U-dual branes and non-geometric string backgrounds

Athanasios Chatzistavrakidis

Institut für Theoretische Physik, Leibniz Universität Hannover

arXiv:1309.2653
With Fridrik F Gautason, George Moutsopoulos and Marco Zagermann

Corfu2013

Motivation

Windows to non-perturbative aspects of string theory:

- ✔ Branes
- ✔ Dualities

Interplay $\rightsquigarrow$ families of non-perturbative extended objects with *unusual* properties.

Motivation

Windows to non-perturbative aspects of string theory:

- ✔ Branes
- ✔ Dualities

Interplay $\rightsquigarrow$ families of non-perturbative extended objects with *unusual* properties.

Flux compactifications: dualities reveal backgrounds with globally ill-defined geometry, "non-geometry".

$$H_{abc} \xleftrightarrow{T_a} f^a{}_{bc} \xleftrightarrow{T_b} Q^{ab}{}_c \xleftrightarrow{T_c} R^{abc}$$

Motivation

Windows to non-perturbative aspects of string theory:

- ✔ Branes
- ✔ Dualities

Interplay $\rightsquigarrow$ families of non-perturbative extended objects with *unusual* properties.

Flux compactifications: dualities reveal backgrounds with globally ill-defined geometry, "non-geometry".

$$H_{abc} \overset{T_a}{\longleftrightarrow} f^a{}_{bc} \overset{T_b}{\longleftrightarrow} Q^{ab}{}_c \overset{T_c}{\longleftrightarrow} R^{abc}$$

The two pictures are related.

Can lead to a better understanding of string theory structure
and of *unconventional* flux vacua.

Zoo of extended objects

String theory contains:

- ✔ strings (F1) $\rightsquigarrow$ couple to Kalb-Ramond 2-form B_2; perturbative.
- ✔ Dp-branes $\rightsquigarrow$ couple to RR forms C_{p+1}; tension $\propto g_s^{-1}$.
- ✔ NS5-branes $\rightsquigarrow$ couple to magnetic dual of Kalb-Ramond B_6; tension $\propto g_s^{-2}$.
- ✔ KK Monopoles $\rightsquigarrow$ couple to KK gauge field; also g_s^{-2}.

Zoo of extended objects

String theory contains:

- ✔ strings (F1) $\rightsquigarrow$ couple to Kalb-Ramond 2-form B_2; perturbative.
- ✔ Dp-branes $\rightsquigarrow$ couple to RR forms C_{p+1}; tension $\propto g_s^{-1}$.
- ✔ NS5-branes $\rightsquigarrow$ couple to magnetic dual of Kalb-Ramond B_6; tension $\propto g_s^{-2}$.
- ✔ KK Monopoles $\rightsquigarrow$ couple to KK gauge field; also g_s^{-2}.

Dualities map branes to branes:

- T-duality (does not mix NSNS and RR sectors):
  - Dp $\overset{T}{\leftrightarrow}$ D(p$\pm$ 1)
  - NS5 $\overset{T}{\leftrightarrow}$ NS5 or KKM
- IIB S-duality (mixes NSNS and RR sectors):
  - D5 $\overset{S}{\leftrightarrow}$ NS5
  - KKM $\overset{S}{\leftrightarrow}$ KKM

Zoo of extended objects

String theory contains:

- ✔ strings (F1) $\rightsquigarrow$ couple to Kalb-Ramond 2-form B_2; perturbative.
- ✔ Dp-branes $\rightsquigarrow$ couple to RR forms C_{p+1}; tension $\propto g_s^{-1}$.
- ✔ NS5-branes $\rightsquigarrow$ couple to magnetic dual of Kalb-Ramond B_6; tension $\propto g_s^{-2}$.
- ✔ KK Monopoles $\rightsquigarrow$ couple to KK gauge field; also g_s^{-2}.

Dualities map branes to branes:

- T-duality (does not mix NSNS and RR sectors):
  - Dp $\overset{T}{\leftrightarrow}$ D(p$\pm$ 1)
  - NS5 $\overset{T}{\leftrightarrow}$ NS5 or KKM
- IIB S-duality (mixes NSNS and RR sectors):
  - D5 $\overset{S}{\leftrightarrow}$ NS5
  - KKM $\overset{S}{\leftrightarrow}$ KKM

Q: Is that all?

More extended objects

Utilizing the full U-duality reveals new families of branes.

[Elitzur, Giveon, Kutasov, Rabinovici '97] [Blau, O'Laughlin '97] [Hull '97]

Co-dimension 2: "Defect" (or "exotic") branes.

[Bergshoeff, Ortin, Riccioni '11] [de Boer, Shigemori '12]

Many objects (especially in lower dimensions). Diversity in:

- ✔ non-perturbativity; tension $\propto g_s^{-\alpha}$ with $\alpha = 1, 2, 3, 4$.
- ✔ special transverse directions; 0,1,2,...,7.
- ✔ monodromy properies; defect branes are generically U-folds.

More extended objects

Utilizing the full U-duality reveals new families of branes.

[Elitzur, Giveon, Kutasov, Rabinovici '97] [Blau, O'Laughlin '97] [Hull '97]

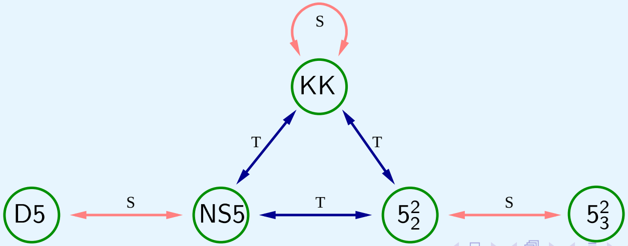
Co-dimension 2: "Defect" (or "exotic") branes.

[Bergshoeff, Ortin, Riccioni '11] [de Boer, Shigemori '12]

Many objects (especially in lower dimensions). Diversity in:

- ✔ non-perturbativity; tension $\propto g_s^{-\alpha}$ with $\alpha = 1, 2, 3, 4$.
- ✔ special transverse directions; 0,1,2,. . . ,7.
- ✔ monodromy properies; defect branes are generically U-folds.

Focus on IIB fivebranes:



Explore:

- ✔ Analogs of DBI action for exotic fivebranes.
- ✔ Couplings to background fields. Exotic branes as sources.
- ✔ Relations to non-geometry.

General picture:

$$
\begin{array}{ccccccccc}
F_{abc} & \overset{S}{\longleftrightarrow} & H_{abc} & \overset{T_a}{\longleftrightarrow} & f^{a}{}_{bc} & \overset{T_b}{\longleftrightarrow} & Q^{ab}{}_{c} & \overset{S}{\longleftrightarrow} & P^{ab}{}_{c} \\
\updownarrow & & \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\
D5 & \overset{S}{\longleftrightarrow} & NS5 & \overset{T}{\longleftrightarrow} & KKM & \overset{T}{\longleftrightarrow} & 5^2_2 & \overset{S}{\longleftrightarrow} & 5^2_3
\end{array}
$$

see also Hassler, Lüst '13

Standard world-volume actions

D5 brane

DBI action: $S_{\mathrm{DBI,D5}} = -T_{\mathrm{D5}} \int \mathrm{d}^6\sigma \; \mathrm{e}^{-\phi} \sqrt{-\det\left(G_{ij} + B_{ij} + F_{ij}\right)}$.

WZ action: $S_{\mathrm{WZ,D5}} = \mu_{\mathrm{D5}} \int \mathrm{e}^{-\mathcal{F}} C\big|_6$ (gauge invariant completion of the magnetic coupling to C_2).
Polyform: $C = \sum C_p$.

As source, modified field eqs. and Bianchi ids.; e.g. $\mathrm{d}F_3 + F_1 \wedge H_3 = *j_{D5}$.

Standard world-volume actions

D5 brane

DBI action: $S_{\mathrm{DBI,D5}} = -T_{\mathrm{D5}} \int \mathrm{d}^6\sigma \; \mathrm{e}^{-\phi} \sqrt{-\det\left(G_{ij} + B_{ij} + F_{ij}\right)}$.

WZ action: $S_{\mathrm{WZ,D5}} = \mu_{\mathrm{D5}} \int \mathrm{e}^{-\mathcal{F}}\, C|_6$ (gauge invariant completion of the magnetic coupling to C_2).
Polyform: $C = \sum C_p$.

As source, modified field eqs. and Bianchi ids.; e.g. $\mathrm{d}F_3 + F_1 \wedge H_3 = * j_{D5}$.

NS5 brane use S-duality see also Eyras, Janssen, Lozano '98

DBI: $S_{\mathrm{DBI,NS5}} = -T_{\mathrm{NS5}} \int \mathrm{d}^6\sigma \mathrm{e}^{-\phi} |\tau| \sqrt{-\det(G_{ij} - |\tau|^{-1} \tilde{\mathcal{F}}_{ij})}$, $\quad \tilde{\mathcal{F}} = C_2 + \mathrm{d}\tilde{A}_1$.

WZ: $S_{\mathrm{WZ,NS5}} = \mu_{\mathrm{NS5}} \int e^{-\tilde{\mathcal{F}}}\, \tilde{C}|_6$, (gauge invariant completion of the magnetic coupling to B_2).
$\rightsquigarrow$ new polyform $\tilde{C} = \frac{C_0}{|\tau|^2} - B_2 - (C_4 - C_2 \wedge B_2) + (B_6 - \frac{1}{2} C_2 \wedge B_2 \wedge B_2)$.

As source, $\mathrm{d}H_3 = * j_{NS5}$ (NSNS source).

Exotic DBI actions

5^2_2 brane use T-dualities @ yz

Strategy:
KK decomposition → T-duality rules → application of duality rules.

$$\begin{aligned} \mathrm{d}s^2 &= \hat{G}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + G_{mn}\eta^m\eta^n, \\ B &= \tfrac{1}{2}\hat{B}_{\mu\nu}\mathrm{d}x^\mu\wedge\mathrm{d}x^\nu + \tfrac{1}{2}B_{mn}\eta^m\wedge\eta^n + (\eta^m - \tfrac{1}{2}A^m)\wedge\theta_m, \end{aligned}$$

where $\eta^m = \mathrm{d}x^m + A^m$ and $\theta_{m\mu} = B_{m\mu} - B_{mn}A^n_\mu$.
Rules:

$$\begin{aligned} G_{mn} &\overset{yz}{\longmapsto} \tilde{G}^{mn} = \frac{\det(G_{mn})}{\det(G_{mn}+B_{mn})}G^{mn}, \\ A^m_\mu &\overset{yz}{\longmapsto} \theta_{m\mu}, \\ \hat{G}_{\mu\nu} &\overset{yz}{\longmapsto} \hat{G}_{\mu\nu}, \\ B_{mn} &\overset{yz}{\longmapsto} \tilde{B}^{mn} = \frac{\det(B_{mn})}{\det(G_{mn}+B_{mn})}(B^{-1})^{mn}, \\ \theta_{m\mu} &\overset{yz}{\longmapsto} A^m_\mu, \\ \hat{B}_{\mu\nu} &\overset{yz}{\longmapsto} \hat{B}_{\mu\nu}. \end{aligned}$$

Similarly for RR sector.

Applying the rules:

$$S_{\mathrm{DBI},5^2_2} = -T_{5^2_2}\int_{\mathcal{M}_{5^2_2}} \mathrm{d}^6\sigma\ \mathrm{e}^{-\phi}|\tilde{\tau}|\sqrt{\det(E_{mn})}$$
$$\times\sqrt{-\det(\hat{G}_{\mu\nu}\partial_i X^\mu \partial_j X^\nu + \tilde{G}^{mn}\tilde{\eta}_{im}\tilde{\eta}_{jn} - |\tilde{\tau}|^{-1}\tilde{\mathcal{F}}_{ij})}.$$

Indices: i: parallel, M: 10D, m: isometries. As usual, $E = G + B$.

Also, modulus $\tilde{\tau} = (C_{yz} - B_{yz}C_0) + i\sqrt{\det(E_{mn})}\mathrm{e}^{-\phi}$,
$\tilde{\eta}_{im} = \partial_i \tilde{X}_m + B_{m\mu}\partial_i X^\mu - B_{mn}A^n_\mu \partial_i X^\mu$,

Gauge invariant 2-form:

$$\tilde{\mathcal{F}}_{ij} = 2\partial_{[i}\tilde{\mathcal{A}}_{j]} + (\zeta_{\mu\nu yz} - \zeta_{\mu\nu}B_{yz})\,\partial_i X^\mu \partial_j X^\nu + 2\epsilon^{nm}\zeta_{\mu m}\partial_{[i}X^\mu \tilde{\eta}_{j]n}$$
$$+\Big(-\epsilon^{mn}\zeta_0 + \tilde{B}^{mn}(\zeta_{yz} - B_{yz}\zeta_0)\Big)\tilde{\eta}_{im}\tilde{\eta}_{jn}.$$

Using S-duality, also obtain the 5^2_3 brane action.

- Similar actions for cases with 1 special direction were studied already in the 90s.

[Bergshoeff, Eyras, Janssen, Lozano, Ortin...]

Couplings of exotic branes

Not so straightforward task... Where does the 5^2_2 couple to?

NS5 couples to B_6. T-duality does not mix NSNS and RR sectors.
$\rightsquigarrow$ it should couple to *some* magnetic dual of B_2.

Our strategy: Use definition of B_6 as magnetic dual of B_2 and the KK decomposition in the η^m basis to find the components of its (double) T-dual that couple to the brane.

$$H_7 = \mathrm{d}B_6 + (\text{RR terms}) = \mathrm{e}^{-2\phi} \star \mathrm{d}B_2.$$

The components of B_6 that couple to the NS5 brane are

$$(1 - P_z)(1 - P_y)B_6, \quad P_y = \mathrm{d}y \wedge \iota_y.$$

We find the rule:

$$(1 - P_z)(1 - P_y)B_6 \stackrel{yz}{\longmapsto} \iota_y \iota_z B_8^{yz}.$$

$\rightsquigarrow$ 5^2_2 couples to the double contraction of an 8-form magnetic dual of B_2.
The same result was predicted before using different methods.

Bergshoeff, Ortin, Riccioni '11

Modified Bianchi identity

As for D5 and NS5 sources, the 5^2_2 should modify the field equations. Consider

$$S = S_{\mathrm{NSNS}} + \mu_{5^2_2} \int \iota_y \iota_z B_8^{yz} .$$

Should express everything in terms of the same variables.

Generalized geometry comes into play.
Generalized metric (general parametrization [cf. Aldazabal, Baron, Marques, Nunez '11]):

$$\mathcal{H} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} + g\beta \\ -g^{-1}B - \beta g & g^{-1} - \beta g \beta \end{pmatrix}, \ \beta = \beta^{ij}\partial_i \wedge \partial_j \rightsquigarrow \text{2-vector.}$$

Rewritten NSNS action (for $B = 0$) [Andriot et al. '11] [cf. Blumenhagen et al.]

$$S_{\mathrm{NSNS}} = \int \mathrm{d}^{10}x \sqrt{-g}\ \mathrm{e}^{-2\tilde{\phi}} \left(\tilde{\mathcal{R}} + 4|\mathrm{d}\tilde{\phi}|^2 - \tfrac{1}{12} Q_M^{NR} Q_{NR}^M \right) + \int \mathrm{d}(\cdots),$$

with $Q_1^2 = \mathrm{d}\beta$, and magnetic dual $\star \mathrm{e}^{-2\phi}\mathrm{d}\beta = \mathrm{d}\beta_8^2$.

Then: $\mathrm{d}(Q_1^{MN} \wedge g_{My}\mathrm{d}y \wedge g_{Nz}\mathrm{d}z) = \star j_6^{5^2_2} \rightsquigarrow$ 5^2_2 is source for Q.

Defect branes are U-folds

The argument: [de Boer, Shigemori '12]

- 3D viewpoint: point-particle states, moduli undergo monodromies (elements of U-duality group $E_{8,8}(\mathbb{Z})$) when transported around branes.
- 10D viewpoint: moduli are components of sugra background fields. $\rightsquigarrow$ monodromies $\rightarrow$ multivalued background fields.

i.e. "non-geometry"; fields cannot be patched locally with diffeos and gauge trafos, need to use dualities as transition functions.

In the flux compactifications language such cases are known as U-folds. [Hull '04]
$\rightsquigarrow$ the connection to non-geometry is better seen in the supergravity description.

Supergravity description

Supergravity solution associated to 5_2^2 brane

[Lozano-Tellechea, Ortin '00]

$$\begin{aligned}
\mathrm{d}s^2 &= H(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2) + HK^{-1}(\mathrm{d}x^{89})^2 + (\mathrm{d}x^{034567})^2, \\
e^{2\phi} &= HK^{-1}, \\
B_2 &= -\theta K^{-1}\mathrm{d}x^{89}, \quad K = H^2 + \theta^2.
\end{aligned}$$

$\theta \to \theta + 2\pi$ leads to problems globally.
E.g. $B_{89}(r, \theta + 2\pi) = -\frac{\theta+2\pi}{H^2+(\theta+2\pi)^2} \neq B_{89}(r,\theta) + \delta B_{89}$.

This is essentially the same situation as for the T-dual of the 3D nilmanifold.
$f^a{}_{bc} \xleftrightarrow{T_b} Q^{ab}{}_c \rightsquigarrow$ T-fold. [Hull '04] [cf. Hassler, Lüst '13]

Explicit expressions

Explicit form of the generalized metric:

$$\mathcal{H}_{5^2_2,(yz)} = \frac{1}{H}\begin{pmatrix} 1 & 0 & 0 & -\theta \\ 0 & 1 & \theta & 0 \\ 0 & \theta & K & 0 \\ -\theta & 0 & 0 & K \end{pmatrix} \rightsquigarrow \qquad \begin{aligned} \mathrm{d}s^2_{yz} &= H^{-1}(\mathrm{d}y^2 + \mathrm{d}z^2), \\ \beta &= -\theta\partial_y \wedge \partial_z. \end{aligned}$$

$\rightsquigarrow$ well-behaved.

Constant Q flux.

Magnetic dual: $\beta_8^{yz} = H\mathrm{d}x^{034567} \wedge \mathrm{d}y \wedge \mathrm{d}z \quad \Rightarrow \quad \iota_y\iota_z\beta_8^{yz} = H\mathrm{d}x^{034567}$.

Remarks:

- Makes no sense to compute B_6 in the non-geometric case.
- In NS5 case, $B_2 = \theta\mathrm{d}y \wedge \mathrm{d}z \quad \Rightarrow \quad B_6 = H\mathrm{d}x^{034567}$.
  Note the correspondence of $B(NS5) - \beta(5^2_2)$ and $B_6(NS5) - \iota_y\iota_z\beta_8^{yz}(5^2_2)$.

RR non-geometry

S-duality mediates non-geometry to the RR sector.

Supergravity solution associated to 5^2_3 brane

$$
\begin{aligned}
\mathrm{d}s^2 &= (HK^{-1})^{\frac{1}{2}}(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2) + (HK^{-1})^{\frac{1}{2}}(\mathrm{d}x^{89})^2 + (HK^{-1})^{-\frac{1}{2}}(\mathrm{d}x^{034567})^2,\\
e^{2\phi} &= (HK^{-1})^{-1},\\
C_2 &= -K^{-1}\theta\mathrm{d}x^{89}, \quad K = H^2 + \theta^2.
\end{aligned}
$$

As before $\theta \to \theta + 2\pi$ leads to $C_{89}(r, \theta + 2\pi) \neq C_{89}(r, \theta) + \delta C_{89}$.

S-dual of the T-fold, ${Q^{ab}}_c \overset{S}{\longleftrightarrow} {P^{ab}}_c \rightsquigarrow$ U-fold. [Aldazabal, Camara, Font, Ibanez '06]

5^2_3 couples to an exotic dual of C_2. [cf. Bergshoeff, Ortin, Riccioni]

Its treatment goes through similar lines, but...
...Extended Generalized Geometry (structure group extended further to U-duality group)
comes into play $\rightsquigarrow$ 2-vector $\gamma = \gamma^{ij}\partial_i \wedge \partial_j$ (cousin of C_2). [Aldazabal, Andres, Camara, Grana]
Flux: ${P_\theta}^{89} = \partial_\theta \gamma^{89}$.

General picture

Inclusion of S-duality enhances the standard flux chain

$$H_{abc} \overset{T_a}{\longleftrightarrow} f^{a}{}_{bc} \overset{T_b}{\longleftrightarrow} Q^{ab}{}_{c}$$

to

$$F_{abc} \overset{S}{\longleftrightarrow} H_{abc} \overset{T_a}{\longleftrightarrow} f^{a}{}_{bc} \overset{T_b}{\longleftrightarrow} Q^{ab}{}_{c} \overset{S}{\longleftrightarrow} P^{ab}{}_{c}$$

General picture

Inclusion of S-duality enhances the standard flux chain

$$H_{abc} \overset{T_a}{\longleftrightarrow} f^a{}_{bc} \overset{T_b}{\longleftrightarrow} Q^{ab}{}_c$$

to

$$F_{abc} \overset{S}{\longleftrightarrow} H_{abc} \overset{T_a}{\longleftrightarrow} f^a{}_{bc} \overset{T_b}{\longleftrightarrow} Q^{ab}{}_c \overset{S}{\longleftrightarrow} P^{ab}{}_c$$

These fluxes are associated to brane sources

$$\begin{array}{ccccccccc}
F_{abc} & \overset{S}{\longleftrightarrow} & H_{abc} & \overset{T_a}{\longleftrightarrow} & f^a{}_{bc} & \overset{T_b}{\longleftrightarrow} & Q^{ab}{}_c & \overset{S}{\longleftrightarrow} & P^{ab}{}_c \\
\updownarrow & & \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\
D5 = 5^0_1 & \overset{S}{\longleftrightarrow} & NS5 = 5^0_2 & \overset{T}{\longleftrightarrow} & KKM = 5^1_2 & \overset{T}{\longleftrightarrow} & 5^2_2 & \overset{S}{\longleftrightarrow} & 5^2_3
\end{array}$$

Summary

Main messages

- ✔ Plethora of non-perturbative objects in string theory due to U-duality.
- ✔ They couple to exotic duals of the standard gauge potentials.
- ✔ Strongly related to non-geometric backgrounds and to modern techniques to study (unconventional but generic) flux vacua.

The study of such situations is very useful
in order to gain a more complete understanding of string vacua.