# Convolutions of Partonic Cross Sections with Splitting Functions at N3LO

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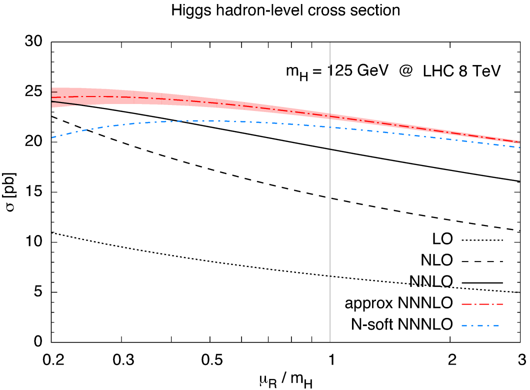
# Motivation

- approximate $N^3LO$ using resummations, high- and low-energy behaviour, . . .

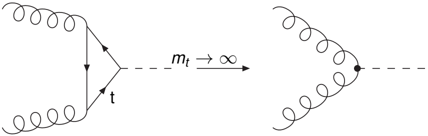



- [Moch, Vogt, '05] ("N-soft"), [Ball, Bonvini, Forte, Marzani, Ridolfi, '13] ("approx")

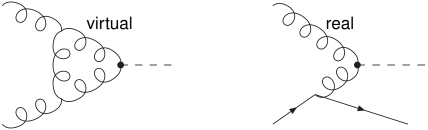
  $\Rightarrow$ show discrepancy

# Real and Virtual Corrections

- LO



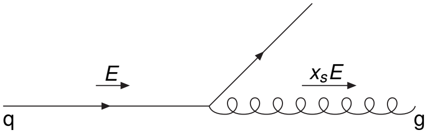

- NLO




- $\sigma^{\text{virtual}} + \sigma^{\text{real}} \neq$ finite $\Rightarrow$ Subtraction terms accounting for collinear emission of the initial states

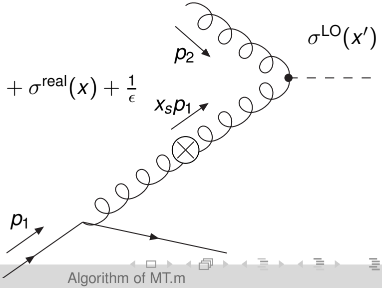
# Initial State Radiation

- Altarelli Parisi splitting functions: $P_{g \leftarrow q}(x_s) =$ Probability for:




- OS-particles, collinearity: $E \to E' = x_s E, p^2 = 0, p'^2 = 0, \vec{p}||\vec{p}'$
  $\Rightarrow p \to p' = x_s p$

$$\sigma^{\text{ren.}}(x) = \sigma^{\text{virtual}}(x) + \sigma^{\text{real}}(x) + \frac{1}{\epsilon}$$




$+ \ldots$

- $x = \frac{m_h^2}{s}$

# What is this $\otimes$ operation?

- $\sigma^{\text{ren.}}(x) \sim P_{g\leftarrow q}(x_s) \otimes \sigma^{\text{LO}}(x')$
- $s = (p_1 + p_2)^2 = 2p_1p_2 \to s' = 2(x_sp_1)p_2 = x_ss$
  $\Rightarrow \frac{m_h^2}{s} = x_s\frac{m_h^2}{s'}$
  $\Rightarrow x = x_sx'$
  $\Rightarrow \otimes \sim \delta(x - x_sx')$
- sum over all $x_s \in [0,1]$ and $x' \in [0,1]$
  $\Rightarrow P_{g\leftarrow q}(x_s) \otimes \sigma^{\text{LO}}(x') := \int_0^1 dx_s \int_0^1 dx' \delta(x - x_sx') P_{g\leftarrow q}(x_s)\sigma^{\text{LO}}(x')$
  $\Rightarrow$ Convolutions of $P_{i\leftarrow j}$ with $\sigma_{ij}$
- all possible convolutions connecting the parton lines, at N3LO e.g.
  $P^{(1)}_{q\leftarrow g} \otimes \frac{\sigma^{\text{NLO}}_{qg}}{x} \otimes P^{(1)}_{g\leftarrow g}$

# Mellin Transform

- $M_n[f(x)] := \int_0^1 dx\, x^{n-1} f(x)$
- $M_n[(f \otimes g)(x)] := M_n[f(x)]\, M_n[g(x)]$
- structure of $\sigma_{ij}(x)$, $P_{ij}(x)$:

$$\left\{\frac{1}{x}, \frac{1}{1-x}, \frac{1}{1+x}\right\} \times \left\{\delta(1-x), \left[\frac{\ln^k(1-x)}{1-x}\right]_+, H_{\vec{w}(4)}(x)\right\}$$

- $M_n\left[\hat{\partial}_x H_1(x)\right] = M_n\left[\left[\frac{1}{1-x}\right]_+\right]$

  $\Rightarrow$ unified treatment of HPLs and Plus Distributions

# Mellin Transforms

## HPLs

$$M_n[1] = \frac{1}{n},$$
$$M_n[H_0(x)] = -\frac{1}{n^2},$$
$$M_n[H_1(x)] = \frac{S_1(n)}{n},$$
$$M_n[H_{-1}(x)] = -\frac{(-1)^n}{n}(S_{-1}(n) + \ln(2)) + \frac{\ln(2)}{n},$$
$$M_n[H_{\vec{w}}(x)] = \ldots$$

## reg. deriv. of HPLs

$$M_n\left[\hat{\partial}_x 1\right] = 1$$
$$M_n\left[\hat{\partial}_x H_1(x)\right] = -S_1(n-1)$$
$$M_n\left[\hat{\partial}_x H_0(x)\right] = \frac{1}{n-1}$$
$$M_n\left[\hat{\partial}_x H_{-1}(x)\right] = (-1)^{n-1}(S_{-1}(n-1) + \ln(2))$$
$$M_n\left[\hat{\partial}_x H_{\vec{w}}(x)\right] = \ldots$$

- HPL tables generated using `harmpol.h` [Remiddi, Vermaseren; '00]

# Algorithm

1. transform expressions to mellin space
2. tabulate Mellin transforms of HPLs up to certain weight; here: 6
   $\Rightarrow$ harmonic sums and transcendental numbers
3. calculate Mellin transforms of regularized derivatives of HPLs
4. solve system for

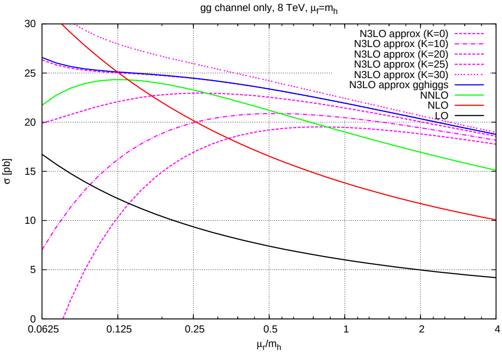
$$\left\{ \frac{1}{n^k}, \frac{S_{\dots}(n)}{n^k}, (-1)^n \frac{S_{\dots}(n)}{n^k} \right\}$$

$\Rightarrow$ inverse Mellin transforms

- implementation in `Mathematica` package `MT.m` [MH, Hoff, Pak, Steinhauser, Ueda, '13]
- using the `HPL.m` package [Maitre; '06, '12]

# New approx. available

- different algorithm and extraction of full explicit scale dependence [Bühler, Lazopoulos, '13] ("approx")




- [Ball, Bonvini, Forte, Marzani, Ridolfi, '13] ("approx gghiggs")
  ⇒ agree for "rescaling factor" K=25

# Status of N3LO higgs prod. at the LHC

- virtual [Baikov, Chetyrkin, Smirnov, Steinhauser, '09], [Gehrmann, Glover, Huber, Ikizlerli, Studerus, '10]
- real, expansion in $(1 - m_h^2/s)$ [Anastasiou, Duhr, Dulat, Mistlberger, '13]
- mixed real-virtual, unknown
- lower order cross sections convoluted with splitting functions [MH, Hoff, Pak, Steinhauser, Ueda, '12], [Bühler, Lazopoulos, '13]