

TECHNISCHE UNIVERSITÄT WIEN Vienna University of Technology

Three- and Four-point correlators of excited bosonic twist fields

Pascal Anastasopoulos

arxiv: 1305.7166 with M. D. Goodsell and R. Richter.

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Plan of the talk

- Motivation
- * D-brane model building
- Three-point correlators
- Four-point correlators
- Conclusions



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- * If the string scale is at a few TeV range and the intersection angle is small, these stringy excitations might be visible at LHC.
- * It is very interesting to study their **decay channels** and their **lifetimes**.
- In order to do so, we have to extend our tools by evaluating the correlation functions of three and four twisted states.



D-brane compactifications

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- * Lets focus more on these states at the intersections.

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* For the quantization we define the commutator/anticommutators:

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* And the three vacua that these states act on:

WS bosonic: $|\theta_{1,2,3}^{ab}\rangle_B$ WS fermionic (NS): $|\theta_{1,2,3}^{ab}\rangle_{NS}$ WS fermionic (R): $|\theta_{1,2,3}^{ab}\rangle_R$



- * Lets consider the NS sector for simplicity.
- * The first states (which are also massless by SUSY) are:

 $\Phi : \prod_{I} \psi_{-\frac{1}{2} + \theta_{I}} | \theta_{1,2,3}^{ab} \rangle_{B \otimes NS}$

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$$\begin{split} \tilde{\Phi} &: \quad \alpha_{-\theta_1} \prod_{I} \psi_{-\frac{1}{2}+\theta_I} |\, \theta_{1,2,3}^{ab} \,\rangle_{B \otimes NS} \\ \tilde{\tilde{\Phi}} &: \quad \alpha_{-\theta_1}^2 \prod_{I} \psi_{-\frac{1}{2}+\theta_I} |\, \theta_{1,2,3}^{ab} \,\rangle_{B \otimes NS} \end{split}$$

$$M^{2} = (1 - \frac{1}{2} \sum_{I} \theta_{I} + \theta_{1}) M_{s}^{2} = \theta_{1} M_{s}^{2}$$
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- These scalars are potentially very light, depending on the intersection angles.
- * The **R** sector gives spacetime fermions, being the super-partners of the above.
- * If the string scale is low, and the angles are small, such states have very low masses.

D-brane worlds

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* In semi-realistic D-brane compactifications, the SM matter fields are massless states, living at the intersections:



- * However, there is a whole tower of stringy excitations with same quantum numbers.
- * The study of the decay channels and the lifetimes of such states is very interesting.

Amplitudes with outgoing excited states

* We wand to compute: the decay amplitudes of two chiral fermions and a scalar:



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- Two difficulties: 1. Vertex operators
 - 2. Bosonic twist field correlator

1. Vertex Operators

For the NS-sector we have the following dictionary *

positive angle θ

$ \theta \rangle_{B \otimes NS}$:	$e^{i\theta H}$
$\alpha_{-\theta} \theta \rangle_{B \otimes NS}$:	$e^{i\theta H}$
$(\alpha_{-\theta})^2 \theta\rangle_{B\otimes NS}$:	$e^{i\theta H}$
$\psi_{-\frac{1}{2}+\theta} \theta\rangle_{B\otimes NS}$:	$e^{i(\theta-$
$\alpha_{-\theta} \psi_{-\frac{1}{2}+\theta} \theta \rangle_{B \otimes NS}$:	$e^{i(\theta - \theta)}$
$(\alpha_{-\theta})^2 \tilde{\psi}_{-\frac{1}{2}+\theta} \theta\rangle_{B\otimes NS}$:	$e^{i(\theta - \theta)}$
-		

 $\begin{array}{c} \sigma_{\theta}^{+} \\ \tau_{\theta}^{+} \\ \omega_{\theta}^{+} \\ \tau_{\theta}^{+} \\ \sigma_{\theta}^{+} \end{array}$ $(1)H \tau_{\theta}^+$ $^{1)H} \omega_{\theta}^+$

negative angle θ

 $\begin{array}{l} |\theta\rangle_{B\otimes NS} & : e^{i\theta H} \sigma_{-\theta}^{-} \\ \alpha_{\theta} |\theta\rangle_{B\otimes NS} & : e^{i\theta H} \tau_{-\theta}^{-} \\ (\alpha_{\theta})^{2} |\theta\rangle_{B\otimes NS} & : e^{i\theta H} \omega_{-\theta}^{-} \\ \psi_{-\frac{1}{2}-\theta} |\theta\rangle_{B\otimes NS} & : e^{i(\theta+1)H} \sigma_{-\theta}^{-} \\ \alpha_{\theta} \psi_{-\frac{1}{2}-\theta} |\theta\rangle_{B\otimes NS} & : e^{i(\theta+1)H} \tau_{-\theta}^{-} \\ (\alpha_{\theta})^{2} \psi_{-\frac{1}{2}-\theta} |\theta\rangle_{B\otimes NS} & : e^{i(\theta+1)H} \omega_{-\theta}^{-} \end{array}$

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$\alpha_{-\theta} \psi_{-\frac{1}{2}+\theta} \theta \rangle_{B\otimes NS}$:	$e^{i(\theta-1)H} \tau_{\theta}$
$(\alpha_{-\theta})^2 \left[\psi_{-\frac{1}{2}+\theta} \right] \theta \rangle_{B \otimes NS}$:	$e^{i(\theta-1)H} \omega_0^{i(\theta-1)H}$
4		

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The σ , τ , ω are twisted bosonic conformal fields.

Anastasopoulos Bianchi Richter

Our setup

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- * Consider three stacks of D-branes within a semi-realistic brane configuration:



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* At the intersections live chiral fermions ψ , $\overline{\psi}$, χ , $\overline{\chi}$, ϕ , $\overline{\phi}$ and their superparteners Ψ , X, Φ .

The vertex operators



* Using our dictionary, we have for example

$$V_{\psi}^{(-1/2)} = \Lambda_{bc}\psi^{\alpha} e^{-\varphi/2} S_{\alpha} \left(\sigma_{\theta_{bc}}^{+} e^{i\left(\theta_{bc}^{1} - \frac{1}{2}\right)H_{1}}\right) \left(\sigma_{\theta_{bc}}^{+} e^{i\left(\theta_{bc}^{2} - \frac{1}{2}\right)H_{2}}\right) \left(\sigma_{-\theta_{bc}}^{-} e^{i\left(\theta_{bc}^{3} + \frac{1}{2}\right)H_{3}}\right) e^{ikX}$$

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* We will need various correlation functions:

$$\langle \tilde{\Phi} \psi \chi \rangle, \langle \tilde{\tilde{\Phi}} \psi \chi \rangle \dots \Longrightarrow$$

$$\begin{array}{ccc} \langle \bar{\psi} \ \psi \ \tilde{\chi} \ \bar{\chi} \rangle, \ \langle \bar{\psi} \ \psi \ \tilde{\chi} \ \bar{\tilde{\chi}} \rangle, \\ & & \\ \langle \bar{\psi} \ \psi \ \tilde{\tilde{\chi}} \ \bar{\chi} \rangle, \dots \end{array} \end{array} \Longrightarrow$$

 $\langle \tau_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\rangle$ $\langle \tau_{\alpha}^{+}(x_{1})\tau_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\rangle$ $\langle \omega_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\rangle \dots$

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2. Correlation functions

* The excited twist fields, τ and ω are not primaries.

 $\langle \tau_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\rangle$ $\langle \tau_{\alpha}^{+}(x_{1})\tau_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\rangle$ $\langle \omega_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\rangle \dots =$

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- * The excited twist fields, τ and ω are not primaries.
- * Thus, we will take a detour: we will evaluating higher correlators with only primaries:

 $\left\langle \partial Z(z)\sigma_{\alpha}(x_1)\sigma_{\beta}(x_2)\sigma_{\gamma}(x_3)...\right\rangle$

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* Next, taking the appropriate limits, like $z \to x_1$, $w \to x_2$ and using the OPE's:

 $\begin{aligned} \partial Z(z) \, \sigma_{\alpha}^{+}(w) &\sim (z-w)^{\alpha-1} \tau_{\alpha}^{+}(w) \\ \partial Z(z) \, \tau_{\alpha}^{+}(w) &\sim (z-w)^{\alpha-1} \omega_{\alpha}^{+}(w) \\ \partial Z(z) \, \omega_{\alpha}^{+}(w) &\sim (z-w)^{\alpha-1} \rho_{\alpha}^{+}(w) \\ \partial Z(z) \, \widetilde{\tau}_{\alpha}^{+}(w) &\sim (z-w)^{-2+\alpha} \sigma_{\alpha}^{+}(w) \\ \partial Z(z) \, \sigma_{\alpha}^{-}(w) &\sim (z-w)^{-\alpha} \tau_{\alpha}^{-}(w) \\ \partial Z(z) \, \widetilde{\tau}_{\alpha}^{-}(w) &\sim (z-w)^{-\alpha} \omega_{\alpha}^{-}(w) \\ \partial Z(z) \, \widetilde{\tau}_{\alpha}^{-}(w) &\sim (z-w)^{-1+\alpha} \sigma_{\alpha}^{-}(w) \\ \partial Z(z) \, \widetilde{\omega}_{\alpha}^{-}(w) &\sim (z-w)^{-1+\alpha} \widetilde{\tau}_{\alpha}^{-}(w) \end{aligned}$

we get our desired correlators:

 $\langle \tau_{\alpha}(x_1)\sigma_{\beta}(x_2)\sigma_{\gamma}(x_3)...\rangle$

 $\begin{array}{l} \partial Z(z) \ \sigma_{\alpha}^{+}(w) \sim (z-w)^{-\alpha} \widetilde{\tau}_{\alpha}^{+}(w) \\ \partial \bar{Z}(z) \ \tau_{\alpha}^{+}(w) \sim (z-w)^{-\alpha-1} \sigma_{\alpha}^{+}(w) \\ \partial \bar{Z}(z) \ \omega_{\alpha}^{+}(w) \sim (z-w)^{-\alpha-1} \tau_{\alpha}^{+}(w) \\ \partial \bar{Z}(z) \ \widetilde{\tau}_{\alpha}^{+}(w) \sim (z-w)^{-\alpha} \widetilde{\omega}_{\alpha}^{+}(w) \\ \partial \bar{Z}(z) \ \sigma_{\alpha}^{-}(w) \sim (z-w)^{\alpha-1} \widetilde{\tau}_{\alpha}^{-}(w) \\ \partial \bar{Z}(z) \ \tau_{\alpha}^{-}(w) \sim (z-w)^{-2+\alpha} \sigma_{\alpha}^{-}(w) \\ \partial \bar{Z}(z) \ \widetilde{\tau}_{\alpha}^{-}(w) \sim (z-w)^{-1-\alpha} \widetilde{\omega}_{\alpha}^{-}(w) \\ \partial \bar{Z}(z) \ \widetilde{\omega}_{\alpha}^{-}(w) \sim (z-w)^{-1-\alpha} \widetilde{\rho}_{\alpha}^{-}(w) \end{array}$

 $\langle \partial Z(z) \partial \bar{Z}(w) \sigma_{\alpha}(x_1) \sigma_{\beta}(x_2) \sigma_{\gamma}(x_3) \dots \rangle$

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 $\partial Z = \partial Z_{cl} + \partial Z_{qu}$ $\partial \overline{Z} = \partial \overline{Z}_{cl} + \partial \overline{Z}_{qu}$.

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* The two parts can be evaluated separately.

Abel Owen

The "classical" part

* The "classical" parts:

 $\frac{\partial Z_{cl}(z)\partial \bar{Z}_{cl}(w) \langle \sigma_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3}) \rangle}{\partial Z_{cl}(z)\partial Z_{cl}(w) \langle \sigma_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3}) \rangle}$ $\frac{\partial \bar{Z}_{cl}(z)\partial \bar{Z}_{cl}(w) \langle \sigma_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3}) \rangle}{\partial \bar{Z}_{cl}(z)\partial \bar{Z}_{cl}(w) \langle \sigma_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3}) \rangle}$

can be computed using the three-point function:

$$\left\langle \sigma_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\right\rangle = \left(2\pi\frac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}\right)^{\frac{1}{4}}x_{12}^{-(1-\alpha)(1-\beta)}x_{13}^{-(1-\alpha)(1-\gamma)}x_{23}^{-(1-\beta)(1-$$

and the classical solutions:

$$\partial Z_{cl}(z) = e^{i\pi(\gamma-1)} z^{\alpha-1} (z-1)^{\beta-1} v_c \frac{\Gamma(1-\gamma)}{\Gamma(\alpha) \Gamma(\beta)}$$
$$\partial \bar{Z}_{cl}(z) = 0 ,$$

Lust Meyr Richter Stieberger Abel Owen Cvetic Richter Weigand

The "quantum" part

* The "quantum" part:

 $g(z,w) \sim \left\langle \partial Z_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_{\alpha}^{+}(x_{1}) \sigma_{\beta}^{+}(x_{2}) \sigma_{\gamma}^{+}(x_{3}) \right\rangle$ $k(z,w) \sim \left\langle \partial Z_{qu}(z) \partial Z_{qu}(w) \sigma_{\alpha}^{+}(x_{1}) \sigma_{\beta}^{+}(x_{2}) \sigma_{\gamma}^{+}(x_{3}) \right\rangle$ $m(z,w) \sim \left\langle \partial \bar{Z}_{qu}(z) \partial \bar{Z}_{qu}(w) \sigma_{\alpha}^{+}(x_{1}) \sigma_{\beta}^{+}(x_{2}) \sigma_{\gamma}^{+}(x_{3}) \right\rangle$

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can be evaluated by using:

* the local behavior

 $\partial Z_{qu}(z)\partial \bar{Z}_{qu}(w) \sim \frac{1}{(z-w)^2}$ $\partial Z_{qu}(z)\partial Z_{qu}(w) \sim \text{regular}$ $\partial \bar{Z}_{qu}(z)\partial \bar{Z}_{qu}(w) \sim \text{regular}$

* the monodromy conditions:

$$e^{i\pi\alpha} \int_{x_1}^{x_2} m(z,w) \, dw - e^{-i\pi\alpha} \int_{x_1}^{x_2} g(z,w) \, dw = 0$$

Cvetic Papadimitriou Abel Owen

The results

* Adding the classical and quantum parts we get the total five-point correlators:

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* Taking various limits, like $z \to x_1, w \to x_2$ or $w \to x_1$ and using the OPE's we get:

$$\begin{split} \left\langle \tau_{\alpha}^{+}(x_{1})\widetilde{\tau}_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\right\rangle &= \alpha \frac{\left(2\pi \frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\right)^{\frac{1}{4}}}{x_{12}^{1+\alpha-(1-\alpha)\beta}x_{13}^{-(1-\alpha)\gamma}x_{23}^{(1+\beta)\gamma}} \sum_{n} e^{-S_{cl}} \\ \left\langle \tau_{\alpha}^{+}(x_{1})\tau_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\right\rangle &= -(2\pi)^{\frac{1}{4}} \left(\frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\right)^{\frac{5}{4}} \\ &\times \sum_{n} \left\{1 - \frac{\sin(\pi\alpha)\sin(\pi\beta)}{\pi\sin(\pi\gamma)}|v_{c}|^{2}\right\} x_{12}^{-\alpha\beta-\alpha-\beta}x_{13}^{-\alpha(2+\gamma)-\gamma+1}x_{23}^{-\beta(2+\gamma)-\gamma+1}e^{-S_{cl}} \\ \left\langle \omega_{\alpha}^{+}(x_{1})\sigma_{\beta}^{+}(x_{2})\sigma_{\gamma}^{+}(x_{3})\right\rangle &= -(2\pi)^{\frac{1}{4}} \left(\frac{\Gamma(1-\alpha)\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\right)^{\frac{5}{4}} \\ &\times \sum_{n} \left\{1 - \frac{\sin(\pi\alpha)\sin(\pi\beta)}{\pi\sin(\pi\gamma)}|v_{c}|^{2}\right\} x_{12}^{-\alpha(2+\beta)}x_{13}^{-\alpha(2+\gamma)}x_{23}^{-\beta(2+\gamma)-2\gamma+2}e^{-S_{cl}} \end{split}$$

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- This displacement is also related to the observed Yukawa coupling hierarchies of the massless fermions.
- This potentially allows one to obtain bounds on the decay rate of light stringy states in terms of observed mass hierarchies.
- * This is a work in progress.



Four-point correlators

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 - 1. we start with the six-point correlators of primary fields:

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- 5. we take various limits and we use the known OPE's.

Four-point results

* Some four-point correlators with:



Four-point results

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 - * **one** twisted field:

$$\left\langle \tau_{\alpha}^{+}(x_{1})\sigma_{\alpha}^{-}(x_{2})\sigma_{\beta}^{+}(x_{3})\sigma_{\beta}^{-}(x_{4})\right\rangle = -x_{12}^{-\alpha(2-\alpha)}x_{34}^{-\beta(1-\beta)}\left(\frac{x_{23}}{x_{13}}\right)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}}\left(\frac{x_{14}}{x_{24}}\right)^{\alpha\beta-\frac{3\alpha}{2}-\frac{\beta}{2}} \times \sqrt{2\pi} e^{-i\pi\alpha}\sum_{p,q}\frac{\left(G_{2}[x]v_{b}+\frac{\sin(\pi\alpha)}{\pi}(1-x)^{\alpha-\beta}B_{1}H_{1}[1-x]e^{i\pi\alpha}v_{a}\right)}{I^{\frac{3}{2}}(x)}e^{-S_{cl}}$$



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- Some four-point correlators with:
 - * one twisted field:

$$\begin{aligned} \pi_{\alpha}^{+}(x_{1})\sigma_{\alpha}^{-}(x_{2})\sigma_{\beta}^{+}(x_{3})\sigma_{\beta}^{-}(x_{4})\rangle &= -x_{12}^{-\alpha(2-\alpha)} x_{34}^{-\beta(1-\beta)} \left(\frac{x_{23}}{x_{13}}\right)^{\alpha\beta-\frac{\alpha}{2}-\frac{\beta}{2}} \left(\frac{x_{14}}{x_{24}}\right)^{\alpha\beta-\frac{3\alpha}{2}-\frac{\beta}{2}} \\ &\times \sqrt{2\pi} \ e^{-i\pi\alpha} \sum_{p,q} \frac{\left(G_{2}[x]v_{b} + \frac{\sin(\pi\alpha)}{\pi}(1-x)^{\alpha-\beta}B_{1}H_{1}[1-x]e^{i\pi\alpha}v_{a}\right)}{I^{\frac{3}{2}}(x)} \ e^{-S_{cl}} \end{aligned}$$

* two twisted fields:

$$\begin{split} &\langle \tau_{\alpha}^{+}(x_{1})\bar{\tau}_{\alpha}^{-}(x_{2}) \sigma_{\beta}^{+}(x_{3})\sigma_{\beta}^{-}(x_{4}) \rangle \\ &= \sqrt{\frac{2\pi}{I(x)}} x_{12}^{\alpha(\alpha-3)} x_{34}^{-\beta(1-\beta)} \left(\frac{x_{14}x_{23}}{x_{13}x_{24}}\right)^{\alpha\beta-\frac{\alpha}{2}-\frac{3\beta}{2}} \sum_{p,q} \left\{ -\alpha(1-x)^{\beta-\alpha} \frac{B_{2}H_{2}[1-x]}{I(x)} \,_{2}F_{1}[-\alpha,\beta,1,x] \right. \\ &+ \frac{B_{2}G_{1}[x]}{I(x)} \left(\alpha H_{2}[1-x] - \beta \,_{2}F_{1}[-\alpha,\beta,1-\alpha+\beta,1-x]\right) + \alpha(1-x)^{\beta-\alpha} \,\frac{2F_{1}[-\alpha,\beta,1,x]}{G_{1}[x]} \\ &- \frac{\left(G_{1}[x]v_{b} + \frac{\sin(\pi\alpha)}{\pi}B_{1}H_{1}[1-x]e^{i\pi\alpha}v_{a}\right) \left(G_{2}[x]v_{b} - \frac{\sin(\pi\alpha)}{\pi}B_{2}H_{2}[1-x]e^{i\pi\alpha}v_{a}\right)}{I^{2}(x)} \right\} e^{-S_{cl}} , \end{split}$$

etc etc...



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- Indeed, from dijet searches for related resonances, since the cross-section is so suppressed we can infer that the bounds on such states will be much less than a TeV.
- * This raises the intriguing prospect that the string scale could be just out of reach of the LHC, but the light stringy states could be hiding in plain sight.

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- * Four-point functions will help in the study of the decay channels of these light stringy states which could be observed at LHC.