Reduction of Couplings and Gauge Yukawa Unification: in Finite Thories and in the MSSM

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CORFU 2013

- What happens as we approach the Planck scale? or just as we go up in energy...
- How do we go from a fundamental theory to field theory as we know it?
- How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- How do particles get their very different masses?
- What is the nature of the Higgs?
- Is there one or many? How this affects all the above?
- Where is the new physics??

Search for understanding relations between parameters

addition of symmetries.

$N = 1$ SUSY GUTs.

Complementary approach: look for RGI relations among couplings at GUT scale $\longrightarrow$ Planck scale

$\Rightarrow$ **reduction of couplings**

resulting theory: less free parameters $\therefore$ more predictive

Gauge Yukawa Unification – GYU

Remarkable: reduction of couplings provides a way to relate two previously unrelated sectors

gauge and Yukawa couplings

Reduction of couplings in third generation provides predictions for quark masses (top and bottom)

Including soft breaking terms gives Higgs masses and SUSY spectrum

Kubo, M.M., Olechowski, Tracas, Zoupanos (1995,1996,1997); Oehme (1995); Kobayashi, Kubo, Raby, Zhang (2005); Gogoladze, Mimura, Nandi (2003,2004); Gogoladze, Li, Senoguz, Shafi, Khalid, Raza (2006,2011); M.M., Tracas, Zoupanos (2013)

Gauge Yukawa Unification in Finite Theories

Dimensionless sector of all-loop finite $SU(5)$ model

$M_{top} \sim 178$ **GeV**
large tan β, heavy SUSY spectrum

Kapetanakis, M.M., Zoupanos, Z.f.Physik (1993)

$M_{top}^{exp} 176 \pm 18$ GeV   found in 1995

$M_{top}^{exp} 173.1 \pm .09$ GeV   2013

Very promising, a more detailed analysis was clearly needed

Higgs mass $\sim 121 - 126$ GeV

$M_H^{exp} 126 \pm 1$ GeV   2012

Heinemeyer M.M., Zoupanos, JHEP, 2007, Phys.Lett.B (2013)

Gauge Yukawa Unification in the MSSM

- Possible to have a reduced system in the third generation compatible with quark masses

large $\tan\beta$, heavy SUSY spectrum

- Higgs mass $\sim 123 - 126$ GeV

M.M., Tracas, Zoupanos, arXiv:1309.0996

Reduction of Couplings

see George Tsamis talk

A RGI relation among couplings $\Phi(g_1, \ldots, g_N) = 0$ satisfies

$$\mu\, d\Phi/d\mu \;=\; \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i \;=\; 0.$$

g_i = coupling, β_i its β function

Finding the $(N-1)$ independent Φ's is equivalent to solve the reduction equations (RE)

$$\beta_g\,(dg_i/dg) = \beta_i\;,$$

$i = 1, \cdots, N$

- Reduced theory: only one independent coupling and its β function
- complete reduction: power series solution of RE

$$g_a = \sum_{n=0} \rho_a^{(n)}\, g^{2n+1}$$

- uniqueness of the solution can be investigated at one-loop
  valid at all loops — Zimmermann, Oehme, Sibold (1984,1985)
- The complete reduction might be too restrictive, one may use fewer Φ's as RGI constraints
- Reduction of couplings is essential for finiteness

finiteness: absence of ∞ renormalizations

$$\Rightarrow \quad \beta^N = 0$$

- SUSY no-renormalization theorems
  - $\Rightarrow$ only study one and two-loops
  - guarantee that is gauge and reparameterization invariant to all loops

Finiteness

A chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g has a superpotential

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k ,$$

Requiring one-loop finiteness $\beta_g^{(1)} = 0 = \gamma_i^{j(1)}$ gives the following conditions:

$$\sum_i T(R_i) = 3C_2(G) , \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i) .$$

$C_2(G)$ quadratic Casimir invariant, $T(R_i)$ Dynkin index of R_i, C_{ijk} Yukawa coup., g gauge coup.

- **restricts the particle content of the models**
- **relates the gauge and Yukawa sectors**

- One-loop finiteness $\Rightarrow$ two-loop finiteness

Jones, Mezincescu and Yao (1984,1985)

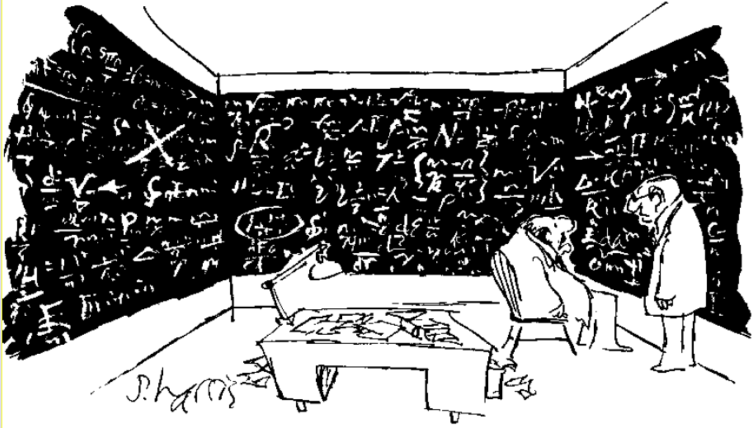
- One-loop finiteness restricts the choice of irreps R_i, as well as the Yukawa couplings
- Cannot be applied to the susy Standard Model (SSM): $C_2[U(1)] = 0$
- The finiteness conditions allow only SSB terms

It is possible to achieve all-loop finiteness $\beta^n = 0$:

Lucchesi, Piguet, Sibold

1. One-loop finiteness conditions must be satisfied
2. The Yukawa couplings must be a formal power series in g, which is solution (isolated and non-degenerate) to the reduction equations

SUSY breaking soft terms



“Whatever happened to *elegant* solutions?”

RGI in the Soft Supersymmetry Breaking Sector

Supersymmetry is essential. It has to be broken, though...

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\, (m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}$$

h trilinear couplings (A), b^{ij} bilinear couplings, m^2 squared scalar masses, M unified gaugino mass

The RGI method has been extended to the SSB of these theories.

- One- and two-loop finiteness conditions for SSB have been known for some time

  Jack, Jones, et al.

- It is also possible to have all-loop RGI relations in the finite and non-finite cases

  Kazakov; Jack, Jones, Pickering

SSB terms depend only on g and the unified gaugino mass M

universality conditions

$$h = -MC, \qquad m^2 \propto M^2, \qquad b \propto M\mu$$

Very appealing! But too restrictive

it leads to phenomenological problems:

- Charge and colour breaking vacua
- Incompatible with radiative electroweak breaking

Brignole, Ibáñez, Muñoz

- The lightest susy particle (LSP) is charged Yoshioka; Kobayashi et al

Possible to relax the universality condition to a sum-rule for the soft scalar masses

$\Rightarrow$ better phenomenology.

Kobayashi, Kubo, M.M., Zoupanos

Soft scalar sum-rule for the finite case

Finiteness implies

$$C^{ijk} = g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n} \Rightarrow h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\, g + O(g^5)$$

If lowest order coefficients $\rho^{ijk}_{(0)}$ and $(m^2)^i_j$ satisfy diagonality relations

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \propto \delta^j_i \,, \qquad (m^2)^i_j = m^2_j \delta^i_j$$

for all p and q.

We find the the following soft scalar-mass sum rule, also to all-loops

for $i,\ j,\ k$ with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(1)}$ is the two-loop correction =0 for universal choice

$$(\ m^2_i + m^2_j + m^2_k\ )/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)} + O(g^4)$$

Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman; Kobayashi, Kubo, Zoupanos

Also satisfied in certain class of orbifold models, where massive states are organized into $N = 4$ supermultiples

Several aspects of Finite Models have been studied

- $SU(5)$ Finite Models studied extensively

  Rabi et al; Kazakov et al; López-Mercader, Quirós et al; M.M, Kapetanakis, Zoupanos; etc

- One of the above coincides with a non-standard Calabi-Yau $SU(5) \times E_8$ Greene et al; Kapetanakis, M.M., Zoupanos
- Finite theory from compactified string model also exists (albeit not good phenomenology) Ibáñez
- Criteria for getting finite theories from branes Hanany, Strassler, Uranga
- $N = 2$ finiteness Frere, Mezincescu and Yao
- Models involving three generations Babu, Enkhbat, Gogoladze
- Some models with $SU(N)^k$ **finite** $\Longleftrightarrow$ **3 generations,** good phenomenology with $SU(3)^3$ Ma, M.M, Zoupanos
- Relation between commutative field theories and finiteness studied Jack and Jones
- Proof of conformal invariance in finite theories Kazakov

$SU(5)$ Finite Models

We study two models with $SU(5)$ gauge group. The matter content is

$$3\,\overline{\mathbf{5}} + 3\,\mathbf{10} + 4\,\{\mathbf{5} + \overline{\mathbf{5}}\} + \mathbf{24}$$

The models are finite to all-loops in the dimensionful and dimensionless sector. In addition:

- The soft scalar masses obey a sum rule
- At the M_{GUT} scale the gauge symmetry is broken and we are left with the MSSM
- At the same time finiteness is broken
- The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\{\mathbf{5} + \overline{\mathbf{5}}\}$ which couple to the third generation

The difference between the two models is the way the Higgses couple to the **24**

Kapetanakis, Mondragón, Zoupanos; Kazakov et al.

The superpotential which describes the two models takes the form

$$
\begin{aligned}
W \;=\; & \sum_{i=1}^{3} \Big[\, \frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \, \overline{H}_i \,\Big] + g_{23}^u \, \mathbf{10}_2 \mathbf{10}_3 H_4 \\
& + g_{23}^d \, \mathbf{10}_2 \overline{\mathbf{5}}_3 \, \overline{H}_4 + g_{32}^d \, \mathbf{10}_3 \overline{\mathbf{5}}_2 \, \overline{H}_4 + \sum_{a=1}^{4} g_a^f \, H_a \, \mathbf{24} \, \overline{H}_a + \frac{g^\lambda}{3} \, (\mathbf{24})^3
\end{aligned}
$$

find isolated and non-degenerate solution to the finiteness conditions

The unique solution implies discrete symmetries
We will do a partial reduction, only third generation

The finiteness relations give at the M_{GUT} scale

Model A

- $g_t^2 = \frac{8}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{6}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 + m_{\mathbf{5}}^2 + m_{\mathbf{10}}^2 = M^2$
- **3 free parameters:**
  M, $m_{\mathbf{5}}^2$ and $m_{\mathbf{10}}^2$

Model B

- $g_t^2 = \frac{4}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{3}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}$
- $m_{\mathbf{5}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}$
- **2 free parameters:**
  M, $m_{\mathbf{5}}^2$

Phenomenology

The gauge symmetry is broken below M_{GUT}, and what remains are boundary conditions of the form $C_i = \kappa_i g$, $h = -MC$ and the sum rule at M_{GUT}, below that is the MSSM.

- Fix the value of $m_\tau \Rightarrow \tan\beta \Rightarrow M_{top}$ and m_{bot}
- We assume a unique susy breaking scale
- The LSP is neutral
- The solutions should be compatible with radiative electroweak breaking
- No fast proton decay

We also

- Allow 5% variation of the Yukawa couplings at GUT scale due to threshold corrections
- Include radiative corrections to bottom and tau, plus resummation **(very important!)**
- Estimate theoretical uncertainties

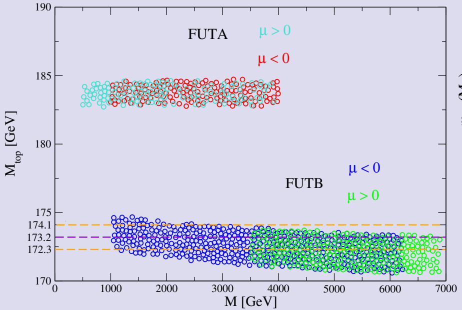
We look for the solutions that satisfy the following constraints:

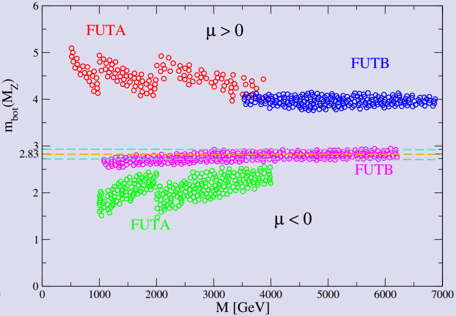
- Right masses for top and bottom
  fact of life — FeynHiggs
- The decay $b \to s\gamma$ — MicroOmegas
  fact of life
- The branching ratio $B_s \to \mu^+\mu^-$ — MicroOmegas
  fact of life
- Cold dark matter density $\Omega_{CDM}h^2$ — MicroOmegas
  loose constraint
- The anomalous magnetic moment of the muon $g - 2$
  see what we get

The lightest MSSM Higgs boson mass
The SUSY spectrum

FeynHiggs, Suspect, FUT

TOP AND BOTTOM MASS





FUTA: $M_{top} \sim 182 \sim 185\ GeV$ FUTB: $M_{top} \sim 172 \sim 174\ GeV$

Theoretical uncertainties $\sim$ 4%

Δb and $\Delta\tau$ included, resummation done

FUTB $\mu < 0$ favoured

New experimental data

- We use the experimental values of M_H to compare with our previous results ($M_H =\sim 121 - 126$ GeV, 2007) and put extra constraints

$$M_H^{exp} = 126 \pm 2 \pm 1$$

  2 GeV theoretical, 1 GeV experimental

- We also use the current experimental value of $B \to \mu^+\mu^-$
  Upper limit October 2012

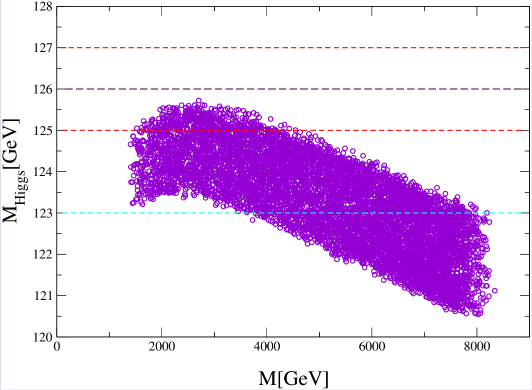
$$\mathrm{BR}(B_s \to \mu^+\mu^-) = 4.5 \times 10^{-9}$$

  Experimental value November 2012

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = (3.2^{+1.4}_{-1.2}(\mathrm{stat})^{+0.5}_{-0.3}(\mathrm{syst})) \times 10^{-9}$$

- We can now restrict (partly) our boundary conditions on M

Higgs mass



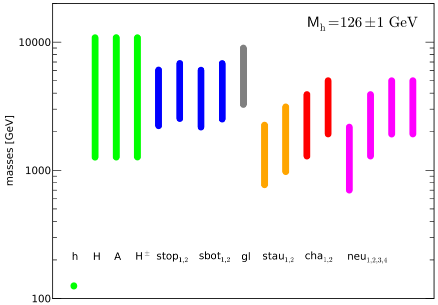
FUTB: $M_{Higgs} = 121 \sim 126\ GeV$

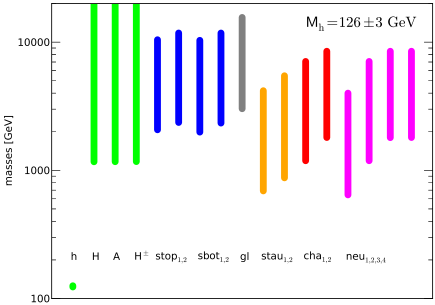
with B Physics constraints

Uncertainties $\pm 3\ GeV$ (FeynHiggs)

Heinemeyer, M.M., Zoupanos (2007); Heinemeyer, M.M., Zoupanos (2013)

S-SPECTRUM





SUSY spectrum with B physics constraints

Challenging for LHC

Results

When confronted with low-energy precision data

only FUTB $\mu < 0$ survives

- $M_{top} \sim 173$ GeV 4% $\quad M^{exp}_{top}\mathrm{exp} = (173.2 \pm 0.9)\mathrm{GeV}$
- $m_{bot}(M_Z) \sim 2.8$ GeV 8 % $\quad m^{exp}_{bot}(M_Z) = (2.83 \pm 0.10)\mathrm{GeV}$
- $M_{Higgs} \sim 122 - 126$ GeV 3 GeV $\quad M^{exp}_{Higgs} = 126 \pm 1$
- $\tan\beta \sim 44 - 46$
- s-spectrum > 500 GeV consistent with the exp bounds

In progress

- 3 families with discrete symmetry
- neutrino masses via $\not{R}$

Reduction of couplings in the MSSM

The superpotential

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2$$

with soft breaking terms,

$$-\mathcal{L}_{SSB} = \sum_\phi m_\phi^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} \frac{1}{2} M_i \lambda_i \lambda_i + \mathrm{h.c} \right]$$
$$+ \left[h_t H_2 Q t^c + h_b H_1 Q b^c + h_\tau H_1 L \tau^c + \mathrm{h.c.} \right],$$

then, reduction of couplings implies

$$\beta_{Y_{t,b,\tau}} = \beta_{g_3} \frac{dY_{t,b,\tau}}{dg_3}$$

Boundary conditions at the unification scale

$$\frac{Y_t^2}{4\pi} = c_1 \frac{g_3^2}{4\pi} + c_2 \left(\frac{g_3^2}{4\pi}\right)^2 \quad (1)$$

$$\frac{Y_b^2}{4\pi} = p_1 \frac{g_3^2}{4\pi} + p_2 \left(\frac{g_3^2}{4\pi}\right)^2 \quad (2)$$

are given by

$$c_1 = \frac{157}{175} + \frac{1}{35}K_\tau = 0.897 + 0.029K_\tau$$

$$p_1 = \frac{143}{175} - \frac{6}{35}K_\tau = 0.817 - 0.171K_\tau$$

$$c_2 = \frac{1}{4\pi}\frac{1457.55 - 84.491K_\tau - 9.66181K_\tau^2 - 0.174927K_\tau^3}{818.943 - 89.2143K_\tau - 2.14286K_\tau^2}$$

$$p_2 = \frac{1}{4\pi}\frac{1402.52 - 223.777K_\tau - 13.9475K_\tau^2 - 0.174927K_\tau^3}{818.943 - 89.2143K_\tau - 2.14286K_\tau^2}$$

where

$$K_\tau = Y_\tau^2/g_3^2$$

Y_τ not reduced, its reduction gives imaginary values

Soft breaking terms

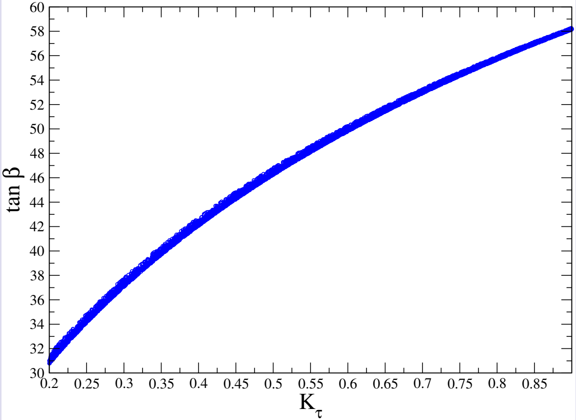
The reduction of couplings in the SSB sector gives the following boundary conditions at the unification scale

$$Y_t^2 = c_1 g_3^2 + c_2 g_3^4/(4\pi) \quad \text{and} \quad Y_b^2 = p_1 g_3^2 + p_2 g_3^4/(4\pi)$$
$$h_{t,b} = -MY_{t,b},$$
$$m_3^2 = -M\mu,$$

$$m_{H_2}^2 + m_Q^2 + m_{t^c}^2 = M^2,$$
$$m_{H_1}^2 + m_Q^2 + m_{b^c}^2 = M^2,$$

M is unified gaugino mass

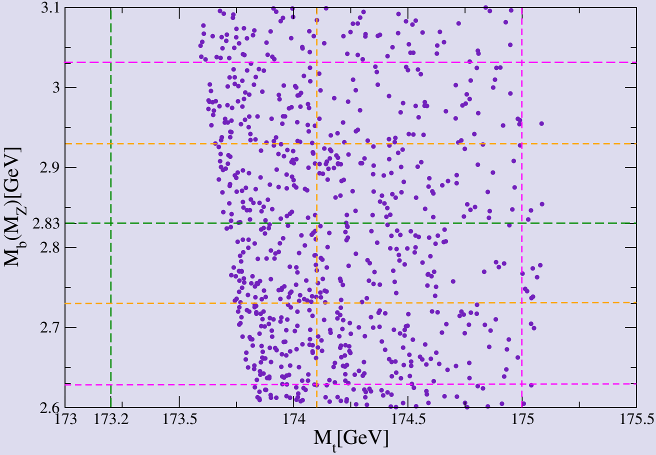
Allowed values of K_τ



Radiative corrections coming from SUSY breaking to the bottom and tau mass can be large (especially to bottom)

They depend on the values of the SUSY masses (M), and $\tan\beta$, and modify the allowed values of $K_\tau = Y_\tau^2/g_3^2$.

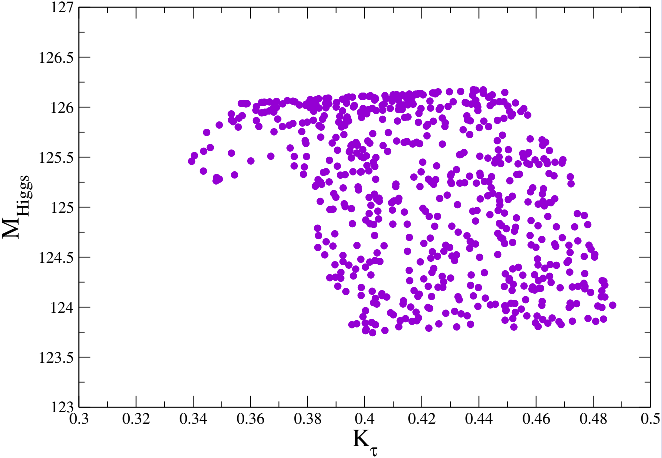
M_{top} vs M_{bot}



Requiring the top and bottom masses within experimental bounds further constrains $K_\tau = Y_\tau^2/g_3^2$ with $\mu < 0$.
No such region exists for $\mu > 0$

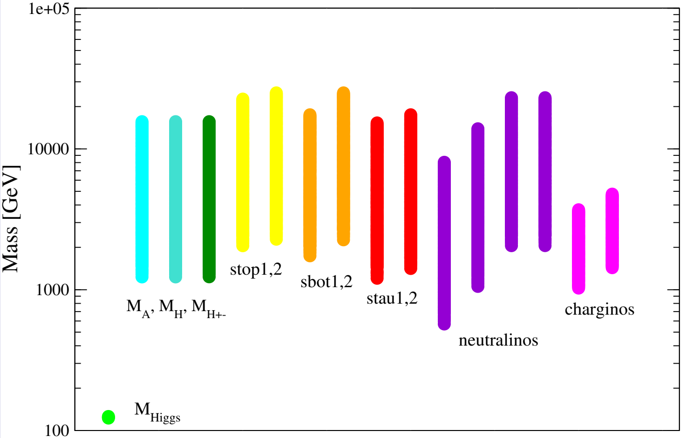
The central value (green dashed lines), 1 and 2σ deviation (orange and magenta lines respectively)

Higgs mass vs K_τ (aka “the bear”)



SUSY spectrum and the Higgs mass, given the GYU conditions constrained by the third generation of quark masses

SUSY spectrum



Heavy spectrum, $1.3\,TeV < M < 10\,TeV$

Will be constrained further by B physics and CDM

Results GYU in MSSM

- Possible to have reduction of couplings in MSSM
- Up to know only attempted in SM or in GUTs
- Reduced system further constrained by phenomenology: compatible with quark masses with $\mu < 0$
- SUSY spectrum, large tan β
- Higgs mass $\sim 123 \sim 126$ *GeV*

Conclusions

- Reduction of couplings: powerful implies Gauge Yukawa Unification
- **Finiteness, interesting and predictive principle $\Rightarrow$ reduces greatly the number of free parameters**
- completely finite theories
  i.e. including the SSB terms, that satisfy the sum rule
- Confronting the $SU(5)$ FUT models with low-energy precision data **does** distinguish among models **FUTB favoured**
- **Possible to have reduction of couplings in MSSM**
- only solutions for $\mu < 0$ compatible with quark masses
- Heavy SUSY spectrum,
- **large** tan β
- **s-spectrum starts above** ~ 500 **GeV**
- **prediction for the Higgs** $M_h \sim 122 - 126$ **GeV**
- **Detailed study of SUSY masses and Higgs decays in progress**