Karlsruhe Institute of Technology

The three-loop β-function for the Higgs self-coupling and the vacuum stability problem in the SM

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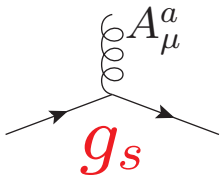
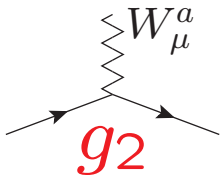
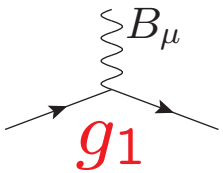
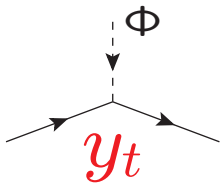
Motivation: Vacuum Stability in the SM

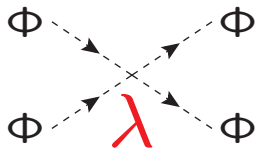
SM interactions:

- gauge couplings: QCD: 

Electroweak: 



- Yukawa couplings and Higgs self-interaction: 

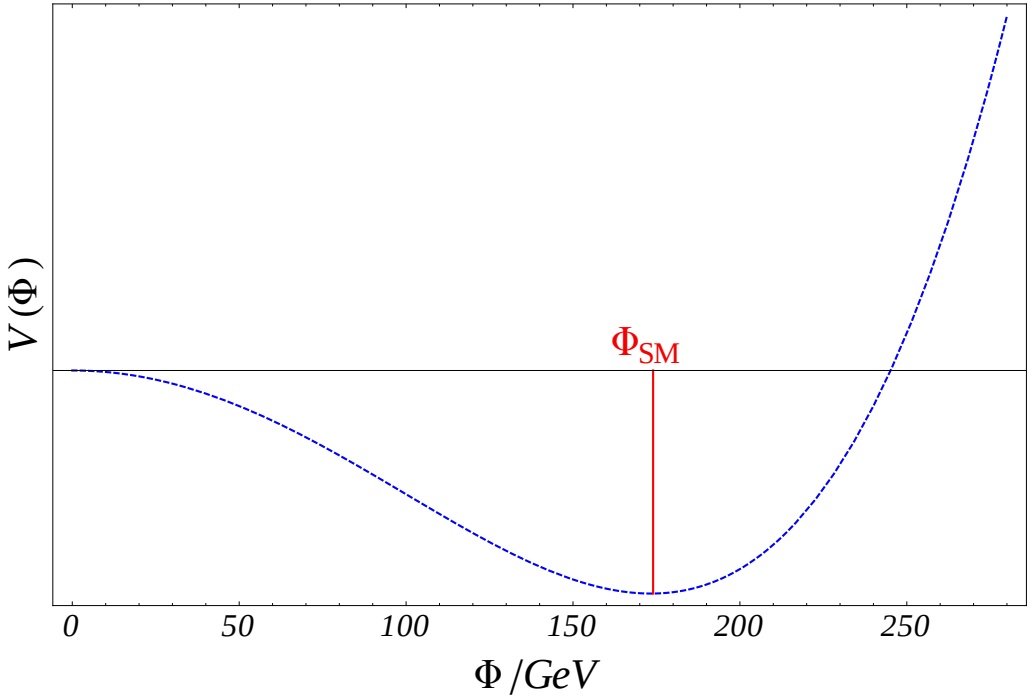


Classical Higgs potential:

$$V(\Phi) = \left(m^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2\right)$$

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \xrightarrow{\text{SSB}} \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix}$$

$$|\Phi_{SM}| = \frac{v}{\sqrt{2}}, \qquad v \approx 246 \text{ GeV}$$

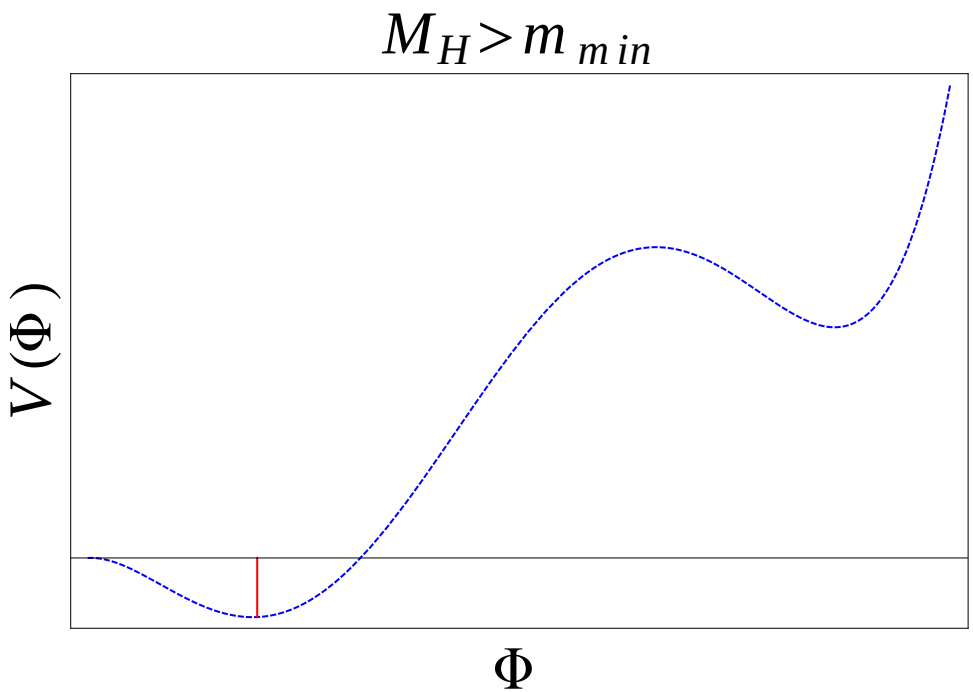


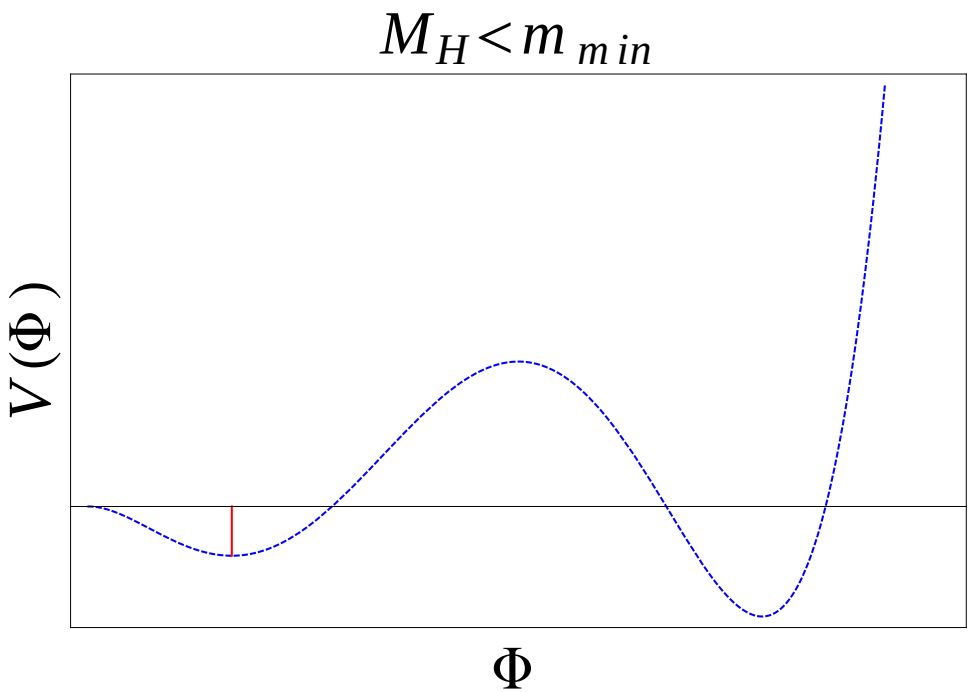
The effective Potential

QFT: Radiative corrections $\Rightarrow$ Evolution of couplings λ, g_1, g_2, g_s, y_t, ... and fields Φ, ...

Higgs potential $\rightarrow$ $\boxed{V_{eff}\,(\lambda(\Lambda), g_i(\Lambda), y_t(\Lambda), \ldots)\,[\Phi(\Lambda)]}$ [Coleman, Weinberg]

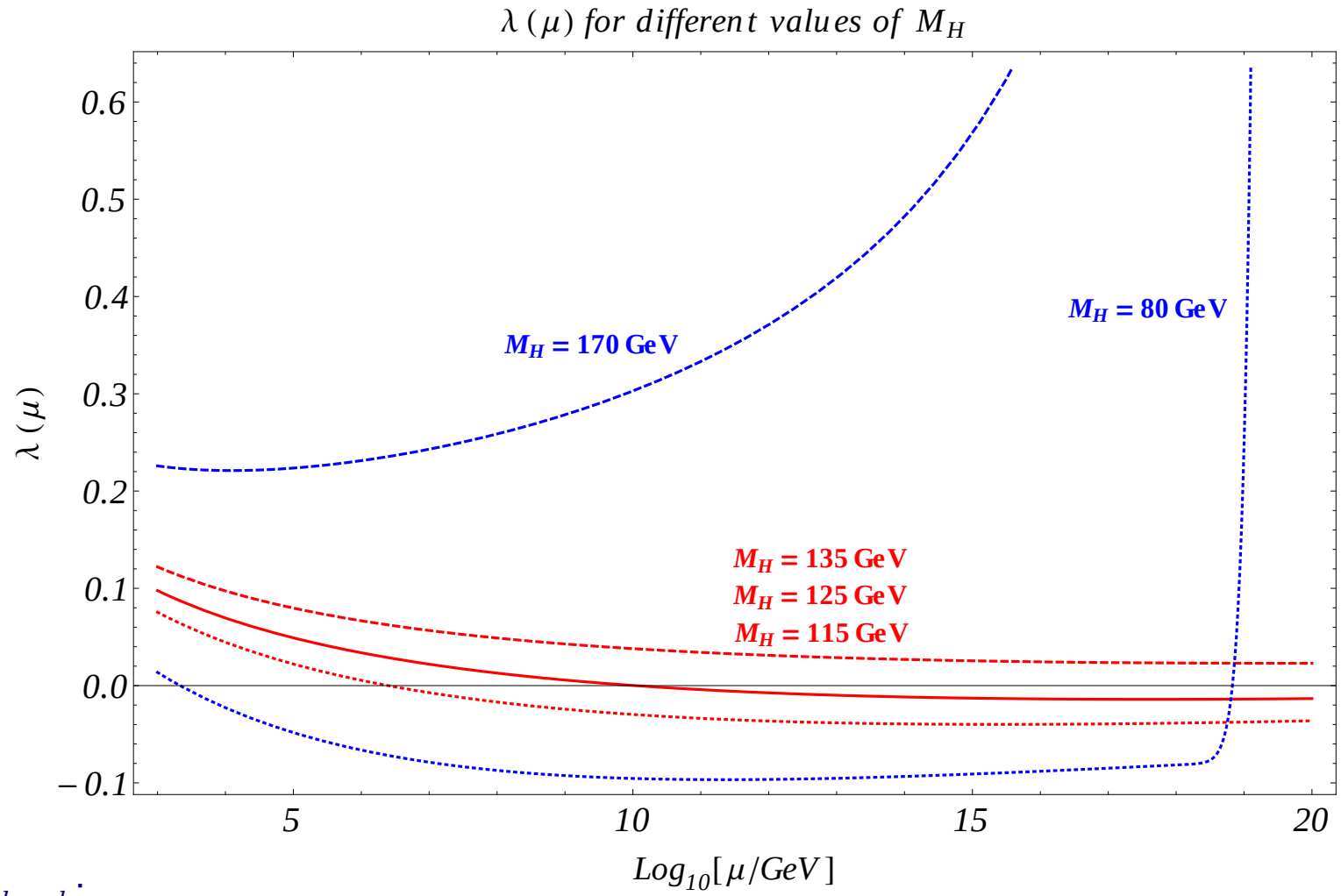
(Λ: scale up to which the SM is valid, starting scale for running e.g. $\mu_0 = M_t$)





For $\Phi \sim \Lambda \gg M_Z$: $\boxed{V_{eff}[\Phi] \approx \lambda(\Lambda)\Phi(\Lambda)^4 + \mathcal{O}(\lambda^2, g_i^2)}$ [Altarelli, Isidori; Ford, Jack, Jones]

Stability of SM vacuum $\Leftrightarrow$ $\lambda(\Lambda) > 0$ [Cabibbo; Sher; Lindner; Ford]



for $\Lambda = M_{Planck}$:

- Upper bound $M_H < m_{max}$: no Landau pole (up to Λ)

  $\boxed{m_{max} \approx 175 \text{ GeV}}$ [Cabibbo, Maiani, Parisi, Petronzio, Lindner, Hambye, Riesselmann]

- Stability bound on the Higgs mass: $M_H > m_{min}$

  $\boxed{m_{min} \approx 129 \pm 3 \text{ GeV}}$ (2011) [Elias-Miro, Espinosa, Giudice, Isidori, Riotto, Strumia]

Evolution of couplings $X \in \{\lambda, g_1, g_2, g_s, y_t, \ldots\}$

β-functions: $\boxed{\mu^2 \frac{d}{d\mu^2} X(\mu^2) = \beta_X[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2), \ldots]}$

$\Rightarrow$ **Coupled system of differential equations with initial conditions:**

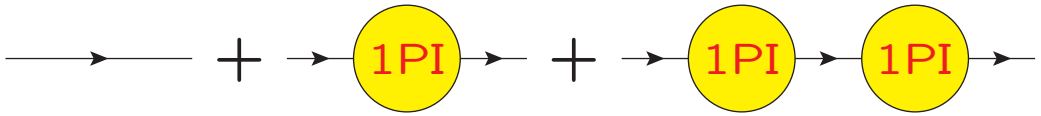
$$\underbrace{\begin{aligned}
\mu^2 \frac{d}{d\mu^2} \lambda(\mu^2) &= \beta_\lambda[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], \\
\mu^2 \frac{d}{d\mu^2} y_t(\mu^2) &= \beta_{y_t}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], \\
\mu^2 \frac{d}{d\mu^2} g_s(\mu^2) &= \beta_{g_s}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], \\
\mu^2 \frac{d}{d\mu^2} g_2(\mu^2) &= \beta_{g_2}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], \\
\mu^2 \frac{d}{d\mu^2} g_1(\mu^2) &= \beta_{g_1}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)],
\end{aligned}}_{\text{Calculated from theory, power series in couplings } \Rightarrow \text{ Theoretical uncertainty}}
\qquad
\underbrace{\begin{aligned}
\lambda(\mu_0^2) &= \lambda_0, \\
y_t(\mu_0^2) &= y_{t\,0}, \\
g_s(\mu_0^2) &= g_{s\,0}, \\
g_2(\mu_0^2) &= g_{2\,0}, \\
g_1(\mu_0^2) &= g_{1\,0}
\end{aligned}}_{\text{Experimental data matched to theoretical } \overline{\text{MS}}\text{-scheme } \Rightarrow \text{ Experimental and theoretical uncertainty}}$$

Starting values for SM couplings

Calculate on-shell propagators to find relations between pole masses $M_t, M_H, \ldots$ and $\overline{\text{MS}}$-parameters

[Sirlin, Zucchini; Hempfling, Kniehl; Jegerlehner et al; Bezrukov et al; Buttazzo et al]

Example: The top propagator



$$+ \ldots = \frac{i}{\not{p} - M_t - \Sigma(\not{p}, M_t, g_i)}$$

pole at $\boxed{\not{p} = M_t}$ $\Rightarrow$ relation between M_t and $m_t^{\overline{\text{MS}}} = \frac{y_t}{\sqrt{2}} v$

Similarly, for Higgs and W propagator $\Rightarrow$ $\boxed{M_t, M_H, M_W \quad \leftrightarrow \quad y_t, \lambda, v}$

From experimental data:		$\overline{\text{MS}}$ parameters:	
$M_t \approx 173.5$ GeV		$g_s(M_t) \approx 1.16$	$y_t(M_t) \approx 0.94$
$M_H \approx 126$ GeV	$\Rightarrow$	$g_2(M_t) \approx 0.65$	$\lambda(M_t) \approx 0.13$
$\alpha_s \approx 0.1184$		$g_1(M_t) \approx 0.36$	$y_b \approx 0.02,\ y_\tau \approx 0.01$

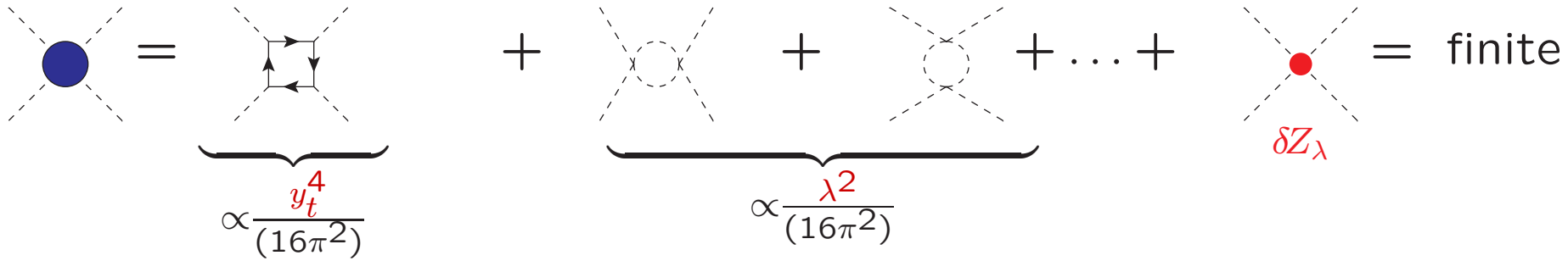
β-functions in the SM

- 1 loop QCD: [Gross, Wilczek (1973); Politzer (1973)]
- 2 loop SM: [Fischler, Hill (1981); Jones (1982); Fischler, Oliensis (1982); Machacek, Vaughn (1983, 1984, 1985); Jack, Osborn (1984, 1985); Ford, Jack, Jones (1992); Luo, Xiao (2003)]
- 3 loop SM:
  - for gauge couplings g_1, g_2, g_s: [Mihaila, Salomon, Steinhauser (2012); Bednyakov, Pikelner, Velizhanin (2012)]
  - for Yukawa couplings y_t, y_b, y_τ, etc.: [Chetyrkin, MZ (2012); Bednyakov, Pikelner, Velizhanin (2013)]
  - for the Higgs self-coupling λ (and the mass parameter m^2): [Chetyrkin, MZ (2012 and 2013); Bednyakov, Pikelner, Velizhanin (2013)]
- 4 loop QCD: [van Ritbergen, Vermaseren, Larin (1997); Czakon (2005)]

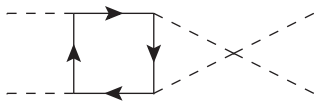
$\lambda_B = (\mu^2)^\varepsilon (\lambda + \delta Z_\lambda)$ with $\delta Z_\lambda = \frac{a_1}{\varepsilon} + \frac{a_2}{\varepsilon^2} + \frac{a_3}{\varepsilon^3} + \ldots$ and $a_i = a_i(\lambda, \{g_i\})$

$$\left(\mu^2 \frac{d}{d\mu^2} \lambda_B = 0\right) \Rightarrow \boxed{\beta_\lambda = \left[\lambda \frac{\partial}{\partial \lambda} + \frac{1}{2} \sum_i g_i \frac{\partial}{\partial g_i} - 1\right] a_1(\lambda, \{g_i\})}$$

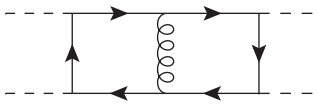
Calculation of $\delta Z_\lambda(\lambda, y_t, g_s, g_2, g_1, \ldots)$



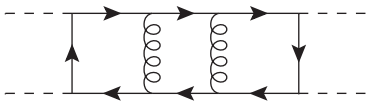
Higher orders:



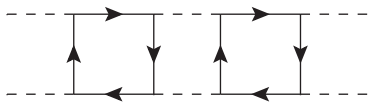
$$\propto \frac{y_t^4 \lambda}{(16\pi^2)^2}$$



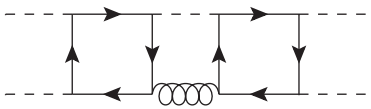
$$\propto \frac{y_t^4 g_s^2}{(16\pi^2)^2}$$



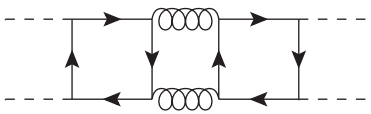
$$\propto \frac{y_t^4 g_s^4}{(16\pi^2)^3}$$



$$\propto \frac{y_t^8}{(16\pi^2)^3}$$



$$\propto \frac{y_t^6 g_s^2}{(16\pi^2)^3}$$



$$\propto \frac{y_t^4 g_s^4}{(16\pi^2)^3}$$

Challenges:

- Huge number of diagrams $\rightarrow \mathcal{O}(10^6)$ at 3 loops
- Treatment of $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ in $D = 4 - 2\varepsilon$ ['t Hooft, Veltman]
- IR divergencies [Chetyrkin, Misiak, Münz]

Results:

$$\boxed{\mu^2 \frac{d}{d\mu^2}\lambda(\mu) = \beta_\lambda = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n}\beta_\lambda^{(n)}}$$ (in the $\overline{\text{MS}}$-scheme)

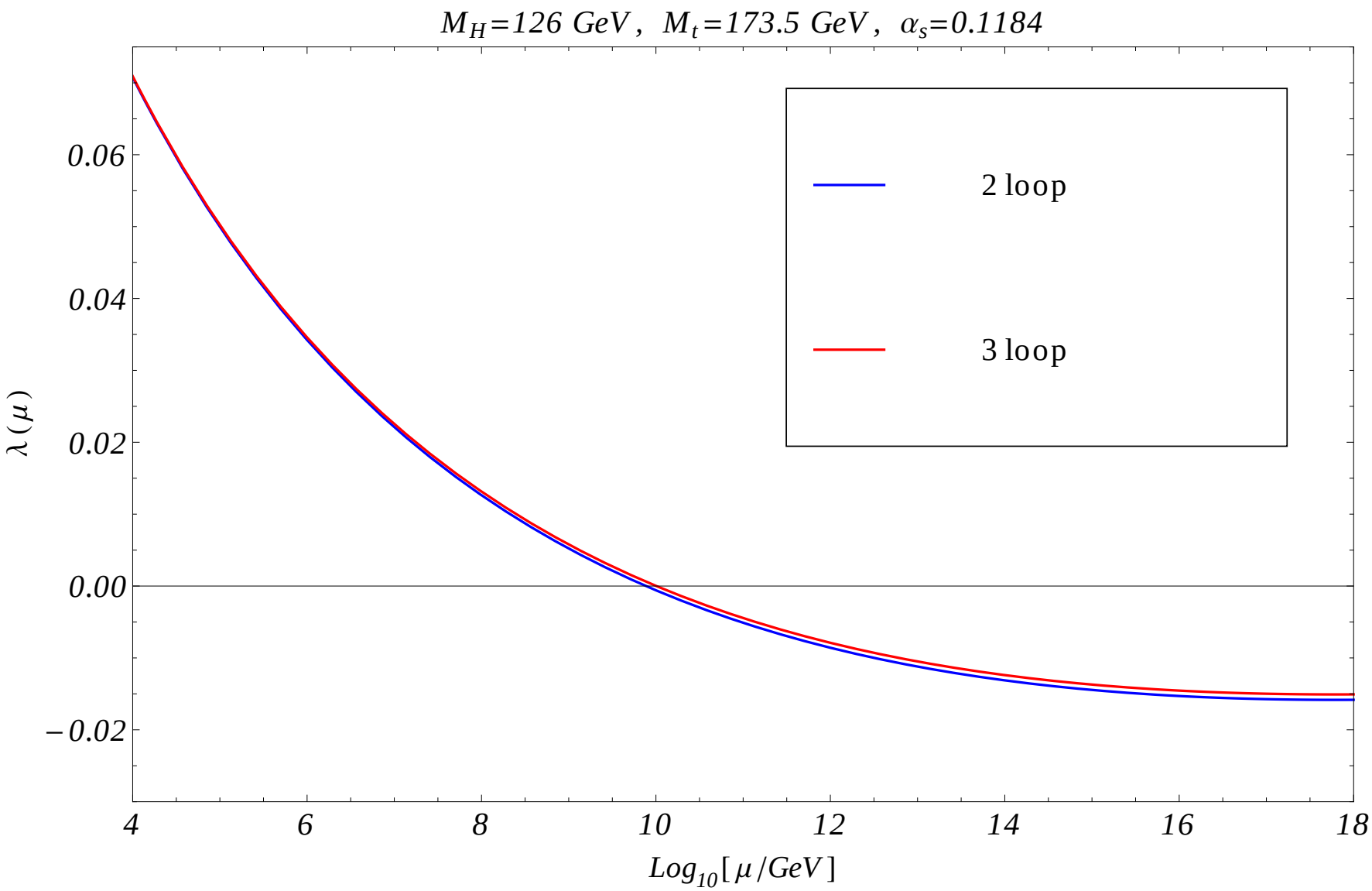
$$\beta_\lambda^{(1)} = -y_t^4 3 + y_t^2\lambda 6 - \lambda g_2^2\frac{9}{2} + \lambda^2 12 + g_2^4\frac{9}{16} - \lambda g_1^2\frac{3}{2} + g_1^2 g_2^2\frac{3}{8} + g_1^4\frac{3}{16} + \lambda y_b^2 6 + \lambda y_\tau^2 2 - y_b^4 3 - y_\tau^4$$

$$\beta_\lambda^{(2)} = -g_s^2 y_t^4 16 + y_t^6 15 + g_s^2 y_t^2\lambda 40 - y_t^2\lambda^2 72 + y_t^2\lambda g_2^2\frac{45}{4} + +\lambda^2 g_2^2 54 - \lambda^3 156 + \ldots$$

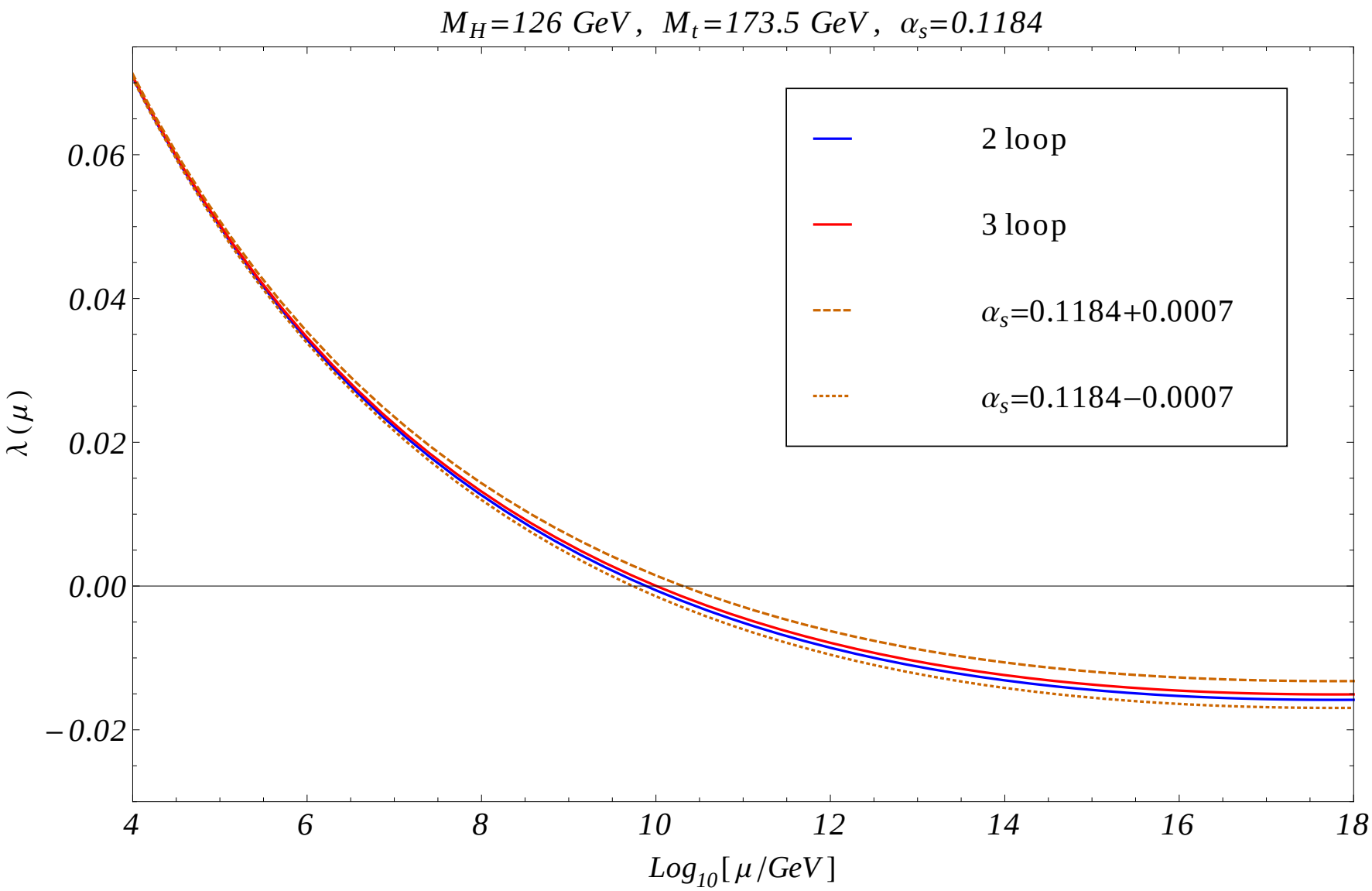
$$\begin{aligned}\beta_\lambda^{(3)} =& g_s^2 y_t^6(-38 + 240\zeta_3) + y_t^8\left(-\frac{1599}{8} - 36\zeta_3\right) + g_s^4 y_t^4\left(-\frac{626}{3} + 32\zeta_3 + 40N_g\right)\\ &+ g_s^2 y_t^4\lambda(895 - 1296\zeta_3) + g_s^4 y_t^2\lambda\left(\frac{1820}{3} - 48\zeta_3 - 64N_g\right) + y_t^4\lambda^2\left(\frac{1719}{2} + 756\zeta_3\right)\\ &+ y_t^6 g_2^2\left(\frac{3411}{32} - 27\zeta_3\right) + y_t^6\lambda\left(\frac{117}{8} - 198\zeta_3\right) + \ldots\end{aligned}$$

JHEP 1206 (2012) 033
JHEP 1304 (2013) 091

Evolution of $\lambda(\mu)$



Evolution of $\lambda(\mu)$



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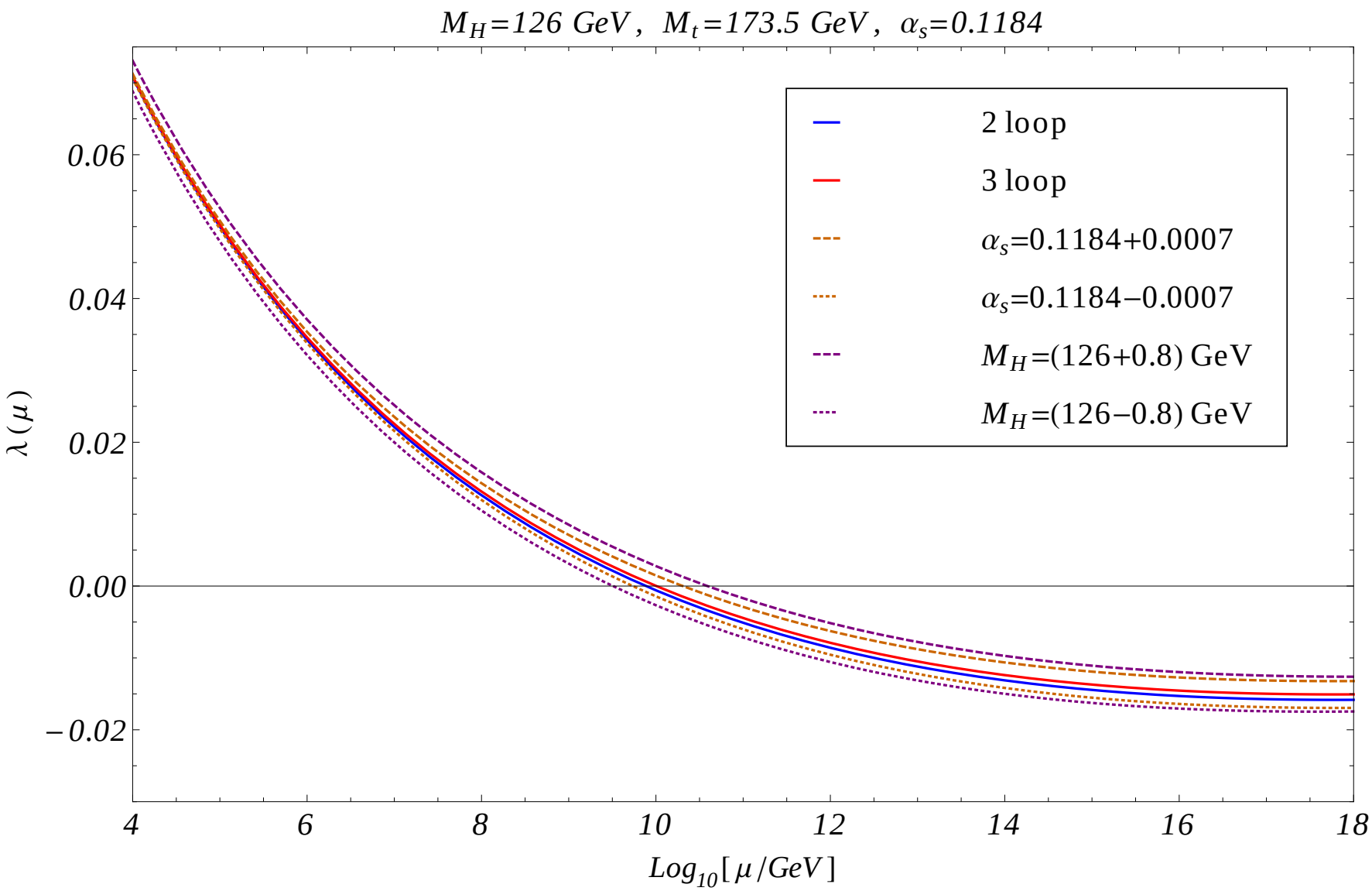
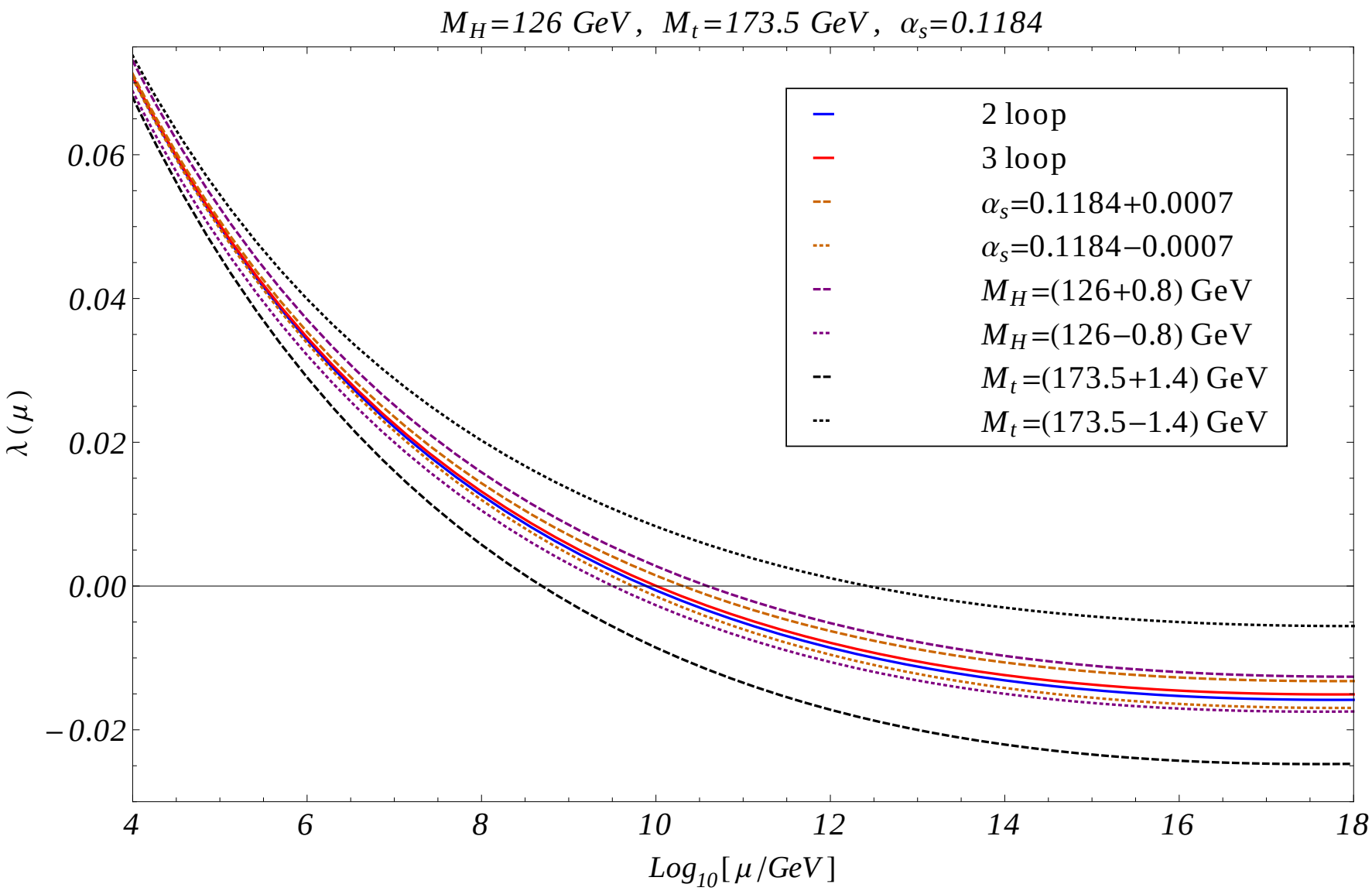
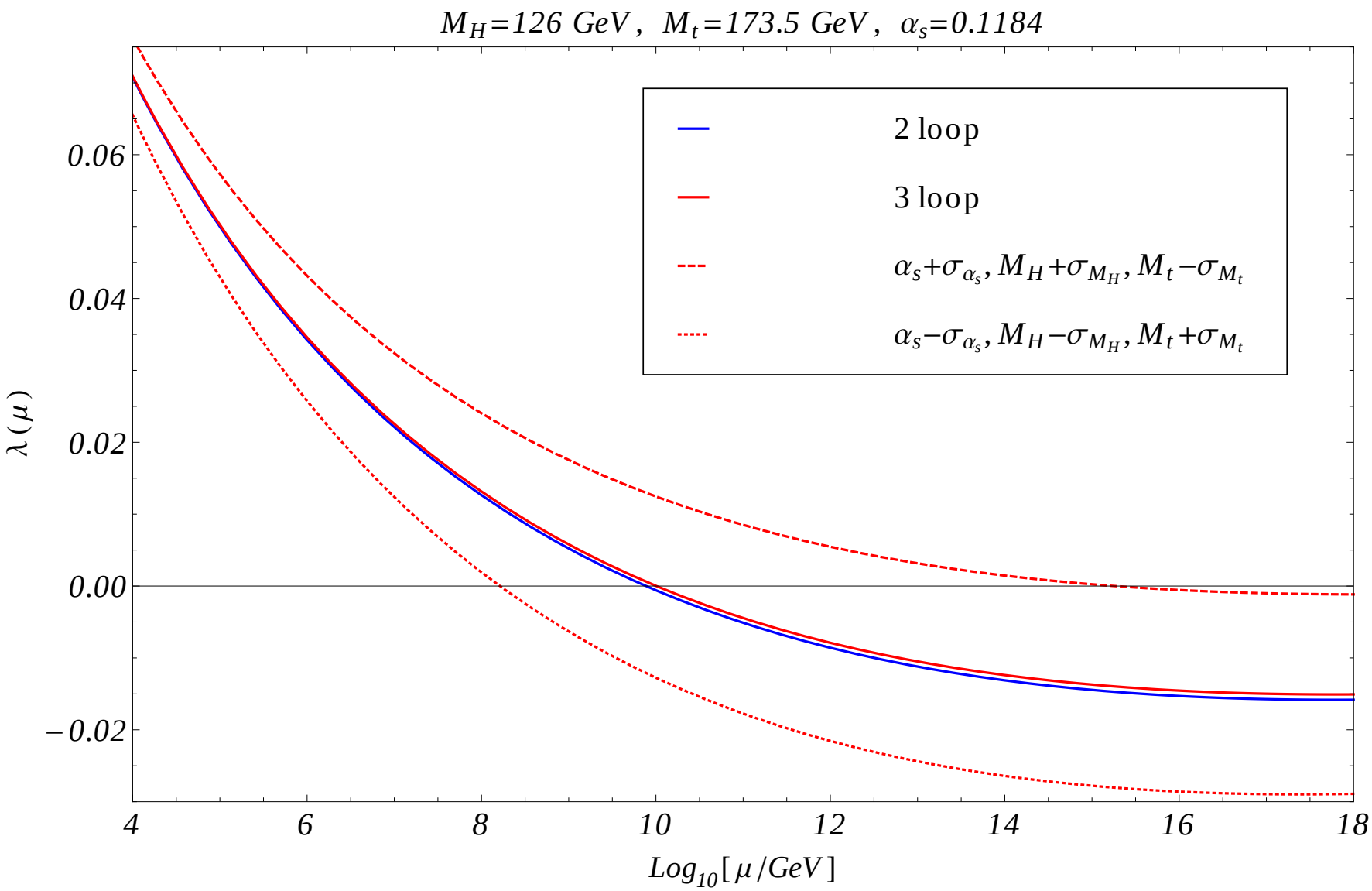


Evolution of $\lambda(\mu)$



Evolution of $\lambda(\mu)$: Combined error



Summary

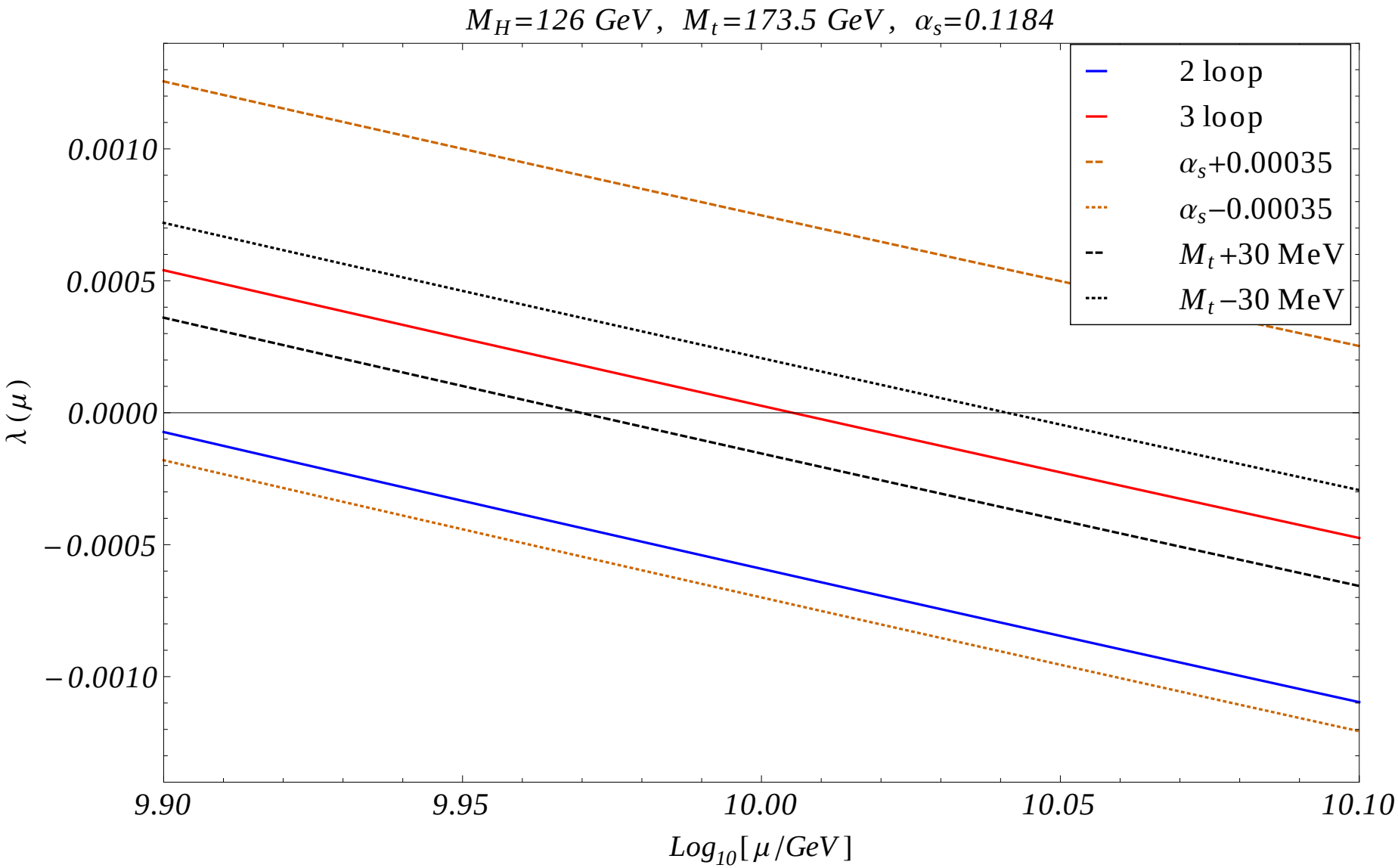
- Stability of SM vacuum $\leftrightarrow$ $\boxed{\lambda > 0}$
  3 loop result improves stability!

- β_λ well convergent for $M_H \approx 126$ GeV
  $\Rightarrow$ Small theory uncertainty from β-function

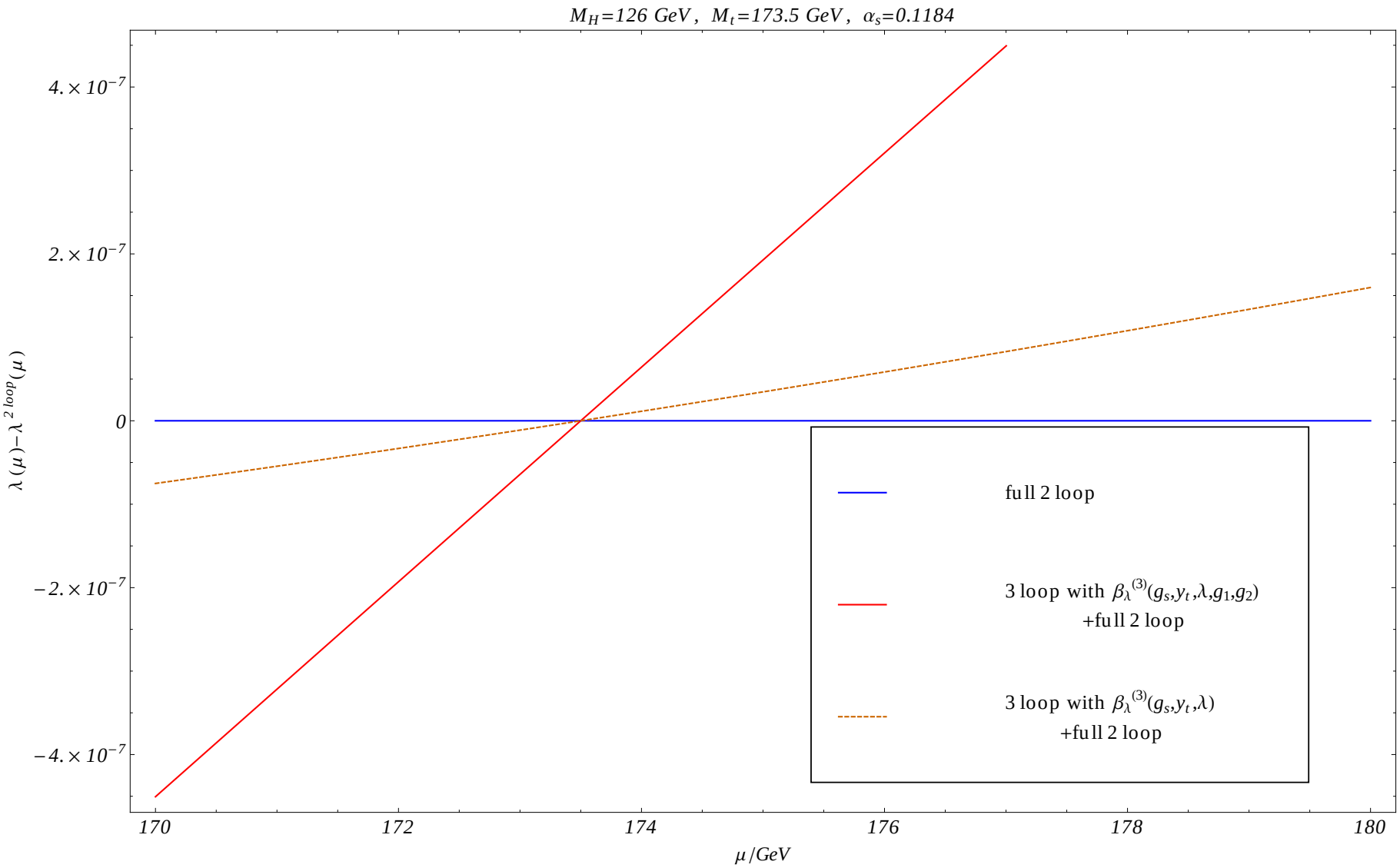
- 3 loop β_λ effect smaller than experimental uncertainty
  for α_s, M_H and mainly M_t

  $\Rightarrow$ The question of vacuum stability remains open!

Evolution of $\lambda(\mu)$: future scenario



Backup: Influence of EW corrections on the evolution of $\lambda(\mu)$



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