezal, Petriello '09

Fully differential NNLO:

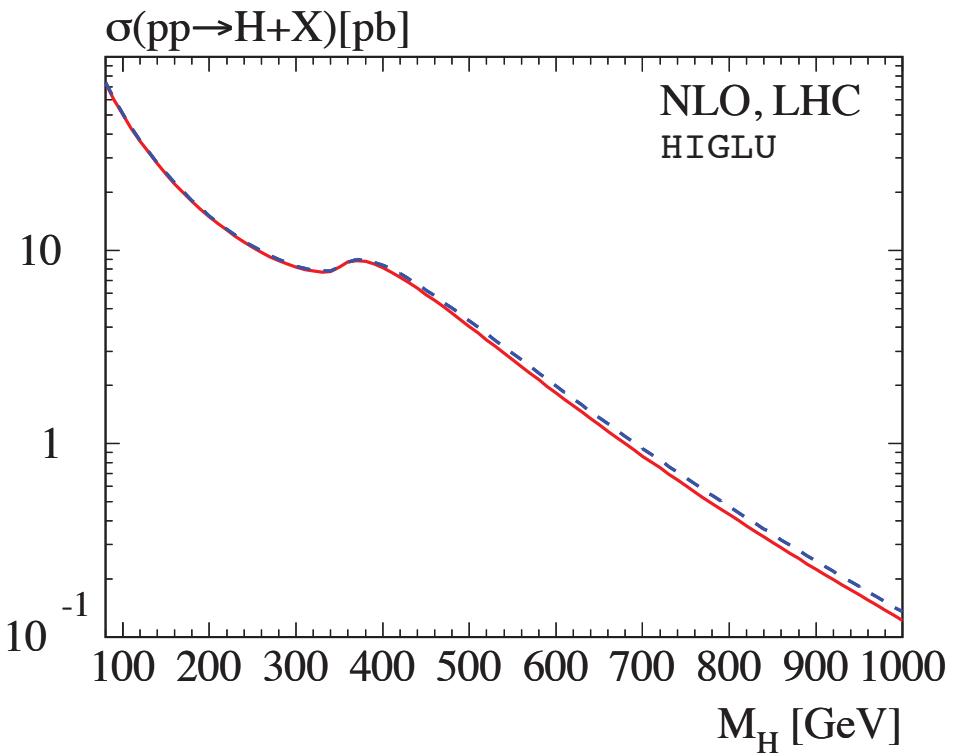
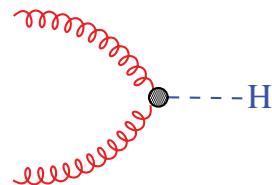
Anastasiou, Melnikov, Petriello '04  
Catani, Grazzini '07

# Effective Theory:



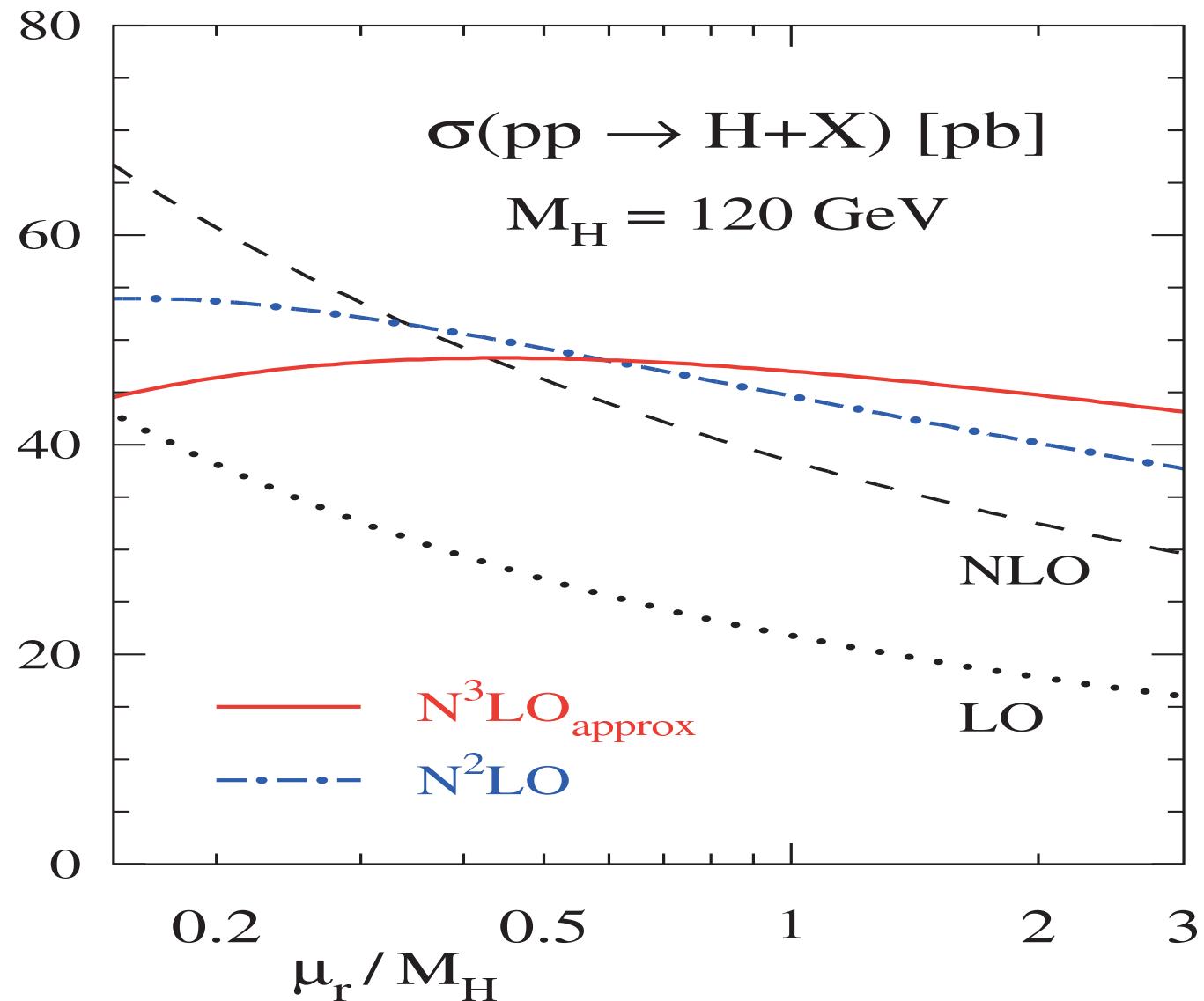
$m_t \gg M_H$

$$C(m_t, \alpha_s) \times$$

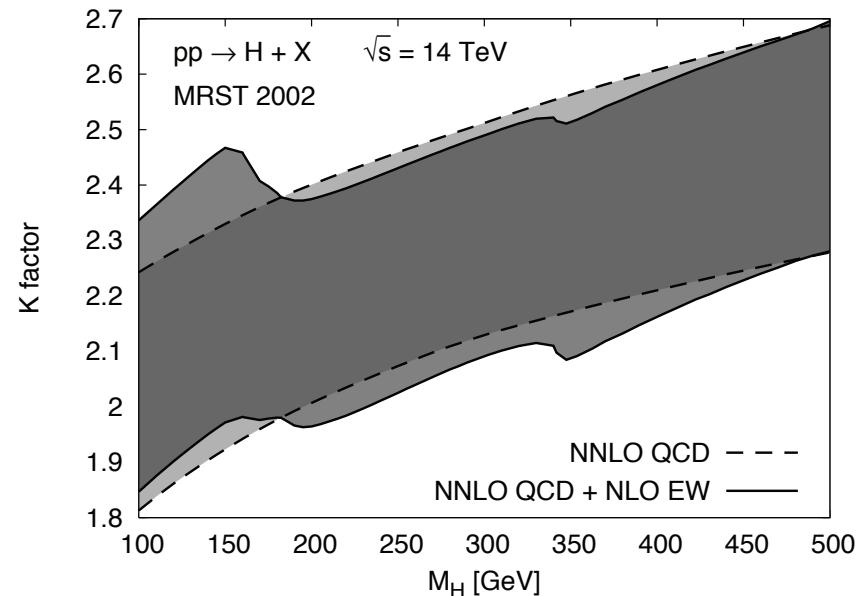
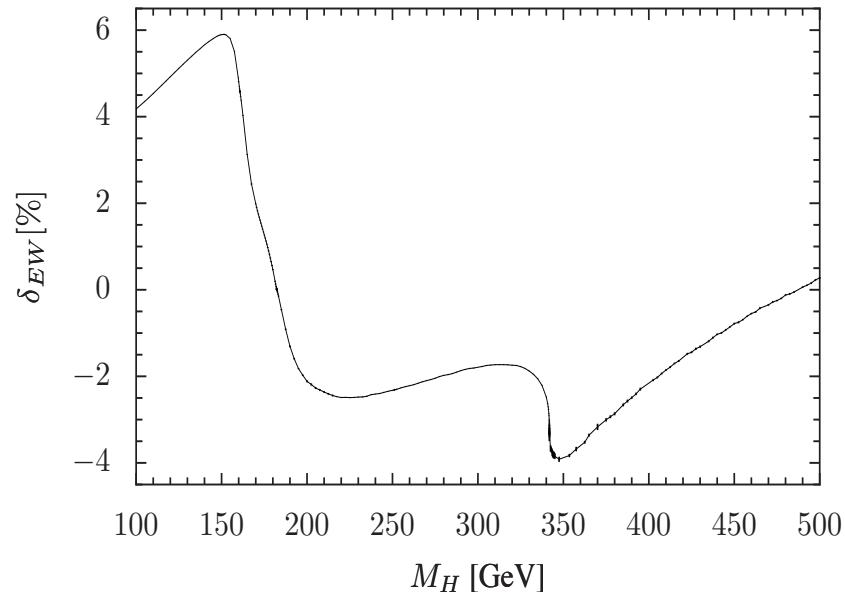


$$\sigma_{\infty}^{\text{HO}} \equiv \sigma^{\text{LO}}(m_t) \left( \frac{\sigma^{\text{HO}}}{\sigma^{\text{LO}}} \right)_{m_t \rightarrow \infty}$$

Moch, Vogt '05



# impact of electroweak contributions at NLO



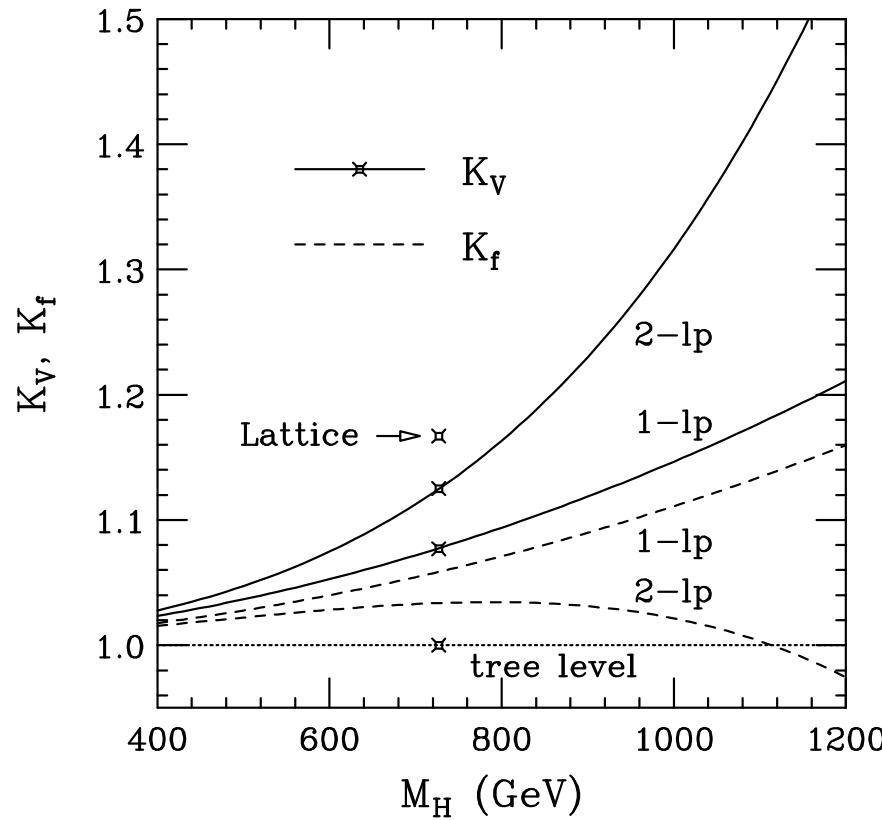
## Theoretical bounds on Higgs boson mass

- perturbativity → upper bound
- unitarity → upper bound
- Landau pole → upper bound
- vacuum stability → lower bound

# perturbativity

decay widths into fermions:  $\Gamma(H \rightarrow f\bar{f}) = \Gamma_{\text{tree}} \cdot K_f$

decay widths into vector bosons:  $\Gamma(H \rightarrow V\bar{V}) = \Gamma_{\text{tree}} \cdot K_V$

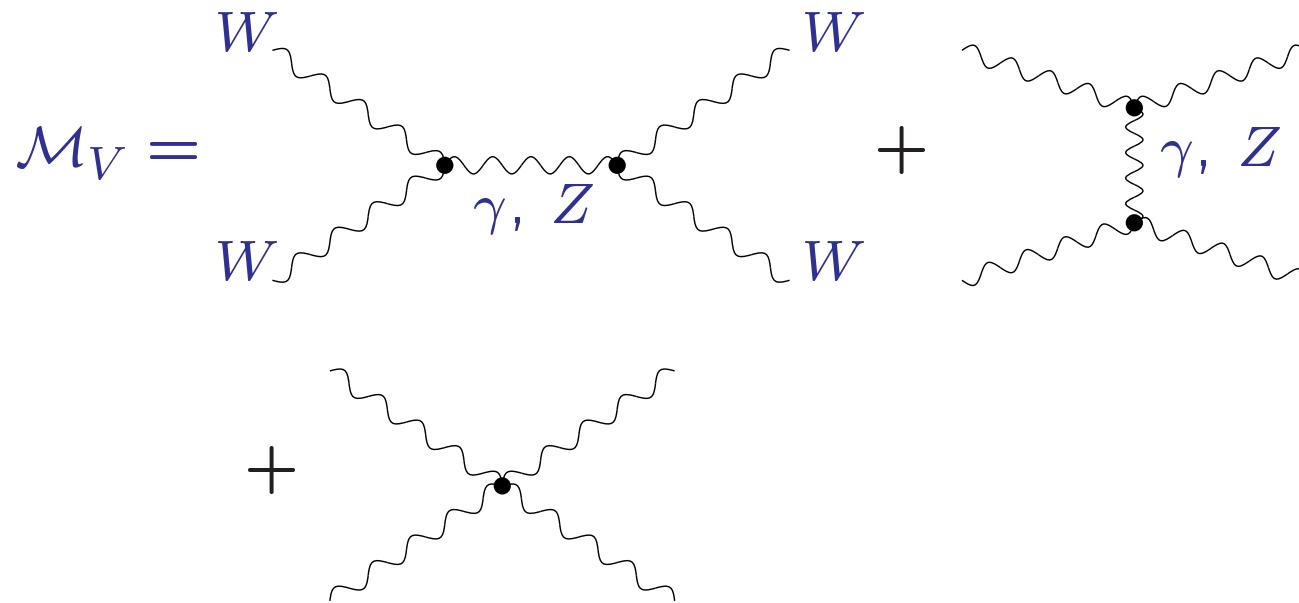


[Ghinculov; Frinck, Kniehl, Riesselmann]

# unitarity

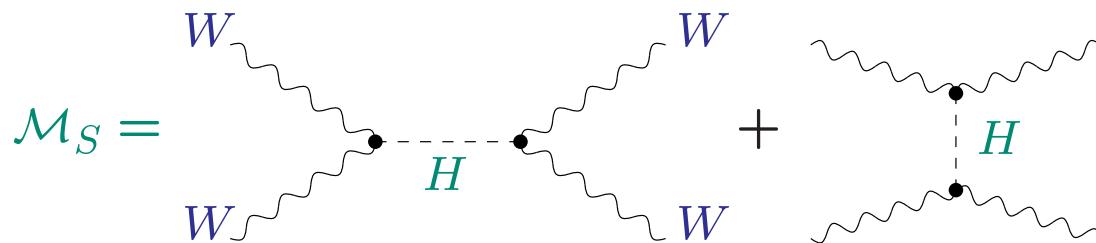
scattering of longitudinally polarized  $W$  bosons:

$$W_L W_L \rightarrow W_L W_L$$



$$= -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1) \quad \text{for } E \rightarrow \infty$$

Extra contribution from scalar particle:



$$= g_{WWH}^2 \frac{E^2}{M_W^4} + \mathcal{O}(1) \quad \text{for } E \rightarrow \infty$$

$$\mathcal{M} = \mathcal{M}_V + \mathcal{M}_S$$

$\Rightarrow$  terms with bad high-energy behavior cancel for

$$g_{WWH} = g M_W$$

for  $s \gg M_W^2$ , with  $t = -\frac{s}{2}(1 - \cos \theta)$ ,

$$\mathcal{M} \approx \frac{M_H^2}{v^2} \left( 2 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_H^2} \right)$$

partial wave expansion:

$$\mathcal{M}(s, t) = 8\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) \color{red}{a_l}$$

unitarity condition:  $|a_l| < 1$

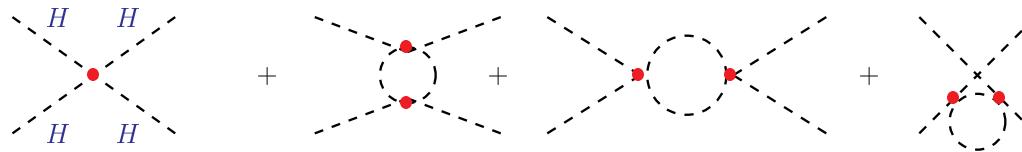
project on  $l = 0$  partial wave:

$$\begin{aligned} \color{red}{a_0} &= \frac{1}{16\pi} \int_{-1}^1 d\cos \theta \mathcal{M}(s, t) \\ &= \frac{M_H^2}{8\pi v^2} \left[ 2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log \left( 1 + \frac{s}{M_H^2} \right) \right] \\ &\approx \frac{M_H^2}{4\pi v^2} \quad \text{for } s \gg M_H^2 \end{aligned}$$

$$a_0 < 1 \quad \Rightarrow \quad M_H < 872 \text{ GeV}$$

# Landau pole

Higgs self coupling is scale dependent,  $\lambda(Q)$



variation with scale  $Q$  described by RGE

$$Q^2 \frac{d\lambda}{dQ^2} = \beta(\lambda) = \frac{3}{4\pi^2} \lambda^2$$

solution:

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^2} \lambda(v) \log \frac{Q^2}{v^2}} \quad \text{with} \quad \lambda(v) = \frac{M_H^2}{2v^2}$$

diverges at scale  $Q = \Lambda_C$  (Landau pole)

$$\Lambda_C = v \exp \left( \frac{4\pi^2 v^2}{3M_H^2} \right)$$

self-coupling diverges at

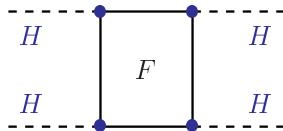
$$\Lambda_C = v \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

maximum Higgs mass by condition  $\Lambda_C > M_H$

$$\Rightarrow M_H < 800 \text{ GeV}$$

## vacuum stability

top-quark Yukawa coupling  $g_t \sim m_t$  contributes to the running Higgs self coupling  $\lambda(Q)$  through top loop  $\sim g_t^4$



variation with scale  $Q$  described by RGE

$$Q^2 \frac{d\lambda}{dQ^2} = \frac{3}{4\pi^2} \left( \lambda^2 - \frac{m_t^4}{v^4} \right)$$

approximate solution:

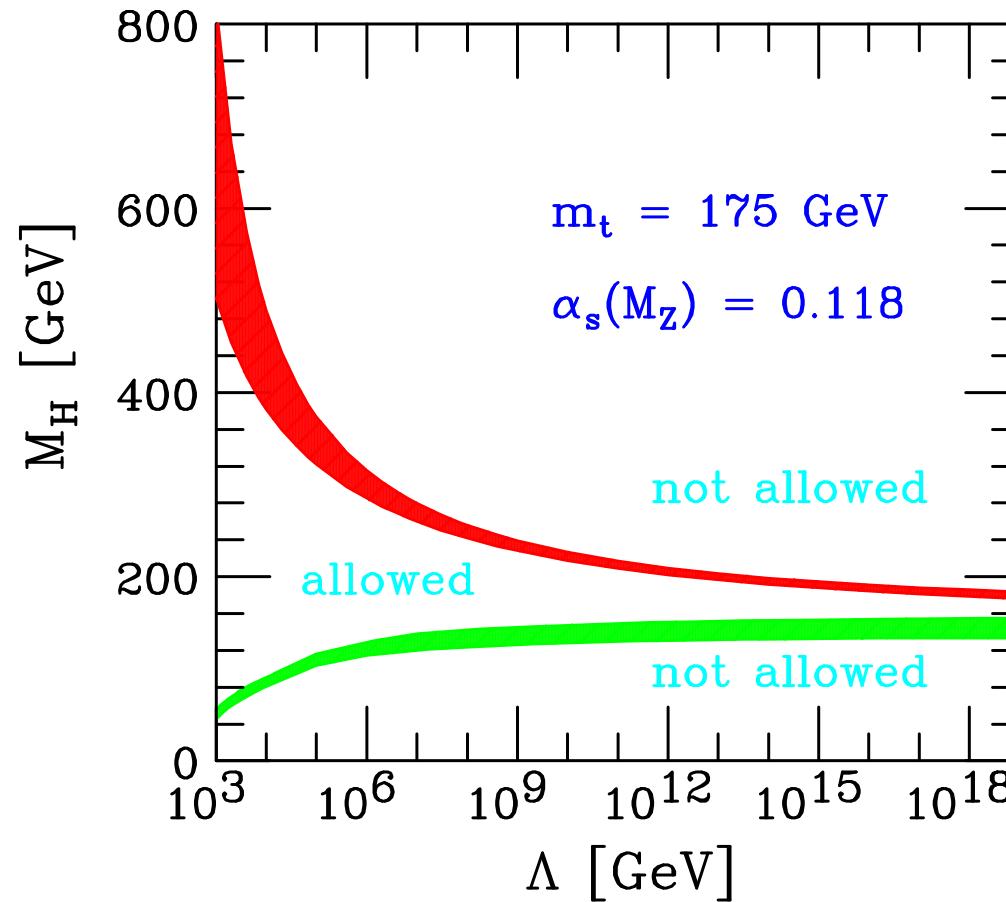
$$\lambda(Q) = \lambda(v) - \frac{3m_t^4}{2\pi^2 v^4} \log \frac{Q}{v}$$

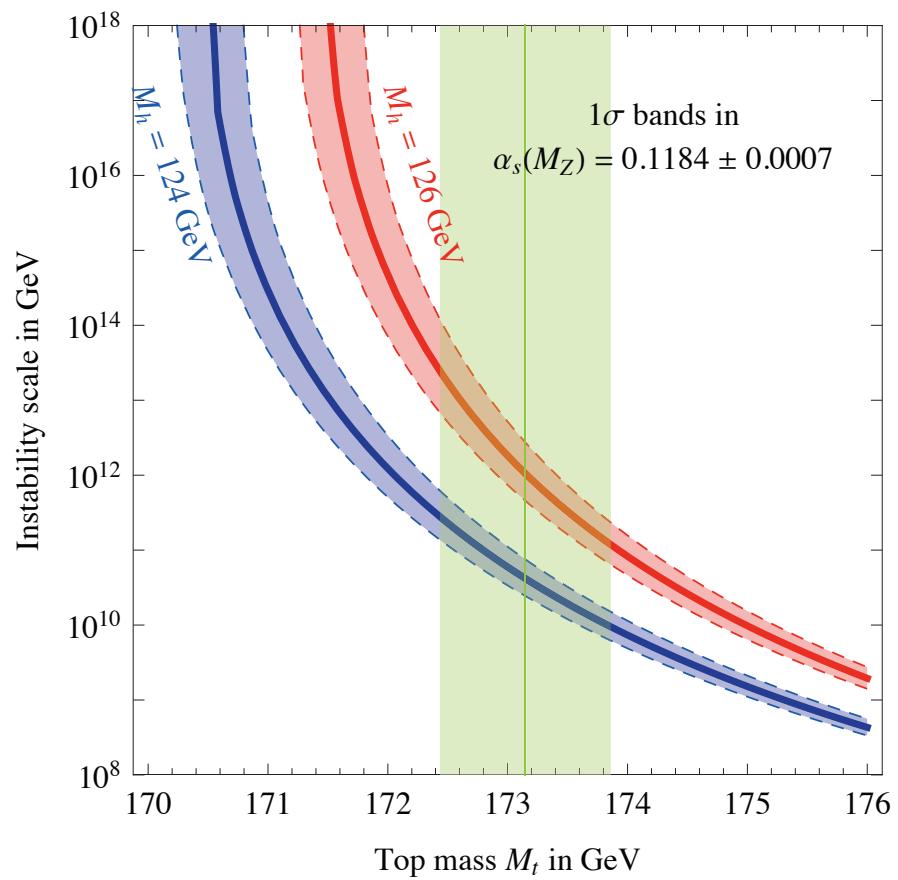
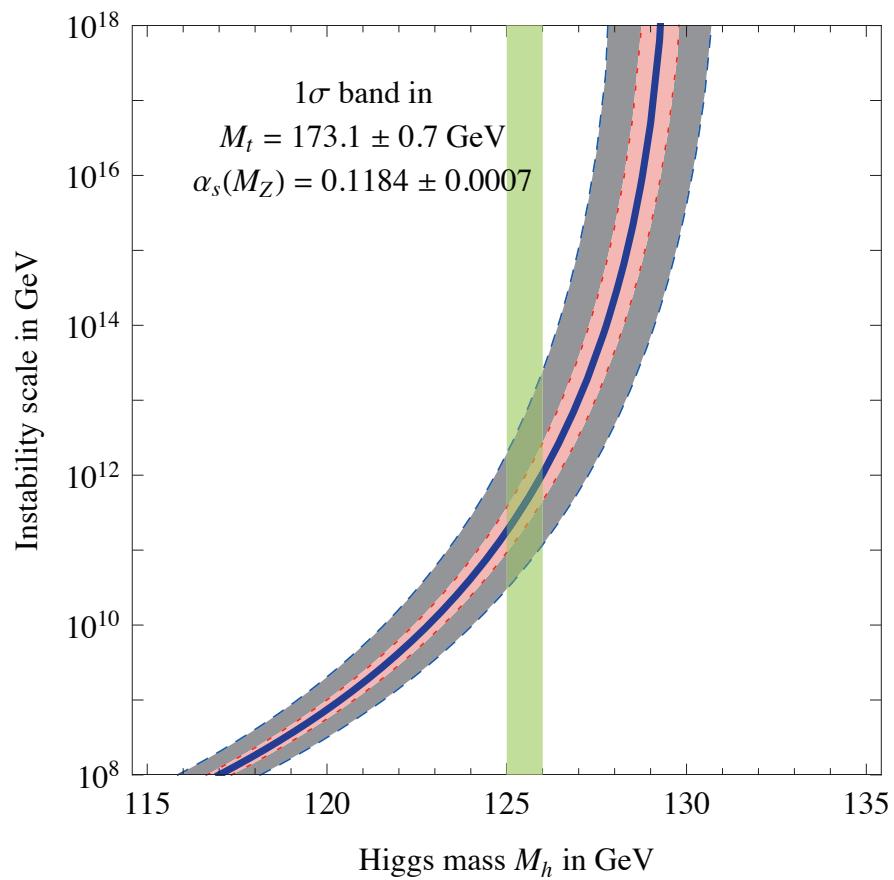
$$\lambda(Q) < 0 \quad \text{for} \quad Q > \Lambda_C \quad \rightarrow \text{vacuum not stable}$$

high value of  $\Lambda_C$  needs  $M_H$  large enough

combined effects:

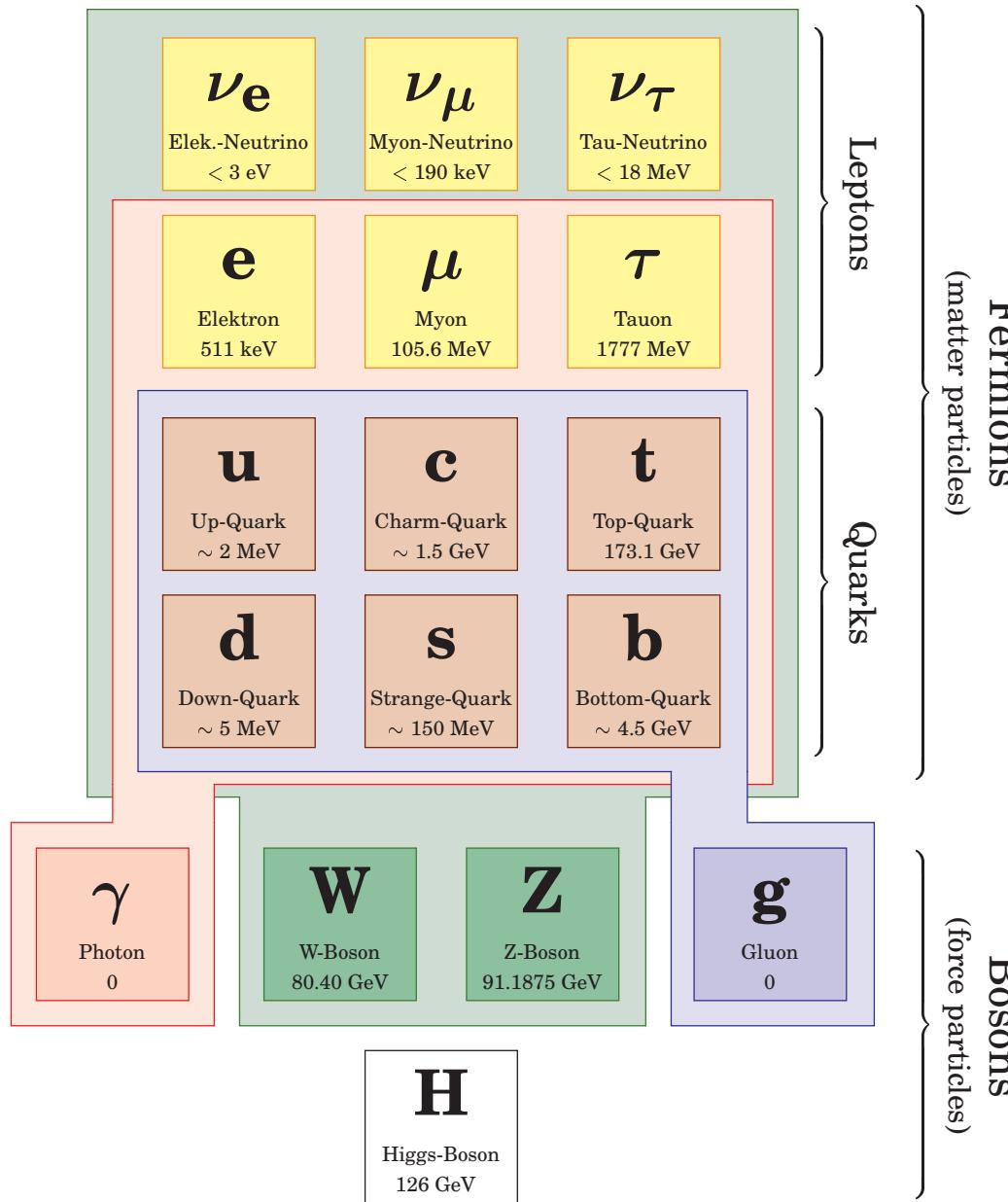
$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2} (12\lambda^2 - 3g_t^4 + 6\lambda g_t^2 + \dots)$$





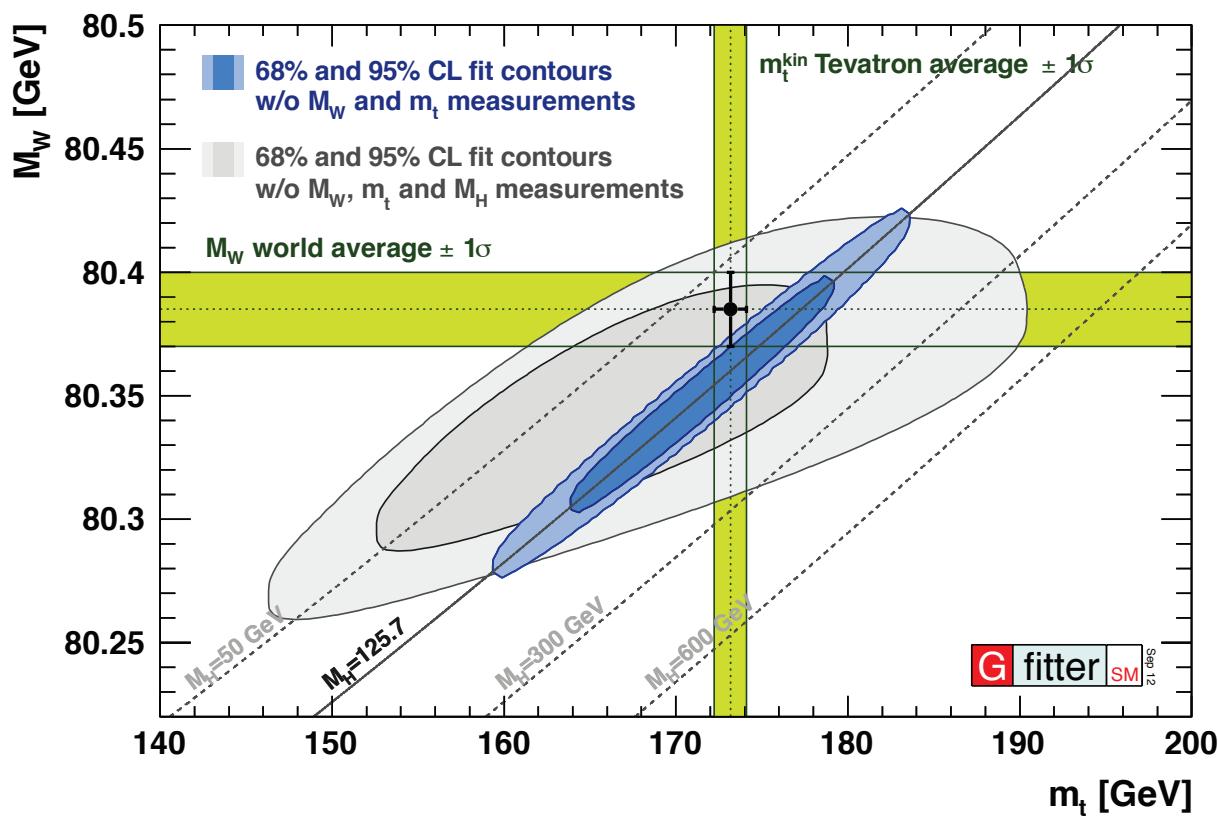
[Degrassi *et al.* 2012]

# Status of the Standard Model



SM input now completey determined  $\Rightarrow$  PO uniquely predicted

	theo	exp
$\sin^2 \theta_{\text{eff}}$	$0.23152 \pm 0.00005 \pm 0.00005$	$0.23153 \pm 0.00016$
$M_W$ (GeV)	$80.361 \pm 0.006 \pm 0.004$	$80.385 \pm 0.015$



## **few observables with not-so-good agreement**

- in general, SM is in overall agreement with data
- yet a few quantities prefer to stand a bit apart ( $\sim 3\sigma$ )
  - the forward-backward asymmetry for b quarks,  
 $A_{\text{FB}}^{b\bar{b}}$  at the Z peak
  - the anomalous magnetic moment of the muon
  - the forward-backward asymmetry for top quarks  
at the Tevatron,  $p\bar{p} \rightarrow t\bar{t}$

no conclusive situation

## SM Higgs:

- $\lambda H^4$  term ad hoc
- Higgs boson mass: free parameter  $\sim \sqrt{\lambda}$
- no a-priori reason for a light Higgs boson

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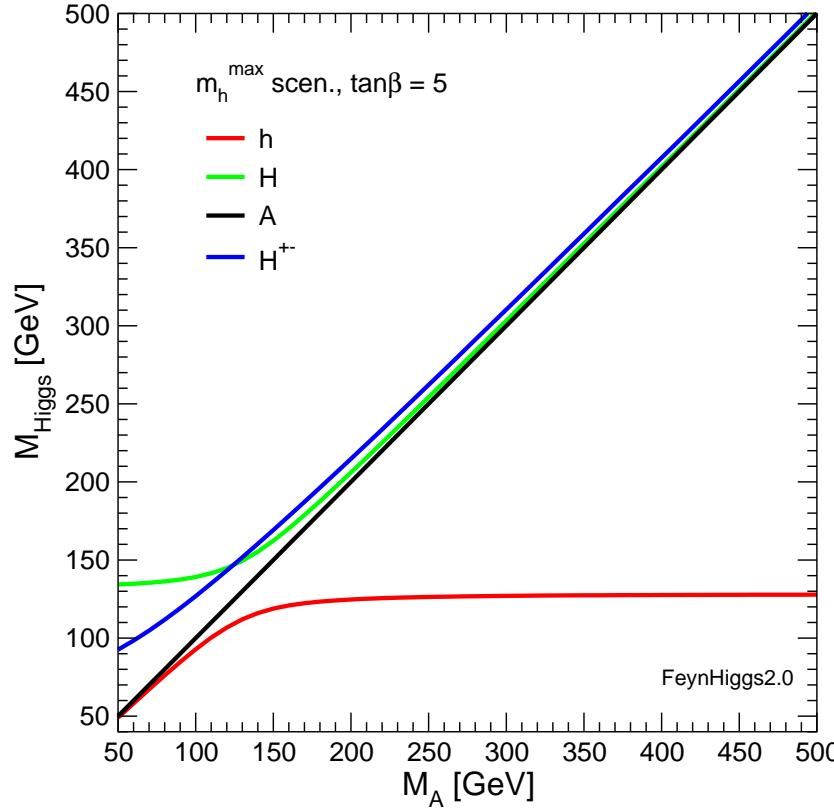
SUSY Standard Model avoids these questions

$$H_2 = \begin{pmatrix} H_2^+ \\ v_2 + H_2^0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} v_1 + H_1^0 \\ H_1^- \end{pmatrix}$$

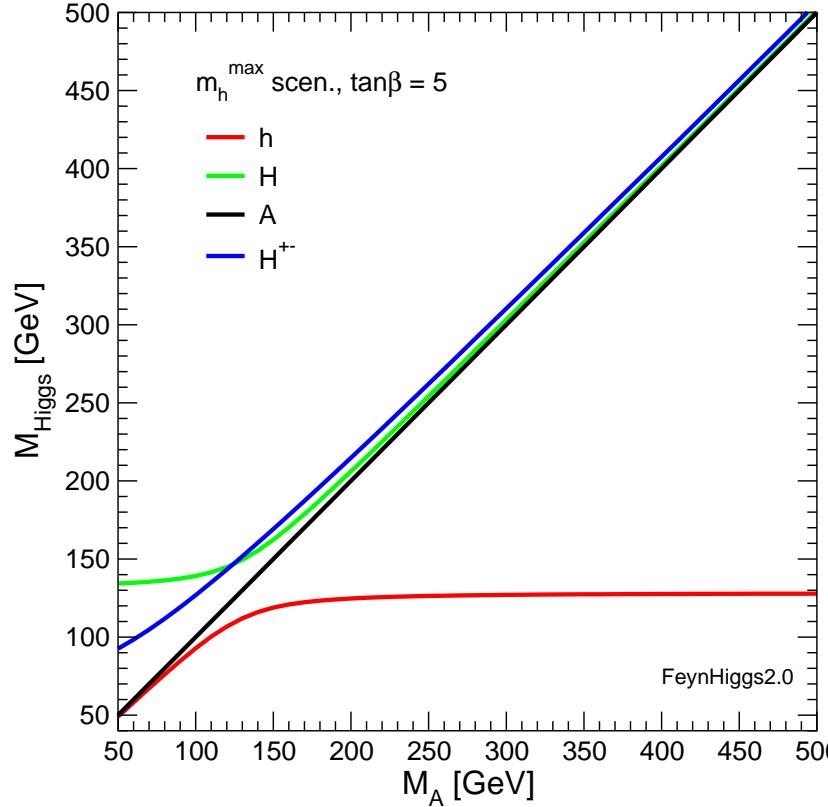
couples to  $u$     couples to  $d$

- SUSY gauge interaction  $\rightarrow H^4$  terms
- self coupling remains weak

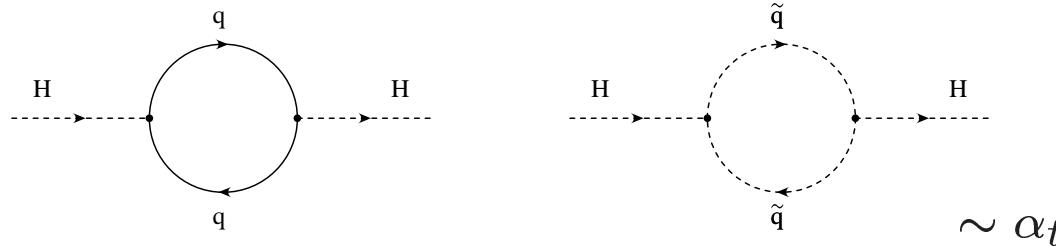
# spectrum of Higgs bosons in the MSSM: $h^0$ , $H^0$ , $A^0$ , $H^\pm$



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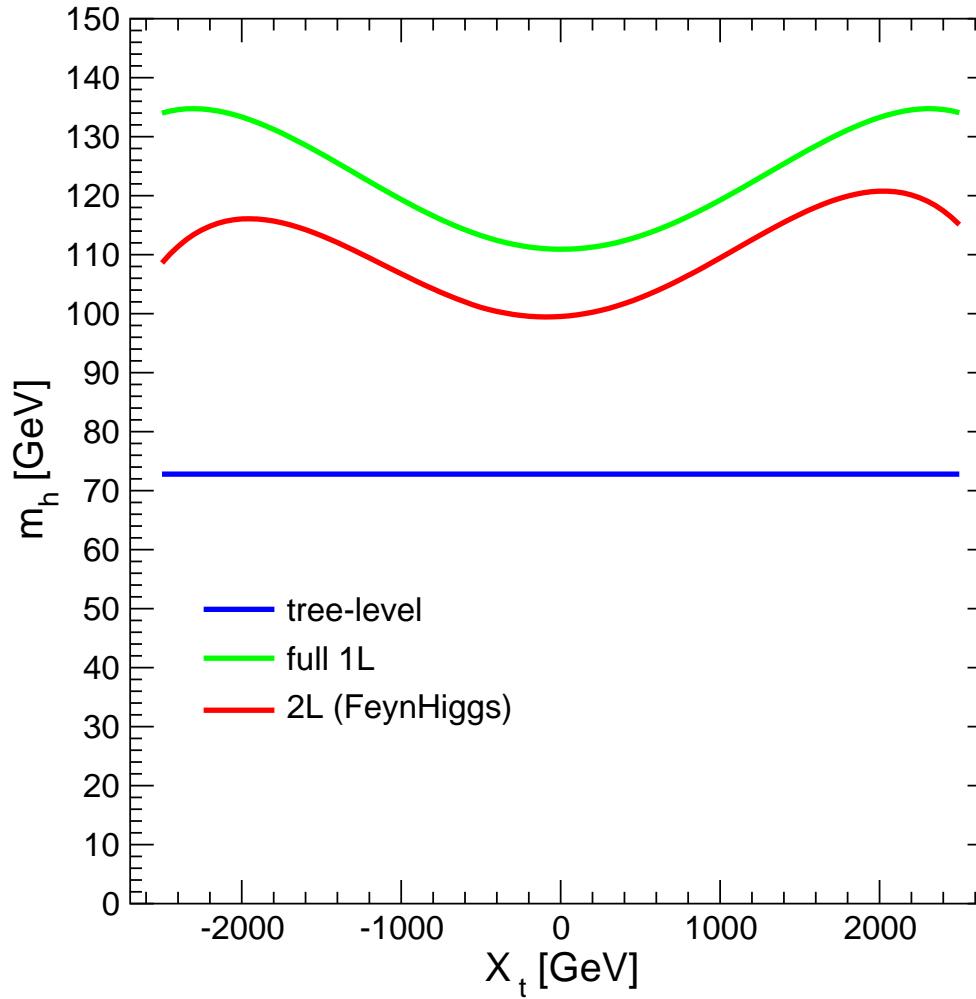


$m_h^0$  strongly influenced by quantum effects, e.g.  $t$ ,  $\tilde{t}$



# sensitivity to mass/mixing parameters

$m_{h^0}$  prediction at different levels of accuracy:



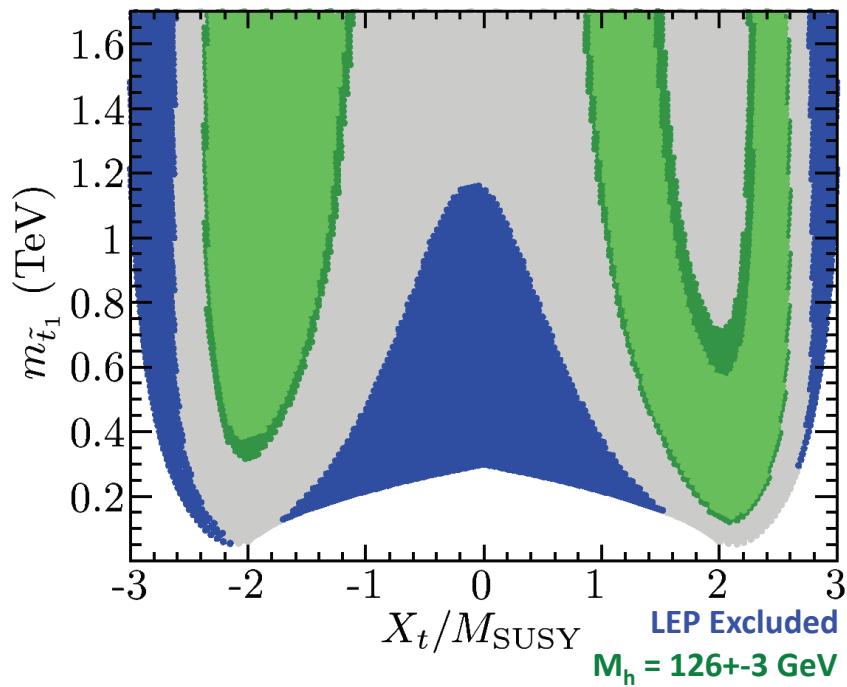
$$\tan \beta = 3, \quad M_{\tilde{Q}} = M_A = 1 \text{ TeV}, \quad m_{\tilde{g}} = 800 \text{ GeV}$$

$X_t$  : top-squark mixing parameter

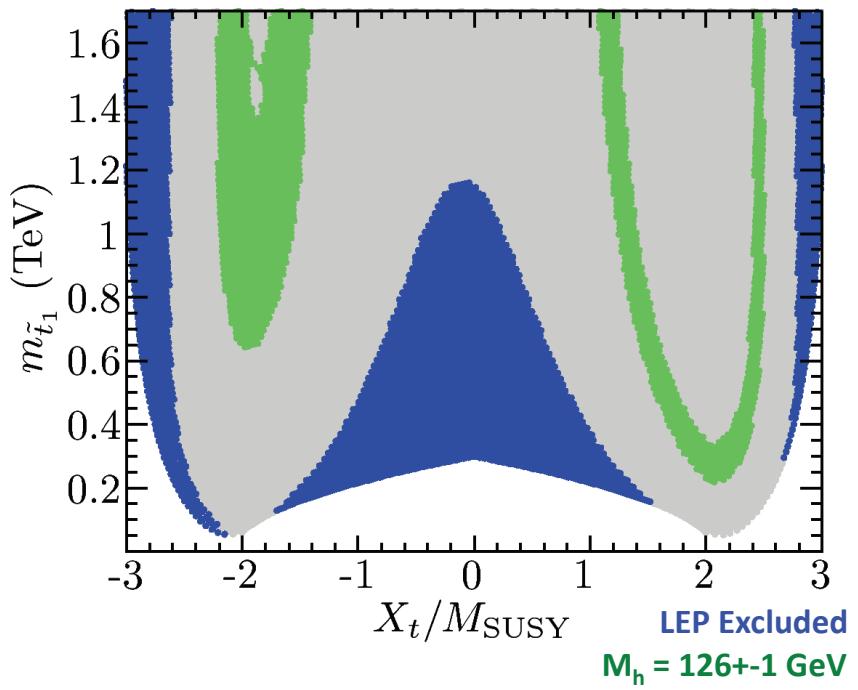
$$X_t = A_t - \mu \cot \beta$$

# allowed region for top-squark mass and mixing

Theory uncertainties included



No theory uncertainties



[Heinemeyer, Staal, Weiglein '12]

compatible with light top-squarks  
ongoing experimental search

# The success of the Standard Model

- impressive confirmation by a huge data sample from low to high energies, no significant deviations
- quantum effects have been established at many  $\sigma$
- perfect indirect and direct determination of the top quark
- now being repeated for the Higgs boson
- new particle around 126 GeV strong candidate for the Higgs boson
- if confirmed: Standard Model closed

Happy End of a successful story ?

# Shortcomings of SM

- no mass terms for neutrinos [introduce  $\nu_R \dots$  ]
- hierarchy problem  $v \ll M_{\text{Pl}}, M_H \ll M_{\text{Pl}}$
- large number of free parameters  $g_1, g_2, v, m_f, V_{\text{CKM}}$
- no further unification of forces
- missing link to gravity
- nature of dark matter?
- baryon asymmetry of the universe?

- next steps with upgraded LHC
  - confirm the Higgs boson properties
  - check versus electroweak precision measurements
  - or find deviations, new structures:
    - more Higgs bosons (doublets, singlet, .. )
    - supersymmetry (minimal or non-minimal)
    - new strong sector, substructure
    - ...

# RESEARCH INSTITUTE

