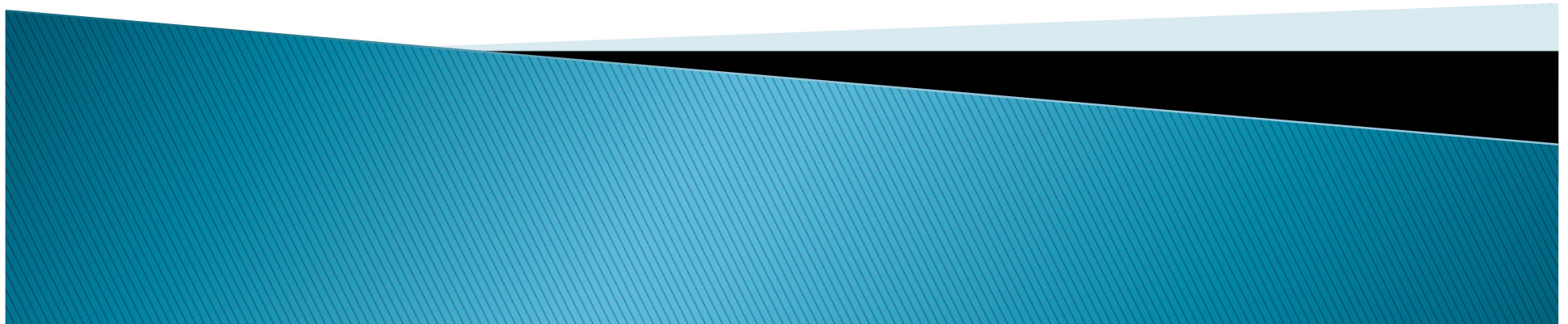


Quantitative Study of Geometrical Scaling in Deep Inelastic Scattering

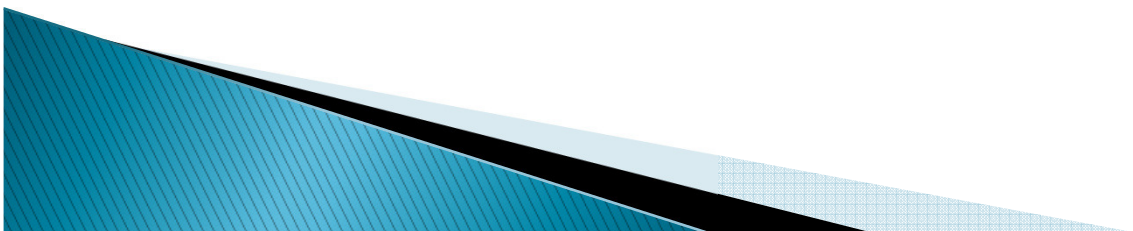
Tomasz Stebel (with Michał Praszalowicz)

M. Smoluchowski Institute of Physics, Jagiellonian University

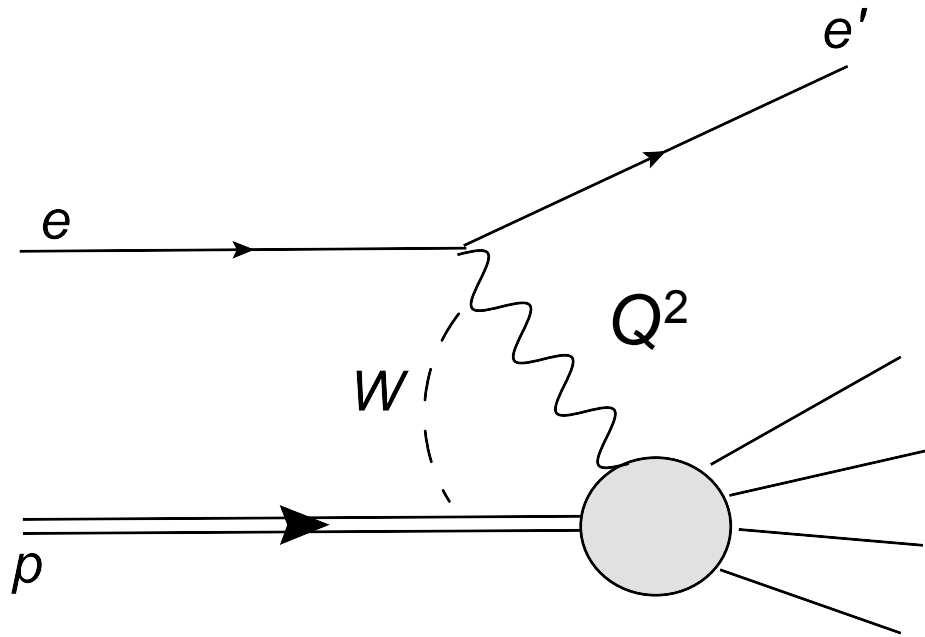


Plan

- ▶ DIS and HERA data.
- ▶ What is Geometrical Scaling (GS) in DIS?
- ▶ Method for GS quantification: χ^2 function.
- ▶ Results for inclusive DIS.
- ▶ Different forms of scaling.
- ▶ Optionally: GS in charm production.



Deep Inelastic Scattering



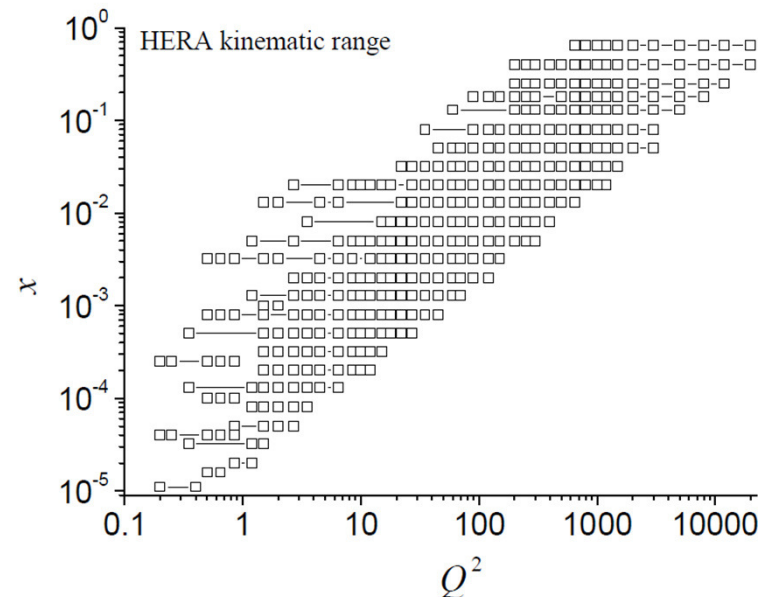
$$x = \frac{Q^2}{Q^2 + W^2 - M_p^2}$$

$$\sigma_{\gamma^* p}(x, Q^2) = 4\pi^2 \alpha_{em} \frac{F_2(x, Q^2)}{Q^2}$$

HERA data for inclusive e^+p

Aaron et al. [H1 and ZEUS Collaborations],
JHEP 1001 (2010) 109

Bin #	Q^2 GeV ²	x	y	$\sigma_r^{+ \text{ave}}$	F_2^{ave}	$\delta_{\text{ave,stat}}$ %	$\delta_{\text{ave,uncor}}$ %	$\delta_{\text{ave,cor}}$ %	$\delta_{\text{ave,exp tot}}$ %	$\delta_{\text{ave,rel}}$ %	$\delta_{\text{ave,gp}}$ %	$\delta_{\text{ave,had}}$ %	$\delta_{\text{ave,tot}}$ %	$\sqrt{s}$ GeV
134	2.7	0.309×10^{-4}	0.864	0.855	—	2.47	2.67	3.33	4.93	0.06	4.62	0.25	6.76	319
135	2.7	0.366×10^{-4}	0.729	0.881	—	2.29	2.29	1.72	3.67	0.09	2.19	0.05	4.27	319
136	2.7	0.409×10^{-4}	0.732	0.975	—	1.54	6.21	1.93	6.68	0.57	4.64	0.61	8.18	301
137	2.7	0.500×10^{-4}	0.599	0.894	0.913	1.06	1.27	1.30	2.11	0.77	2.43	0.54	3.35	301
138	2.7	0.500×10^{-4}	0.534	0.882	0.901	1.51	1.65	0.93	2.42	0.17	0.41	0.10	2.46	319
139	2.7	0.800×10^{-4}	0.374	0.892	0.899	3.28	1.00	1.81	3.87	0.39	0.65	0.75	4.02	301
140	2.7	0.800×10^{-4}	0.333	0.886	0.892	0.72	1.03	0.73	1.45	0.29	0.40	0.30	1.56	319
141	2.7	0.130×10^{-3}	0.205	0.804	0.806	0.62	0.87	0.69	1.28	0.40	-0.02	0.24	1.36	319
142	2.7	0.200×10^{-3}	0.122	0.780	0.780	0.76	1.61	0.82	1.06	0.20	0.28	0.10	2.01	319



Geometrical Scaling in DIS

$$\sigma_{\gamma^*p}(x, Q^2) = \sigma_{\gamma^*p}(\tau), \quad \tau = \frac{Q^2}{Q_s^2(x)}$$

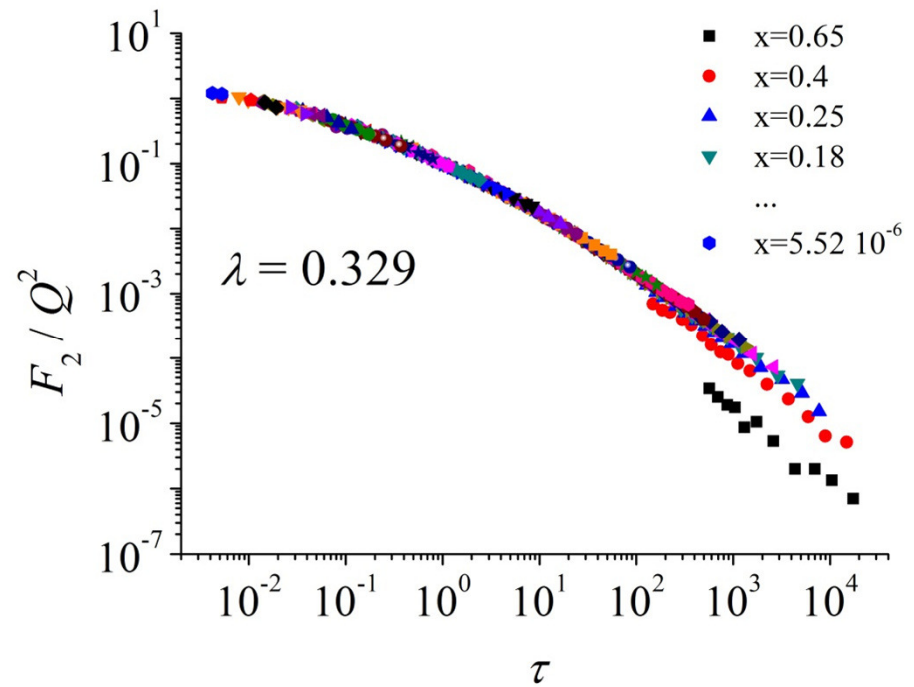
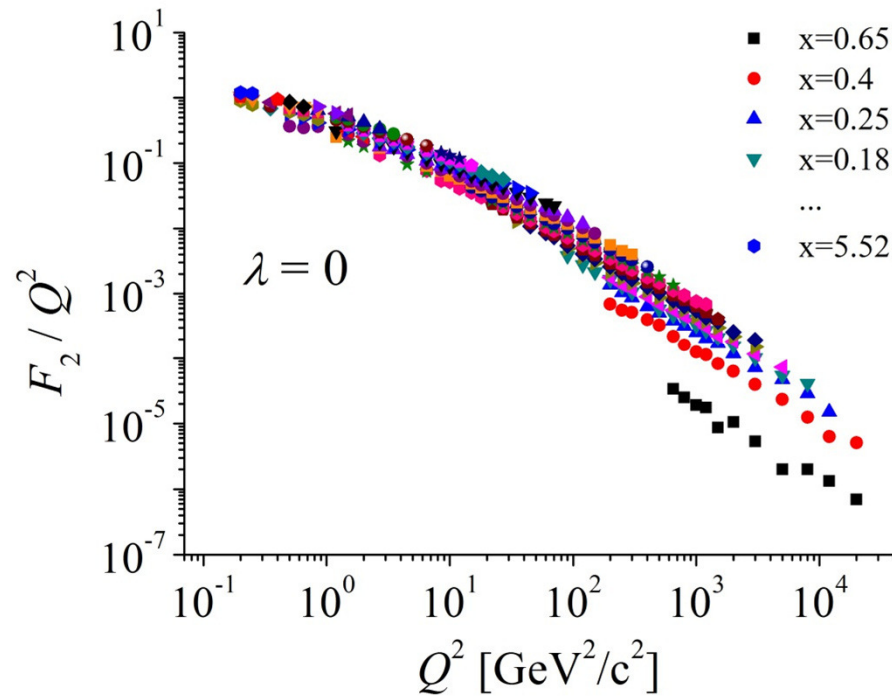
Golec-Biernat – Wusthoff (GBW) model:

$$Q_s^2(x) = Q_0^2 \left(\frac{x}{x_0} \right)^{-\lambda}$$

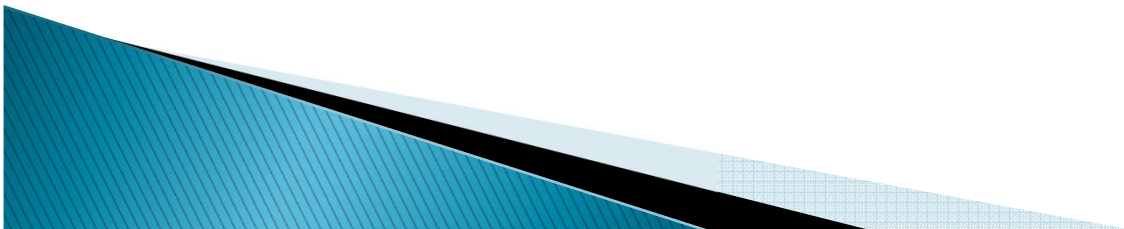
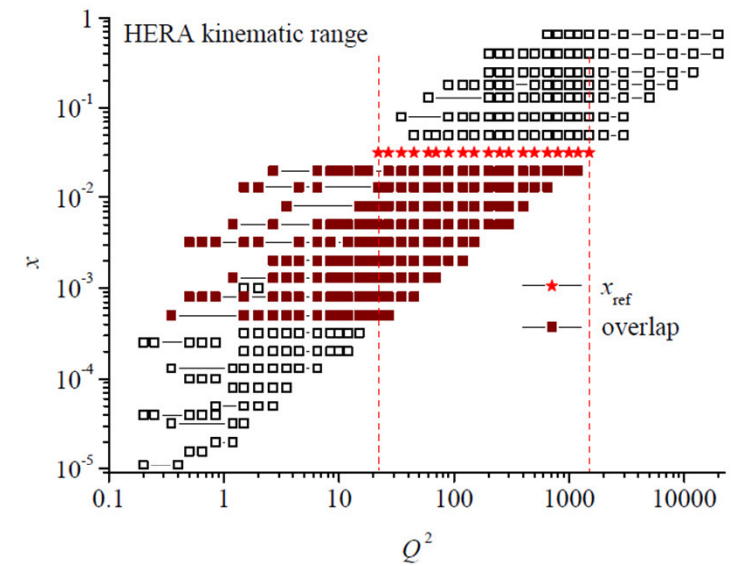
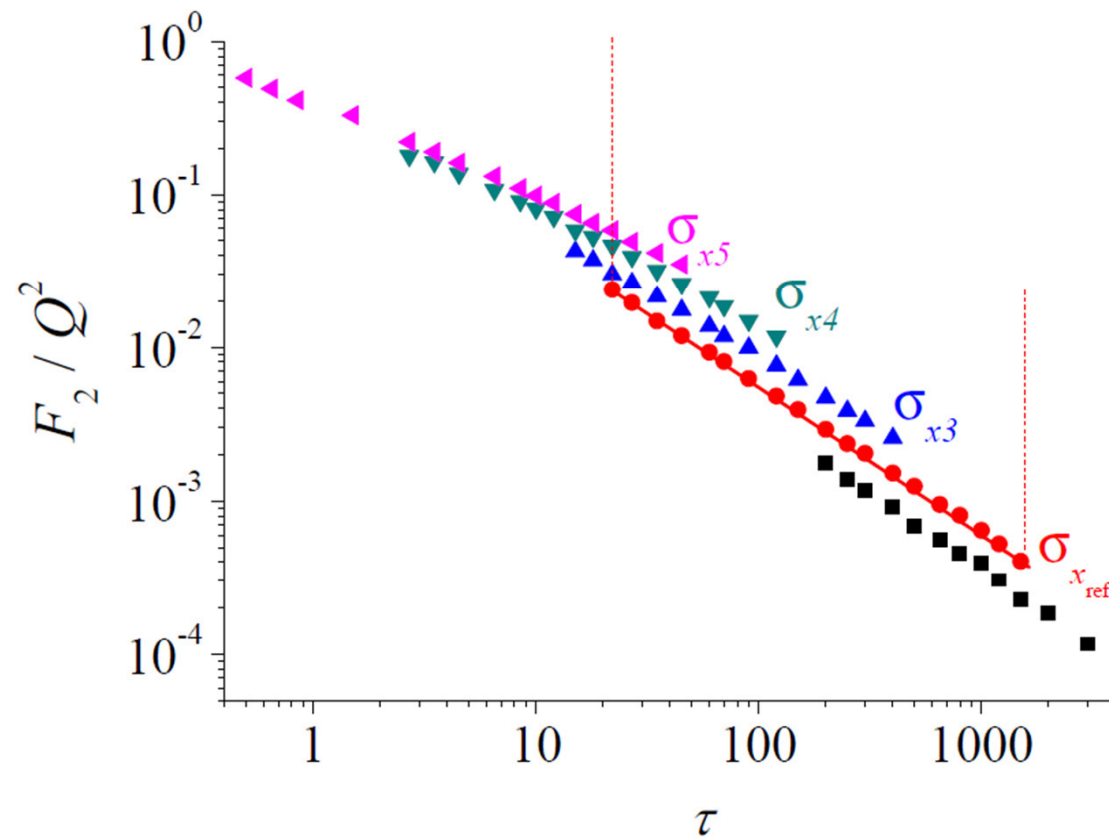
Our choice: $Q_0 = 1 \text{ GeV}/c$ and $x_0 = 1$.

$$\tau = Q^2 x^\lambda$$

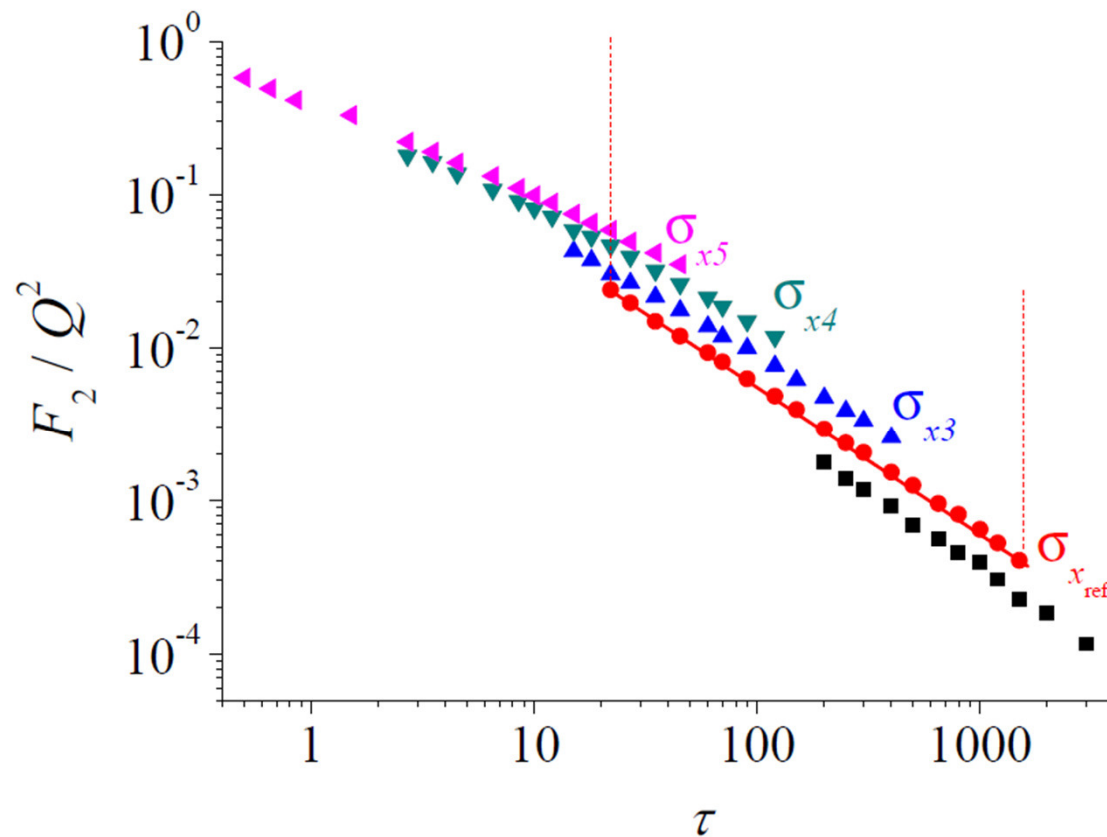
Geometrical Scaling in DIS



Ratios of spectra



Ratios of spectra



$$R_{x3, x_{ref}}(\tau_k, \lambda) = \frac{\sigma_{x3}(\tau_k, \lambda)}{\sigma_{x_{ref}}(\tau_k, \lambda)}$$

$$R_{x4, x_{ref}}(\tau_k, \lambda) = \frac{\sigma_{x4}(\tau_k, \lambda)}{\sigma_{x_{ref}}(\tau_k, \lambda)}$$

$$R_{x5, x_{ref}}(\tau_k, \lambda) = \frac{\sigma_{x5}(\tau_k, \lambda)}{\sigma_{x_{ref}}(\tau_k, \lambda)}$$

For each pair (x, x_{ref}) we have ratios:

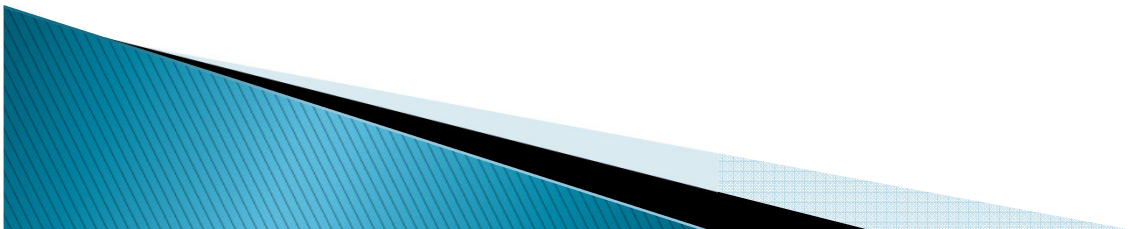
$$\{ R_{x, x_{ref}}(\tau_1, \lambda), R_{x, x_{ref}}(\tau_2, \lambda), \dots, R_{x, x_{ref}}(\tau_4, \lambda) \}$$

χ^2 function

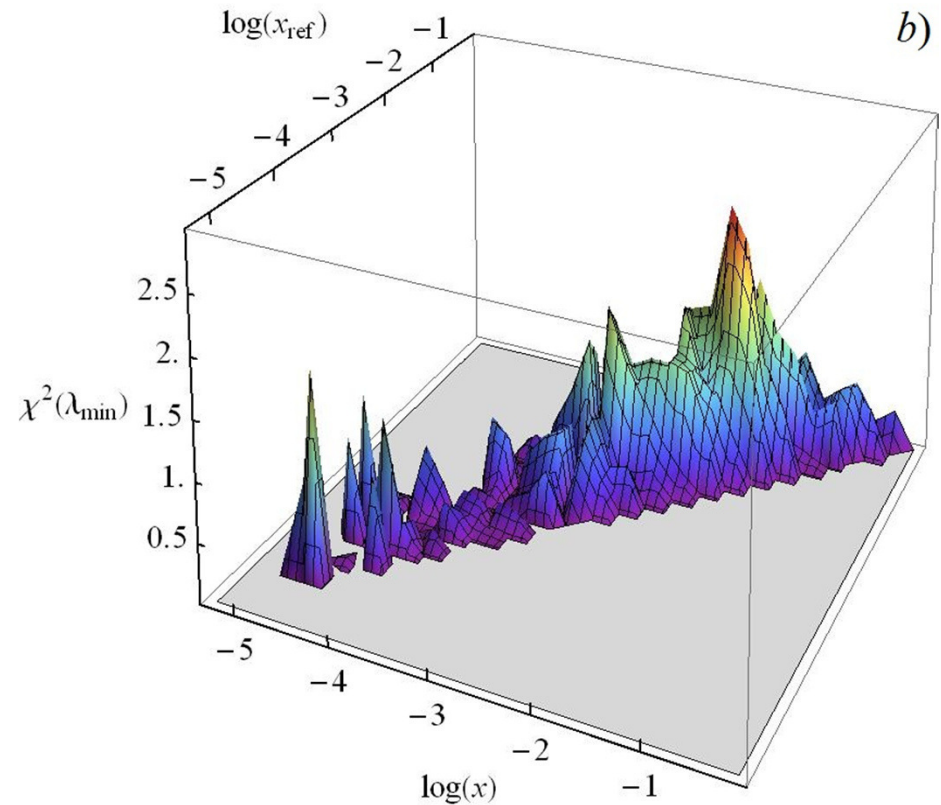
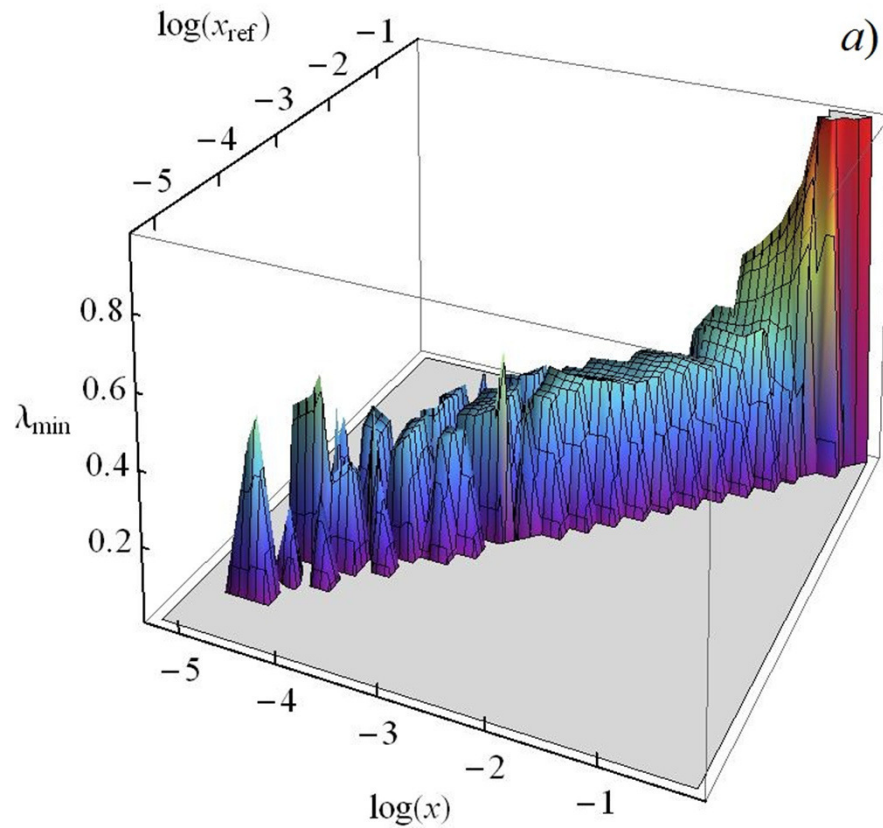
$$\chi_{x,x_{ref}}^2(\lambda) = \frac{1}{N_{x,x_{ref}} - 1} \sum_{k \in x} \frac{\left(R_{x,x_{ref}}(\tau_k, \lambda) - 1\right)^2}{\Delta R_{x,x_{ref}}(\tau_k, \lambda)^2}$$

Best value of λ :

$$\chi_{x,x_{ref}}^2(\lambda) \xrightarrow{\text{minimization}} \lambda_{\min}(x, x_{ref})$$

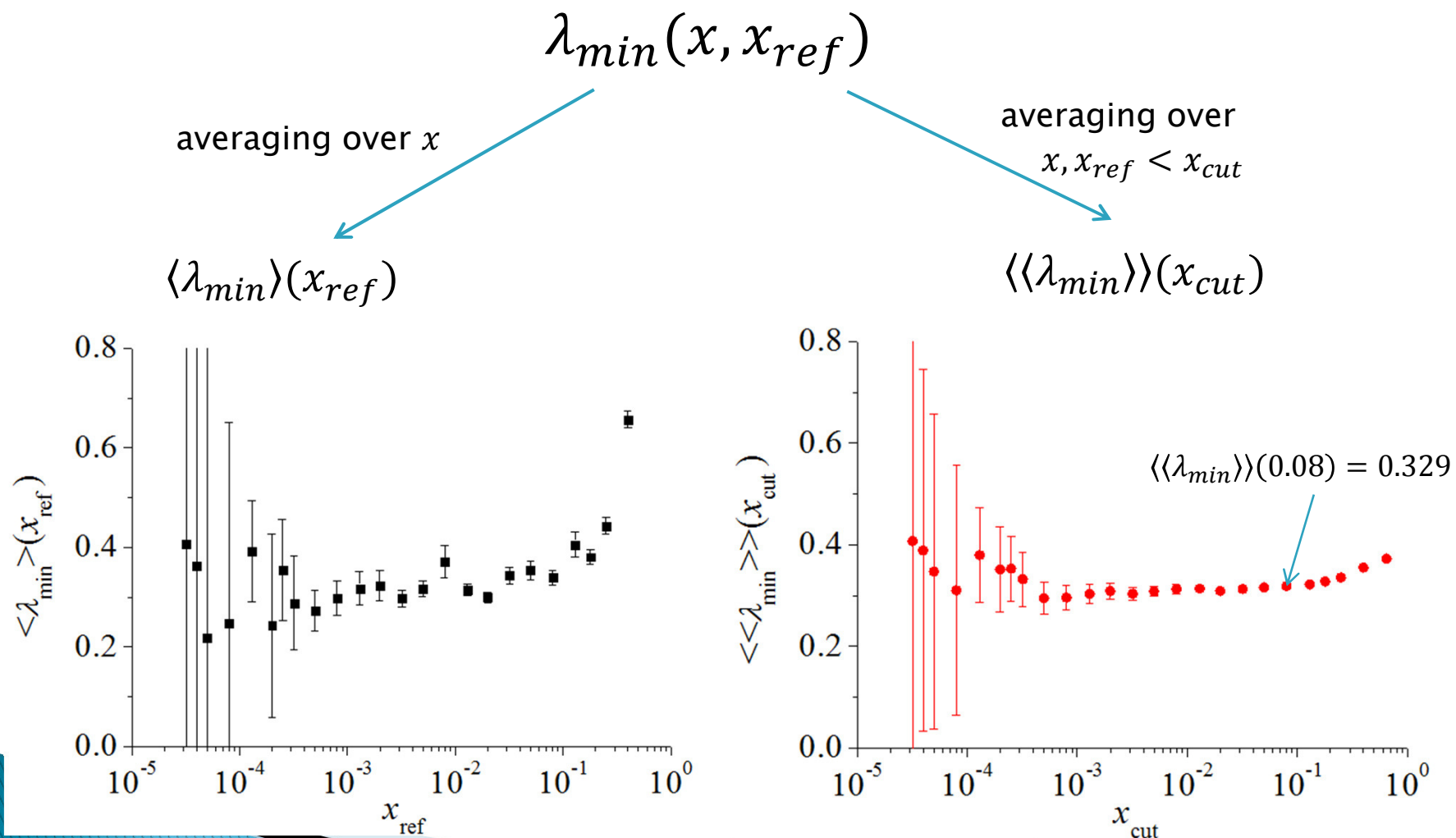


Result for λ_{min} and $\chi^2_{x,x_{ref}}(\lambda_{min})$

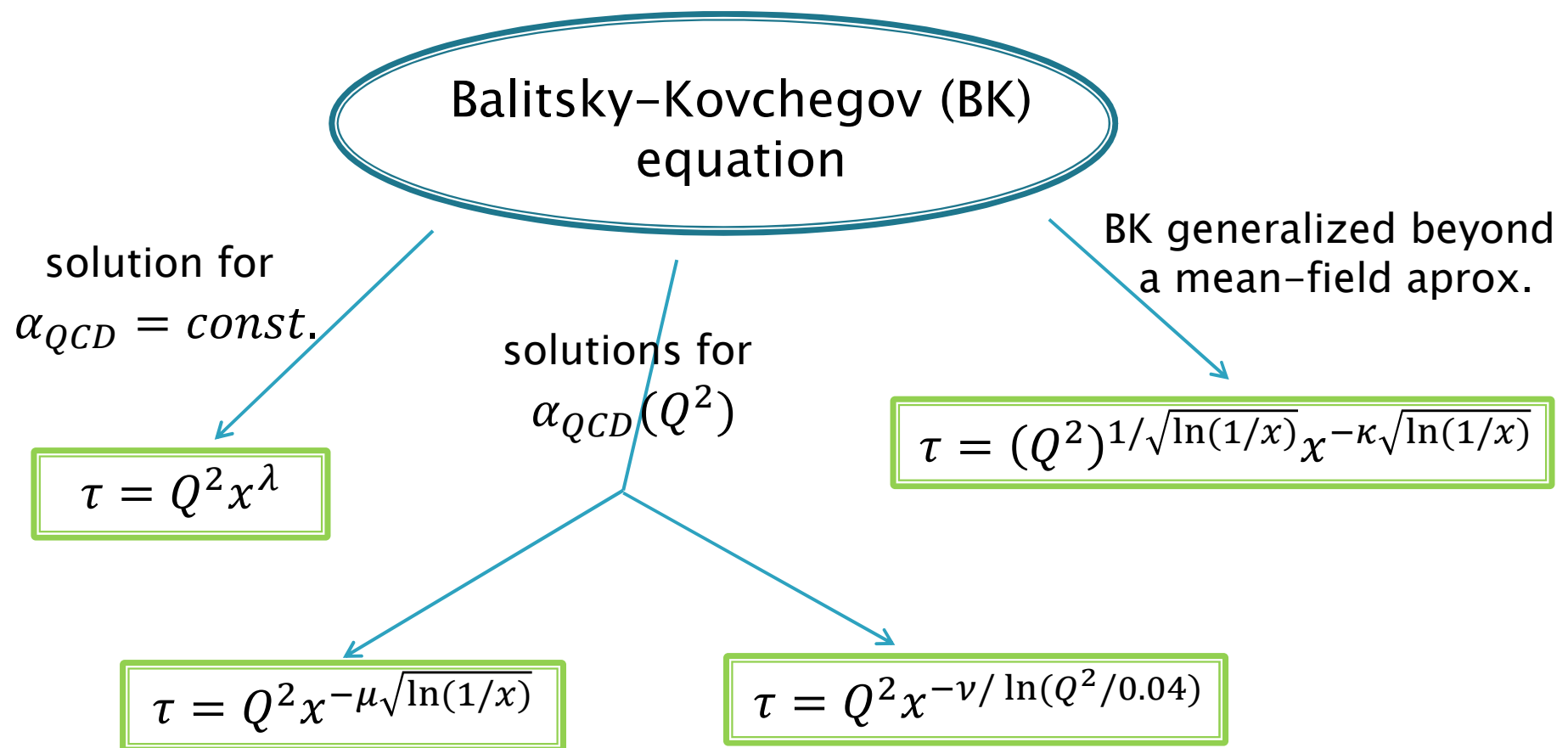


M. Praszalowicz and TS, JHEP 03 (2013) 090

Averages of $\lambda_{min}(x, x_{ref})$



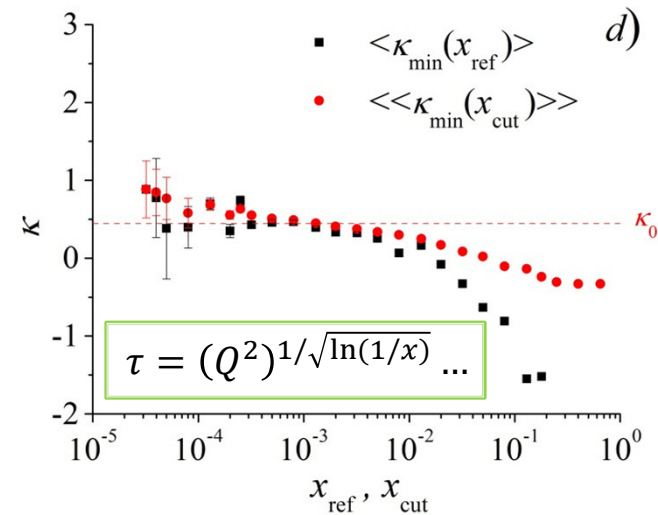
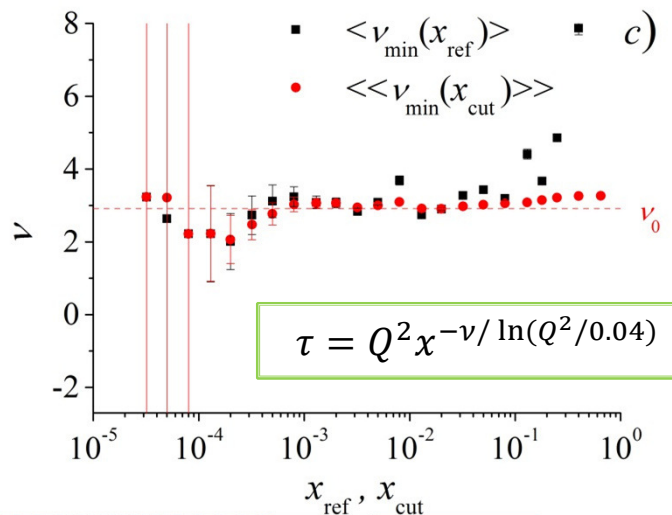
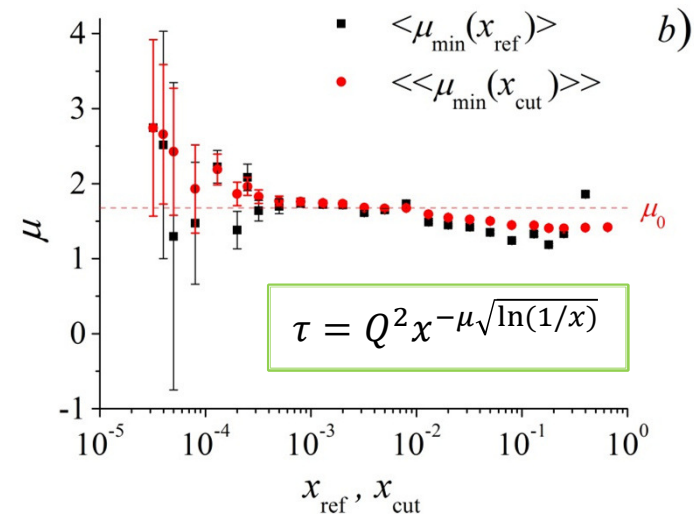
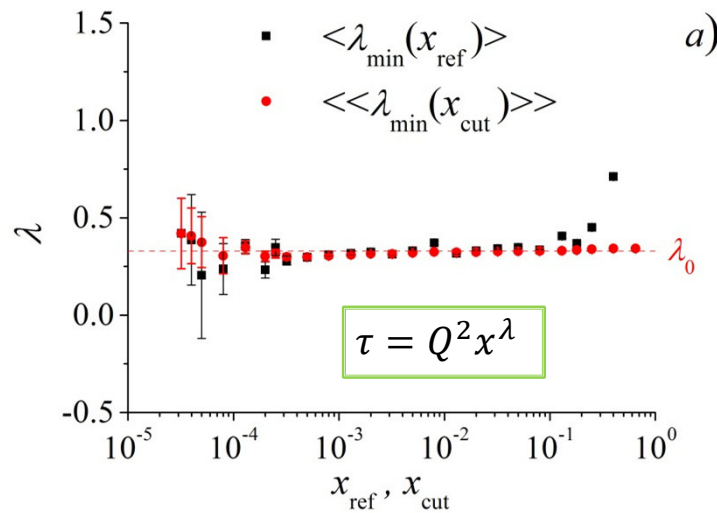
Different forms of scaling variable



S. Munier and R.B. Peschanski, *Phys. Rev. Lett.*
91 (2003) 232001

Different forms of scaling variable

M. Praszalowicz and TS, JHEP 04 (2013) 169



Charm production

$$\sigma^{c\bar{c}}(x, Q^2) = 4\pi^2 \alpha_{em} \frac{F_2^{c\bar{c}}(x, Q^2)}{Q^2}$$

Geometrical scaling (similar like for inclusive DIS):

$$\sigma^{c\bar{c}}(x, Q^2) = \sigma^{c\bar{c}}(\tau), \quad \tau = Q^2 x^\lambda$$

Problem:

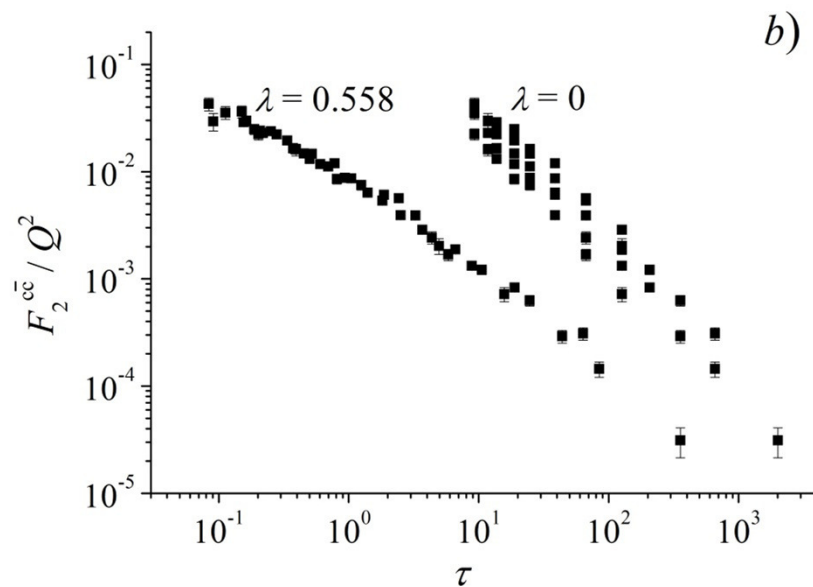
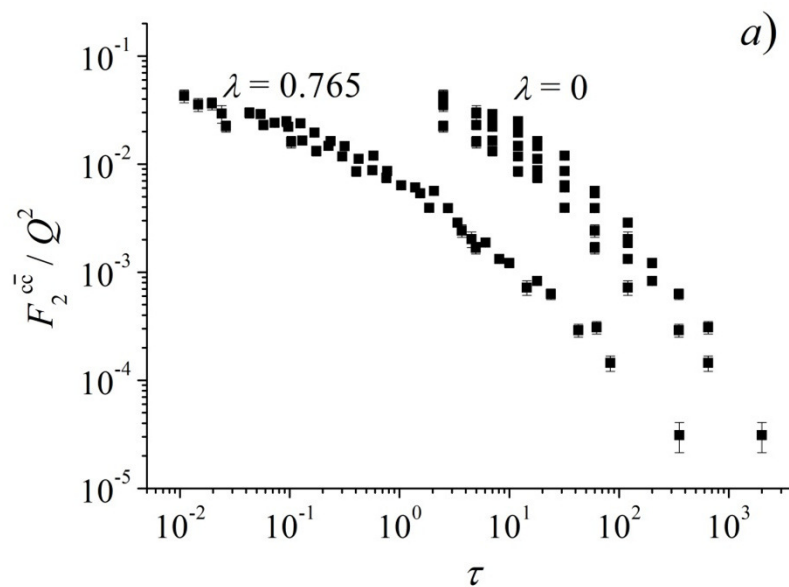
There is an additional mass scale $m_c \approx 1.3$ GeV.

Charm production

Solution:

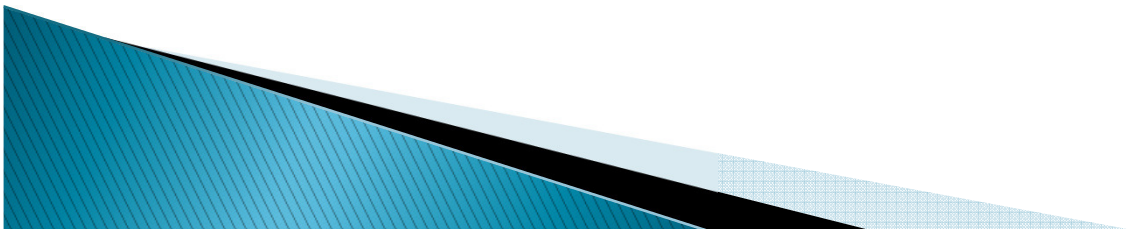
Modify scaling variable:

$$\tau = Q^2 x^\lambda \longrightarrow \tau = \left(1 + \frac{4m_c^2}{Q^2}\right)^{1+\lambda} Q^2 x^\lambda$$



Summary

- ▶ Standard (the simplest) GS holds up to $x \sim 0.1$.
- ▶ More complicated forms of scaling have worse quality.
- ▶ Residual GS in charm production is present. One can improve scaling quality by modification of the scaling variable (inclusion of the quark mass).



Thank you

