Grand symmetry, spectral action and the Higgs mass

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- **128** $= 32 \times 4 = 2 \times (2 \times 4)^2$ where

  - 32 is the number of particles per generation in the SM:
    (2 quarks $\times$ 3 colours $+$ 2 leptons) $\times$ 2 chiralities $=$ 16 $+$ antiparticles,

  - $4 = 2^{\frac{4}{2}}$ is the dimension of the spinor representation on a 4 dimensional manifold.

Spectral triple: $*$-algebra $\mathcal{A}$, faithful representation on $\mathcal{H}$, operator D on $\mathcal{H}$ such that $[D, a]$ is bounded and $a[D - \lambda\mathbb{I}]^{-1}$ is compact for all $a \in \mathcal{A}$ and $\lambda \notin \mathrm{Sp}\, D$.

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Theorem Connes 1996-2008-2013

$\mathcal{M}$ compact Riemann spin manifold, then $(C^\infty(\mathcal{M}), L^2(\mathcal{M}, S), \partial\!\!\!/)$ is a spectral triple.

$(\mathcal{A}, \mathcal{H}, D)$ a spectral triple with $\mathcal{A}$ unital commutative, then there exists a compact Riemannian spin manifold $\mathcal{M}$ such that $\mathcal{A} = C^\infty(\mathcal{M})$.

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Standard model: product of a manifold by a finite dimensional spectral triple

$$\mathcal{A}_{sm} = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}), \qquad \mathcal{H}_F = \mathbb{C}^{96=3\times 32}$$

D_F : fermions masses, Yukama coupling, neutrino mixing matrix

$$\mathcal{A} = C^\infty(\mathcal{M}) \otimes \mathcal{A}_{sm}, \quad \mathcal{H} = L^2(\mathcal{M}, S) \otimes \mathcal{H}_F, \quad D = \partial\!\!\!/ \otimes \mathbb{I}_F + \gamma^5 \otimes D_F.$$

Spectral action Tr $f(\frac{D^2}{\Lambda})$ yields the Lagrangian of the SM minimally coupled with Einstein-Hilbert action.

The new field σ and the Higgs mass

Spectral action requires a unique unification scale. With $\Lambda = 10^{17}$GeV, the running of the Higgs quartic selfcoupling λ_H under the big desert hypothesis yields

$$m_H \simeq 170 \text{ GeV}.$$

† Elias-Miro, Espinosa, Guidice, Lee and Sturmia, *Stabilization of the Electroweak Vacuum by a Scalar Threshold effect*, JHEP **1206** (2012) 031;
Degrassi, Di Vita, Elias-Miro, Espinosa, Guidice, Isidori and A. Sturmia, *Higgs mass and Vacuum Stability in the SM at NNLO*, arXiv:1205.6497;
Chian-Shu Chen and Yong Tang, *Vacuum Stability, Neutrinos and Dark matter*, JHEP **1204** (2012) 019;
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A new scalar field σ - that lives at high energy and gives mass to the neutrinos - has been introduced by phenomenologists[†] to solve some instability due to the low mass of the Higgs (radiative corrections may drive λ_H negative and destabilize the electroweak vacuum):

$$V(H, \sigma) = \frac{1}{4}(\lambda_H H^4 + \lambda_\sigma \sigma^4 + 2\lambda_{H\sigma} H^2 \sigma^2).$$

As a bonus, it pulls m_H back to 126 GeV.

Resilience of the spectral SM, Chamseddine, Connes 2012

Is σ natural in NCG, or is it just an artifact for solving the model ?

[†] Elias-Miro, Espinosa, Guidice, Lee and Sturmia, *Stabilization of the Electroweak Vacuum by a Scalar Threshold effect*, JHEP **1206** (2012) 031; Degrassi, Di Vita, Elias-Miro, Espinosa, Guidice, Isidori and A. Sturmia, *Higgs mass and Vacuum Stability in the SM at NNLO*, arXiv:1205.6497; Chian-Shu Chen and Yong Tang, *Vacuum Stability, Neutrinos and Dark matter*, JHEP **1204** (2012) 019; Oleg Lebedev, *On Stability of the Higgs Potential and the Higgs Portal*, JHEP, arXiv:1203.0156.

Gauge fields and the first order condition

The gauge fields of the SM (including the Higgs) are obtained by fluctuation of the metric

$$[D, a] = [\partial\!\!\!/ \otimes \mathbb{I} + \gamma^5 \otimes D_F, f^i \otimes m_i],$$

allowing to turn the constant components of D_F into fields on the manifold $\mathcal{M}$.

The field σ is obtained by turning the entry corresponding to the neutrino mass into a field. Unfortunately, the first order conditon

$$[[D, a], JbJ^{-1}] = 0 \quad \forall a, b \in \mathcal{A}$$

prevents to do so by fluctuation of the metric. Indeed, for D_M the Dirac with only the neutrino mass,

$$[[D_M, a], JbJ^{-1}] = 0 \quad \forall a, b \in \mathcal{A}_{sm} \Longrightarrow [D_M, a] = 0.$$

The finite dimensional algebras of the SM

Connes, Chamseddine, Marcolli: the finite dimensional algebra is of the form

$$M_a(\mathbb{H}) \oplus M_{2a}(\mathbb{C}) \quad a \in \mathbb{N}$$

and acts on an Hilbert space of dimension $d = 2 \times (2 \times a)^2$.

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  Grading condition $[\Gamma, a] = 0$ (coming from the orientability axiom) imposes

$$\mathcal{A}_F = M_2(\mathbb{H}) \oplus M_4(\mathbb{C}) \longrightarrow \mathcal{A}_{LR} = \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_4(\mathbb{C}).$$

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  1^{st}-order condition without neutrino mass ($D_F = D_0$) further imposes

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  1^{st}-order condition with neutrino mass $(D_F = D_0 + D_M)$ finally gives

$$\mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C}) \oplus \mathbb{C}. \longrightarrow \mathbb{H}_L \oplus \mathbb{C}' \oplus M_3(\mathbb{C}) \oplus \mathbb{C}$$

  with $\mathbb{C} = \mathbb{C}'$. Hence the the reduction

$$\mathcal{A}_F \to \mathcal{A}_{sm} = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}).$$

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- a=4: $d = 128 =$ dimension 4×32 of the total Hilbert space for 1 generation:

$\mathcal{H} = L^2(\mathcal{M}, S) \otimes \mathcal{H}_F = L^2(\mathcal{M}) \otimes \mathrm{H}_F$ where $\mathrm{H}_F = \mathbb{C}^4 \otimes \mathcal{H}_F = \mathbb{C}^4 \otimes \mathbb{C}^{32} = \mathbb{C}^{128}$.

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By mixing the spin

$$s = l, r, \quad \dot{s} = \dot{0}, \dot{1}$$

and the internal

$$C = p, a \quad \alpha = u_R, d_R, u_L, d_L \ (I = 1, 2, 3), \ e_R, \nu_R, e_L, \nu_L \ (I = 0)$$

degrees of freedom, the Hilbert space $\mathcal{H}$ of the standard model allows to represent the grand algebra

$$C^\infty(\mathcal{M}) \otimes \mathcal{A}_G \text{ where } \mathcal{A}_G = M_4(\mathbb{H}) \oplus M_8(\mathbb{C})$$

without touching the particle contents of the SM, and in a way satisfying the order 0 condition (coming from the reality axiom)

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- $M_2(\mathbb{H})$ and $M_4(\mathbb{C})$ now have a **non-diagonal** action on the spin indices $s, \dot{s}$.

Reduction of the grand algebra: emergence of geometry

The Dirac operator is unchanged

$$D = \partial\!\!\!/ \otimes \mathbb{I}_{\mathcal{H}_F} + \gamma^5\, \mathbb{I}_{L^2(\mathcal{M},S)} \otimes D_F = \partial_\mu \otimes \gamma^\mu_{s\dot{s}}\, \mathbb{I}^{CI\alpha} + \mathbb{I}_{L^2(\mathcal{M})} \otimes \gamma^5_{s\dot{s}}\, D_F^{CI\alpha},$$

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- We start with a topological phase. The metric structure emerges dynamically (work in progress), by the first order condition of the free Dirac operator.
- In the SM, one deals with a product of spectral triples, so the 1st-order condition for the free Dirac is automatically satisfied.

Reduction of the grand algebra bis: fiat neutrino !

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A solution of the 1^{st}-order condition of the Majorana mass only $(\mathbb{I}_{L^2(\mathcal{M})} \otimes D_M^{Cl\alpha})$ is

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- By inverting the order of the reductions, one can generate the field σ by a fluctuation of D_M, respecting the 1$^{\text{st}}$-order condition imposed by D_M:

Proposition — Devastato, Lizzi, Martinetti 2013

For $a \in \mathcal{A}''_G, \quad [\mathbb{I}_{L^2(\mathcal{M})} \otimes D_M^{Cl\,\alpha}, a]$ is not necessarily zero.

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- σ does not satisfy the 1st-order condition imposed by the free Dirac operator: σ is the "Higgs field" corresponding to the (geometrical, possibly dynamical) symmetry breaking $\mathcal{A}''_G \rightarrow \mathcal{A}_{sm}$.

Conclusion

Almost simultaneously, Chamseddine, Connes and van Suijlekom proposed a definition of "inner fluctuation without first order condition".

Starting with $M_2(\mathbb{H}) \oplus M_4(\mathbb{C})$, they generate the field σ, and retrieve the 1^{st}-order condition dynamically, by minimizing the spectral action.

- The grand symmetry allows to dissociate the Majorana 1^{st}-order condition from the free Dirac 1^{st}-order condition.

The phenomenological consequences are under investigation.

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