

# The $S_3$ flavour symmetry: Quarks, Leptons and Higgs sector\*

A. Mondragón

Instituto de Física, UNAM

*13th Hellenic School and Workshop on Elementary Particle Physics and Gravity  
Corfu, Greece*

*31 August-11 September 2013, Greece.*

---

\*Work supported by DGAPA PAPIIT IN 113712 and CONACYT-MEXICO 132059

# Contents

- Flavour permutational symmetry
- A minimal  $S_3$ -invariant extension of the Standard Model
- Masses and mixings in the quark sector
- Masses and mixings in the leptonic sector
- The neutrino mass spectrum
- Summary and conclusions

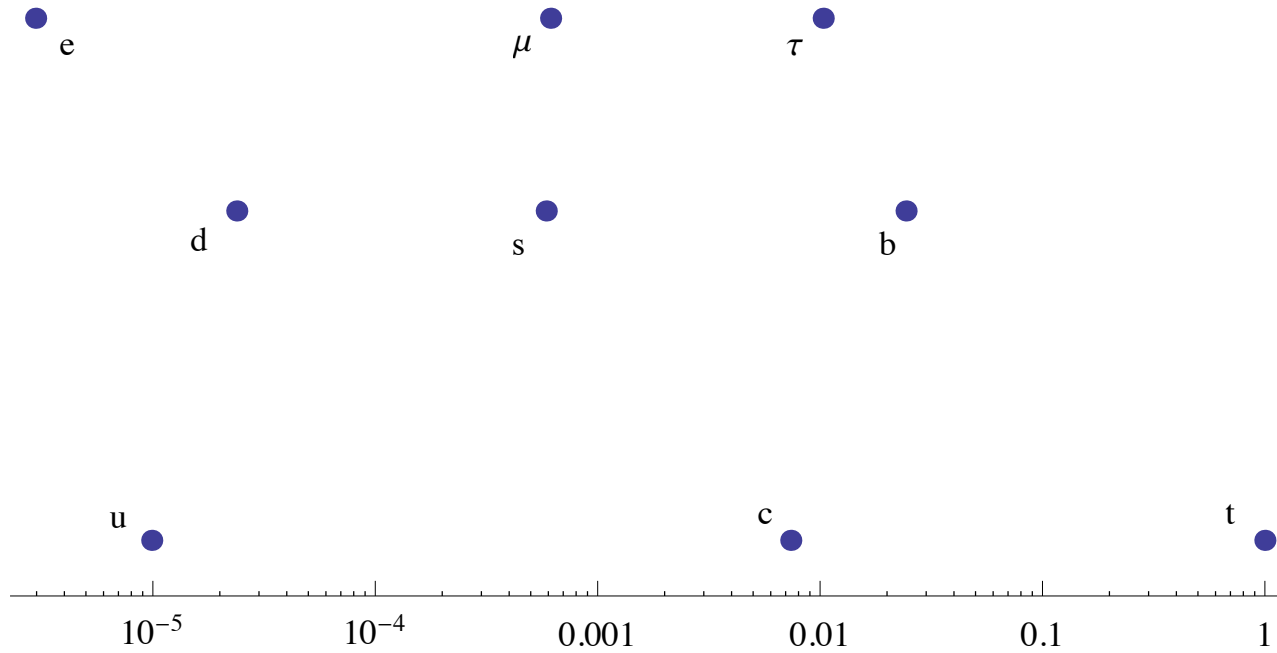
# Philosophy

## How do we choose a flavour symmetry?

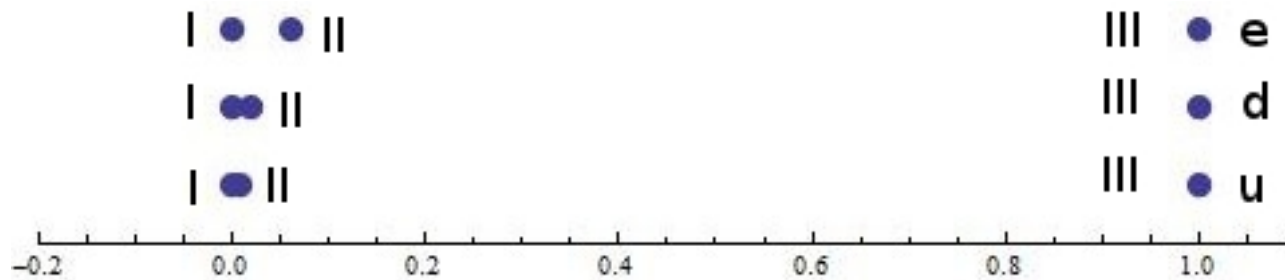
Different points of view. We will:

- Find the smallest possible flavour symmetry suggested by the data
- Follow it to the end
- Compare with the data

## Masses in a logarithmic plot



## Plot of fermion mass ratios

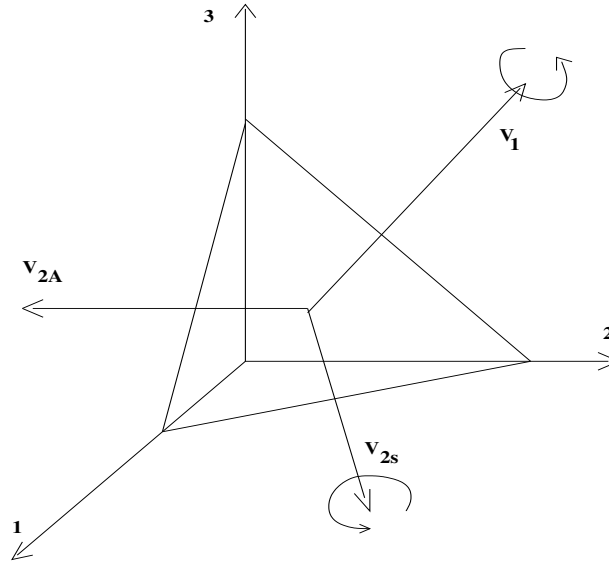


Fundamental fermions normalized by the heaviest of each type

suggests  $2 \oplus 1$  structure

Also, prior to electroweak symmetry breaking all three families are indistinguishable

## The $S_3$ symmetry group: permutation of 3 objects.



Assignment of fermion and Higgs fields to irreps of  $S_3$

$$\phi \rightarrow F = F(\phi_1, \phi_2, \phi_3)$$

F is a  $S_3$  reducible representation  $\mathbf{1}_s \oplus \mathbf{2}$  of  $S_3$

$$F_S = \frac{1}{\sqrt{3}}(\phi_1 + \phi_2 + \phi_3)$$

$$F_D = \begin{pmatrix} \frac{1}{2}(\phi_1 - \phi_2) \\ \frac{1}{\sqrt{6}}(\phi_1 + \phi_2 - 2\phi_3) \end{pmatrix}$$

## Some references to other works with an $S_3$ symmetry

- S. Pakvasa et al, Phys. Lett. 73B, 61 (1978)
- E. Derman, Phys. Rev. D19, 317 (1979)
- D. Wyler, Phys. Rev. D19, 330 (1979)
- R. Yahalom, Phys. Rev. D29, 536 (1984)
- A. Mondragón et al, Phys. Rev. D59, 093009, (1999)
- J. Kubo, A. Mondragón, et al, Prog. Theor. Phys. 109, 795 (2003)
- J. Kubo et al, Phys. Rev. D70, 036007 (2004)
- S. Chen, M. Frigerio and E. Ma, Phys. Rev. D70, 073008 (2004)
- A. Mondragón et al, Phys. Rev. D76, 076003, (2007)
- S. Kaneko et al, hep-ph/0703250, (2007)
- D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)
- T. Teshima et al, Phys.Rev. D84 (2011) 016003 Phys.Rev. D85 105013 (2012)
- F. González Canales, A&M. Mondragón Fort. der Physik 61, Issue 4-5 (2013)
- H.B. Benaoum, Phys. RevD.87.073010 (2013)
- E. Ma and B. Melic, arXiv:1303.6928
- F. González Canales, A. &M Mondragón, U. Saldaña, L. Velasco, arXiv:1304.6644
- R. Jora et al, Int.J.Mod.Phys. A28 (2013) 1350028
- S. Pakvasa and H. Sugawara, Phys. Lett. 73B, 61 (1978)
- E. Derman, Phys. Rev. D19, 317 (1979)
- D. Wyler, Phys. Rev. D19, 330 (1979)
- R. Yahalom, Phys. Rev. D29, 536 (1984)
- Y. Koide, Phys. Rev. D60, 077301 (1999)
- J. Kubo et al, Phys. Rev. D70, 036007 (2004)
- S. Chen et al, Phys. Rev. D70, 073008 (2004)
- O. Félix-Beltrán, M.M., et al, J.Phys.Conf.Ser. 171, 012028 (2009)
- D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)
- G. Bhattacharyya et al, Phys. Rev. D83, 011701 (2011)
- D. Meloni, JHEP 1205 (2012) 124
- S. Dev et al, Phys.Lett. B708 (2012) 284-289
- S. Zhou, Phys.Lett. B704 (2011) 291-295
- There are many more, I apologize for those not included.

# Facts

Some aspects of the flavour problem:

- Quark masses vastly different

$$m_u : m_c : m_t \approx 10^{-6} : 10^{-3} : 1, \quad m_d : m_s : m_b \approx 10^{-4} : 10^{-2} : 1,$$
$$m_e : m_\mu : m_\tau \approx 10^{-5} : 10^{-2} : 1.$$

- Quark weak mixing angles:

- $\theta_{12} \approx 13.0^\circ$
- $\theta_{23} \approx 2.4^\circ$
- $\theta_{13} \approx 0.2^\circ$

- Lepton masses not known (only difference of squared masses), but extremely small

- Lepton weak mixing angles, best fit of recent experimental data

- $\Theta_{12} \approx 33.9^\circ$
- $\Theta_{23} \approx 46.1^\circ$
- $\Theta_{13} \approx 9.2^\circ$  (Inv Hier  $9.45^\circ$ )  
 $\Rightarrow \Theta_{13} \neq 0$

- CP-violation occurs in the weak sector



## Mass matrices

After spontaneous gauge symmetry breaking

$$\langle H_{D1} \rangle = \langle H_{D2} \rangle \neq 0 \quad \text{and} \quad \langle H_3 \rangle \neq 0$$

and

$$\langle H_3 \rangle^2 + \langle H_{D1} \rangle^2 + \langle H_{D2} \rangle^2 \approx \left( \frac{246}{2} \text{GeV} \right)^2$$

Then, the Yukawa interactions yield mass matrices for all fermions of the generic form

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}$$

The Majorana masses for  $\nu_L$  are obtained from the see-saw mechanism

$$M_{\nu_L} = M_{\nu_D} \tilde{\mathbf{M}}^{-1} (M_{\nu_D})^T \quad \text{with} \quad \tilde{\mathbf{M}} = \text{diag}(M_1, M_2, M_3)$$

## Mixing matrices

The mass matrices are diagonalized by unitary matrices

$$U_{d(u,e)L}^\dagger \mathbf{M}_{d(u,e)} \mathbf{U}_{d(u,e)R} = \text{diag}\left(m_{d(u,e)}, m_{s(c,\mu)}, m_{b(t,\tau)}\right)$$

and

$$U_\nu^T M_\nu U_\nu = \text{diag}\left(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\right)$$

The masses can be complex, and so,  $U_{eL}$  is such that

$$U_{eL}^\dagger M_e M_e^\dagger U_{eL} = \text{diag}\left(|m_e|^2, |m_\mu|^2, |m_\tau|^2\right), \quad \textit{etc.}$$

The quark mixing matrix is

$$V_{CKM} = U_{uL}^\dagger U_{dL}$$

and, the neutrino mixing matrix is

$$\mathbf{V}_{PMNS} = U_{eL}^\dagger U_\nu$$

## Quark mass matrices

The quark mass matrices are obtained from  $M^{generic}$  by means of a  $45^\circ$  rotation and a shift of the mass origin

$$\hat{M}_q = U_{45^\circ}^\dagger M^{generic} U_{45^\circ} - \mu_o \mathbf{1}$$

After reparametrizing  $\hat{M}_q$  in terms of its eigenvalues, we get

$$\hat{M}_{u,d} \begin{pmatrix} 0 & \sqrt{\frac{\tilde{m}_1^{(u,d)} \tilde{m}_2^{(u,d)}}{1-\delta_{(u,d)}}} e^{i\phi} & 0 \\ \sqrt{\frac{\tilde{m}_1^{(u,d)} \tilde{m}_2^{(u,d)}}{1-\delta_{(u,d)}}} e^{-i\phi} & \tilde{m}_1^{(u,d)} - \tilde{m}_2^{(u,d)} + \delta_{(u,d)} & \sqrt{\frac{\delta_{(u,d)}}{1-\delta_{(u,d)}}} \xi_1^{(u,d)} \xi_2^{(u,d)} \\ 0 & \sqrt{\frac{\delta_{(u,d)}}{1-\delta_{(u,d)}}} \xi_1^{(u,d)} \xi_2^{(u,d)} & 1 - \delta_{(u,d)} \end{pmatrix}$$

$$\tilde{m}_i^{(u,d)} = \frac{m_i^{(u,d)}}{m_3}, \quad \xi_1^{(u,d)} = 1 - \tilde{m}_i^{(u,d)} - \delta_{(u,d)}, \quad \xi_2^{(u,d)} = 1 + \tilde{m}_2^{(u,d)} = 1 + \tilde{m}_2^{(u,d)} - \delta_{(u,d)}$$

## The matrix $U_q$

The quark mass matrices may be brought to diagonal form by means of a unitary transformation

$$U_q = P_q Q_q$$

$$M_q = (P_q O_q) M_{q,diag} (O_q^T P_q)$$

$P_q$  is a diagonal matrix of phases and

$$O_q = \begin{pmatrix} \left( \tilde{m}_{2q} \xi_1 / D_1 \right)^{1/2} & - \left( \tilde{m}_{1q} \xi_2 / D_2 \right)^{1/2} & \left( \frac{\tilde{m}_{1q} \tilde{m}_{2q} \delta_q}{D_3} \right)^{1/2} \\ \left[ \tilde{m}_{1q} (1 - \delta_q) \xi_1 / D_1 \right]^{1/2} & \left[ \tilde{m}_{2q} (1 - \delta_q) \xi_2 / D_2 \right]^{1/2} & \left[ (1 - \delta_q) \delta_q / D_3 \right]^{1/2} \\ - \left[ \tilde{m}_{1q} \xi_2 \xi_3 / D_1 \right]^{1/2} & - \left[ \tilde{m}_{2q} \xi_1 \xi_3 / D_2 \right]^{1/2} & \left[ \xi_1 \xi_2 / D_3 \right]^{1/2} \end{pmatrix}$$

$$\tilde{m}_{iq} = \frac{m_{iq}}{m_{3q}}, \quad \xi_1 = 1 - \tilde{m}_{1q} - \delta_q, \quad \xi_2 = 1 + \tilde{m}_{2q} - \delta_q$$

$$D_1 = (1 - \delta_q) (\tilde{m}_{1q} + \tilde{m}_{2q}) (1 - \tilde{m}_{1q})$$

$$D_2 = (1 - \delta_q) (\tilde{m}_{1q} + \tilde{m}_{2q}) (1 + \tilde{m}_{2q}),$$

$$D_3 = (1 - \delta_q) (1 - \tilde{m}_{1q}) (1 + \tilde{m}_{2q})$$

$$q = u, d$$

## The quark mixing matrix $V_{CKM}$

After the diagonalization of the mass matrices  $M_q$ , one obtains the mixing matrix as

$$\mathbf{V}_{CKM}^{th} = \mathbf{O}_u^T \mathbf{P}^{u-d} \mathbf{O}_d$$

where

$$\mathbf{P}^{u-d} = \text{diag}[1, e^{i\phi}, e^{i\phi}], \quad \phi = \phi_u - \phi_d$$

In this way, the elements of the mixing matrix  $V_{CKM}$  are written as functions of the quark mass ratios and two free parameters. For example:

$$\begin{aligned} \mathbf{V}_{us}^{th} = & - \left( \frac{\tilde{m}_c (1 - \tilde{m}_u - \delta_u) \tilde{m}_d (1 + \tilde{m}_s - \delta_q)}{(1 - \delta_u) (1 - \tilde{m}_u) (\tilde{m}_c + \tilde{m}_u) (1 - \delta_q) (1 + \tilde{m}_s) (m_s + m_d)} \right)^{1/2} \\ & + \left( \frac{\tilde{m}_u \tilde{m}_s}{(1 - \tilde{m}_u) (\tilde{m}_c + \tilde{m}_u) (\tilde{m}_d + \tilde{m}_s)} \right)^{1/2} \left\{ \left( \frac{(1 - \tilde{m}_u - \delta_u) (1 + \tilde{m}_s - \delta_d)}{(1 + \tilde{m}_s)} \right)^{1/2} \right. \\ & \left. + \left( \frac{(1 + \tilde{m}_c - \delta_u) \delta_u (1 - \tilde{m}_d - \delta_d) \delta_d}{(1 - \delta_u) (1 - \delta_d) (1 + \tilde{m}_s)} \right)^{1/2} \right\} e^{i\phi} \end{aligned}$$

## $V_{CKM}^{th}$ from $\chi^2$ minimization

We construct the  $\chi^2$  function as (in collaboration with F. González Canales, M. Mondragón, U.J. Saldaña Salazar and L. Velasco-Sevilla)

$$\chi^2 = \frac{(|V_{ud}^{th}| - |V_{ud}|)^2}{\sigma_{V_{ud}}^2} + \frac{(|V_{us}^{th}| - |V_{us}|)^2}{\sigma_{V_{us}}^2} + \frac{(|V_{ub}^{th}| - |V_{ub}|)^2}{\sigma_{V_{ub}}^2} + \frac{(|V_{cd}^{th}| - |V_{cd}|)^2}{\sigma_{V_{cd}}^2}$$

In the fitting procedure, the mass ratios in  $V_{CKM}^{th}$  are not free parameters

We took values for  $\tilde{m}_i = m_i/m_{3q}$  in their allowed  $3\sigma$  regions

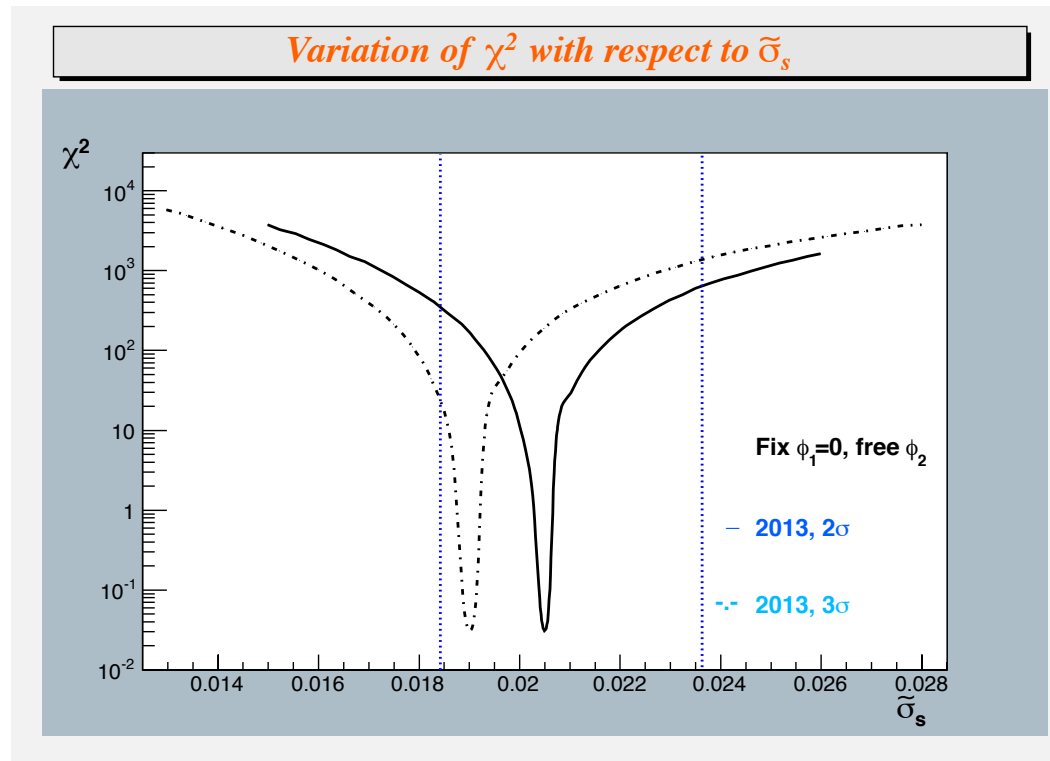
	2012
$\tilde{m}_u (M_Z)$	$0.0000077 \pm 0.0000030$
$\tilde{m}_c (M_Z)$	$0.0036 \pm 0.00017$
$\tilde{m}_d (M_Z)$	$0.00097 \pm 0.00017$
$\tilde{m}_s (M_Z)$	$0.0210 \pm 0.0026$

Table 1: Values of the mass ratios at  $M_Z$ .

We fitted our theoretical expressions to the following experimental values

$$\begin{aligned} |V_{ud}^{PDG}| &= 0.97425 \pm 0.00022, & |V_{us}^{PDG}| &= 0.2252 \pm 0.0009, \\ |V_{ub}^{PDG}| &= 0.00415 \pm 0.00049, & |V_{cd}^{PDG}| &= 0.230 \pm 0.111, \end{aligned}$$

## Results from a $\chi^2$ fit of $V_{CKM}^{th}$ to $V_{CKM}^{exp}$



We find an excellent agreement between our theoretical  $V_{CKM}^{th}$  and  $V_{CKM}^{PDG}$

$$\chi^2 = 3.10 \times 10^{-1}/4 \text{ for the fit using the central values}$$

$$\chi^2 = 4/4 \text{ for the fit using the values with restricted precision}$$

## The leptonic sector

To achieve a further reduction of the number of parameters, in the leptonic sector we introduce an additional discrete  $Z_2$  symmetry

−	+
$H_I, \nu_{3R}$	$H_S, L_3, L_I, e_{3R}, e_{IR}, \nu_{IR}$

then,

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0$$

Hence, the leptonic mass matrices are

$$M_e = \begin{pmatrix} \mu_2^e & \mu_2^e & \mu_5^e \\ \mu_2^e & -\mu_2^e & \mu_5^e \\ \mu_4^e & \mu_4^e & 0 \end{pmatrix} \quad \text{and} \quad M_{\nu D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}$$



## The Mass Matrix of the charged leptons as function of its eigenvalues

The mass matrix of the charged leptons is

$$M_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 \end{pmatrix}.$$

$$x = m_e/m_\mu, \tilde{m}_\mu = m_\mu/m_\tau \text{ and } \tilde{m}_e = m_e/m_\tau$$

This expression is accurate to order  $10^{-9}$  in units of the  $\tau$  mass

There are no free parameters in  $\mathbf{M}_e$  other than the Dirac Phase  $\delta_D!!$

## The Unitary Matrix $U_{eL}$

The unitary matrix  $U_{eL}$  is calculated from

$$U_{eL}^\dagger M_e M_e^\dagger U_{eL} = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)$$

We find

$$U_{eL} = \Phi_{eL} O_{eL}, \quad \Phi_{eL} = \text{diag}[1, 1, e^{i\delta_D}]$$

and

$$O_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} x \frac{(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -x \frac{(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2)\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2}\tilde{m}_e\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix},$$

$$x = m_e/m_\mu, \quad \tilde{m}_\mu = m_\mu/m_\tau \quad \text{and} \quad \tilde{m}_e = m_e/m_\tau$$

## The neutrino mass matrix I

The Majorana masses for  $\nu_L$  are obtained from the see-saw mechanism

$$\mathbf{M}_\nu = \mathbf{M}_{\nu D} \tilde{\mathbf{M}}_R^{-1} \mathbf{M}_{\nu D}^T$$

with

$$\tilde{\mathbf{M}}_R = \text{diag}[M_1, M_2, M_3] \quad M_1 \neq M_2 \neq M_3$$

and

$$\mathbf{M}_{\nu D} = \begin{pmatrix} \mu_2 & \mu_2 & 0 \\ \mu_2 & -\mu_2 & 0 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}$$

Then

$$M_\nu = \begin{pmatrix} \frac{2\mu_2^2}{M} & \lambda \frac{2\mu_2^2}{M} & \frac{2\mu_2\mu_4}{M} \\ \lambda \frac{2\mu_2^2}{M} & \frac{2\mu_2^2}{M} & \lambda \frac{2\mu_2\mu_4}{M} \\ \frac{2\mu_2\mu_4}{M} & \lambda \frac{2\mu_2\mu_4}{M} & \frac{2\mu_4^2}{M} + \frac{\mu_3^2}{M} \end{pmatrix}.$$

$$\frac{1}{\bar{M}} = \frac{1}{2} \left( \frac{1}{M_1} + \frac{1}{M_2} \right) \quad \text{and} \quad \lambda = \frac{M_2 - M_1}{M_1 + M_1}$$

## The neutrino mass matrix II

When  $\bar{M}$  is real, the phases in  $M_\nu$  may be factorized out as

$$M_\nu = P_\nu \bar{M}_\nu P_\nu \quad (1)$$

where

$$P_\nu = e^{i\phi} \text{diag}[1, 1, e^{i\delta_\nu}] \quad (2)$$

Then, the real symmetric matrix  $\bar{M}_\nu$  may be diagonalized by means of an orthogonal matrix  $O_\nu$

$$M_\nu = P_\nu O_\nu [\text{diag}(|m_{1\nu}|, |m_{2\nu}|, |m_{3\nu}|)] O^T P_\nu \quad (3)$$

Therefore, the unitary matrix

$$U_\nu = P_\nu O_\nu \quad (4)$$

diagonalizes the neutrino mass matrix  $M_\nu$ .

## The neutrino mass matrix III

The reduced neutrino mass matrix  $\bar{M}_\nu$  is reparametrized in terms of the neutrino masses  $|m_i|$  and two free parameters  $\mu_o$  and  $d$ , as

$$\bar{M}_\nu = \begin{pmatrix} \mu_o + \frac{1}{2}d & \frac{1}{2}d & \frac{1}{\sqrt{2}}(a + c) \\ \frac{1}{2}d & \mu_o + \frac{1}{2}d & -\frac{1}{\sqrt{2}}(a - c) \\ \frac{1}{\sqrt{2}}(a + c) & -\frac{1}{\sqrt{2}}(a - c) & b + \mu_o \end{pmatrix}$$

where

$$a^2 = -\frac{(|m_1| - \mu_o)(|m_2| - \mu_o)(|m_3| - \mu_o)}{d}, \quad \mu_o = \frac{2|\mu_2^2|}{\bar{M}}(1 - \lambda)$$

$$b = |m_1| + |m_2| + |m_3| - d - 3\mu_o \quad d = \lambda \frac{2|\mu_2|^2}{\bar{M}}$$

$$c^2 = \frac{1}{d}[(d - |m_1|)(d - |m_2|)(d - |m_3|)]$$

## The Orthogonal Matrix $O_\nu$

The real orthogonal matrix that diagonalizes  $\bar{M}_\nu$  and  $M_\nu$  is obtained from the eigenvectors of  $\bar{M}_\nu$

$$O_\nu = \left[ |m_1 \rangle, |m_2 \rangle, |m_3 \rangle \right]$$

$$|m_i \rangle = \begin{pmatrix} \left[ \frac{-\sigma_{i+1}\sigma_{i+2}(\sigma_i-d)}{d(\sigma_i-\sigma_{i+1})(\sigma_i-\sigma_{i+2})} \right]^{1/2} \\ \left[ \frac{\sigma_i(\sigma_i-d)}{(\sigma_i-\sigma_{i+1})(\sigma_i-\sigma_{i+2})} \right]^{1/2} \\ \left[ \frac{\sigma_i(d-\sigma_{i+1})(d-\sigma_{i+2})}{d(\sigma_i-\sigma_{i+1})(\sigma_i-\sigma_{i+2})} \right]^{1/2} \end{pmatrix}, \quad i = 1, 2, 3, \dots \text{mod}(3)$$

where  $\sigma_i = (m_i - \mu_o)$ .

The unitary matrix  $U_\nu$  that diagonalizes  $M_\nu$  is

$$U_\nu = P_\nu O_\nu$$

## The neutrino mixing matrix I

$$V_{PMNS}^{th} = U_{eL}^\dagger U_\nu.$$

The theoretical mixing matrix  $V_{PMNS}^{th}$  is

$$V_{PMNS}^{th} = O_{eL}^T P_{\nu-e} O_\nu K$$

where  $P_{\nu-e}$  is a diagonal matrix of phases

$$P_{\nu-e} = \text{diag}[1, 1, e^{i(\delta_\nu - \delta_e)}]$$

Hence

$$\left(V_{PMNS}^{th}\right)_{ij} = \sum_{r=1}^3 \left(O_{eL}\right)_{ri} \left(P_{\nu-e}\right)_r \left(O_\nu\right)_{rj}$$

From a comparison of  $V_{PMNS}^{PDG}$  with  $V_{PMNS}^{th}$ , we obtain the neutrino mixing angles as functions of the lepton masses.

## The neutrino mixing matrix II

From a comparison of  $V_{PMNS}^{th}$  with  $V_{PMNS}^{PDG}$ , we obtain the neutrino mixing angles as function of the lepton masses

$$\sin^2 \theta_{12}^\nu = \frac{|(V_{PMNS})_{12}|^2}{1 - |(V_{PMNS})_{13}|^2}$$

$$\sin^2 \theta_{23}^\nu = \frac{|(V_{PMNS})_{23}|^2}{1 - |(V_{PMNS})_{13}|^2}$$

$$\sin^2 \theta_{13}^\nu = |(V_{PMNS})_{13}|^2$$



## Neutrino Mixing Angles I

The solar angle  $\theta_{12}$  is strongly dependent on the neutrino masses but depends only very weakly on the charged lepton masses

$$\tan \theta_{12}^2 = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu_3}|^2)^{1/2} - |m_{\nu_3}|}{(\Delta m_{13}^2 + |m_{\nu_3}|^2 \cos^2 \phi_\nu)^{1/2} + |m_{\nu_3}|}.$$

the numerical value of  $\tan^2 \theta_{12}$  fixes the origin and scale of the neutrino masses.

The atmospheric mixing angle  $\theta_{23}$  depends mostly on the charged lepton masses

$$\sin \theta_{23} \approx \frac{1}{\sqrt{2}} \frac{1 - 2\tilde{m}_\mu^2 + \tilde{m}_\mu^4}{\sqrt{1 - 4\tilde{m}_\mu^2 + x^2 + 6\tilde{m}_\mu^4}} = 0.7071$$

$$x = m_e/m_\mu = 4.84 \times 10^{-3}, \quad \tilde{m}_\mu = m_\mu/m_\tau = 5.95 \times 10^{-2}$$

## Neutrino Mixing Angles II

The reactor mixing angle  $\theta_{13}$  is mostly determined by the interplay of the  $S_3$  symmetry and the mass splitting of the right-handed neutrinos in the see saw mechanism plus a very small contribution from the charged leptons,

$$\sin \theta_{13} \approx \frac{2(\lambda\mu)m_{\nu_3}}{m_{\nu_1} - m_{\nu_2}} \left(1 - \sqrt{\frac{(m_{\nu_2} - m_{\nu_3})}{m_{\nu_3} - m_{\nu_1}}}\right) \left(\cos \eta - \sqrt{\left(1 - \frac{m_{\nu_1}}{m_{\nu_3}}\right) \left(\frac{m_{\nu_2}}{m_{\nu_3}} - \frac{m_{\nu_1}}{m_{\nu_3}}\right)} \sin \eta\right) \\ + \frac{1}{\sqrt{2}} \frac{m_e}{m_\mu} \frac{(1 + 4(\frac{m_e}{m_\mu})^2 - (\frac{m_\mu}{m_\tau})^4)}{\sqrt{1 + (\frac{m_\mu}{m_\tau})^2 + 5(\frac{m_e}{m_\mu})^2 - (\frac{m_\mu}{m_\tau})^4}}$$

where

$$\cos \eta = \sqrt{\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_2} - m_{\nu_1}}}, \quad \sin \eta = \sqrt{\frac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_1}}}$$

We get

$$\sin \theta_{13}^{th} \approx 0.17 \quad \text{with} \quad (\lambda\mu) \approx 0.02$$

## Neutrino Mixing Angles: Theory vs Experiment

The most recent experimental values of the neutrino mixing angles  $\theta_{13}$  and  $\theta_{23}$  (D.V. Forero, M. Tórtola and J.W.F. Valle, arXiv: 1205.4018 v4 [hep-ph] 24 Oct 2012)

$$\bar{\theta}_{13}^{o \text{ exp}} = 9.02 \pm 1.2 \rightarrow \sin^2 \bar{\theta}_{13}^{o \text{ exp}} = 0.0246_{-0.0027}^{+0.0029};$$

$$\bar{\theta}_{23}^{o \text{ exp}} = 50.8 \pm 1.5 \quad \sin^2 \bar{\theta}_{23}^{o \text{ exp}} = 0.6 \pm 0.029$$

Our theoretical predictions (A.Mondragón, M. Mondragón and E. Peinado Phys. Rev. D **76**, 076003 (2007), F. González Canales and A. Mondragón, J. Phys: Conference Series **387** (2012) 012008 and F. González Canales, A. Mondragón and M. Mondragón, Fortschr. Phys. (2012)/DOI 10.1002/prop.201200121 )

$$\theta_{13}^{o \text{ th}} = 9.6 \pm 0.07 \quad \sin^2 \theta_{13}^{o \text{ th}} = 0.028 \pm 0.002$$

$$\theta_{23}^{o \text{ th}} = 44.97 \pm 1.2 \quad \sin^2 \theta_{23}^{o \text{ th}} = 0.50 \pm 0.02$$

in very good agreement with the latest experimental values !!!

## The neutrino mass spectrum I

In the present model, the experimental restriction

$$|\Delta m_{21}^2| < |\Delta m_{23}^2|$$

implies an inverted neutrino mass spectrum  $m_{\nu_3} < m_{\nu_1}, m_{\nu_2}$

From our previous expressions for  $\tan \theta_{12}$

$$|m_{\nu_3}| = \frac{\sqrt{\Delta m_{13}^2}}{2 \cos \phi_\nu \tan \theta_{12}} \frac{1 - \tan^4 \theta_{12} + r^2}{\sqrt{1 + \tan^2 \theta_{12}} \sqrt{1 + \tan^2 \theta_{12} + r^2}},$$

where  $r = \Delta m_{21}^2 / \Delta m_{23}^2$ .

Then, the mass  $|m_{\nu_3}|$  is approximately given by

$$|m_{\nu_3}| \approx \frac{1}{2} \frac{\sqrt{\Delta m_{13}^2}}{\tan \theta_{12}} (1 - \tan^2 \theta_{12})$$

## Neutrino mass spectrum II

- We wrote the neutrino mass differences,  $m_{\nu_i} - m_{\nu_j}$ , in terms of the differences of the squared masses  $\Delta_{ij}^2 = m_{\nu_i}^2 - m_{\nu_j}^2$  and one of the neutrino masses, say  $m_{\nu_3}$ .
- The mass  $m_{\nu_2}$  was taken as a free parameter in the fitting of our formula for  $\tan \theta_{12}$  to the experimental value
- with

$$\Delta m_{21}^2 = 7.6 \times 10^{-5} eV^2 \quad \Delta m_{13}^2 = 2.4 \times 10^{-3} eV^2$$

and

$$\tan \theta_{12} = 0.696$$

we get

$$|m_{\nu_3}| \approx 0.019 eV \implies |m_{\nu_2}| \approx 0.053 eV \quad \text{and} \quad |m_{\nu_1}| \approx 0.052 eV$$

- The neutrino mass spectrum has an inverted hierarchy of masses

# FCNC I

In the Standard Model the FCNC at tree level are suppressed by the GIM mechanism.

Models with more than one Higgs  $SU(2)$  doublet have tree level FCNC due to the exchange of scalar fields. The mass matrix written in terms of the Yukawa couplings is

$$\mathcal{M}_Y^e = Y_w^{E1} H_1^0 + Y_w^{E2} H_2^0,$$

FCNC processes:

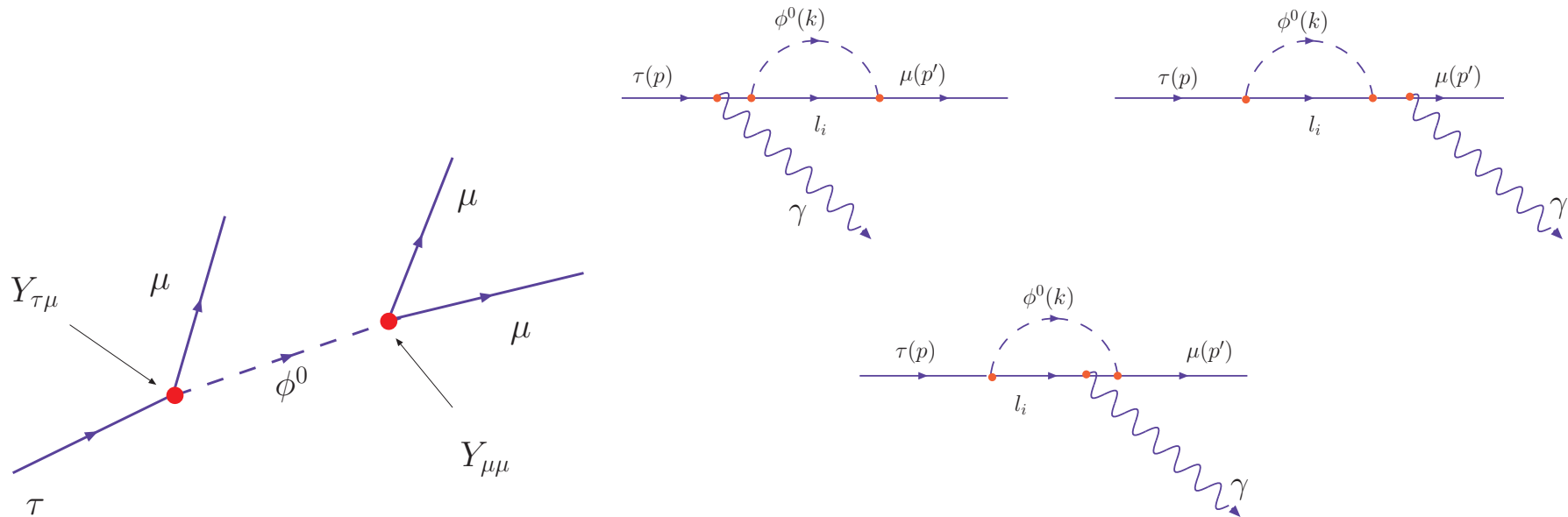


Figure 1: The diagram in the left contributes to the process  $\tau^- \rightarrow 3\mu$ . The three diagrams in the right contribute to the process  $\tau \rightarrow \mu\gamma$ .

## The Yukawa matrices

The Yukawa matrices in the weak basis are

$$Y_w^{E1} = \frac{m_\tau}{v_1} \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 & 0 \end{pmatrix}$$

and

$$Y_w^{E2} = \frac{m_\tau}{v_2} \begin{pmatrix} \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ 0 & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 \end{pmatrix}.$$

## Yukawa matrices in the mass representation

The Yukawa matrices in the mass basis defined by

$$\tilde{Y}_m^{EI} = U_{eL}^\dagger Y_w^{EI} U_{eR}$$

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m,$$

and

$$\tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m,$$

all off diagonal terms give rise to FCNC processes!!



## Branching ratios

We define the partial branching ratio (only leptonic decays)

$$Br(\tau \rightarrow \mu e^+ e^-) = \frac{\Gamma(\tau \rightarrow \mu e^+ e^-)}{\Gamma(\tau \rightarrow e \nu \bar{\nu}) + \Gamma(\tau \rightarrow \mu \nu \bar{\nu})}, \quad \Gamma(\tau \rightarrow \mu e^+ e^-) \approx \frac{m_\tau^5}{3 \times 2^{10} \pi^3} \frac{(Y_{\tau\mu}^{1,2} Y_{ee'}^{1,2})^2}{M_{H_{1,2}}^4}$$

thus

$$Br(\tau \rightarrow \mu e^+ e^-) \approx \frac{9}{4} \left( \frac{m_e m_\mu}{m_\tau^2} \right)^2 \left( \frac{m_\tau}{M_{H_{1,2}}} \right)^4,$$

Similar computations lead to

$$Br(\tau \rightarrow e \gamma) \approx \frac{3\alpha}{8\pi} \left( \frac{m_\mu}{M_H} \right)^4,$$

$$Br(\tau \rightarrow \mu \gamma) \approx \frac{3\alpha}{128\pi} \left( \frac{m_\mu}{m_\tau} \right)^2 \left( \frac{m_\tau}{M_H} \right)^4,$$

$$Br(\tau \rightarrow 3\mu) \approx \frac{9}{64} \left( \frac{m_\mu}{M_H} \right)^4,$$

$$Br(\mu \rightarrow 3e) \approx 18 \left( \frac{m_e m_\mu}{m_\tau^2} \right)^2 \left( \frac{m_\tau}{M_H} \right)^4,$$

$$Br(\mu \rightarrow e \gamma) \approx \frac{27\alpha}{64\pi} \left( \frac{m_e}{m_\mu} \right)^4 \left( \frac{m_\tau}{M_H} \right)^4.$$

## Numerical results

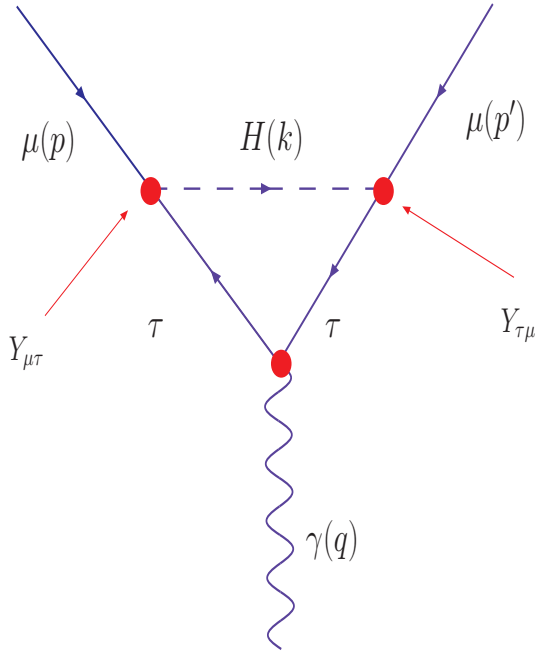
Table 2: Leptonic processes via FCNC

FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \rightarrow 3\mu$	$8.43 \times 10^{-14}$	$5.3 \times 10^{-8}$	B. Aubert <i>et al.</i> (2007)
$\tau \rightarrow \mu e^+ e^-$	$3.15 \times 10^{-17}$	$8 \times 10^{-8}$	B. Aubert <i>et al.</i> (2007)
$\tau \rightarrow \mu\gamma$	$9.24 \times 10^{-15}$	$6.8 \times 10^{-8}$	B. Aubert <i>et al.</i> (2005)
$\tau \rightarrow e\gamma$	$5.22 \times 10^{-16}$	$1.1 \times 10^{-11}$	B. Aubert <i>et al.</i> (2006)
$\mu \rightarrow 3e$	$2.53 \times 10^{-16}$	$1 \times 10^{-12}$	U. Bellgardt <i>et al.</i> (1998)
$\mu \rightarrow e\gamma$	$2.42 \times 10^{-20}$	$1.2 \times 10^{-11}$	M. L. Brooks <i>et al.</i> (1999)

Small FCNC processes mediating non-standard quark-neutrino interactions could be important in the theoretical description of the **gravitational core collapse and shock generation** in the explosion stage of a supernova

## Muon Anomalous Magnetic Moment

The anomalous magnetic moment of the muon is related to the gyromagnetic ratio by



$$a_\mu = \frac{\mu_\mu}{\mu_B} - 1 = \frac{1}{2}(g_\mu - 2)$$

In models with more than one Higgs  $SU(2)$  doublet, the exchange of flavour changing neutral scalars also contribute to the anomalous magnetic moment of the muon

$$\delta a_\mu^{(H)} = \frac{Y_{\mu\tau} Y_{\tau\mu} m_\mu m_\tau}{16\pi^2 M_H^2} \left( \log \left( \frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right)$$

From our results:  $Y_{\mu\tau} Y_{\tau\mu} = \frac{m_\mu m_\tau}{4v_1 v_2}$

$$\delta a_\mu^{(H)} = \frac{m_\tau^2}{(246 \text{ GeV})^2} \frac{(2 + \tan^2 \beta)}{32\pi^2} \frac{m_\mu^2}{M_H^2} \left( \log \left( \frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right), \quad \tan \beta = \frac{v_s}{v_1}$$

From the experimental upper bound on  $(\mu \rightarrow 3e)$ , we get  $\tan \beta \leq 14$ , Hence

$$\delta a_\mu = 1.7 \times 10^{-10}$$

## Contribution to the anomaly of the muon's magnetic moment

The difference between the experimental value and the Standard Model prediction for the anomaly is

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (28.7 \pm 9.1) \times 10^{-10}$$

$$\Delta a_\mu \sim 3\sigma \text{ (three standard deviations) !!}$$

But, the uncertainty in the computation of higher order hadronic effects is large

$$\delta a_\mu^{LBL}(3, had) \approx 1.59 \times 10^{-9}; \quad \delta a_\mu^{VP}(3, had) \approx -1.82 \times 10^{-9}$$

$$\frac{\delta a_\mu^{(H)}}{\Delta a_\mu} \approx \frac{1.7}{28} \approx 6\% \quad \text{and} \quad \delta a_\mu^{(H)} < \delta a_\mu(3, had)$$

The contribution of the exchange of flavour changing scalars to the anomaly of the muon's magnetic moment,  $\delta a_\mu^{(H)}$ , is small but not negligible, and it is compatible with the best, state of the art, measurements and theoretical predictions.

## Summary

- By introducing three  $SU(2)_L$  Higgs doublet fields, in the theory, we extended the concept of flavour and generations to the Higgs sector and formulated a minimal  $S_3$ —invariant Extension of the Standard Model.
- The mass matrices of quarks and leptons are reparametrized in terms of their eigenvalues and the mixing matrices are computed.
- In the case of quarks, the agreement between our  $V_{CKM}^{th}$  (with only two free parameters) and  $V^{PDG}$  is excellent
- In the case of neutrinos, the solar mixing angle,  $\theta_{12}$ , fixes the scale and origin of the neutrino mass spectrum which has an inverted mass hierarchy with values
$$|m_{\nu_2}| \approx 0.053eV, \quad |m_{\nu_1}| \approx 0.052eV, \quad |m_{\nu_3}| \approx 0.019eV$$
- In the leptonic sector, the  $S_3 \times Z_2$  symmetry implies a non vanishing and sizeable reactor mixing angle,  $\theta_{13}^{oth} = 9.6 \pm 0.07$  in excellent agreement with the most recent experimental results.
- The fit of  $\sin^2 \theta_{13}^{th}$  to the  $\sin^2 \theta_{13}^{exp}$  breaks the mass degeneracy of the right handed neutrinos