arXiv [hep-ph]: 1304.6644, 1210.0288 + to appear

Mexican Grants: PAPIIT-IN113712 and CONACyT-132059

In collaboration with:
A. Mondragón, M. Mondragón,
L. Velasco-Sevilla,
F. González-Canales

A 4H doublets S3 flavour model

U. J. Saldaña-Salazar
(IF-UNAM)

13th Hellenic Summer School,
Corfu, Greece, September 09, 2013

Petitions

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What is the mean objective of my talk?

To make you aware of the reality behind the purpose of flavour models.

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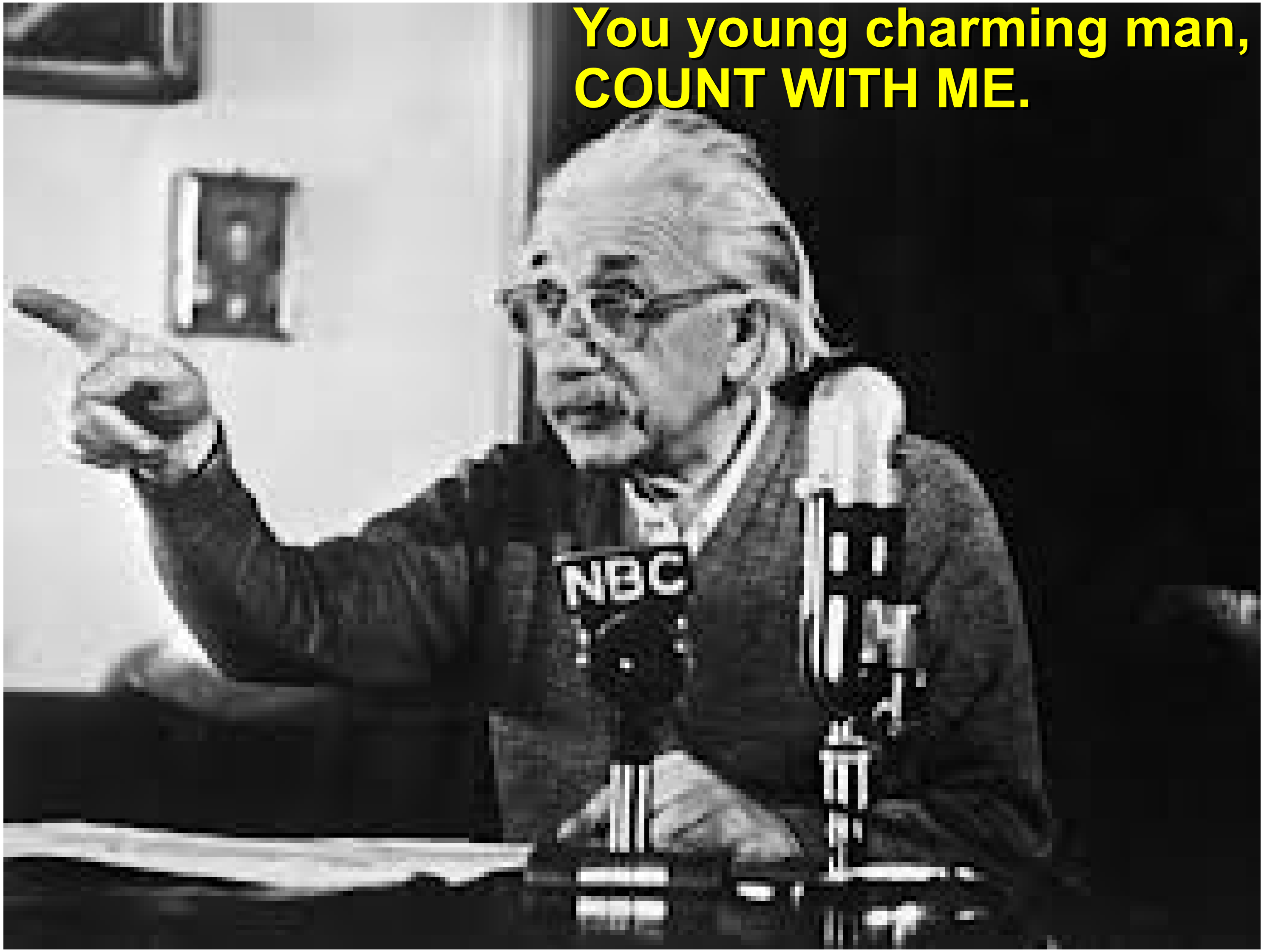
PLEASE. I will certainly make mistakes or even have wrong arguments, STOP ME whenever you notice them. I WILL BE GREATLY AND ETERNALLY PLEASED WITH YOU.


You young charming man,
COUNT WITH ME.
NBC

Outline

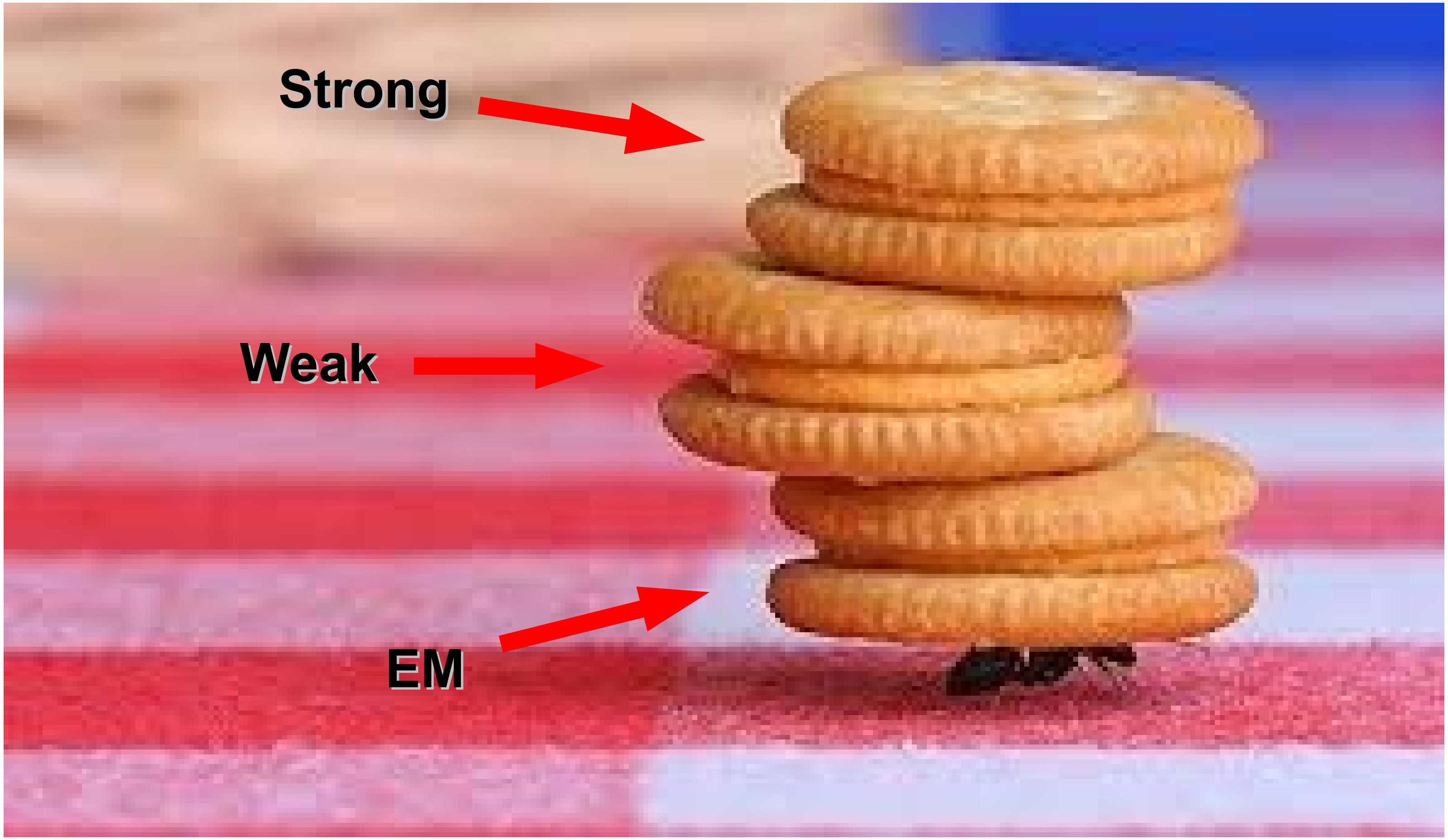
Lots of motivations

The model

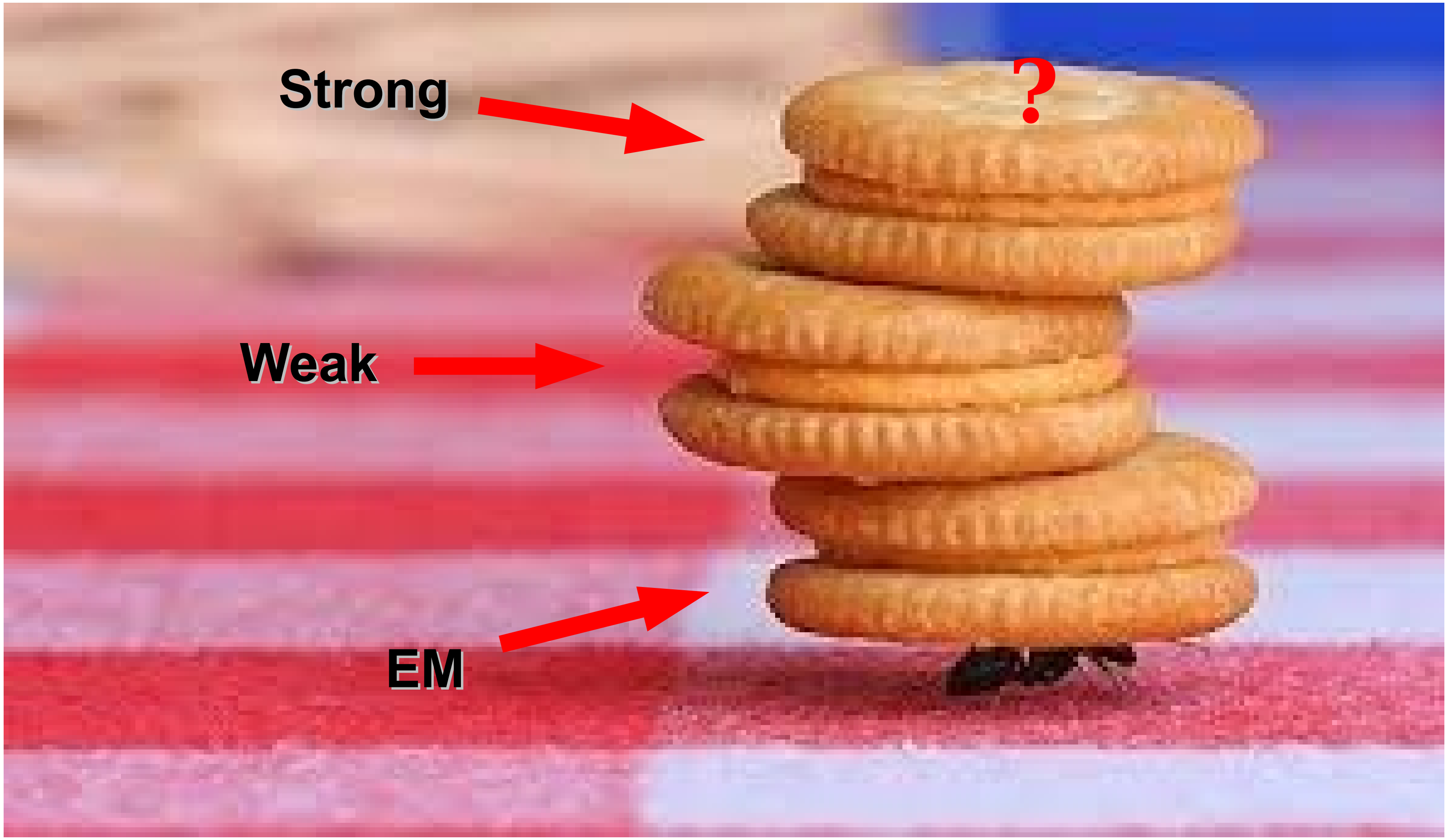
What do we get?

What do we want to do?

The Standard Model

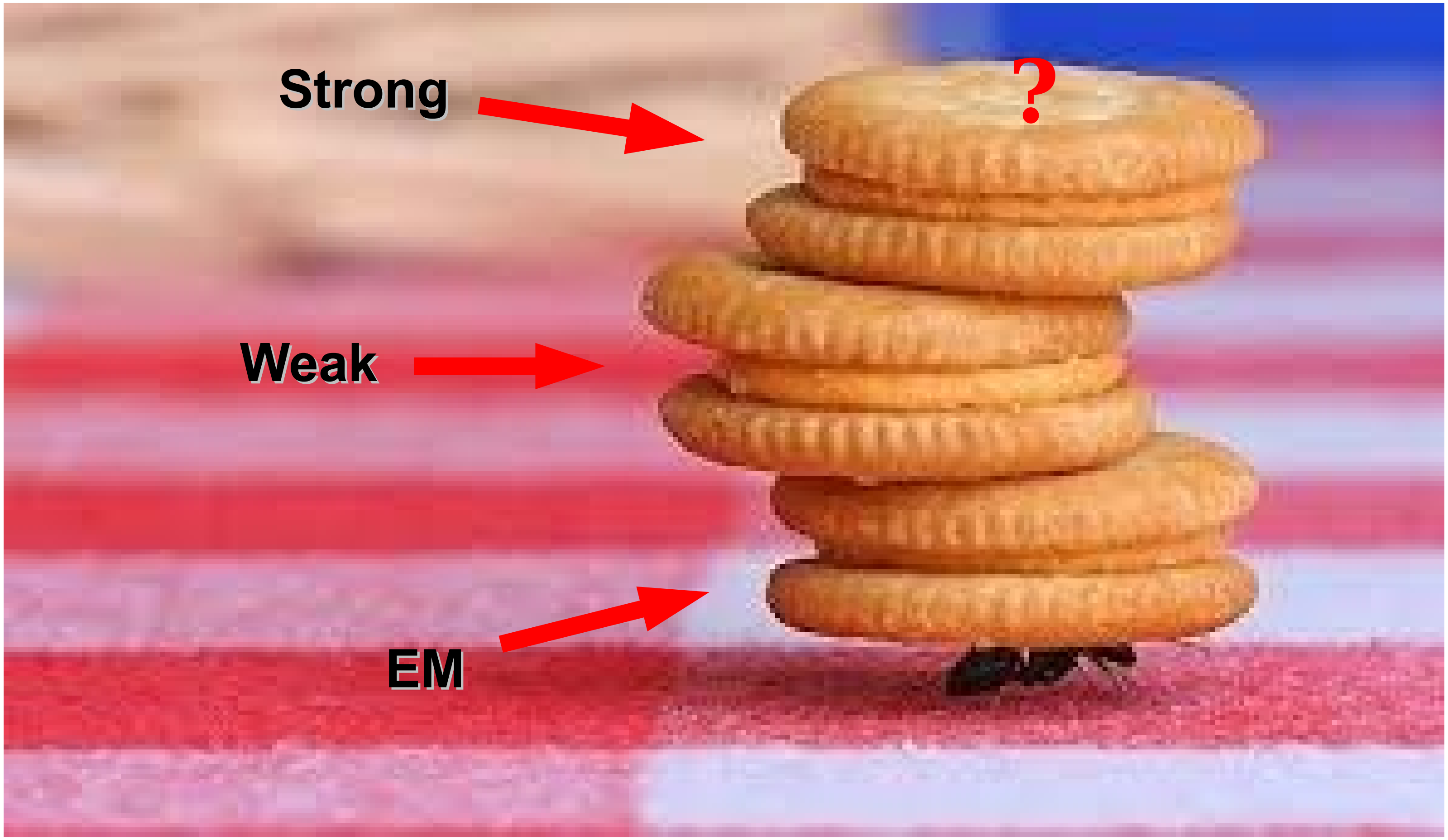


The Standard Model



The Standard Model


HANNA BARBERA PRESENTS
ATOM ANT
in:
MUSCLE MAGIC
HBR
HANNA BARBERA
ORIGINAL T.V. STARS
SONGS:
ATOM ANT
MUSCLE MAGIC
FIT AS A FIDDLE
UP AND ATOM
FEATURING SONGS BY THE HANNA-BARBERA SINGERS
1965 HANNA-BARBERA PRODUCTIONS INC.
CARTOON SERIES HLP 2041



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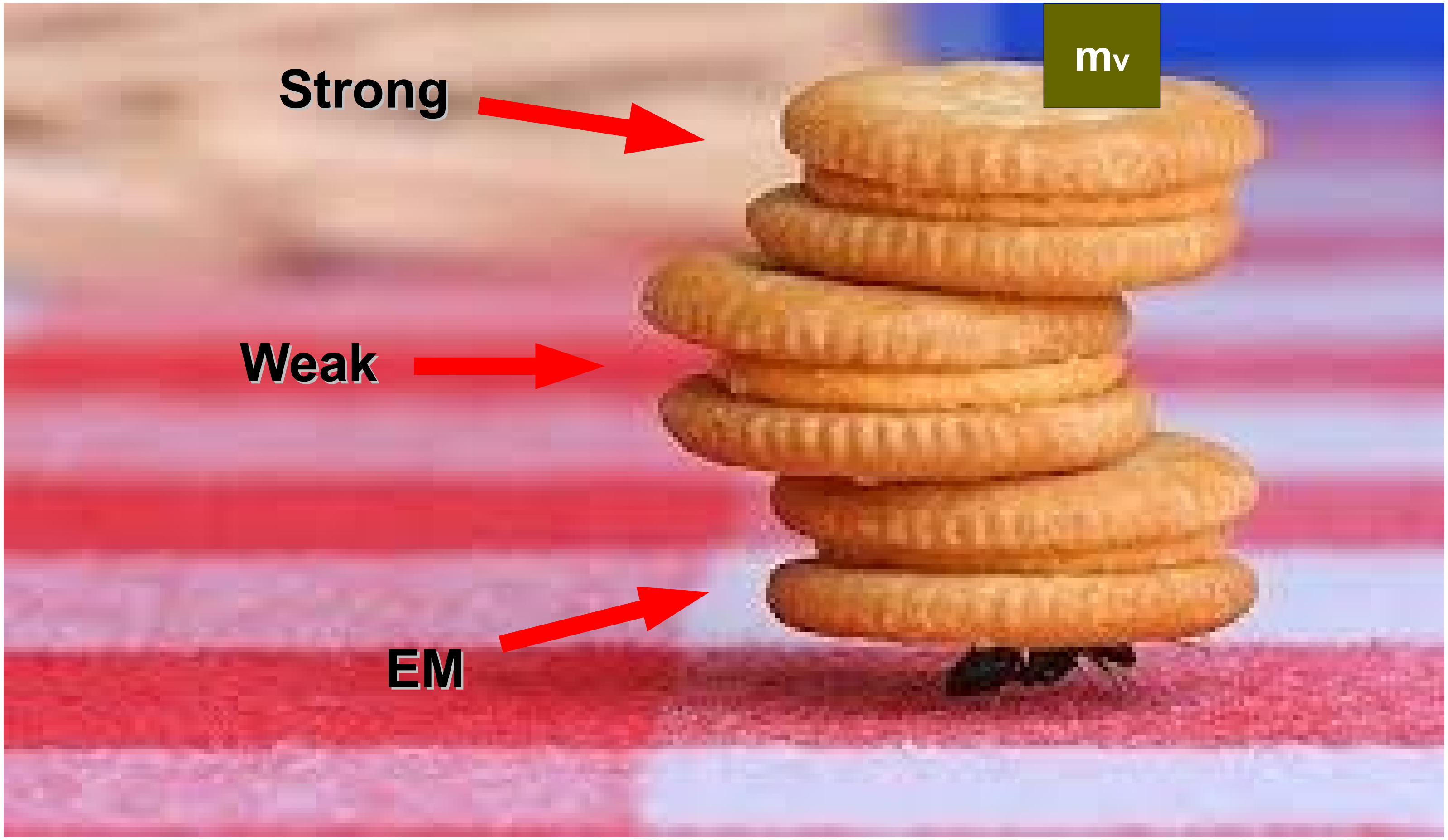

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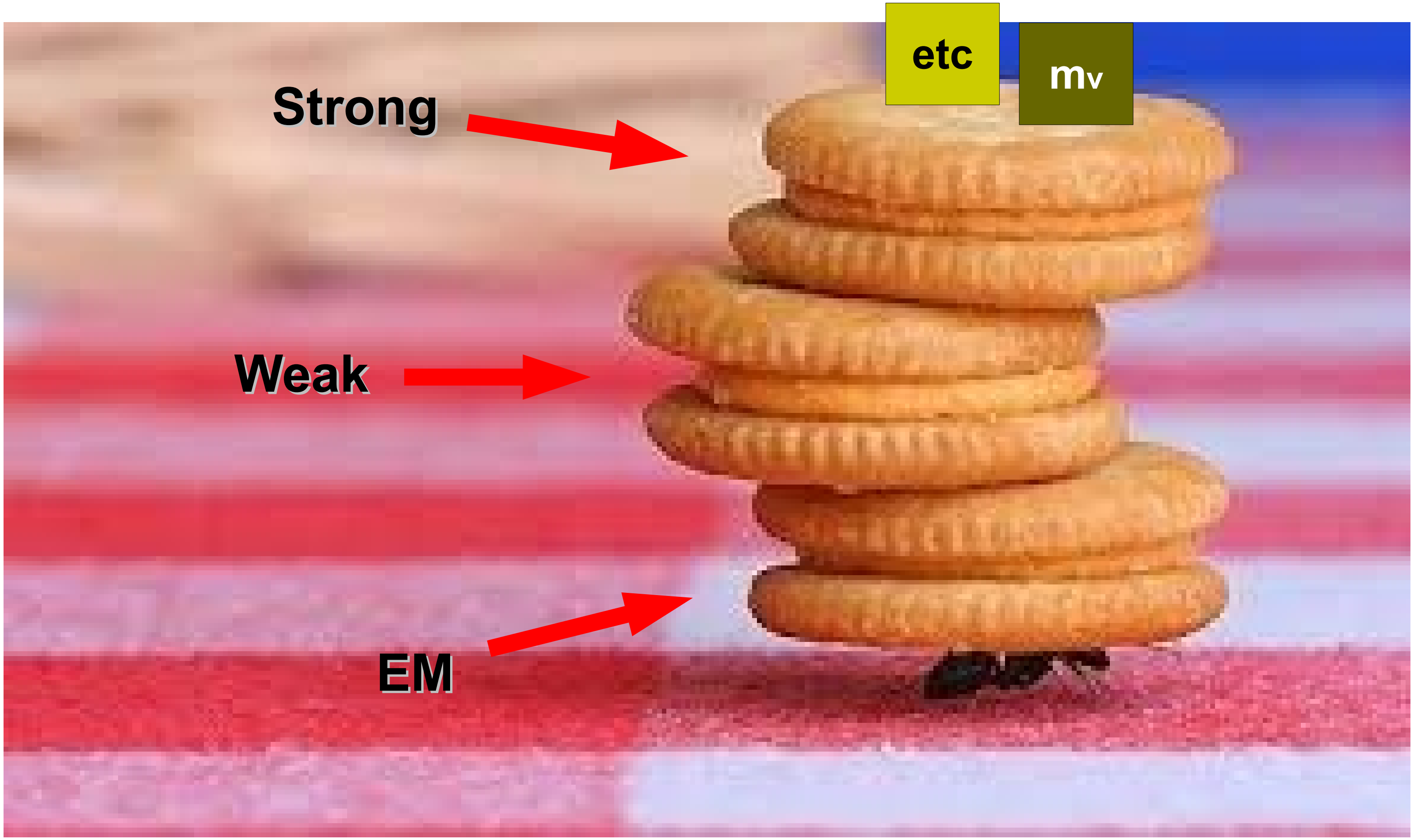

The Standard Model

3 Flavour Dogmas introduced in the 70's, soon after the creation and early success of the S.M.

Dogma 1 – Neutrinos are strictly massless!

Dogma 2 – No Z-mediated FCNC at tree level

Dogma 3 – No Higgs mediated FCNC at tree level.

Question: We all know the "Fate" of Dogma N°1
Will the other two Dogmas have the same "Fate" as Dogma 1.

Branco's Talk

15

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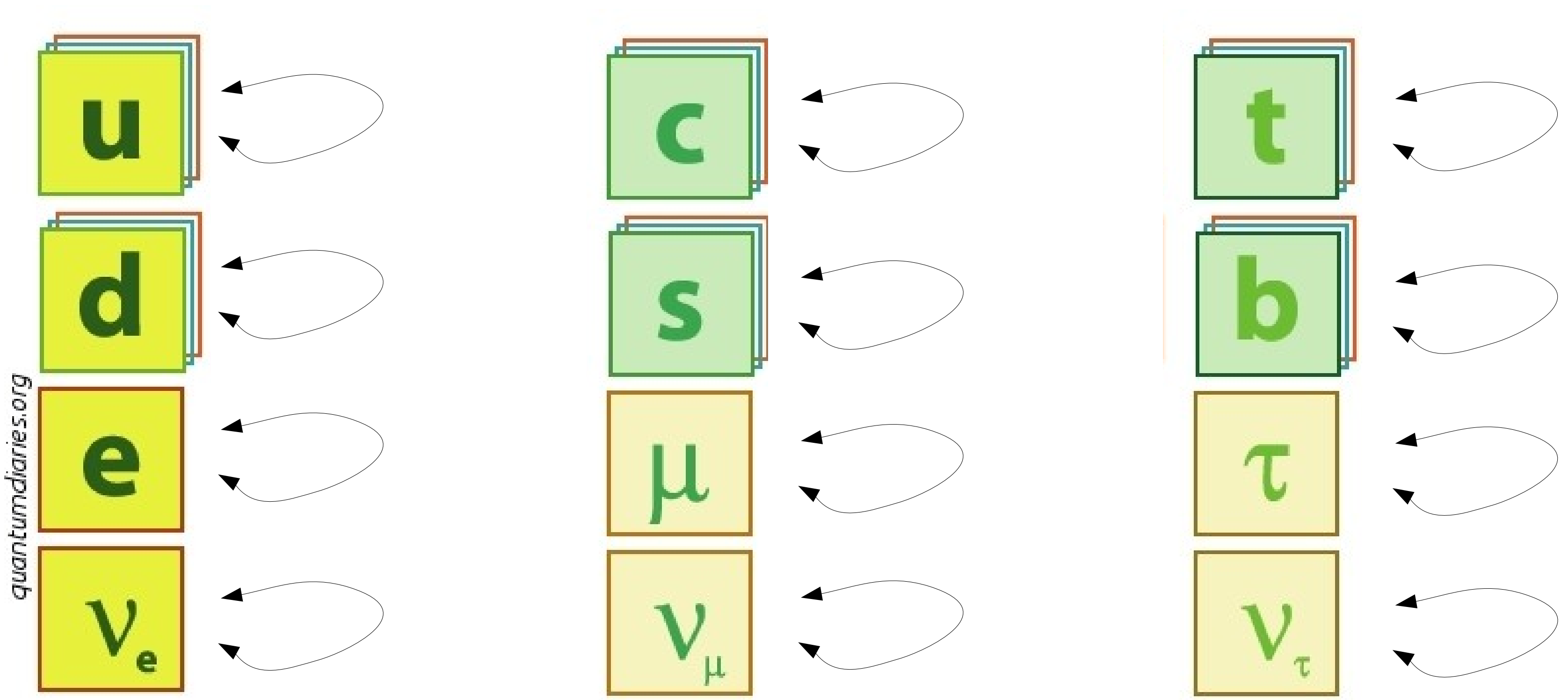
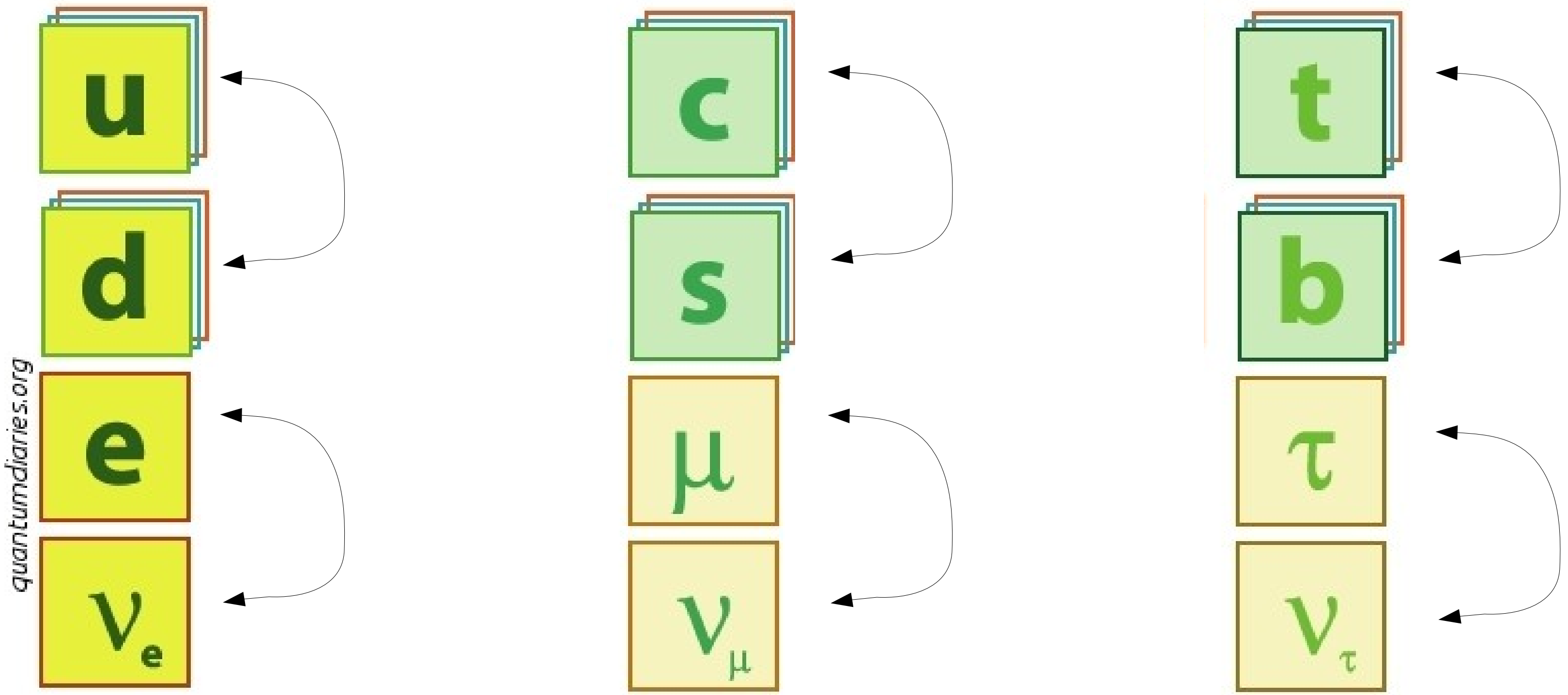
Dogma 4:

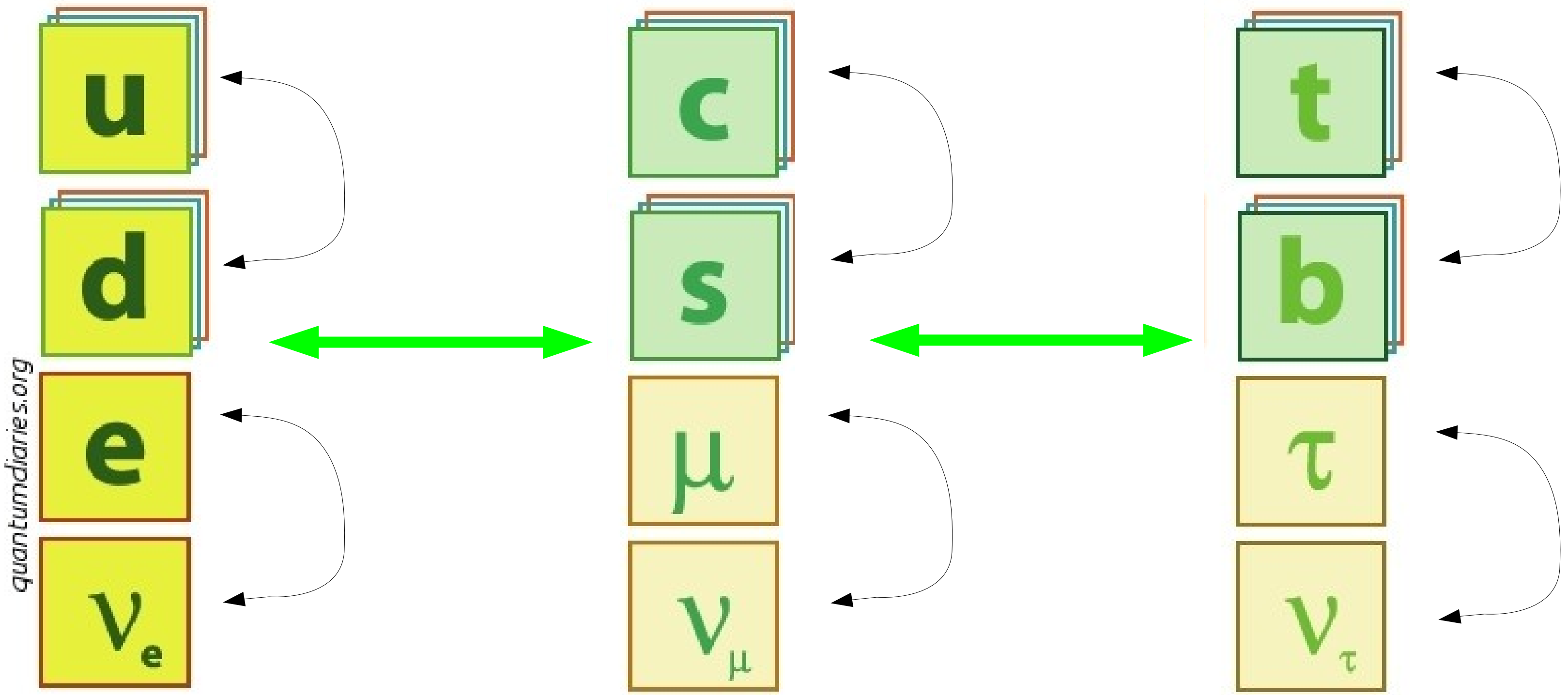
"The three independent magnitudes and the one phase of the mixing matrix are arbitrary parameters of the SM"



$$Z^0, \gamma, G_a$$



$W^{\pm}$ (Weak-Basis)



$W^{\pm}$ (Weak-Basis)

$$\mathrm{L}_f = i\overline{\psi}_{f,i}\gamma_\mu \mathcal{D}_f^\mu \psi_{f,i}$$

$$G_F = U(3)_L^Q \otimes U(3)_R^u \otimes U(3)_R^d \otimes U(3)_L^l \otimes U(3)_R^e \otimes U(3)_R^\nu$$

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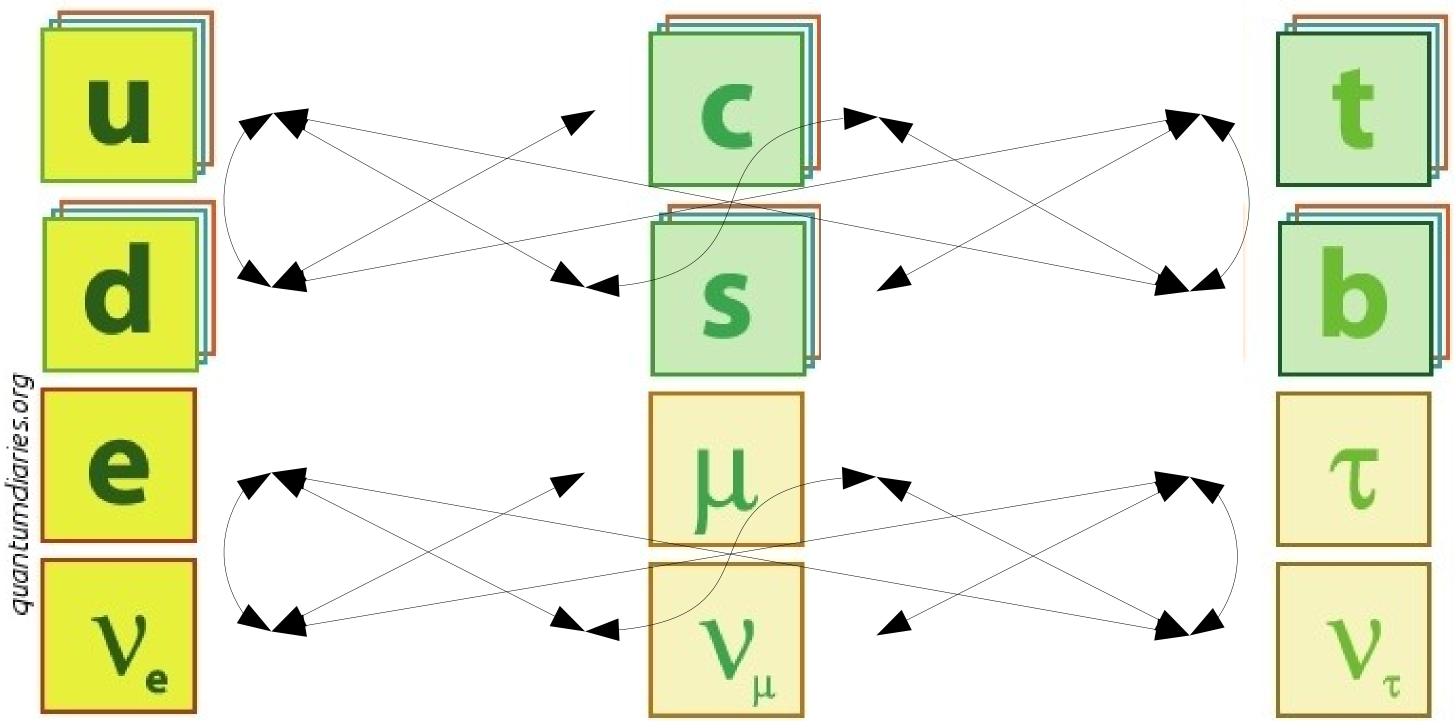
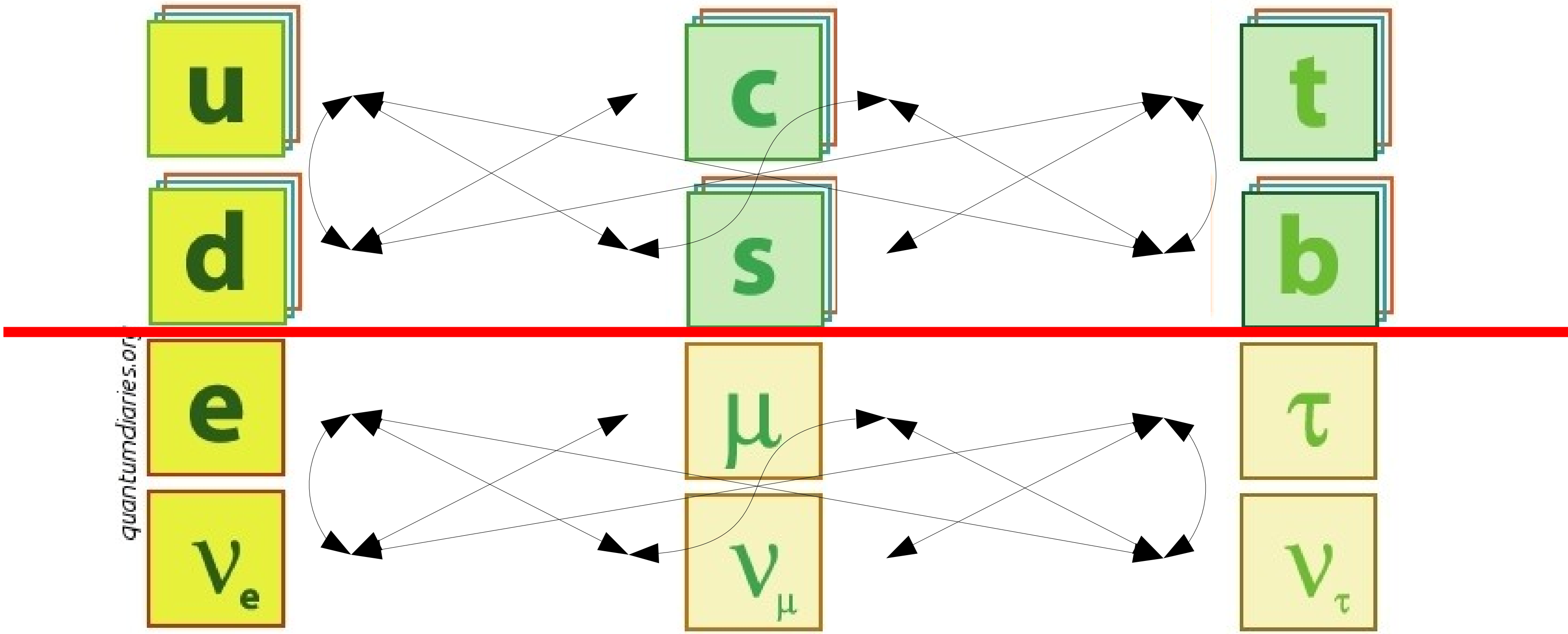


$W^{\pm}$ (Mass-Basis)



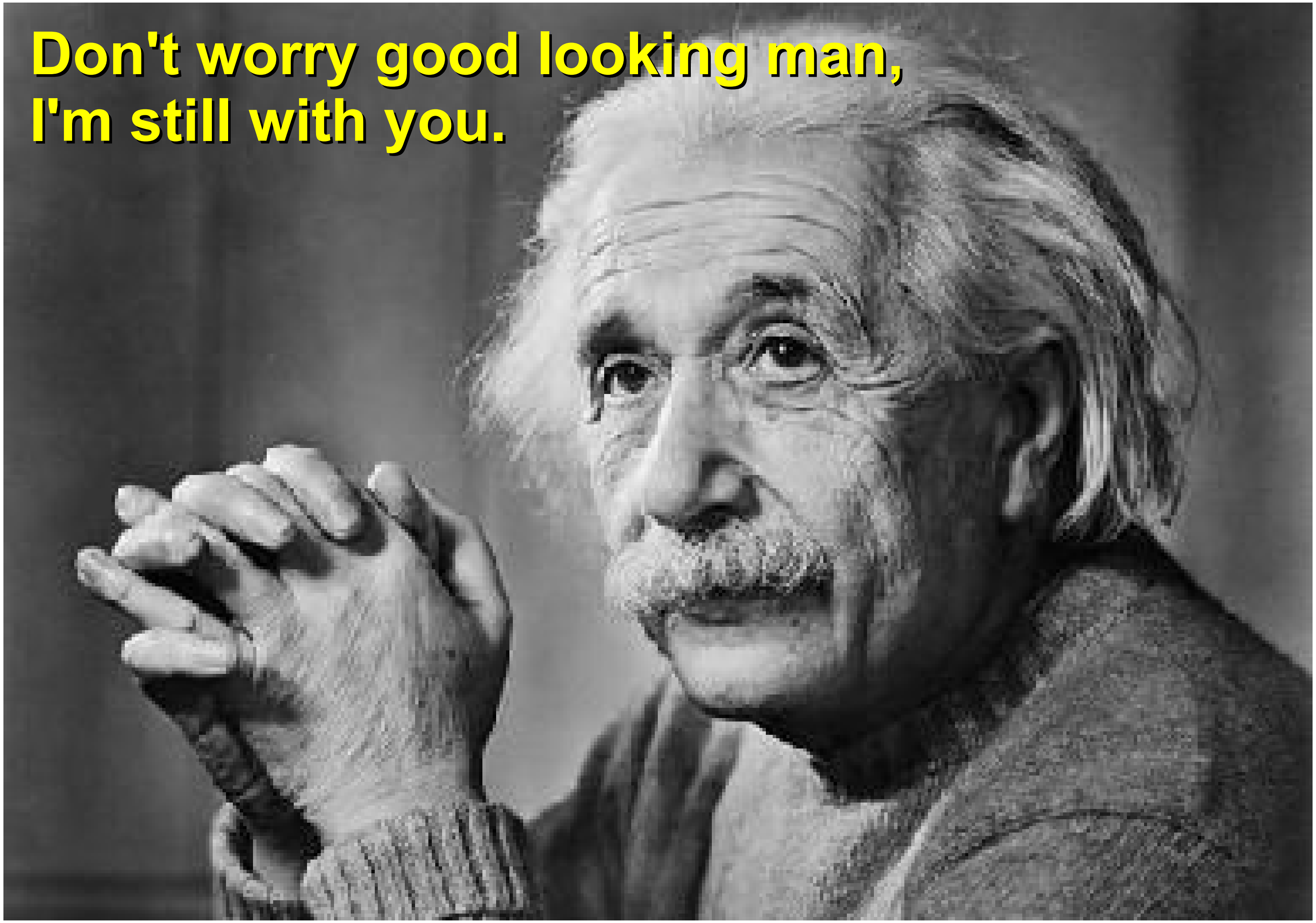
$W^{\pm}$ (Mass-Basis)

$$\mathrm{L}_f = i\overline{\psi}_{f,i}\gamma_\mu \mathcal{D}^\mu_f \psi_{f,i} - \sum_{ij} Y_f^{ij}\overline{\psi}^L_{f,i} H \psi^R_{f,i} + h.c.$$

$$\boxed{\mathrm{G}_F \rightarrow U(1)_B \otimes U(1)_L}$$

Yukawa param. - # Broken G. = # Phys. Param.

$$\boxed{4(\ 9 + 9\ ) \text{ - } [6(9) \text{ - } 2] = 2\ (10)}$$


Don't worry good looking man,
I'm still with you.

However!
At the same time...

In 1968,

$$\theta_C \approx \sqrt{m_d/m_s}$$

Gatto, Sartori, and Tonin, Phys. Lett. B28, 128. (1968)
Cabibbo and Maiani, Phys. Lett. B28, 131. (1968)

Later generalized to the case of three families,

Pagels; Weinberg; Wilczek and Zee; Fritzsch; Ebrahim;
Mohapatra and Senjanovic; Pakvasa and Sugawara;
Derman; Wyler; Frere; Yahalom
Li; Branco, Lavoura and Mota; Cheng and Sher;
Branco and Mota

Let us illustrate this.

Two family case (Quark sector)

$$\mathrm{M}_q = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$$

- Real matrix

$$\overline{M}_q = O M_q O^T$$

- No Right-handed WCC

$$\mathrm{U}^R_{CKM} = O^u_R {O^d_R}^T$$

Symmetric real mass matrix,

$$\alpha_{12} = \alpha_{21} \longrightarrow \mathrm{M}_q = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{12} & \alpha_{22} \end{pmatrix}$$

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A simultaneous rotation in the quark sector,

$$\mathrm{M}_q = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$$

$$\boxed{\mathrm{U}_{CKM} = O_L^u R_u R_d^T {O_L^d}^T}$$

$$\boxed{\mathrm{R}_u R_d^T = 1}$$

produces,

$$\mathrm{M'}_q \equiv R M_q R^T = \begin{pmatrix} 0 & \alpha'_{12} \\ \alpha'_{12} & \alpha'_{22} \end{pmatrix}$$

May be written as,

$$\mathrm{M'}_q = \begin{pmatrix} 0 & \pm\sqrt{-Det[M'_q]} \\ \pm\sqrt{-Det[M'_q]} & Tr[M'_q] \end{pmatrix}$$

by use of,

$$\mathrm{Tr}[\mathrm{M'}_q] = \alpha'_{22} = m_1 + m_2$$

$$\mathrm{Det}[\mathrm{M'}_q] = -{\alpha'_{12}}^2 = m_1 m_2$$

Then, the mixing matrix,

$$\mathrm{U}_{CKM} = O_L^u {O_L^d}^T$$

can be, finally, expressed in terms of masses

$$U_{CKM} = \frac{1}{\sqrt{(1+\frac{m_u}{m_c})(1+\frac{m_d}{m_s})}} \times$$

$$\begin{pmatrix} 1 \pm \sqrt{\frac{m_u m_d}{m_c m_s}} & -(\mp\sqrt{\frac{m_u}{m_c}} \pm \sqrt{\frac{m_d}{m_s}}) \\ \pm\sqrt{\frac{m_d}{m_s}} \mp \sqrt{\frac{m_u}{m_c}} & 1 \pm \sqrt{\frac{m_u m_d}{m_c m_s}} \end{pmatrix}$$

Which may be written as,

$$U_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & -\lambda \\ \lambda & 1-\frac{1}{2}\lambda^2 \end{pmatrix} + O(\lambda^3)$$

with

$$\lambda = \pm\sqrt{\frac{m_d}{m_s}} \mp \sqrt{\frac{m_u}{m_c}}$$

Plugging the mass values,

$$U_{CKM} \approx \begin{pmatrix} 0.97 & -0.22 \\ 0.22 & 0.97 \end{pmatrix}$$

But! Mixing parameters are described as arbitrary parameters!!

M. Cirelli's Talk


KEEP
CALM
AND
THINK
DIFFERENT
www.tame-geek.co.uk

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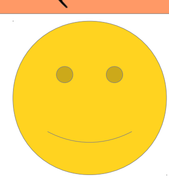

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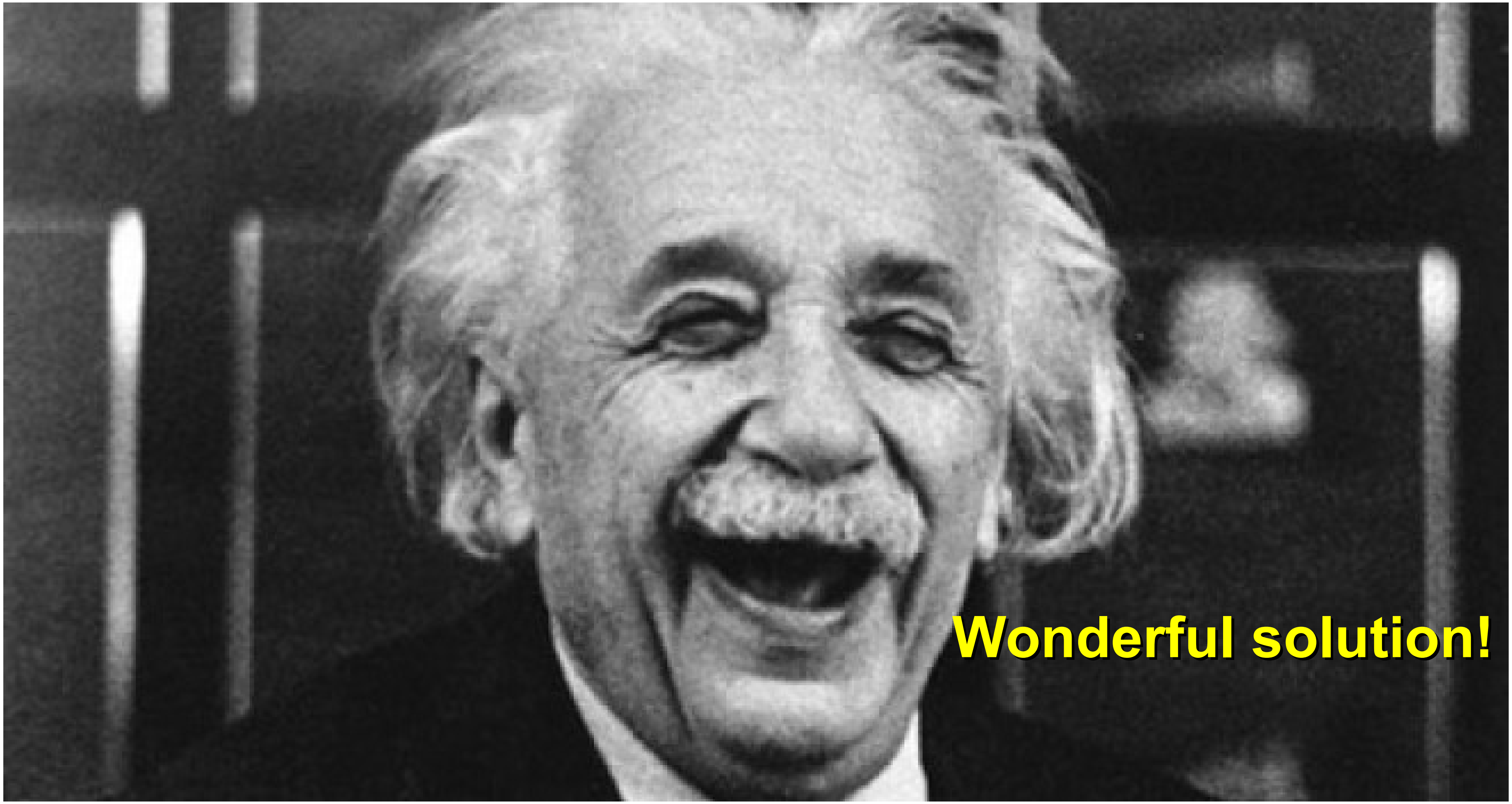
Flavour symmetry

U. J. Saldaña-Salazar, (IF-UNAM)


Wonderful solution!

The model

But,

- Which symmetry?
- Where to search for it?
- What facts could we already know?

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- What facts could we already know?

But,

- Which symmetry?
- Where to search for it?
- What facts do we already know?

- What facts could we already know?

1. Discrete and a subgroup of U(3).
2. It acts independently on each fermion species.
3. It acts independently on each handedness.

$$\mathrm{L}_f = i\overline{\psi}_{f,i}\gamma_\mu \mathcal{D}_f^\mu \psi_{f,i}$$

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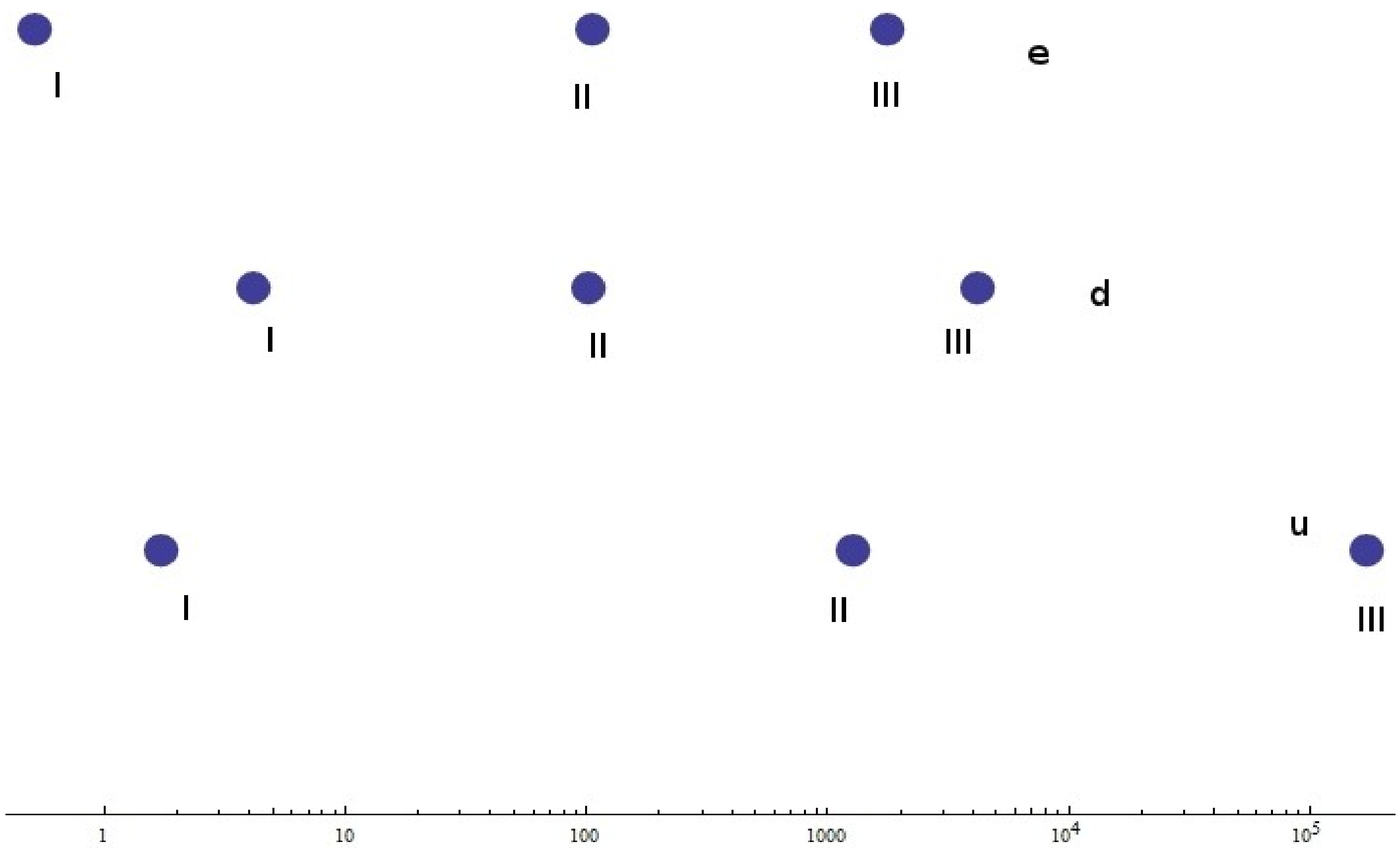
- Where to search for it?

  1. Mass spectra
  2. In the kinetic terms of the Lagrangian


I
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1000
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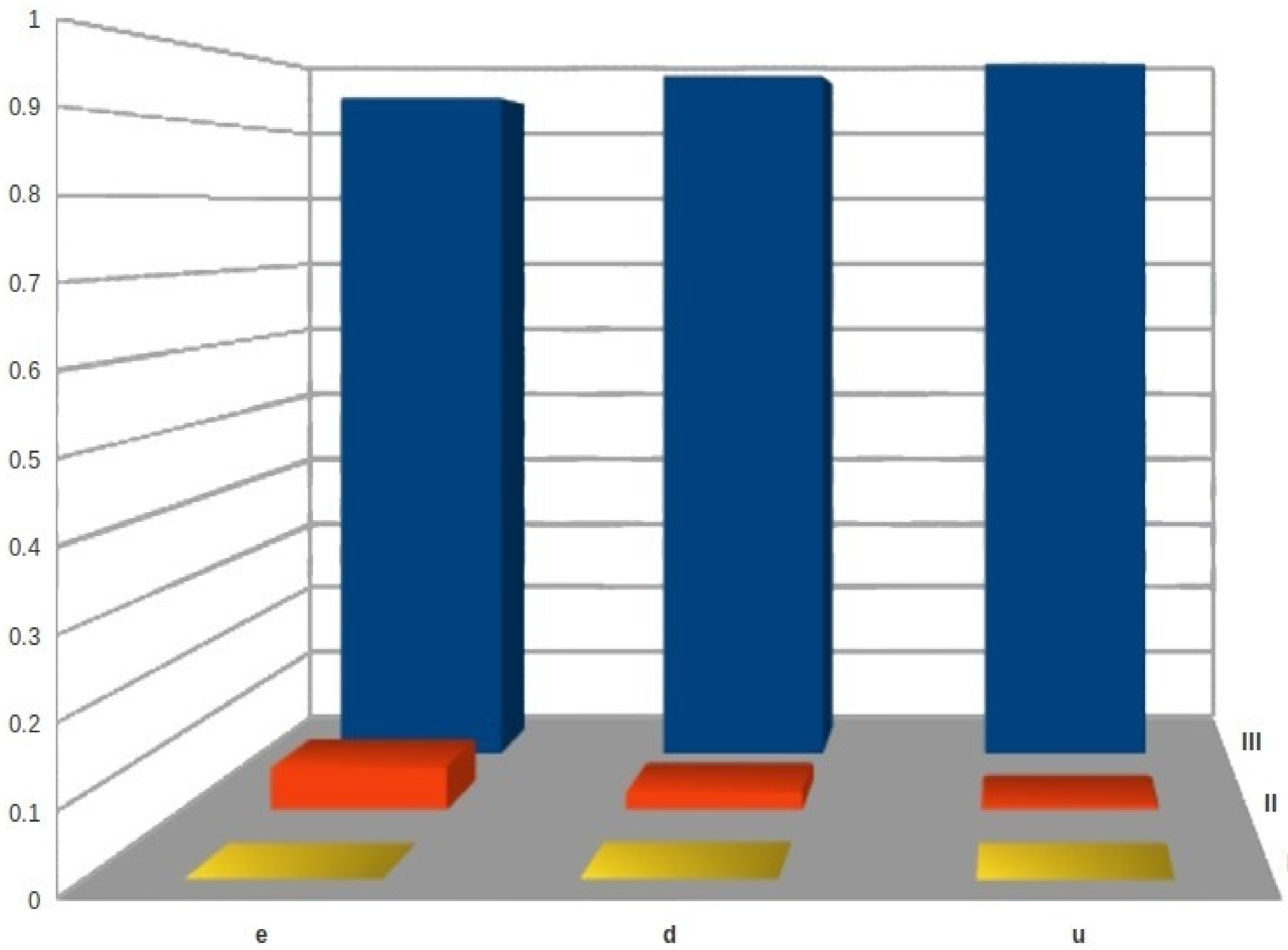
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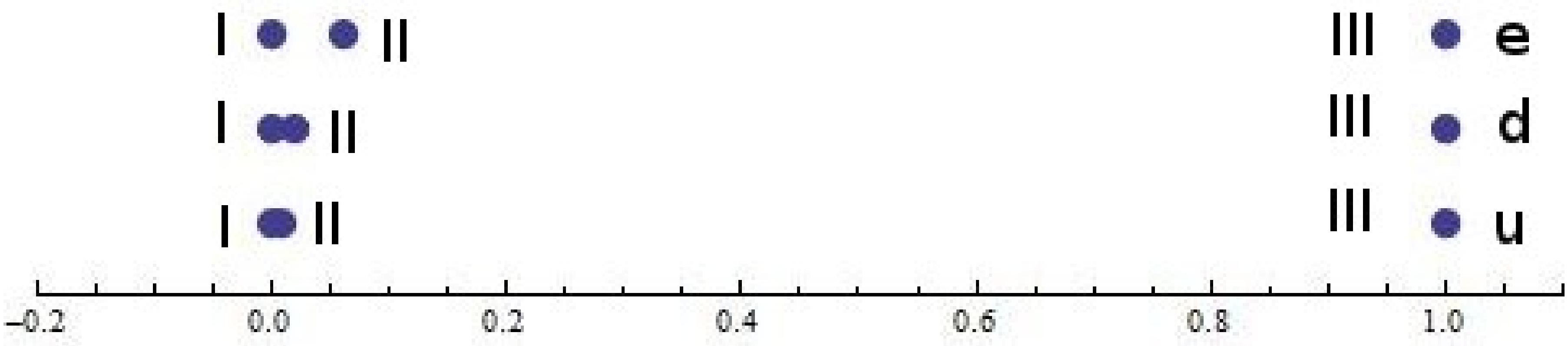
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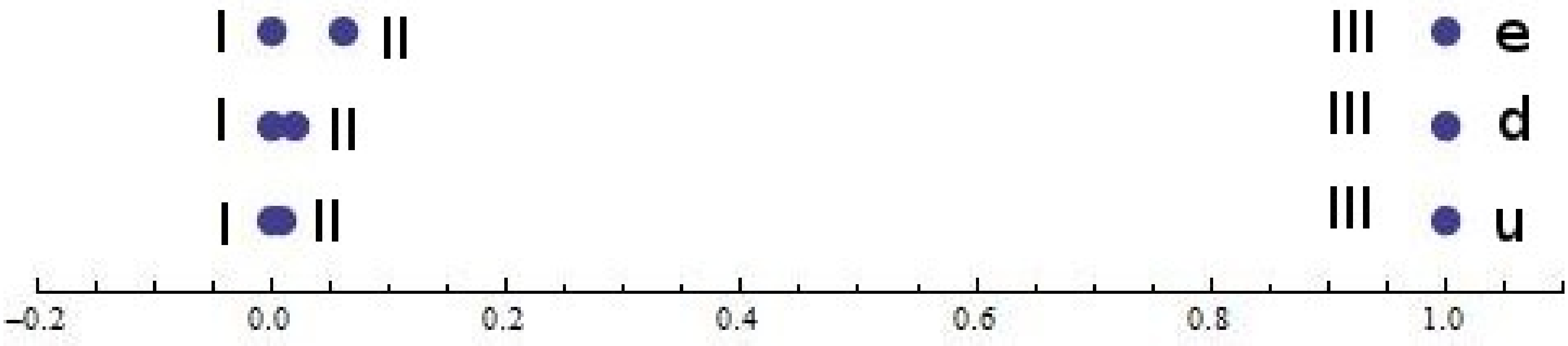
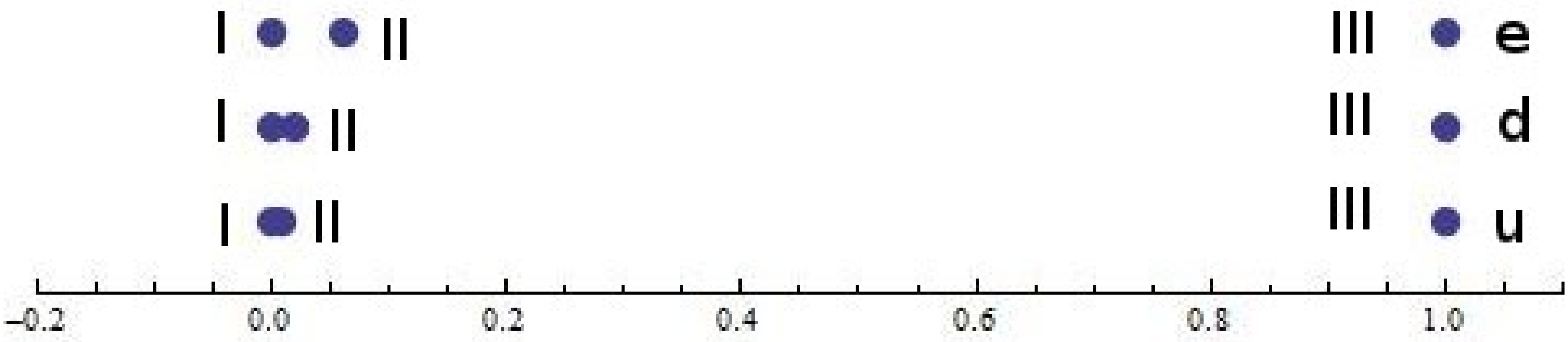
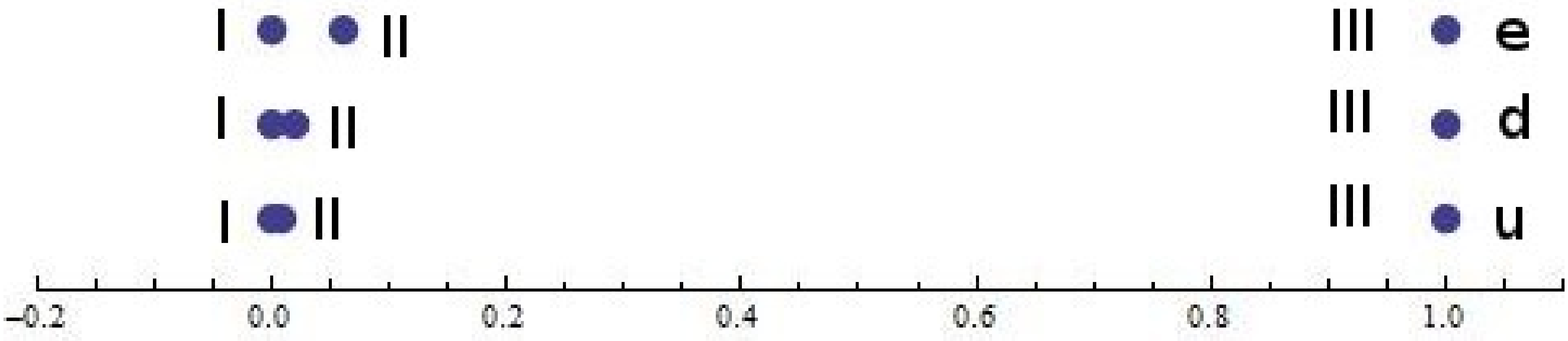


➔2 + 1



➔2 + 1

➔A hidden symmetry after the EWSB



➔2 + 1

➔A hidden symmetry after the EWSB

➔Non-Abelian Symmetry Group

➢ Which symmetry?

➢ Which symmetry?

1. A non-Abelian discrete subgroup of U(3)
2. A hidden symmetry after the EWSB
3. $3 \rightarrow 2 + 1$

- Where to search for it?

1. Mass spectra
2. In the kinetic terms of the Lagrangian

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$$\boxed{\mathrm{L'}_f = L_f(1 \leftrightarrow 2 \leftrightarrow 3)}$$

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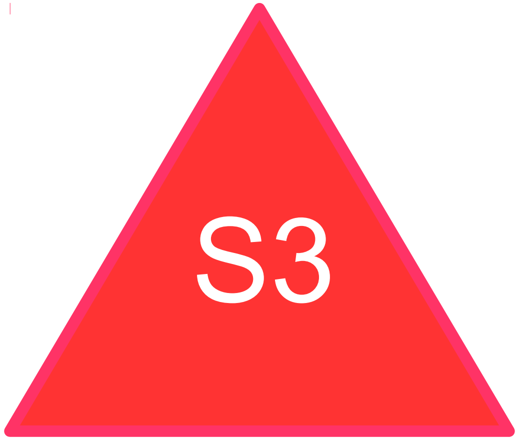

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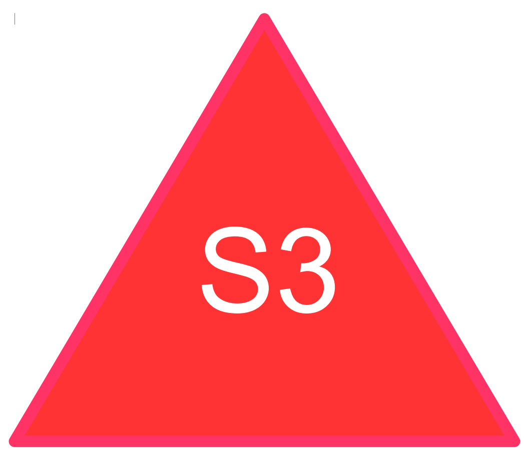
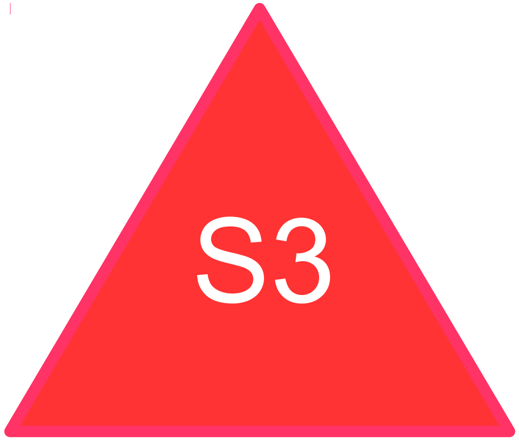
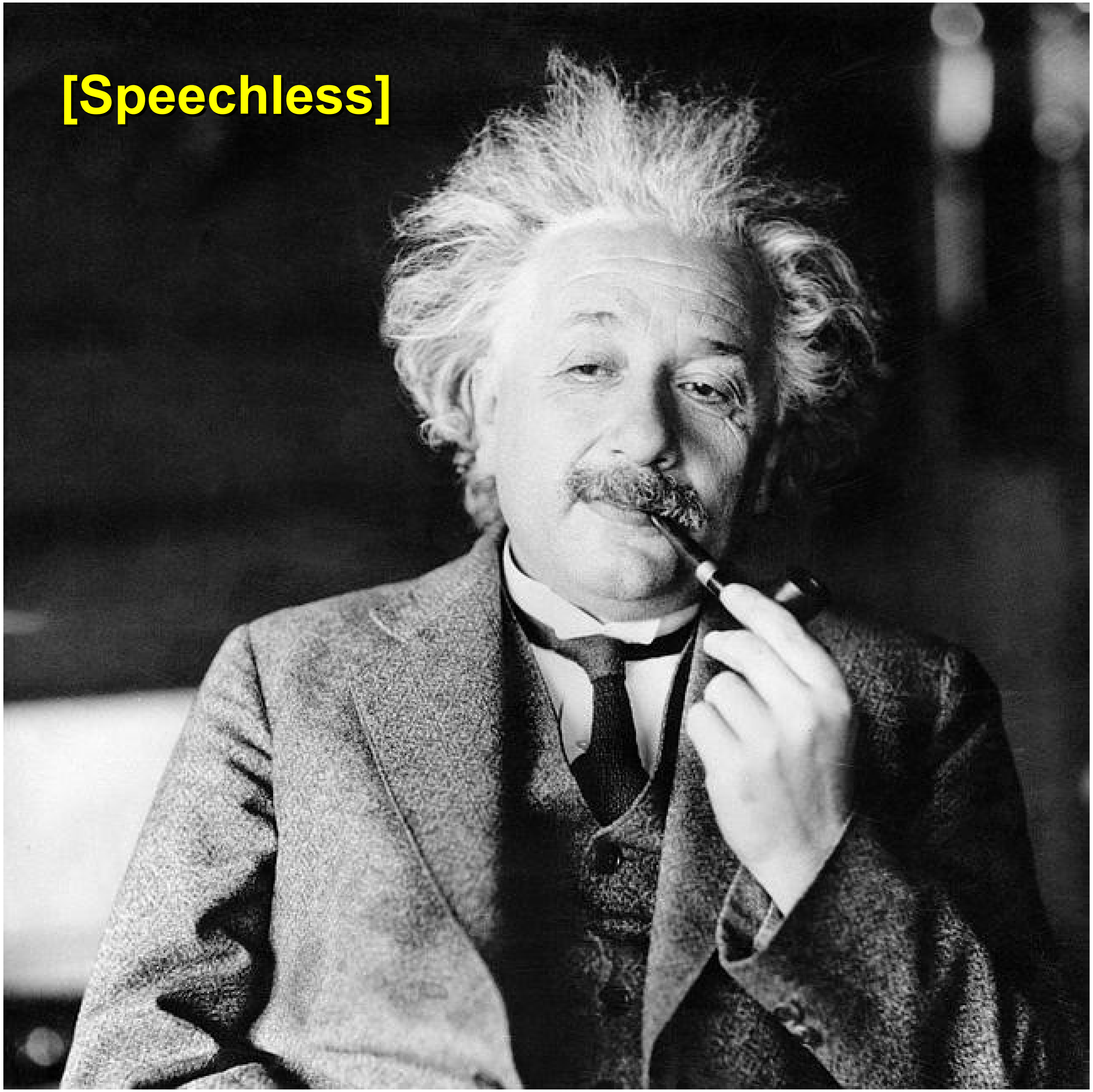

S3

- ✔ A non-Abelian discrete subgroup of U(3)
- ✔ Hidden after the EWSB
- ✔ $3 \rightarrow 2 + 1$




[Speechless]

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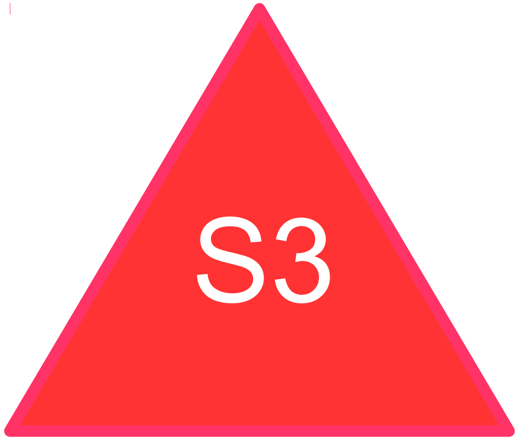
$$\mathrm{L}_Y = -\sum_{ij} Y_f^{ij}\overline{\psi}^L_{f,i} H \psi^R_{f,i}$$

$$\boxed{\mathrm{L'}_Y \overset{?}{=} L_Y(1 \leftrightarrow 2 \leftrightarrow 3)}$$



1 Higgs $\longrightarrow$

$$m_1 = m_2 \neq m_3$$

$$V = I_{3\times 3}$$

$\longrightarrow$ **S3**

- Mondragón et al, Phys. Rev. D59, 093009
- Mondragón et al, Phys. Rev. D61, 113002
- Barranco et al, Phys. Rev. D82, 073010
- Feruglio et al, Nucl. Phys. B800, 77
- Kobayashi et al, Phys. Rev. D78, 115006
- Jora et al, Phys. Rev. D80, 093007
- Xing et al, Phys. Lett. B690, 204
- ...

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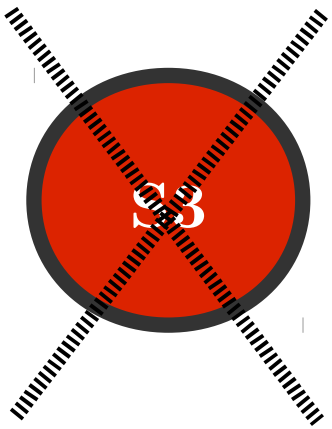
$\longrightarrow$



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DEAD
END

15

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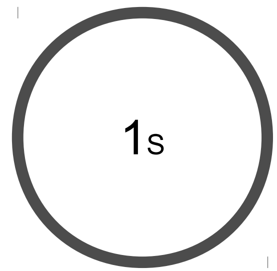
Dogma 1 – Neutrinos are strictly massless!

Dogma 2 – No Z-mediated FCNC at tree level

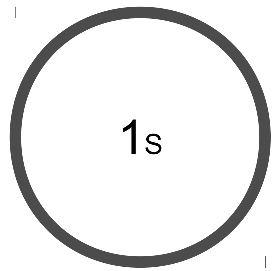
Dogma 3 – No Higgs mediated FCNC at tree level.

Question: We all know the "Fate" of Dogma N°1

Will the other two Dogmas have the same "Fate" as Dogma 1.

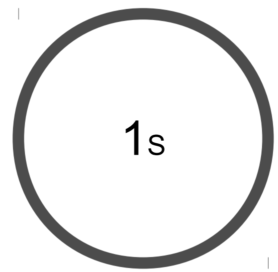


SM Higgs



SM Higgs

+



SM Higgs

+

3 Higgs

4 Higgs $\longrightarrow$ $m_1 < m_2 << m_3$, $V = V_{exp}$ $\longrightarrow$ **S3**

S3 hidden + **4H**:

Yahalom (1984)

**Mondragon et al, arXiv:1304.6644

S3 hidden + **3H**:

Pakvasa and Sugawara (1975); Derman (1978); Wyler (1979); Frere (1979); Yahalom (1984); Ma (1991); Hall and Murayama (1995); Koide (1999); Lavoura (2000); Kubo, Mondragón, Rodríguez-Jauregui (2003); Kubo, Okada, and Sakamaki (2005); Kaneko et al (2007); Gonzalez Canales, Mondragón, and Mondragón (2012); Teshima and Okomura (2011); Bhattacharyya, Leser, and Pas (2011); Teshima (2012);...

4 Higgs $\longrightarrow$

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$$\mathrm{V} = \mathrm{V}_{exp}$$

$\longrightarrow$ **S3**

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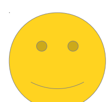
S3 hidden + **3H**: 

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Up to now...

Standard Model +
Massive neutrinos +
3 more Higgses +
S3

Matter content

Fermions I II III (u,d,l,ν_D)

2 1_S

Higgses I II IV III

2 1_A 1_S

What do we get?

Yahalom, Hall and Murayama

$$\mathcal{M}^f_{S_3} = \begin{pmatrix} \sqrt{2}Y^f_2 v_S + Y^f_3 w_2 & Y^f_3 w_1 + \sqrt{2}Y^f_4 v_A & \sqrt{2}Y^f_5 w_1 \\ Y^f_3 w_1 - \sqrt{2}Y^f_4 v_A & \sqrt{2}Y^f_2 v_S - Y^f_3 w_2 & \sqrt{2}Y^f_5 w_2 \\ \sqrt{2}Y^f_6 w_1 & \sqrt{2}Y^f_6 w_2 & 2Y^f_1 v_S \end{pmatrix}$$

$$w_1 \equiv \langle 0|H_{1W}|0\rangle, \;\; w_2 \equiv \langle 0|H_{2W}|0\rangle, \;\; v_S \equiv \langle 0|H_{SW}|0\rangle, \;\; \text{and} \;\; v_A \equiv \langle 0|H_{AW}|0\rangle,$$

Yahalom, Hall and Murayama

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$$\mathrm{N}_{L,f} = M_f M_f^\dagger$$

$$\mathrm{N}_{R,f} = M_f^\dagger M_f$$

Yahalom, Hall and Murayama

$$\mathcal{M}^f_{S_3} = \begin{pmatrix} \sqrt{2}Y_2^f v_S + Y_3^f w_2 & Y_3^f w_1 + \sqrt{2}Y_4^f v_A & \sqrt{2}Y_5^f w_1 \\ Y_3^f w_1 - \sqrt{2}Y_4^f v_A & \sqrt{2}Y_2^f v_S - Y_3^f w_2 & \sqrt{2}Y_5^f w_2 \\ \sqrt{2}Y_6^f w_1 & \sqrt{2}Y_6^f w_2 & 2Y_1^f v_S \end{pmatrix}$$

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Hermiticity

$$\mathrm{M}_f = M_f^\dagger$$

See
D. Emmanuel-Costa's talk

$$\hat{\mathcal{M}}_f^{S_3} = \begin{pmatrix} |\mu_1^f| + |\mu_2^f| c^2(1-3t^2) & |\mu_2^f| sc(3-t^2) + i|\mu_5^f| & 0 \\ |\mu_2^f| sc(3-t^2) - i|\mu_5^f| & |\mu_1^f| - |\mu_2^f| c^2(1-3t^2) & \mu_7^f/c \\ 0 & \mu_7^{f\,*}/c & |\mu_3^f| \end{pmatrix}$$

- Hermiticity
- 2x2 Rotation
-

See
D. Emmanuel-Costa's talk

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- The angle should satisfy:

$$\tan(\alpha) = \frac{w_1}{w_2}$$

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$$\mathcal{U}\mathcal{A}_{n\times n}\mathcal{U}^{-1} = diag(a_1, a_2, ..., a_n),$$

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$$\Delta\mathcal{U}\mathcal{U}^{-1} + \mathcal{U}\overline{\mathcal{A}}_{n\times n}\mathcal{U}^{-1} = diag(a_1, a_2, ..., a_n),$$

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$$\mathcal{U}\overline{\mathcal{A}}_{n\times n}\mathcal{U}^{-1} = diag(a_1 - \Delta, a_2 - \Delta, ..., a_n - \Delta),$$

4HDM: $G_{SM} \otimes S_3$

Name	$\mathcal{F}_L$	$\mathcal{F}_R$	Mass matrix (FB)	Possible mass textures
A	$\mathbf{2},\ 1_\mathbf{S}$	$\mathbf{2},\ 1_\mathbf{S}$	$\begin{pmatrix} \mu_1^f+\mu_2^f & \mu_4^f+\mu_5^f & \mu_6^f \\ \mu_4^f-\mu_5^f & \mu_1^f-\mu_2^f & \mu_7^f \\ \mu_8^f & \mu_9^f & \mu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & \mu_2^f sc\,(3-t^2)+\mu_5^f & 0 \\ \mu_2^f sc\,(3-t^2)-\mu_5^f & -2\mu_2^f c^2\,(1-3t^2) & \mu_7^f/c \\ 0 & \mu_7^{f\,*}/c & \mu_3^f-\mu_1^f-\mu_2^f c^2(1-3t^2) \end{pmatrix}$
A'				$\begin{pmatrix} 0 & \frac{2}{\sqrt{3}}\mu_2^f+\mu_5^f & 0 \\ \frac{2}{\sqrt{3}}\mu_2^f-\mu_5^f & 0 & \frac{2}{\sqrt{3}}\mu_7^f \\ 0 & \frac{2}{\sqrt{3}}\mu_8^f & \mu_3^f-\mu_1^f \end{pmatrix}$
B	$\mathbf{2},\ 1_\mathbf{A}$	$\mathbf{2},\ 1_\mathbf{A}$	$\begin{pmatrix} \mu_1^f+\mu_2^f & \mu_4^f+\mu_5^f & \mu_7^f \\ \mu_4^f-\mu_5^f & \mu_1^f-\mu_2^f & -\mu_6^f \\ \mu_9^f & -\mu_8^f & \mu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & -\mu_4^f c^2\,(1-3t^2)+\mu_5^f & 0 \\ -\mu_4^f c^2\,(1-3t^2)-\mu_5^f & 2\mu_4^f cs\,(3-t^2) & -\mu_6^f/c \\ 0 & -\mu_6^{f\,*}/c & \mu_3^f-\mu_1^f+\mu_4^f sc(3-t^2) \end{pmatrix}$
B'				$\begin{pmatrix} 0 & -2\mu_4^f+\mu_5^f & 0 \\ -2\mu_4^f-\mu_5^f & 0 & -2\mu_6^f \\ 0 & -2\mu_8^f & \mu_3^f-\mu_1^f \end{pmatrix}$

Mondragón et al,
arXiv:1304.6644

C	$\mathbf{2},\ 1_{\mathbf{A}}$	$\mathbf{2},\ 1_{\mathbf{S}}$	$\begin{pmatrix} \mu_1^f+\mu_2^f & \mu_4^f+\mu_5^f & \mu_6^f \\ \mu_4^f-\mu_5^f & \mu_1^f-\mu_2^f & \mu_7^f \\ \mu_9^f & -\mu_8^f & \nu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & A_{12} & 0 \\ A_{21} & A_{22} & A_{23} \\ A_{32} & 0 & A_{33}(\mu_3^f \to \nu_3^f) \end{pmatrix}$,	$\begin{pmatrix} 0 & B_{12} & B_{23} \\ B_{21} & B_{22} & 0 \\ 0 & B_{32} & B_{33}(\mu_3^f \to \nu_3^f) \end{pmatrix}$
C'				$\begin{pmatrix} 0 & A'_{12} & 0 \\ A'_{21} & 0 & A'_{23} \\ A'_{32} & 0 & A'_{33}(\mu_3^f \to \nu_3^f) \end{pmatrix}$,	$\begin{pmatrix} 0 & B'_{12} & B'_{23} \\ B'_{21} & 0 & 0 \\ 0 & B'_{32} & B'_{33}(\mu_3^f \to \nu_3^f) \end{pmatrix}$
D	$\mathbf{2},\ 1_{\mathbf{S}}$	$\mathbf{2},\ 1_{\mathbf{A}}$	$\begin{pmatrix} \mu_1^f+\mu_2^f & \mu_4^f+\mu_5^f & \mu_7^f \\ \mu_4^f-\mu_5^f & \mu_1^f-\mu_2^f & -\mu_6^f \\ \mu_8^f & \mu_9^f & \nu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & A_{12} & A_{23} \\ A_{21} & A_{22} & 0 \\ 0 & A_{32} & A_{33}(\mu_3^f \to \nu_3^f) \end{pmatrix}$,	$\begin{pmatrix} 0 & B_{12} & 0 \\ B_{21} & B_{22} & B_{23} \\ B_{32} & 0 & B_{33}(\mu_3^f \to \nu_3^f) \end{pmatrix}$
D'				$\begin{pmatrix} 0 & A'_{12} & A'_{23} \\ A'_{21} & 0 & 0 \\ 0 & A'_{32} & A'_{33}(\mu_3^f \to \nu_3^f) \end{pmatrix}$,	$\begin{pmatrix} 0 & B'_{12} & 0 \\ B'_{21} & 0 & B'_{23} \\ B'_{32} & 0 & B'_{33}(\mu_3^f \to \nu_3^f) \end{pmatrix}$

Table 4: Mass matrices of S_3-invariant flavour models with four Higgs $SU(2)_L$ doublets which are assigned to all the S_3 irreducible representations, $\mathcal{H}_{AW} \oplus \mathcal{H}_{SW} \oplus \mathcal{H}_{DW}$. We denote $\mu_1^f \equiv \sqrt{2}Y_2^f v_S$, $\mu_2^f \equiv Y_3^f w_2$, $\mu_3^f \equiv 2Y_1^f v_S$, $\mu_4^f \equiv Y_3^f w_1$, $\mu_5^f \equiv \sqrt{2}Y_4^f v_A$, $\mu_6^f \equiv \sqrt{2}Y_5^f w_1$, $\mu_7^f \equiv \sqrt{2}Y_5^f w_2$, $\mu_8^f \equiv \sqrt{2}Y_6^f w_1$, $\mu_9^f \equiv \sqrt{2}Y_6^f w_2$, and $\nu_3^f \equiv 2Y_1^f v_A$.

Mondragón et al,
arXiv:1304.6644

By use of

$$\begin{aligned}
\mathrm{Tr}[\bar{\mathcal{M}}^f_{Hier}] &= \widetilde{\sigma}^f_1 - \widetilde{\sigma}^f_2 + 1 \; , \\
\mathrm{Det}[\bar{\mathcal{M}}^f_{Hier}] &= -\widetilde{\sigma}^f_1 \widetilde{\sigma}^f_2 \; , \\
\mathrm{Tr}[(\bar{\mathcal{M}}^f_{Hier})^2] &= (\widetilde{\sigma}^f_1)^2 + (\widetilde{\sigma}^f_2)^2 + 1
\end{aligned}$$

where

$$m^f_i = \mu^f_0 + \sigma^f_i$$

We obtain

$$\bar{\mathcal{M}}_f^{S_3} = \begin{pmatrix} 0 & \sqrt{\frac{\widetilde{\sigma}_1^f \widetilde{\sigma}_2^f}{1-\delta_f}} & 0 \\ \sqrt{\frac{\widetilde{\sigma}_1^f \widetilde{\sigma}_2^f}{1-\delta_f}} & \widetilde{\sigma}_1^f - \widetilde{\sigma}_2^f + \delta_f & \sqrt{\frac{\delta_f}{1-\delta_f}\xi_1^f \xi_2^f} \\ 0 & \sqrt{\frac{\delta_f}{1-\delta_f}\xi_1^f \xi_2^f} & 1-\delta_f \end{pmatrix}$$

$$\xi_1^f \equiv 1 - \widetilde{\sigma}_1^f - \delta_f, \quad \xi_2^f \equiv 1 + \widetilde{\sigma}_2^f - \delta_f$$

with

$$\tan\phi_{1f} = \sqrt{\frac{F_{\phi_1}(\delta_f, \widetilde{\sigma}_i{}^f)}{G_{\phi_1}(\delta_f, \widetilde{\sigma}_i{}^f)}},$$

where

$$\begin{aligned} F_{\phi_1}(\delta_f, \widetilde{\sigma}_i{}^f) &= (\delta_f^3 - [1 - 2(\widetilde{\sigma}_1^f - \widetilde{\sigma}_2^f)]\delta_f^2 + (\widetilde{\sigma}_1^f - \widetilde{\sigma}_2^f)(\widetilde{\sigma}_1^f - \widetilde{\sigma}_2^f - 2)\delta_f \\ &- (\widetilde{\sigma}_1^f - \widetilde{\sigma}_2^f)^2)\tan^2\theta(3 - \tan^2\theta)^2 + 4\widetilde{\sigma}_1^f\widetilde{\sigma}_2^f(1 - 3\tan^2\theta)^2, \\ G_{\phi_1}(\delta_f, \widetilde{\sigma}_i^f) &= (1 - \delta_f)(\widetilde{\sigma}_1^f - \widetilde{\sigma}_2^f + \delta_f)^2\tan^2\theta(3 - \tan^2\theta)^2 \end{aligned}$$

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$$
\begin{aligned}
V_{ud}^{th} &= \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_s\xi_1^u\xi_1^d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_d}{\mathcal{D}_{1u}\mathcal{D}_{1d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_1^u\xi_1^d} + \sqrt{\delta_u\delta_d\xi_2^u\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1}, \\
V_{us}^{th} &= -\sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_d\xi_1^u\xi_2^d}{\mathcal{D}_{1u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_s}{\mathcal{D}_{1u}\mathcal{D}_{2d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_1^u\xi_2^d} + \sqrt{\delta_u\delta_d\xi_2^u\xi_1^d}e^{i\phi_2}\right)e^{i\phi_1}, \\
V_{ub}^{th} &= \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_d\widetilde{\sigma}_s\delta_d\xi_1^u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}} + \sqrt{\frac{\widetilde{\sigma}_u}{\mathcal{D}_{1u}\mathcal{D}_{3d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\delta_d\xi_1^u} - \sqrt{\delta_u\xi_2^u\xi_1^d\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1}, \\
V_{cd}^{th} &= -\sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_s\xi_2^u\xi_1^d}{\mathcal{D}_{2u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_d}{\mathcal{D}_{2u}\mathcal{D}_{1d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_2^u\xi_1^d} + \sqrt{\delta_u\delta_d\xi_1^u\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1}, \\
V_{cs}^{th} &= \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_d\xi_2^u\xi_2^d}{\mathcal{D}_{2u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{\sigma}_c\widetilde{\sigma}_s}{\mathcal{D}_{2u}\mathcal{D}_{2d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\xi_2^u\xi_2^d} + \sqrt{\delta_u\delta_d\xi_1^u\xi_1^d}e^{i\phi_2}\right)e^{i\phi_1}, \qquad (38) \\
V_{cb}^{th} &= -\sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_d\widetilde{\sigma}_s\delta_d\xi_2^u}{\mathcal{D}_{2u}\mathcal{D}_{3d}}} + \sqrt{\frac{\widetilde{\sigma}_c}{\mathcal{D}_{2u}\mathcal{D}_{3d}}}\left(\sqrt{(1-\delta_u)(1-\delta_d)\,\delta_d\xi_2^u} - \sqrt{\delta_u\xi_1^u\xi_1^d\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1}, \\
V_{td}^{th} &= \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_c\widetilde{\sigma}_s\delta_u\xi_1^d}{\mathcal{D}_{3u}\mathcal{D}_{1d}}} + \sqrt{\frac{\widetilde{\sigma}_d}{\mathcal{D}_{3u}\mathcal{D}_{1d}}}\left(\sqrt{\delta_u(1-\delta_u)(1-\delta_d)\,\xi_1^d} - \sqrt{\delta_d\xi_1^u\xi_2^u\xi_2^d}e^{i\phi_2}\right)e^{i\phi_1}, \\
V_{ts}^{th} &= -\sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_c\widetilde{\sigma}_d\delta_u\xi_2^d}{\mathcal{D}_{3u}\mathcal{D}_{2d}}} + \sqrt{\frac{\widetilde{\sigma}_s}{\mathcal{D}_{3u}\mathcal{D}_{2d}}}\left(\sqrt{\delta_u(1-\delta_u)(1-\delta_d)\,\xi_2^d} - \sqrt{\delta_d\xi_1^u\xi_2^u\xi_1^d}e^{i\phi_2}\right)e^{i\phi_1}, \\
V_{tb}^{th} &= \sqrt{\frac{\widetilde{\sigma}_u\widetilde{\sigma}_c\widetilde{\sigma}_d\widetilde{\sigma}_s\delta_u\delta_d}{\mathcal{D}_{3u}\mathcal{D}_{3d}}} + \left(\sqrt{\frac{\xi_1^u\xi_2^u\xi_1^d\xi_2^d}{\mathcal{D}_{3u}\mathcal{D}_{3d}}} + \sqrt{\frac{\delta_u\delta_d(1-\delta_u)(1-\delta_d)}{\mathcal{D}_{3u}\mathcal{D}_{3d}}}e^{i\phi_2}\right)e^{i\phi_1},
\end{aligned}
$$

Mondragón et al, arXiv:1304.6644

For all Dirac Fermions.

Mondragon and Rodriguez-Jauregui

$$
\begin{aligned}
\xi_1^{u,d} &= 1 - \widetilde{\sigma}_{u,d} - \delta_{u,d}, \quad \xi_2^{u,d} = 1 + \widetilde{\sigma}_{c,s} - \delta_{u,d}, \\
\mathcal{D}_{1(u,d)} &= (1 - \delta_{u,d})(\widetilde{\sigma}_{u,d} + \widetilde{\sigma}_{c,s})(1 - \widetilde{\sigma}_{u,d}), \\
\mathcal{D}_{2(u,d)} &= (1 - \delta_{u,d})(\widetilde{\sigma}_{u,d} + \widetilde{\sigma}_{c,s})(1 + \widetilde{\sigma}_{c,s}), \\
\mathcal{D}_{3(u,d)} &= (1 - \delta_{u,d})(1 - \widetilde{\sigma}_{u,d})(1 + \widetilde{\sigma}_{c,s}).
\end{aligned}
$$

Mondragón et al, arXiv:1304.6644

$$U_{CKM} = \begin{pmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{pmatrix}$$

$$U_{CKM} = \begin{pmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{pmatrix}$$

$$\mathrm{m}_u = 0.0018GeV,\ m_c = 1.275GeV,\ m_t = 172.85GeV$$

$$\mathrm{m}_d = 0.0051GeV,\ m_s = 0.095GeV,\ m_b = 4.65GeV$$

PDG 2012

$$\tan\theta = 1$$

$$\phi_1 = \pi/2 \quad \phi_2 = 0$$

$$U_{CKM} = \begin{pmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{pmatrix}$$

$$\mathrm{m}_u = 0.0018 GeV,\ m_c = 1.275 GeV,\ m_t = 172.85 GeV$$

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PDG 2012

$$\boxed{\begin{array}{l} \delta_u = 0.00737 \\ \delta_d = 0.01933 \end{array}}$$

$$\tan\theta = 1$$

$$\phi_1 = \pi/2 \quad \phi_2 = 0$$

$$|U^{th}_{CKM}| = \begin{pmatrix} 0.97352 & 0.22854 & 0.00205 \\ 0.22821 & 0.97219 & 0.05248 \\ 0.012497 & 0.05101 & 0.99862 \end{pmatrix}$$

$$\mathrm{m}_u = 0.0018GeV,\ m_c = 1.275GeV,\ m_t = 172.85GeV$$

$$\mathrm{m}_d = 0.0051GeV,\ m_s = 0.095GeV,\ m_b = 4.65GeV$$

PDG 2012

$$\boxed{\begin{array}{l} \delta_u = 0.00737 \\ \delta_d = 0.01933 \end{array}} \quad \begin{array}{ll} \tan\theta = 1 & \mu_0 = 0 \\ \phi_1 = \pi/2 & \phi_2 = 0 \end{array}$$

$$|U_{CKM}^{th}| = \begin{pmatrix} 0.97352 & 0.22854 & 0.00205 \\ 0.22821 & 0.97219 & 0.05248 \\ 0.012497 & 0.05101 & 0.99862 \end{pmatrix}$$

$$|U_{CKM}^{exp}| = \begin{pmatrix} 0.97427 & 0.22534 & 0.00351 \\ 0.22520 & 0.97344 & 0.0412 \\ 0.00867 & 0.0404 & 0.999146 \end{pmatrix}$$

PDG 2012

Summary and Remarks

The theoretical framework
of the SM + few ingredients
is in very well shape!

Summary and Remarks

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2) These relations can be obtained by means of a *flavour symmetry*.

3 a) The mass spectra and the kinetic terms of the SM Lagrangian could be suggesting already which flavour symmetry (*S3*)...

Summary and Remarks (cont'd)

3 b) and moreover, a multiHiggs isodoublet scenario (*in this case: 3 more*).

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3 b) and moreover, a multiHiggs isodoublet scenario (*in this case: 3 more*).

4) FCNC's are expected to be small due to the strong hierarchy in the masses. (to be published)

5) A more realistic model needs the full analysis of the S3-invariant Higgs pot. (work in progress)

Thanks for
Your attention!

"It's funny how day by day, nothing changes. But when you look back everything is different."

-Calvin & Hobbes

Any questions?

Backup spartans

PHYSICAL REVIEW D **82,** 073010 (2010)

Universal mass matrix for quarks and leptons and *CP* violation

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The measurements of the neutrino and quark mixing angles satisfy the empirical relations called quark-lepton complementarity. These empirical relations suggest the existence of a correlation between the mixing matrices of leptons and quarks. In this work, we examine the possibility that this correlation between the mixing angles of quarks and leptons originates in the similar hierarchy of quarks and charged lepton masses and the seesaw mechanism type I, that gives mass to the Majorana neutrinos. We assume that the similar mass hierarchies of charged lepton and quark masses allows us to represent all the mass matrices of Dirac fermions in terms of a universal form with four texture zeroes.

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8.2 Fitting procedure

We construct the χ^2 function as

$$\chi^2 = \frac{\left(|V_{ud}^{\rm th}| - |V_{ud}|\right)^2}{\sigma_{V_{ud}}^2} + \frac{\left(|V_{us}^{\rm th}| - |V_{us}|\right)^2}{\sigma_{V_{us}}^2} + \frac{\left(|V_{ub}^{\rm th}| - |V_{ub}|\right)^2}{\sigma_{V_{ub}}^2} + \frac{\left(\mathcal{J}_q^{\rm th} - \mathcal{J}_q\right)^2}{\sigma_{\mathcal{J}_q}^2}, \tag{46}$$

where the quantities with super-index "th" are the complete expressions for the CKM elements, as given by the S_3 models, and those without, the experimental quantities along with their uncertainty $\sigma_{V_{ij}}$. We consider the following experimental CKM values

$$\begin{array}{llll} 2011: & |V_{ud}| = 0.97428 \pm 0.00015, & 2012: & |V_{ud}| = 0.97427 \pm 0.00015, \\ & |V_{us}| = 0.2253 \pm 0.007, & & |V_{us}| = 0.2253 \pm 0.007, \\ & |V_{ub}| = 0.00347 \pm 0.00014, & & |V_{ub}| = 0.00351 \pm 0.00015, \\ & \mathcal{J}_q = (2.91 \pm 0.155) \times 10^{-5}, & & \mathcal{J}_q = (2.96 \pm 0.18) \times 10^{-5}\ , \end{array} \tag{47}$$

If we want to use just the CKM experimental values, without assuming unitarity conditions, we can use again the three best measured values of the CKM matrix V_{ud}, V_{us}, and V_{ub}, and additionally V_{cd}. The third row of the CKM matrix is not accurately measured and, except for V_{tb}, there is not any direct information about such elements. We take then the following, completely uncorrelated, experimental values:

$$\begin{aligned}
&|V_{ud}| = 0.97425 \pm 0.00022, \quad |V_{us}| = 0.2252 \pm 0.0009, \quad |V_{ub}| = 0.00415 \pm 0.00049,\\
&|V_{cd}| = 0.230 \pm 0.011.
\end{aligned} \tag{54}$$

Then construct the χ^2 function

$$\chi^2 = \frac{\left(|V_{ud}^{\mathrm{th}}| - |V_{ud}|\right)^2}{\sigma_{V_{ud}}^2} + \frac{\left(|V_{us}^{\mathrm{th}}| - |V_{us}|\right)^2}{\sigma_{V_{us}}^2} + \frac{\left(|V_{ub}^{\mathrm{th}}| - |V_{ub}|\right)^2}{\sigma_{V_{ub}}^2} + \frac{\left(|V_{cd}^{\mathrm{th}}| - |V_{cd}|\right)^2}{\sigma_{V_{cd}}^2}. \tag{55}$$

Parameter	Central value	χ^2	Values with restricted precision	χ^2
Fit using the 2012 values of the parameters $\widetilde{m}_i$				
$\widetilde{\sigma_u}\,(M_Z)$	2.08977×10^{-6}		$(2.09 \pm 0.19) \times 10^{-6}$	
$\widetilde{\sigma}_c\,(M_Z)$	3.93180×10^{-3}		$(3.93 \pm 0.007) \times 10^{-3}$	
$\widetilde{\sigma}_d\,(M_Z)$	1.35949×10^{-3}		$(1.36 \pm 0.004) \times 10^{-3}$	
$\widetilde{\sigma}_s\,(M_Z)$	2.08443×10^{-2}		$(2.08 \pm 0.02) \times 10^{-2}$	
δ_u	3.96726×10^{-2}		$(3.97 \pm 0.35) \times 10^{-2}$	
δ_d	5.29260×10^{-2}		$(5.29 \pm 041) \times 10^{-2}$	
$\cos\phi_2$	8.48776×10^{-1}		$(8.49 \pm 0.22) \times 10^{-1}$	
		3.3×10^{-4}		3.9×10^{-1}
Fit using the 2012 values of the parameters $\widetilde{m}_i$ (with $\widetilde{m}_s^{th}$)				
$\widetilde{\sigma_u}\,(M_Z)$	2.17737×10^{-6}		$(2.18 \pm 0.35) \times 10^{-6}$	
$\widetilde{\sigma}_c\,(M_Z)$	3.94×10^{-3}		$(3.94 \pm 0.007) \times 10^{-3}$	
$\widetilde{\sigma}_d\,(M_Z)$	1.19392×10^{-3}		$(1.19 \pm 0.009) \times 10^{-3}$	
$\widetilde{\sigma}_s\,(M_Z)$	1.82432×10^{-2}		$(1.82 \pm 0.02) \times 10^{-3}$	
δ_u	6.12747×10^{-2}		$(6.13 \pm 0.41) \times 10^{-2}$	
δ_d	8.36979×10^{-2}		$(8.37 \pm 0.64 \times 10^{-2}$	
$\cos\phi_2$	9.23028×10^{-1}		$(9.23 \pm 0.11) \times 10^{-1}$	
		3.3×10^{-4}		7.3×10^{-2}
Fit using $\lvert V_{cd}\rvert$ and the 2013 values of the parameters with $\widetilde{m}_s^{th}$				
$\widetilde{\sigma_u}\,(M_Z)$	1.63792×10^{-6}		$(1.64 \pm 0.16) \times 10^{-6}$	
$\widetilde{\sigma}_c\,(M_Z)$	3.58675×10^{-3}		$(3.59 \pm 0.78) \times 10^{-3}$	
$\widetilde{\sigma}_d\,(M_Z)$	1.21487×10^{-3}		$(1.21 \pm 0.88) \times 10^{-3}$	
$\widetilde{\sigma}_s\,(M_Z)$	1.89366×10^{-2}		$(1.90 \pm 0.81) \times 10^{-3}$	
δ_u	9.98871×10^{-2}		$(1.00 \pm 0.5) \times 10^{-2}$	
δ_d	5.37158×10^{-2}		$(5.37 \pm 7.19 \times 10^{-2}$	
$\cos\phi_2$	7.82382×10^{-1}		$(7.82 \pm 9.35) \times 10^{-1}$	
		3.1×10^{-2}		4

$$\mathcal{M} = \begin{pmatrix} 0 & A & 0 \\ A^* & |B| & C \\ 0 & C^* & |D| \end{pmatrix} \quad \text{(Two texture zeroes)}$$

$$\mathcal{M} = \begin{pmatrix} 0 & A & 0 \\ \pm A & 0 & B_1 \\ 0 & B_2 & C \end{pmatrix} \quad \text{(Nearest Neighbour Interaction)}$$