

NEUTRINO PHYSICS

Lecture I :

- Historical introduction: neutrinos in the Standard Model
- Neutrino masses and mixing : Majorana versus Dirac
- Neutrino oscillations in vacuum and in matter

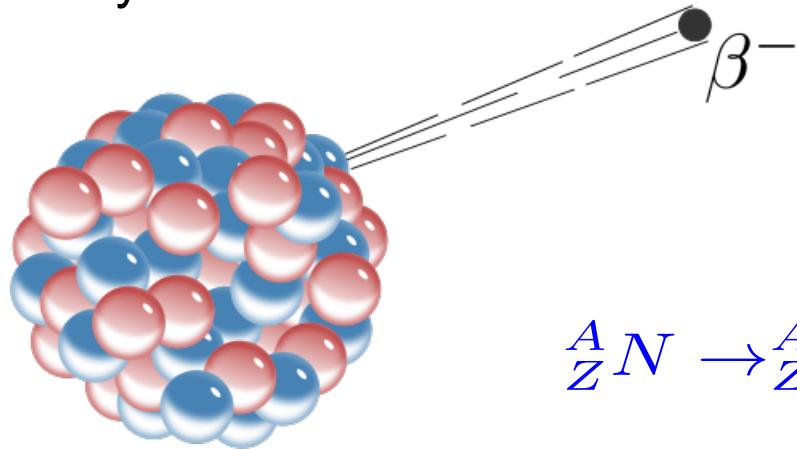
Lecture II:

- The standard 3ν scenario and a few outliers...
- Prospects in neutrino physics

Neutrino: the dark particle

1900 Radioactivity: Becquerel, M & P Curie, Rutherford....

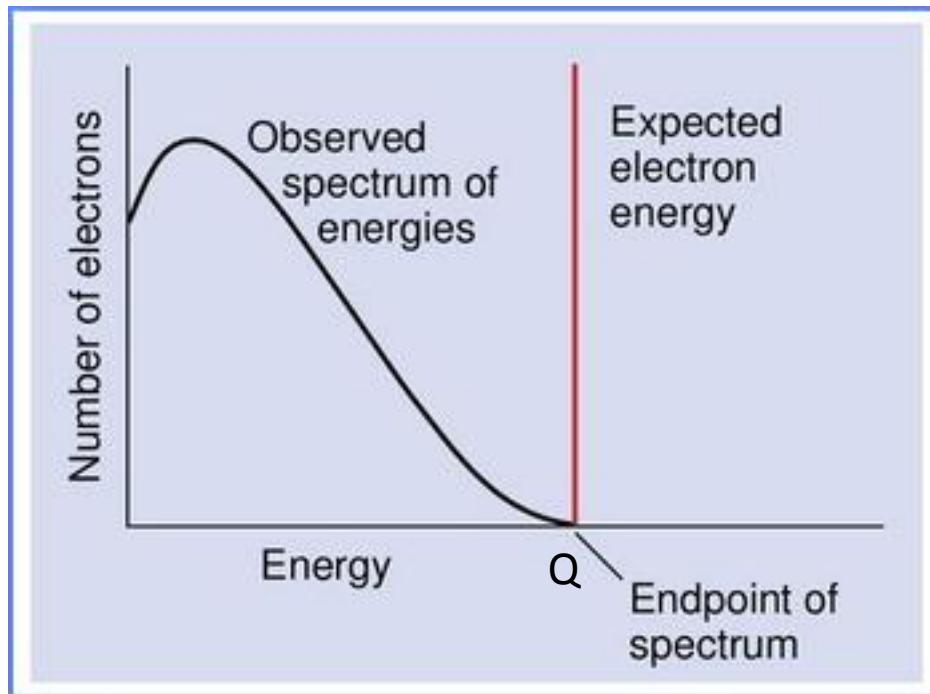
β^- decay



Energy conservation: $E_{\text{electron}} \simeq (M_N - M_{N'})c^2 = Q = \text{constante}$

1911/1914

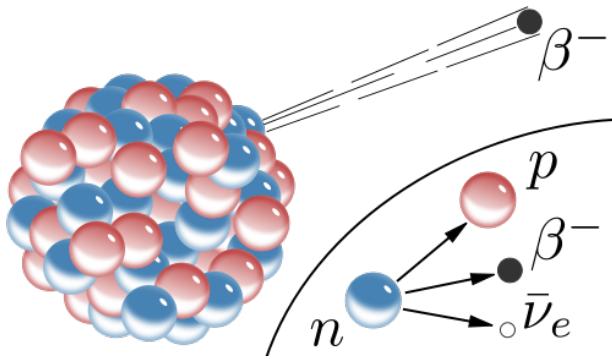
Electron spectrum:



Meitner, Hahn
(Nobel 1944 only him!)



Chadwick (Nobel 1935)



1930



Dear Radioactive Ladies and Gentlemen,

Pauli (Nobel 1945)

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin $1/2$ and obey the exclusion principle, and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball on the night from December 6 to 7....

1933: Solvay's conference

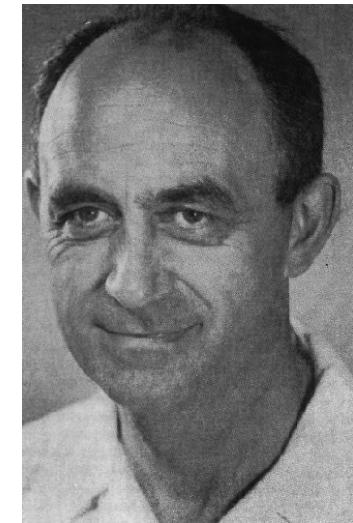
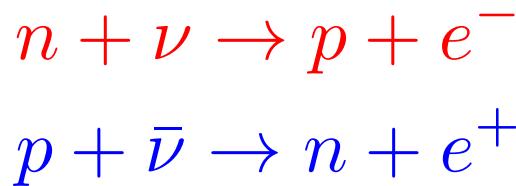
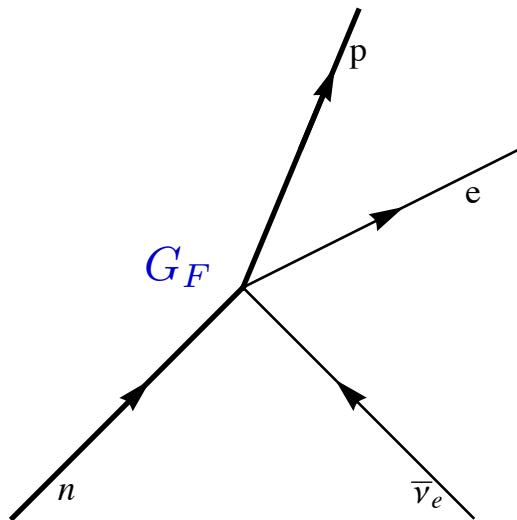
The neutron was discovered in 1932 by Chadwick ...



"... their mass can not be very much more than the electron mass. In order to distinguish them from heavy neutrons, mister Fermi has proposed to name them "neutrinos". It is possible that the proper mass of neutrinos be zero... It seems to me plausible that neutrinos have a spin 1/2... We know nothing about the interaction of neutrinos with the other particles of matter and with photons: the hypothesis that they have a magnetic moment seems to me not funded at all."

W. Pauli

1934: Theory of beta decay



E. Fermi
(Nobel 1938)

Nature did not publish his article: “contained speculations too remote from reality to be of interest to the reader...”

Bethe-Peierls (1934): compute the neutrino cross section using this theory

$$\sigma \simeq 10^{-44} \text{ cm}^2, E(\bar{\nu}) = 2 \text{ MeV}$$

“there is not practically possible way of detecting a neutrino”

How to detect them ?

$$\lambda \simeq \frac{1}{n\sigma}$$

$$\lambda|_{\text{water}} \simeq 1.5 \times 10^{21} \text{ cm} \simeq 1600 \text{ Light Years}$$

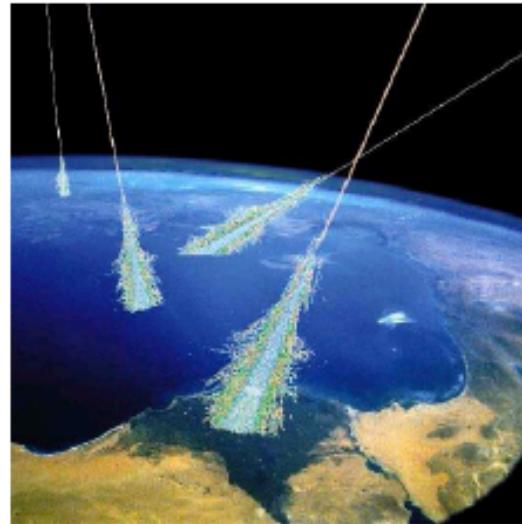
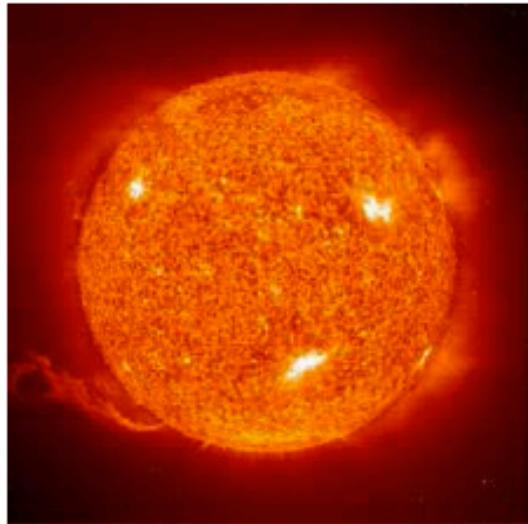
$$\lambda|_{\text{interstelar}} \simeq 10^{44} \text{ cm} \simeq 10^{26} \text{ Light Years}$$

“I have done a terrible thing. I have postulated a particle that cannot be detected”

W. Pauli

“Not even wrong”

Revealing Pauli's dark matter was just a question of time and ingenuity...



Enjoyable reading "Neutrinos" by F. Close

How to detect them?

1946 Pontecorvo

Not so desperate...



Бруно Понтеорво

$$\begin{aligned}N_{CC} &= \Phi_\nu \times \sigma \times \text{Numero de blancos} \times \Delta T \\&= \Phi_\nu (cm^{-2}s^{-1}) \times 10^{-44} cm^2 \times N_{\text{Avogadro}} \times \text{Detector mass(gr)} \times 10^5 s \times \# \text{dias}\end{aligned}$$

In a **1000kg** detector, a **10^{10} v/s** a few events per day

Needs a reaction where the final isotope is radioactive with a proper lifetime

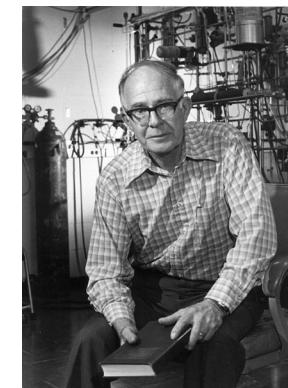


Then the (by-then) recently invented nuclear reactors could be this source...

Reactors: $\sim 10^{20}$ /second!



1955 Davies



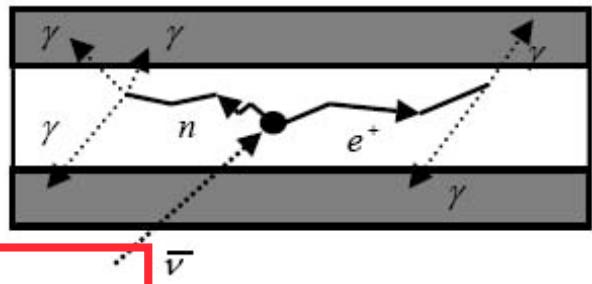
Built a 4000 liter detector, but did not see a thing...

$(10^{11}/\text{s}@\text{100 meters})$

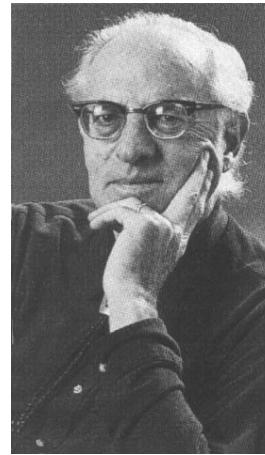


1956 (anti)neutrino detection

Poltergeist project

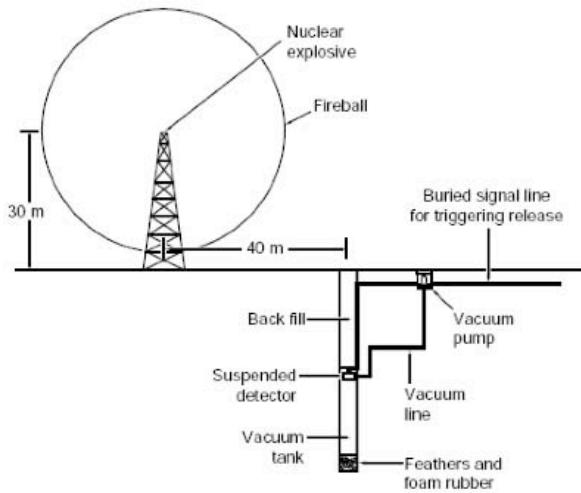


Scintillator
H₂O + CdCl₂
Scintillator



Reines Nobel 95

Cowan (died 74)



First idea: put the detector close to a nuclear explosion !

Finally used the reactor Savannah River to discover the anti-neutrino

The flavour of neutrinos

1937 μ discovered in cosmic rays

1947 Pontecorvo

Is a heavy version of the electron and not the nuclear agent (pion)



Бруно Понтекорво

$$\pi \rightarrow \mu \bar{\nu}_\mu$$

1959 Pontecorvo

The neutrino that accompanies the μ is different to that in beta decay

Neutrino cross section in Fermi theory grows with energy: he proposes the first experiment with a neutrino beam !

Neutrino Flavour

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$



Lederman

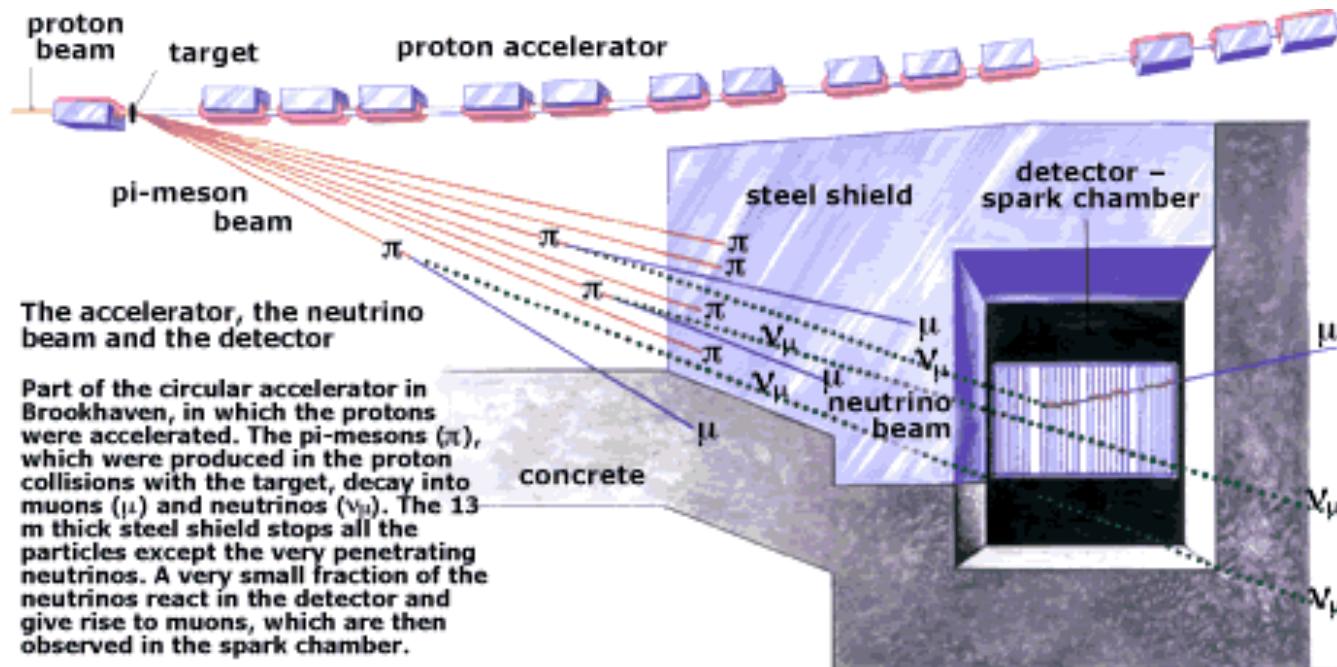


Schwartz



Steinberger

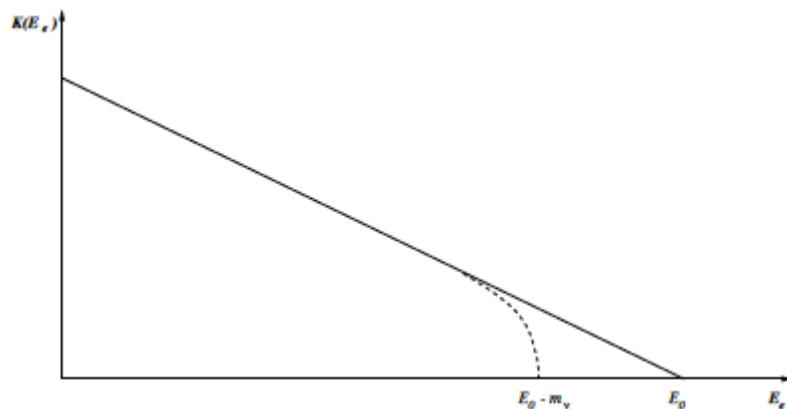
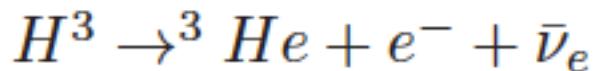
Nobel 1988



Based on a drawing in Scientific American, March 1963.

Kinematical effects of neutrino mass

Most stringent from Tritium beta-decay



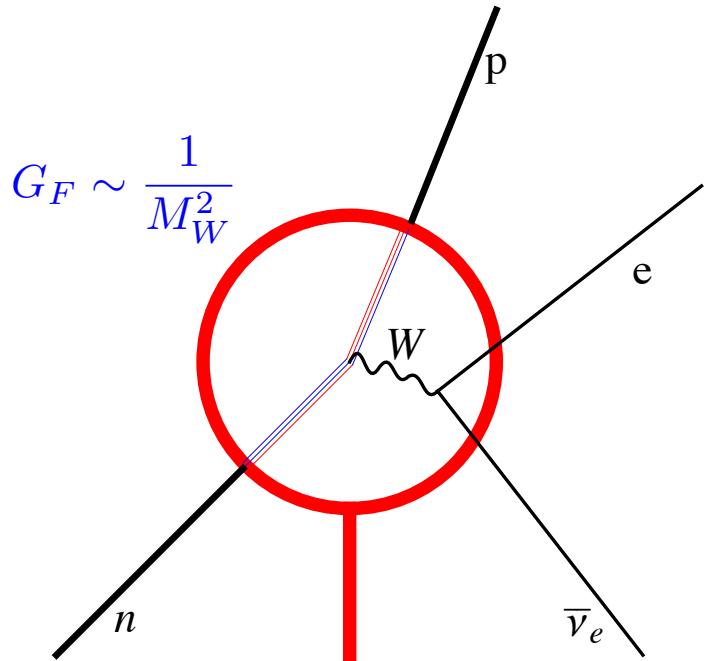
$m_{\nu_e} < 2.2\text{eV}$ (Mainz-Troitsk)

$m_{\nu_\mu} < 170\text{keV}$ (PSI: $\pi^+ \rightarrow \mu^+ \nu_\mu$)

$m_{\nu_\tau} < 18.2\text{MeV}$ (LEP: $\tau^- \rightarrow 5\pi\nu_\tau$)

Standard Model neutrinos assumed massless

Neutrinos in the Standard Model



$$SU(3) \times SU(2) \times U(1)_Y$$

$(1, 2)_{-\frac{1}{2}}$	$(3, 2)_{-\frac{1}{6}}$	$(1, 1)_{-1}$	$(3, 1)_{-\frac{2}{3}}$	$(3, 1)_{-\frac{1}{3}}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$ $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$ $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad \begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$		e_R μ_R τ_R	u_R^i c_R^i t_R^i	d_R^i s_R^i b_R^i

Left-handed



Right-handed



Charged currents: CC

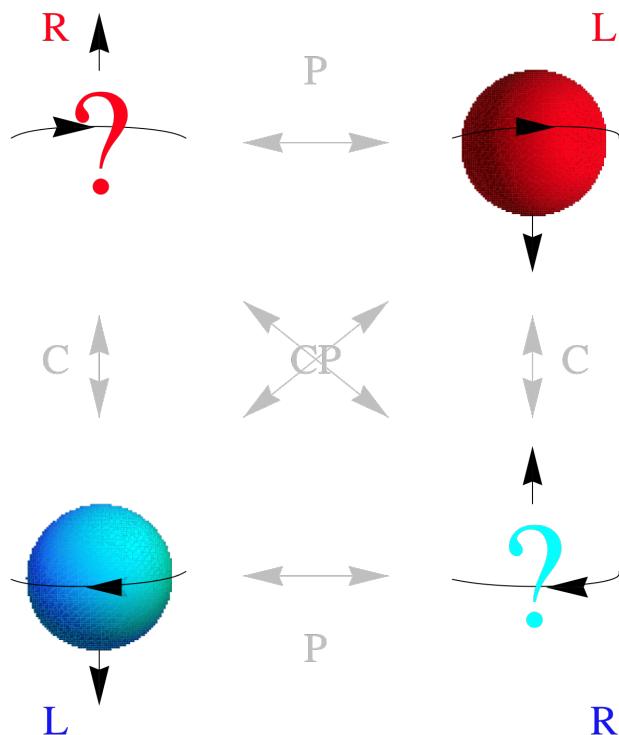
Weyl fermions

$$\nu = \frac{1 - \gamma_5}{2} \nu \simeq \frac{1}{2} \left(1 - \underbrace{\frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}}_{\text{Helicity}} \right) \nu + \mathcal{O}(v/c)$$

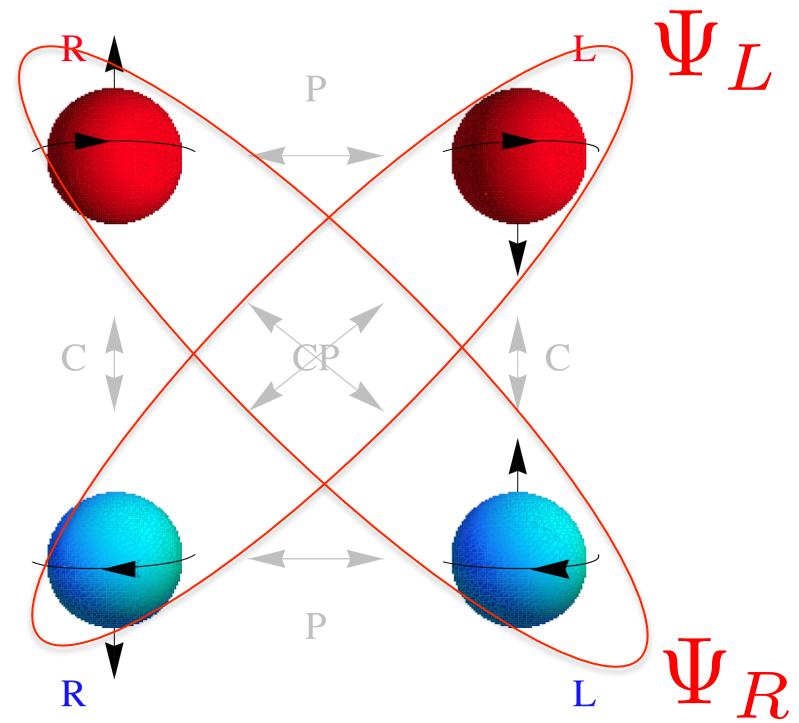
$$\Psi_{L/R} \equiv P_{L/R} \Psi$$

$$P_{L/R} \equiv \frac{1 \mp \gamma_5}{2}$$

Weyl fermion= 2-component spinor
 (Minimal spin $\frac{1}{2}$)

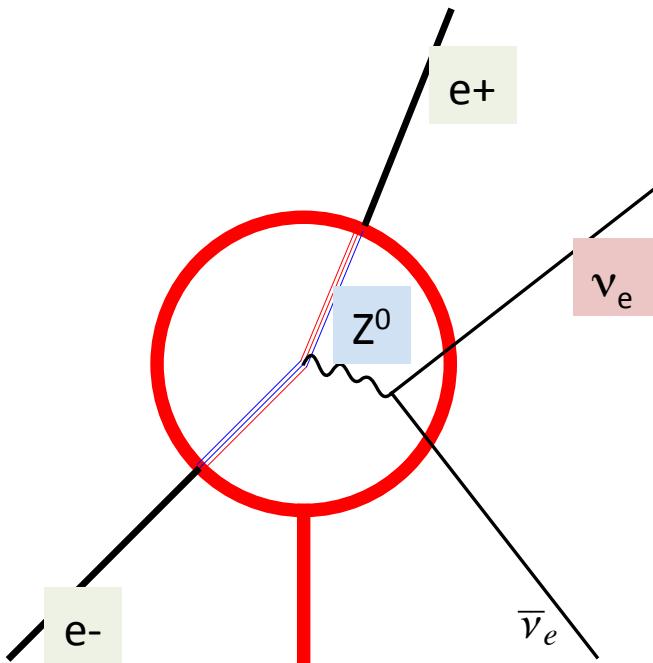


Dirac fermion= 4-component spinor
 (Minimal spin $\frac{1}{2} + \text{Parity}$)



Weyl fermion field = negative helicity particle + positive helicity anti-particle

Neutrinos in the Standard Model



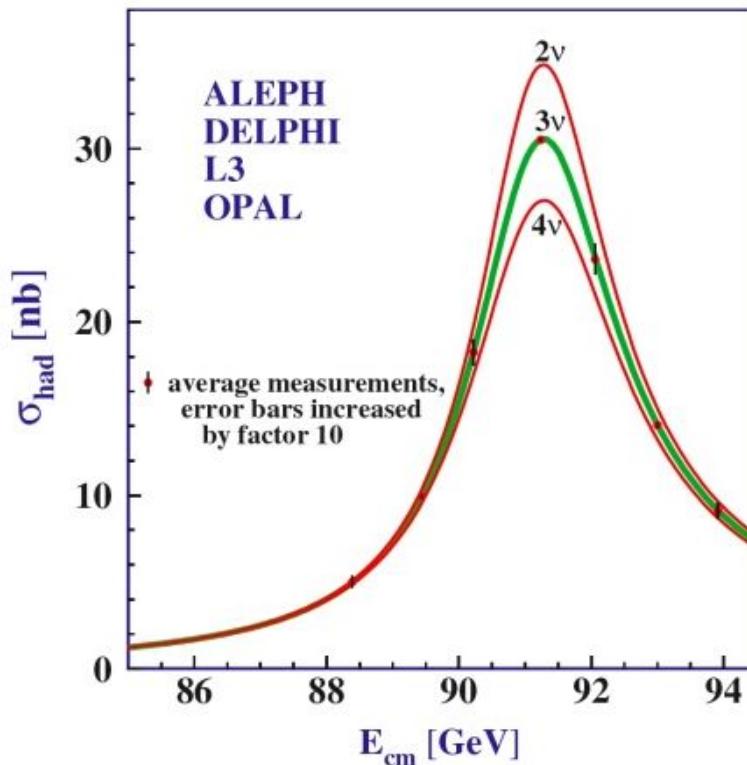
Neutral currents: NC

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{\nu\bar{\nu}}} = 2.984 \pm 0.008$$

At LEP:

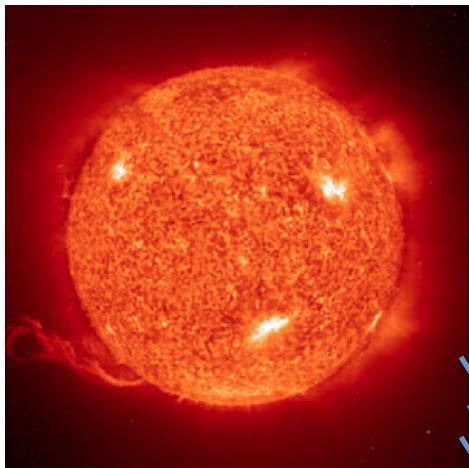


Only three neutrinos were found

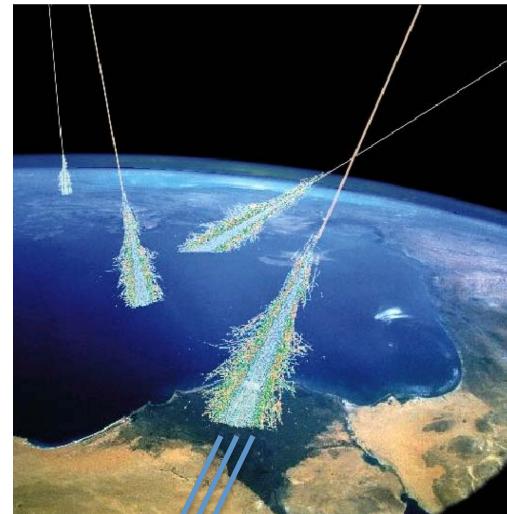


Ubiquitous Neutrinos

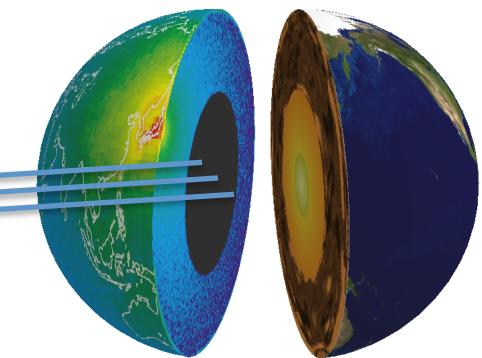
They are everywhere...



Sun: $5 \times 10^{12}/\text{second}$

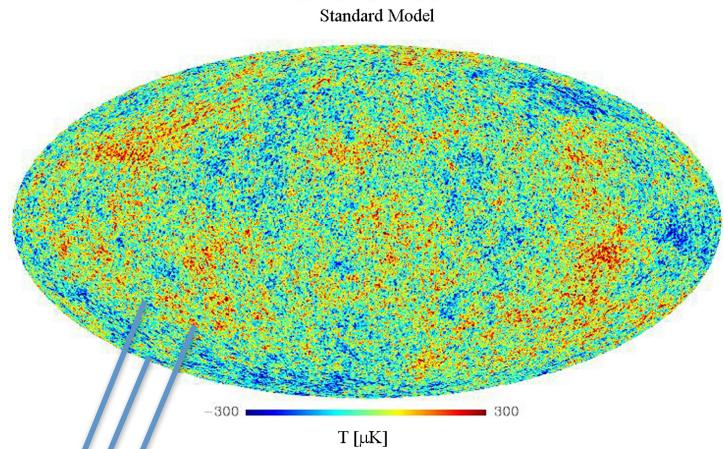


Atmosphere: $\sim 20/\text{second}$



Earth: $\sim 10^9/\text{second}$

Ubiquitous Neutrinos



Simulation showing the distribution on the sky of temperature fluctuations in the Cosmic Microwave Background with neutrinos as in the Standard Model.

Big Bang: $\sim 2 \times 10^{12}/\text{second}$

Supernova 1987: $\sim 10^{12}/\text{second}$

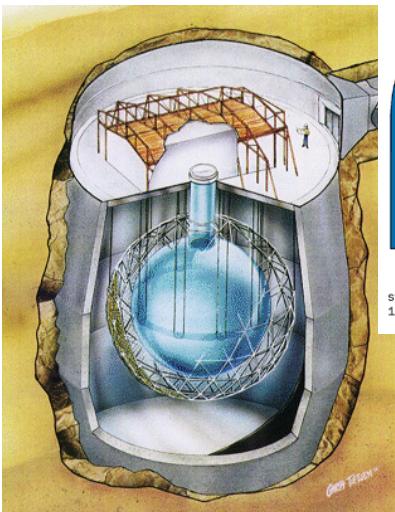
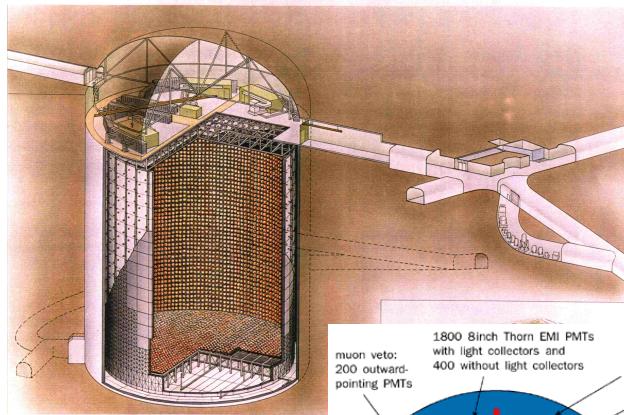
@168000 Light years!
 10^8 farther from Earth



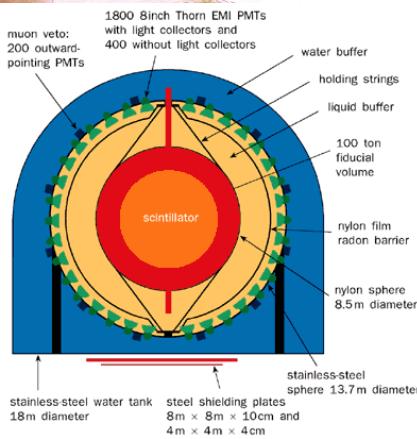
Using many of these sources, and others man-made, a decade of revolutionary neutrino experiments have demonstrated that neutrinos are not quite standard, because they have a tiny mass & massive neutrinos require new dofs!

Wark's talk

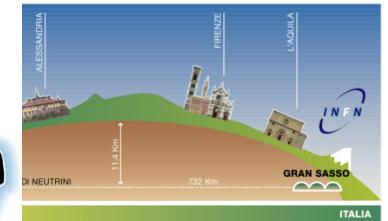
SuperKamiokande



SNO
Borexino



MINOS, Opera



...and more



Massive (free) fermions ?

Dirac fermion of mass m:

$$-\mathcal{L}_m^{\text{Dirac}} = m\bar{\psi}\psi = m(\overline{\psi_L + \psi_R})(\psi_L + \psi_R) = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$



Majorana fermion of mass m (Weyl representation)

$$-\mathcal{L}_m^{\text{Majorana}} = \frac{m}{2}\overline{\psi^c}\psi + \frac{m}{2}\overline{\psi}\psi^c \equiv \frac{m}{2}\psi^T C\psi + \frac{m}{2}\bar{\psi}C\bar{\psi}^T,$$

$$\psi^c \equiv C\bar{\psi}^T = C\gamma_0\psi^* \quad C = i\gamma_2\gamma_0$$

- ✓ Non-zero for Weyl fermion: $\Psi = P_L\Psi \rightarrow \Psi^T C \Psi = \Psi_L^T i\sigma_2 \Psi_L$
- ✓ Lorentz invariant
- ✓ Massive fermion: dispersion relation $E^2 - \mathbf{p}^2 = m^2$

Massive fermions & Weak Interactions ?

Dirac fermion of mass m:

$$-\mathcal{L}_m^{\text{Dirac}} = m\bar{\psi}\psi = m(\overline{\psi_L + \psi_R})(\psi_L + \psi_R) = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

Breaks SU(2)xU(1) gauge invariance!

Majorana fermion of mass m (Weyl representation)

$$-\mathcal{L}_m^{\text{Majorana}} = \frac{m}{2}\overline{\psi^c}\psi + \frac{m}{2}\overline{\psi}\psi^c \equiv \frac{m}{2}\psi^T C\psi - \frac{m}{2}\overline{\psi}C\overline{\psi}^T,$$

No gauge/global symmetry of ψ possible!

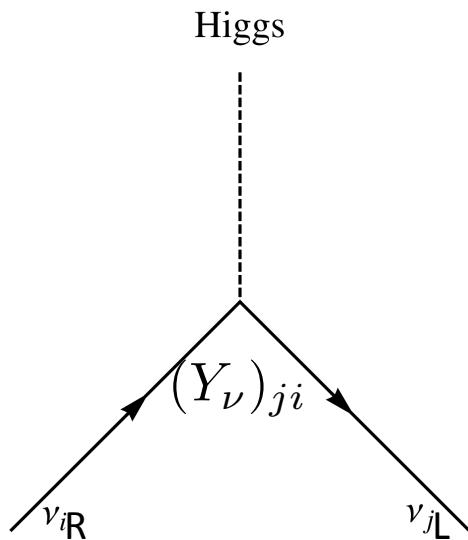
Spontaneous symmetry breaking can induce Dirac masses for all fermions but Majorana masses only for neutrinos !

Massive Dirac neutrinos & SSB ?

$$\tilde{\phi} \equiv \sigma_2 \phi^*, \quad \tilde{\phi} : (1, 2, -\frac{1}{2}), \quad \langle \tilde{\phi} \rangle = \begin{pmatrix} \frac{v}{2} \\ 0 \end{pmatrix}$$

Massive Dirac neutrino

$$-\mathcal{L}_m^{\text{Dirac}} = Y_\nu \underbrace{\bar{L}}_{(1,1,0)} \underbrace{\tilde{\phi}}_{(1,1,0)} \underbrace{\nu_R}_{(1,1,0)} + h.c \rightarrow SSB \rightarrow Y_\nu \bar{\nu}_L \frac{v}{\sqrt{2}} \nu_R + h.c.$$



$$m_\nu = Y_\nu \frac{v}{\sqrt{2}}$$

Massive Majorana neutrinos & SSB ?

$$\tilde{\phi} \equiv \sigma_2 \phi^*, \quad \tilde{\phi} : (1, 2, -\frac{1}{2}), \quad \langle \tilde{\phi} \rangle = \begin{pmatrix} \frac{v}{2} \\ 0 \end{pmatrix}$$

Massive Majorana neutrino

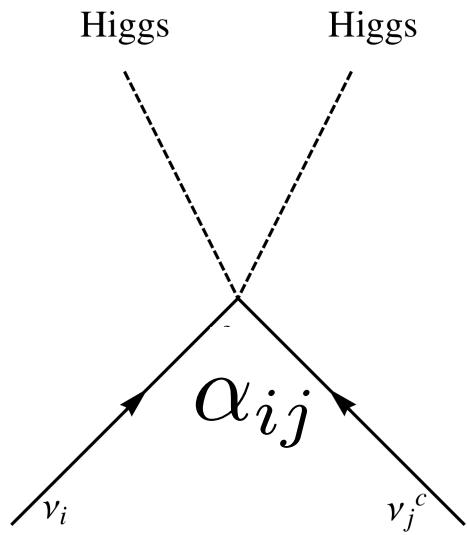
$$-\mathcal{L}^{\text{Majorana}} = \alpha \bar{L} \tilde{\phi} C \tilde{\phi}^T \bar{L}^T + h.c. \rightarrow SSB \rightarrow \alpha \frac{v^2}{2} \bar{\nu}_L C \bar{\nu}_L^T + h.c.$$

Weinberg's operator

$$m_\nu = \alpha \frac{v^2}{2}$$

$$[\alpha] = -1$$

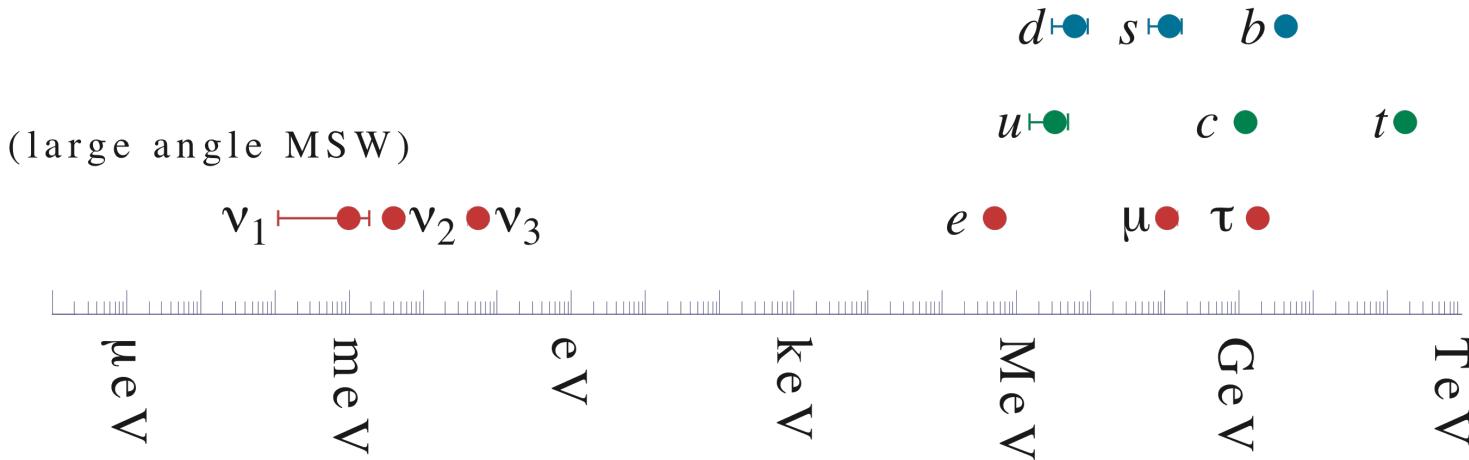
$$\alpha = \frac{Y_\nu}{\Lambda}$$



Implies the existence of a new physics scale unrelated to v !

Massive Majorana neutrinos & SSB ?

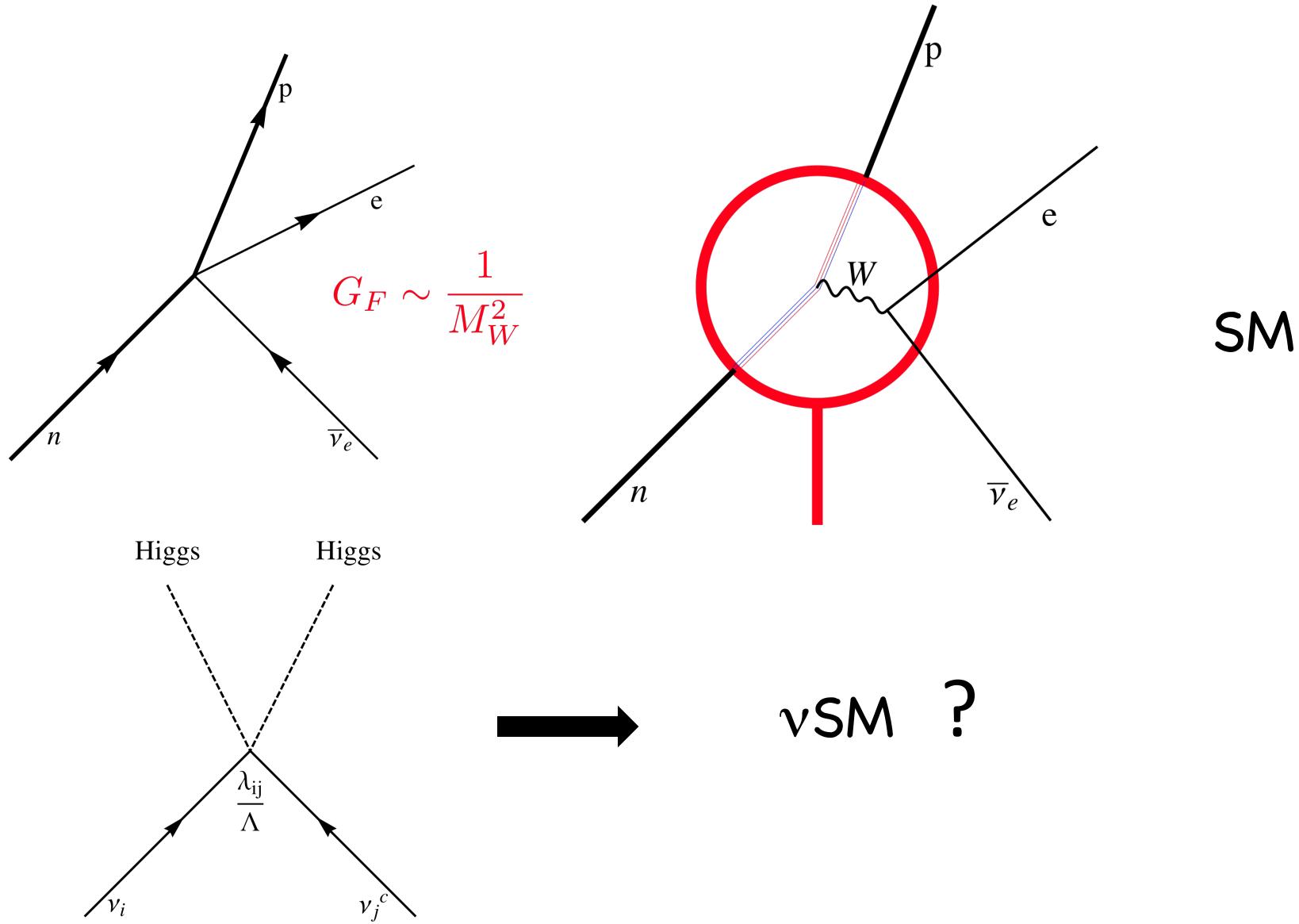
If $\Lambda \gg v$ natural explanation for the smallness of neutrino mass



$$m_f(\text{charged}) \sim Yv, \quad m_\nu \sim Y \frac{v^2}{\Lambda} \sim m_f \frac{v}{\Lambda}$$

Lepton number is not conserved -> a new mechanism to explain the matter/antimatter asymmetry emerges

Majorana neutrinos imply a new Standard Model



Effective Theories of Neutrino Masses (model-independent)

If $\Lambda \gg v$ low-energy effects should be well described by an effective field theory:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{\alpha_i}{\Lambda} O_i^{d=5} + \sum_i \frac{\beta_i}{\Lambda^2} O_i^{d=6} + \dots$$

Weinberg; Buchmuller, Wyler;...

O_i^d built from SM fields satisfying the gauge symmetries

Only one with $d=5$: Weinberg's operator or neutrino masses !

$$O^{d=5} = \bar{L} \tilde{\Phi} C \tilde{\Phi}^T \bar{L}^T + h.c.$$

Neutrino masses & lepton mixing (Dirac)

Are generic complex matrices in flavour space

$$-\mathcal{L}_m^{lepton} = \bar{\nu}_{Li} \underbrace{(M_\nu)_{ij}}_{3 \times n_R} \nu_{Rj} + \bar{l}_{Li} \underbrace{(M_l)_{ij}}_{3 \times 3} l_{Rj} + h.c.$$

$$M_\nu = U_\nu^\dagger \text{Diag}(m_1, m_2, m_3) V_\nu, \quad M_l = U_l^\dagger \text{Diag}(m_e, m_\mu, m_\tau) V_l$$

In the mass eigenbasis

$$\mathcal{L}_{\text{gauge-lepton}} \supset -\frac{g}{\sqrt{2}} \bar{l}'_{Li} \underbrace{(U_l^\dagger U_\nu)_{ij}}_{U_{PMNS}} \gamma_\mu W_\mu^- \nu'_{Lj} + h.c.$$

Pontecorvo-Maki-Nakagawa-Sakata

$U_{PMNS}(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$ unitary matrix analogous to CKM

Why only one phase ?

Counting physical parameters in lepton mixing (Dirac)

physical parameters = # parameters in Yukawas
- # parameters in field redefinitions
+ # parameters of exact symmetries

		Field Redef.	Symmetries	Physical
	Y_v, Y_l	$U_L(n) \times U_{lR}(n) \times U_{vR}(n)$	$U(1)_L$	
Moduli	$2 n^2$	$3 (n^2 - n)/2$	0	$n^2/2 + 3 n/2$
Phases	$2 n^2$	$3 (n^2 + n)/2$	1	$n^2/2 - 3 n/2 + 1$

Moduli = $2n$ masses + $n(n-1)/2$ angles For $n=3$: 3 angles, 1 phase

Neutrino masses & lepton mixing (Majorana)

Are generic complex matrices in flavour space

$$-\mathcal{L}_m^{lepton} = \frac{1}{2} \bar{\nu}_{Li} (M_\nu)_{ij} \nu_{Lj}^c + \bar{l}_{Li} (M_l)_{ij} l_{Rj} + h.c.$$

$$M_\nu^T = M_\nu \rightarrow M_\nu = U_\nu^T \text{Diag}(m_1, m_2, m_3) U_\nu$$

In the mass eigenbasis

$$\mathcal{L}_{\text{gauge-lepton}} \supset -\frac{g}{\sqrt{2}} \bar{l}'_{Li} \underbrace{(U_l^\dagger U_\nu)_{ij}}_{U_{PMNS}} \gamma_\mu W_\mu^- \nu'_{Lj} + h.c.$$

$U_{PMNS}(\theta_{12}, \theta_{13}, \theta_{23}, \delta, \alpha_1, \alpha_2)$ depends on three phases

Counting physical parameters in lepton mixing (Majorana)

physical parameters = # parameters in Yukawas
- # parameters in field redefinitions
+ # parameters of field redefinitions of exact symmetries

	Yukawas	Field. Red.	Symmetries	Physical
	α_v, Y_I	$U_L(n) \times U_{IR}(n)$	0	
Moduli	$n(n+1)/2 + n^2$	$n^2 - n$	0	$n^2/2 + 3n/2$
Phases	$n(n+1)/2 + n^2$	$n^2 + n$	0	$n(n-1)/2$

Moduli = 2n masses + n (n-1)/2 angles For n=3: 3 angles, 3 phases

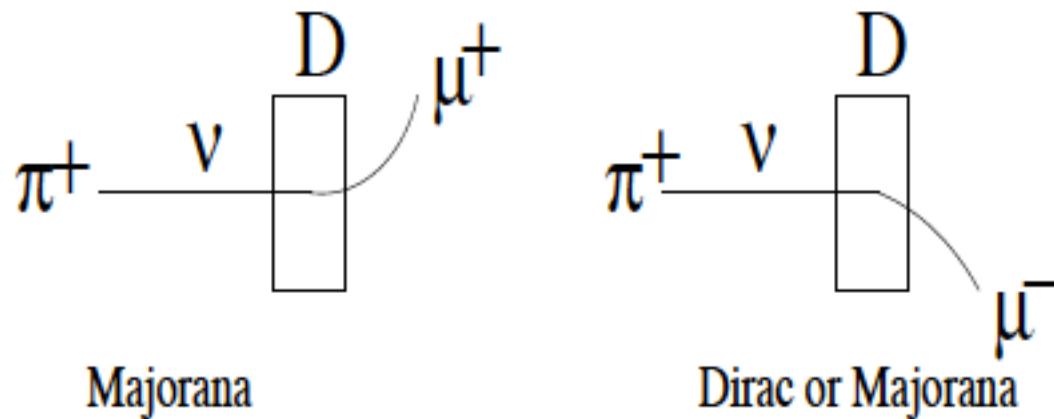
Majorana versus Dirac

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}$$

$c_{ij} \equiv \cos \theta_{ij}$ $s_{ij} \equiv \sin \theta_{ij}$

Majorana phases

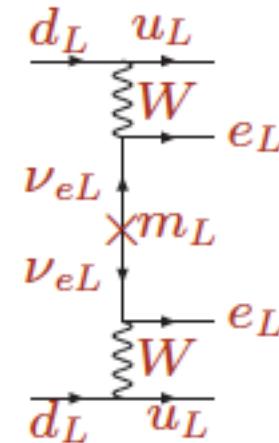
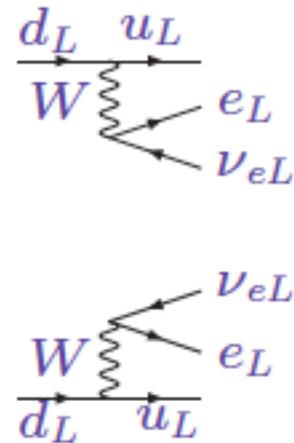
In principle clear experimental signatures



In practice these processes are extremely rare: suppressed by $\left(\frac{m_\nu}{E}\right)^2$

Neutrinoless double- β decay

Best hope is neutrinoless double- β decay



$$T_{2\beta 2\nu} \sim 10^{18} - 10^{21} \text{ years} \quad T_{2\beta 0\nu}^{-1} \sim \left(\frac{m_\nu}{E}\right)^2 10^9 T_{2\beta 2\nu}^{-1}$$

If neutrinos are Majorana this process must be there at some level

Neutrinoless double- β decay

$$T_{2\beta 0\nu}^{-1} \simeq \underbrace{G^{0\nu}}_{\text{Phase}} \underbrace{\left| M^{0\nu} \right|^2}_{\text{NuclearM.E.}} \underbrace{\left| \sum_i \left(V_{MNS}^{ei} \right)^2 m_i \right|^2}_{|m_{ee}|^2}$$

Present bounds:

Sarazin 2012

Isotope	$T_{1/2}^{2\nu}$ (yr)	Experiment	$T_{1/2}^{0\nu}$ (yr) (90% C.L.)	Experiment	$\langle m_{ee} \rangle$ (eV)	Min.	Max.
^{48}Ca	$4.2^{+2.1}_{-1.0} 10^{19}$	NEMO-3	$5.8 10^{22}$	CANDLES [111]	3.55	0.01	
^{76}Ge	$1.5 \pm 0.1 10^{21}$	HDM	$1.9 10^{25}$	HDM [46]	0.2	0.4	GERDA '13
^{82}Se	$9.0 \pm 0.7 10^{19}$	NEMO-3	$3.2 10^{23}$	NEMO-3 [40]	0.85	2.08	
^{96}Zr	$2.0 \pm 0.3 10^{19}$	NEMO-3	$9.2 10^{21}$	NEMO-3 [35]	3.97	14.39	
^{100}Mo	$7.1 \pm 0.4 10^{18}$	NEMO-3	$1.0 10^{24}$	NEMO-3 [40]	0.31	0.79	
^{116}Cd	$3.0 \pm 0.2 10^{19}$	NEMO-3	$1.7 10^{23}$	SOLOTVINO [81]	1.22	2.30	
^{130}Te	$0.7 \pm 0.1 10^{21}$	NEMO-3	$2.8 10^{24}$	CUORICINO [65]	0.27	0.57	
^{136}Xe	$2.38 \pm 0.14 10^{21}$	Kamland	$5.7 10^{24}$	Kamland-Zen [93]			
^{150}Nd	$7.8 \pm 0.7 10^{18}$	NEMO-3	$1.8 10^{22}$	NEMO-3 [37]	2.35	8.65	
^{136}Xe				EXO-Kamland '12	0.12	0.25	

Klapdor et al claim has been excluded by GERDA

Neutrino oscillations

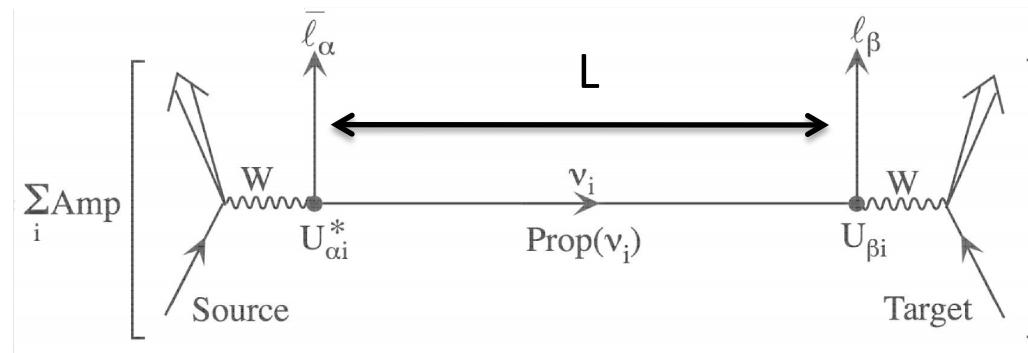
1968 Pontecorvo

If neutrinos are massive

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \dots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



A neutrino experiment is an interferometer in flavour space, because neutrinos are so weakly interacting that can keep coherence over very long distances !



ν_i travel at different velocities in vacuum: neutrino oscillations

Neutrino oscillations

Many ways to derive the oscillation probability master formula

Quantum mechanics with neutrinos as plane waves

Quantum mechanics with neutrinos as wave packets

Quantum Field Theory \leftrightarrow neutrinos as intermediate states

The basic ingredients:

- ✓ Uncertainty in momentum at production & detection (they must be better localized than baseline)
- ✓ Coherence of mass eigenstates over macroscopic distances

Neutrino oscillations in QM (plane waves)

$$|\nu_\alpha(t_0)\rangle = \sum_i U_{\alpha i}^* |\nu_i(\mathbf{p})\rangle, \quad \hat{H} |\nu_i(\mathbf{p})\rangle = E_i(\mathbf{p}) |\nu_i(\mathbf{p})\rangle, \quad \mathbf{p}^2 + m_i^2 = E_i^2(\mathbf{p})$$

↓ time evolution

$$|\nu_\alpha(t)\rangle = \sum_i U_{\alpha i}^* e^{-iE_i(\mathbf{p})(t-t_0)} |\nu_i(\mathbf{p})\rangle$$

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta)(t) &= |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \sum_i U_{\beta i} U_{\alpha i}^* e^{-iE_i(t-t_0)} \right|^2 \\ &= \sum_{i,j} e^{-i(E_i - E_j)(t-t_0)} U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} \end{aligned}$$

$$E_i(\mathbf{p}) - E_j(\mathbf{p}) \simeq \frac{1}{2} \frac{m_i^2 - m_j^2}{|\mathbf{p}|} + \mathcal{O}(m^4) \quad L \simeq t - t_0, v_i \simeq c$$

$$P(\nu_\alpha \rightarrow \nu_\beta)(L) \simeq \sum_{i,j} e^{i \frac{\Delta m_{ji}^2 L}{2E}} U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j}$$

Neutrino Oscillation

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{ij} U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} e^{-i \frac{(m_i^2 - m_j^2)L}{2E}}$$

$\alpha \neq \beta$ appearance probability
 $\alpha = \beta$ disappearance or survival probability

$$L_{osc} \sim \frac{E}{m_i^2 - m_j^2}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \underbrace{2 \sum_{i < j} \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] + \sum_{i=j} |U_{\alpha i}|^2 |U_{\beta i}|^2}_{\delta_{\alpha\beta}}$$

CP-even

$$- 4 \sum_{i < j} \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin^2 \left[\frac{\Delta m_{ji}^2 L}{4E} \right]$$

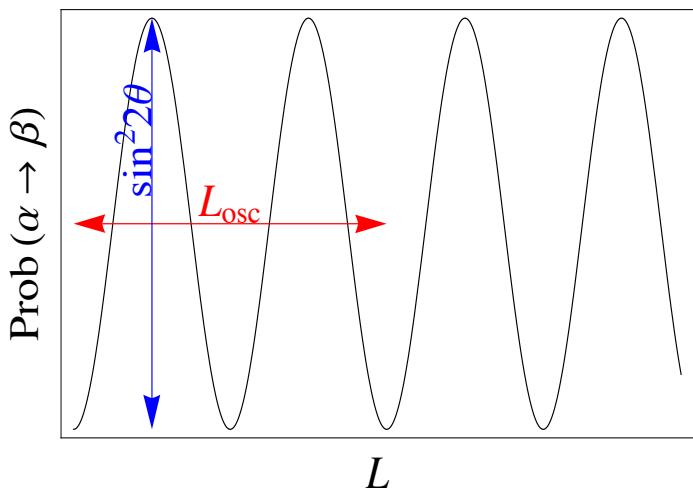
CP-odd

$$- 2 \sum_{i < j} \text{Im}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin \left[\frac{\Delta m_{ji}^2 L}{2E} \right]$$

Neutrino Oscillation: 2ν

Only one oscillation frequency, $U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 (eV^2) L (km)}{E (GeV)} \right)$$



$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - P(\nu_\alpha \rightarrow \nu_\beta)$$

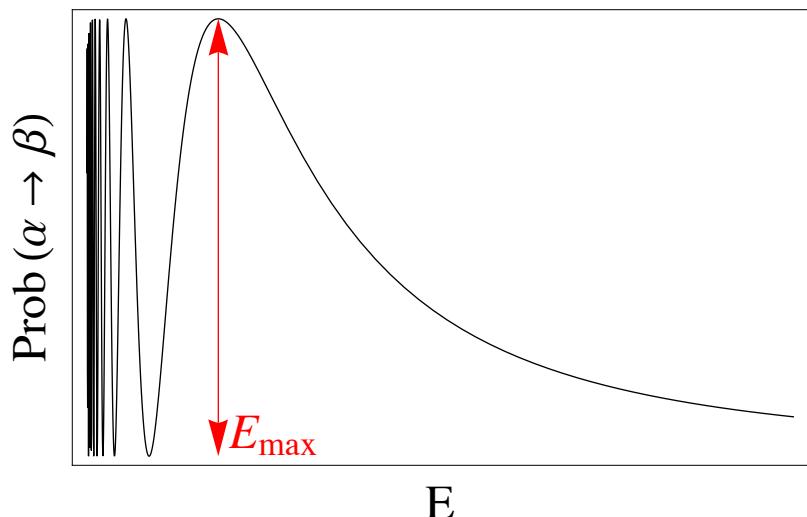
$$L_{osc}(km) = \frac{\pi}{1.27} \frac{E(GeV)}{\Delta m^2(eV^2)}$$

Neutrino Oscillation: 2ν

Only one oscillation frequency, $U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 (eV^2) L(km)}{E(GeV)} \right)$$

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - P(\nu_\alpha \rightarrow \nu_\beta)$$



$$E_{max}(GeV) = 1.27 \frac{\Delta m^2 (eV^2) L(km)}{\pi/2}$$

L , E dependence give Δm^2 amplitude of oscillation gives θ

Optimal experiment: $\frac{E}{L} \sim \Delta m^2$

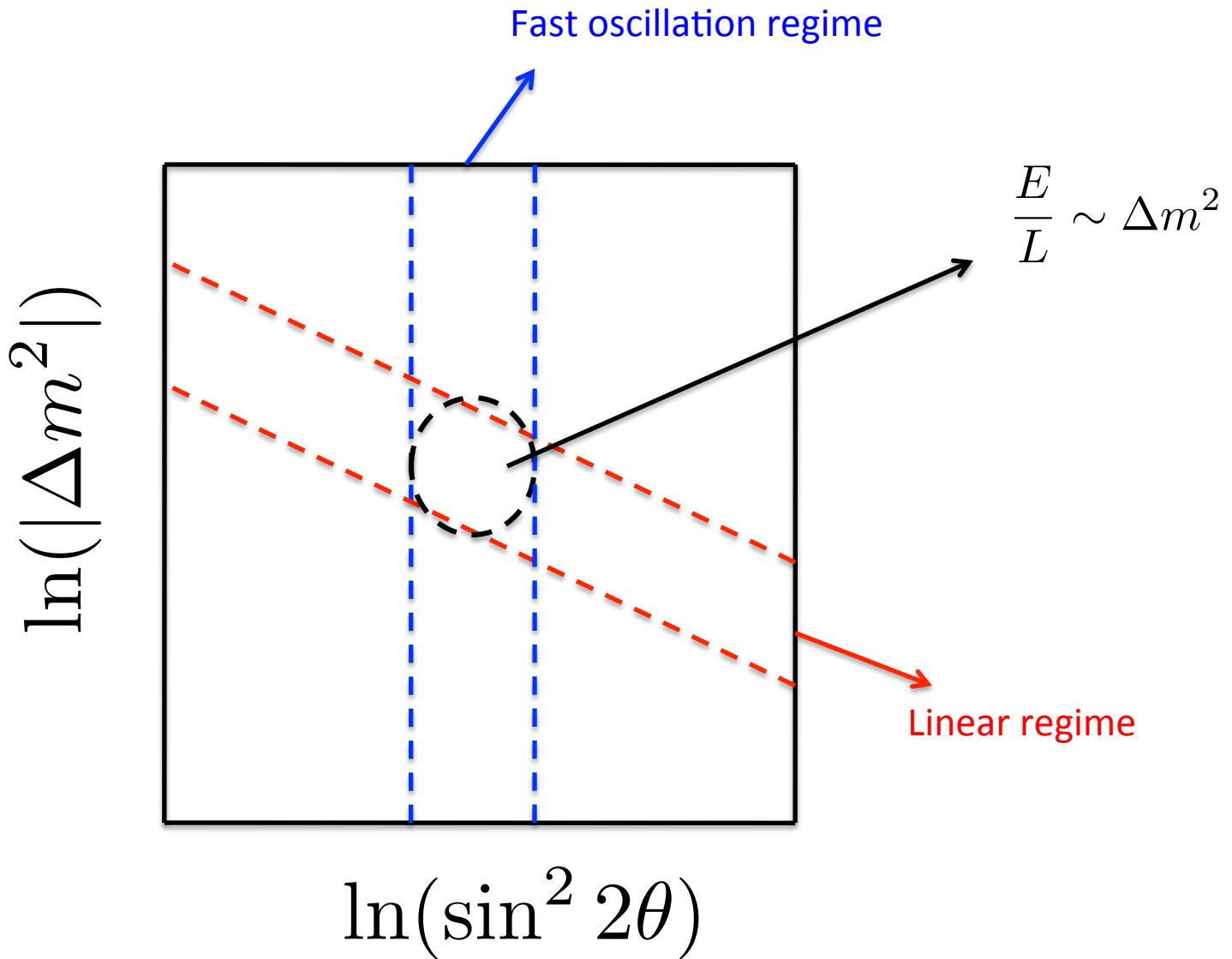
$\frac{E}{L} \gg \Delta m^2$ Oscillation suppressed

$$P(\nu_\alpha \rightarrow \nu_\beta) \propto \sin^2 2\theta (\Delta m^2)^2$$

$\frac{E}{L} \ll \Delta m^2$ Fast oscillation regime

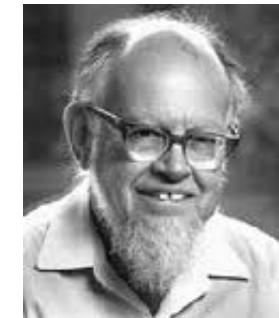
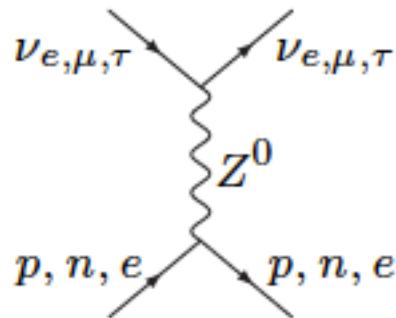
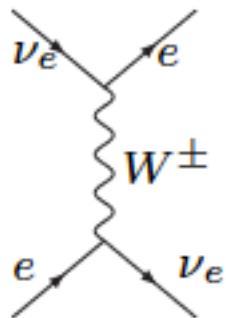
$$P(\nu_\alpha \rightarrow \nu_\beta) \simeq \sin^2 2\theta \left\langle \sin^2 \frac{\Delta m^2 L}{4E} \right\rangle \simeq \frac{1}{2} \sin^2 2\theta = |U_{\alpha 1}^* U_{\beta 1}|^2 + |U_{\alpha 2}^* U_{\beta 2}|^2$$

Equivalent to incoherent propagation: sensitivity to mass splitting is lost



Neutrino Oscillations in matter

Many neutrino oscillation experiments involve neutrinos propagating in matter (Earth for atmospheric neutrinos or accelerator experiments, Sun for solar neutrinos)



Wolfenstein

Index of refraction (coherent forward scattering) can strongly affect the oscillation probability

$$\begin{aligned} \mathcal{H}_{CC} &= \frac{G_F}{\sqrt{2}} [\bar{e} \gamma_\mu (1 - \gamma_5) \nu_e] [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) e] = \frac{G_F}{\sqrt{2}} [\bar{e} \gamma_\mu (1 - \gamma_5) e] [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) \nu_e] \\ \langle \bar{e} \gamma_\mu P_L e \rangle_{\text{unpol. medium}} &= \delta_{\mu 0} \frac{N_e}{2} \end{aligned}$$

Neutrino propagation in matter

$$\langle \mathcal{H}_{CC} + \mathcal{H}_{NC} \rangle_{\text{medium}} = \sqrt{2} G_F \bar{\nu} \gamma_0 \begin{pmatrix} N_e - \frac{N_n}{2} & & \\ & -\frac{N_n}{2} & \\ & & -\frac{N_n}{2} \end{pmatrix} \nu \equiv \bar{\nu} \gamma_0 V_m \nu$$

$$\mathcal{L} \simeq \bar{\nu} (i\partial - M_\nu - \gamma_0 V_m) \nu + \dots$$

$$\mathcal{O}(V_m^2, M_\nu^2 V_m)$$

$$E^2 - \mathbf{p}^2 = \pm 2 V_m E + M_\nu^2$$

Earth: $V_m \simeq 10^{-13} eV \rightarrow 2V_m E \simeq 10^{-4} eV^2 \left[\frac{E}{1GeV} \right]$

Sun: $V_m \simeq 10^{-12} eV \rightarrow 2V_m E \simeq 10^{-6} eV^2 \left[\frac{E}{1MeV} \right]$

Neutrino oscillations in constant matter

Effective mixing angles and masses depend on energy

$$\begin{pmatrix} \tilde{m}_1^2 & 0 & 0 \\ 0 & \tilde{m}_2^2 & 0 \\ 0 & 0 & \tilde{m}_3^2 \end{pmatrix} = \tilde{U}_{\text{PMNS}}^\dagger \left(M_\nu^2 \pm 2E \begin{pmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau \end{pmatrix} \right) \tilde{U}_{\text{PMNS}}$$

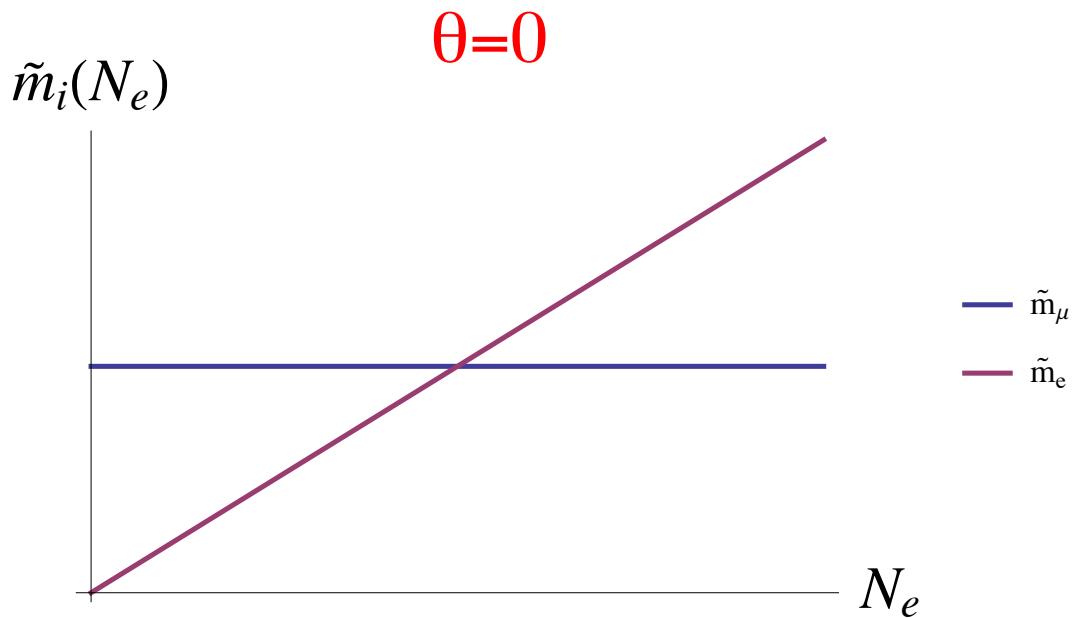
For two families (- neutrinos, + antineutrinos):

$$\sin^2 2\tilde{\theta} = \frac{(\Delta m^2 \sin 2\theta)^2}{(\Delta m^2 \cos 2\theta \pm 2\sqrt{2} G_F E N_e)^2 + (\Delta m^2 \sin 2\theta)^2}$$
$$\Delta \tilde{m}^2 = \sqrt{(\Delta m^2 \cos 2\theta \pm 2\sqrt{2} E G_F N_e)^2 + (\Delta m^2 \sin 2\theta)^2}$$

$$\Delta m^2 \cos 2\theta \pm 2\sqrt{2} G_F E N_e = 0 \quad \sin^2 2\tilde{\theta} = 1, \quad \Delta \tilde{m}^2 = \Delta m^2 \sin 2\theta$$

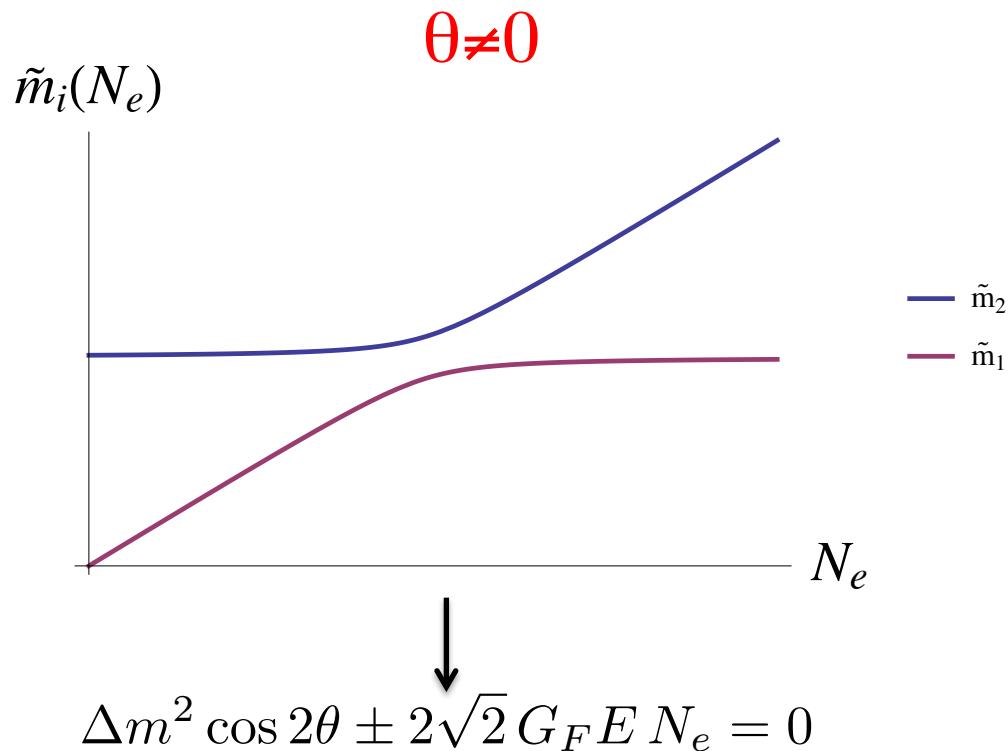
MSW resonance

Mikheyev, Smirnov '85



MSW resonance

Mikheyev, Smirnov '85



MSW Resonance:

- Only for ν or $\bar{\nu}$, not both
- Only for one sign of $\Delta m^2 \cos 2\theta$

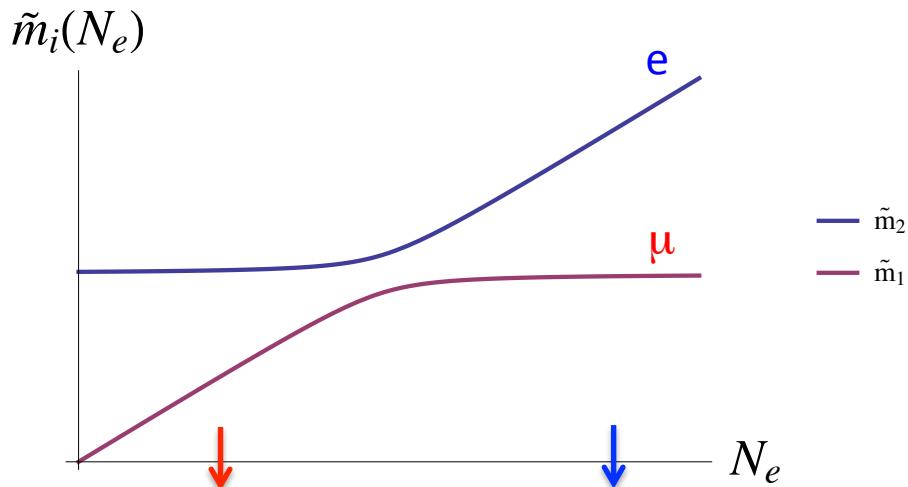
Neutrinos in variable matter

Solar neutrinos propagate in variable matter:

$$N_e(r) \propto N_e(0)e^{-r/R}$$

If the variation is slow enough: **adiabatic approximation** (if a state is at $r=0$ in an eigenstate $\tilde{m}_i^2(0)$ it remains in the i -th eigenstate until it exits the sun)

$$P(\nu_e \rightarrow \nu_e) = \sum_i |\langle \nu_e | \tilde{\nu}_i(\infty) \rangle|^2 |\langle \tilde{\nu}_i(0) | \nu_e \rangle|^2$$

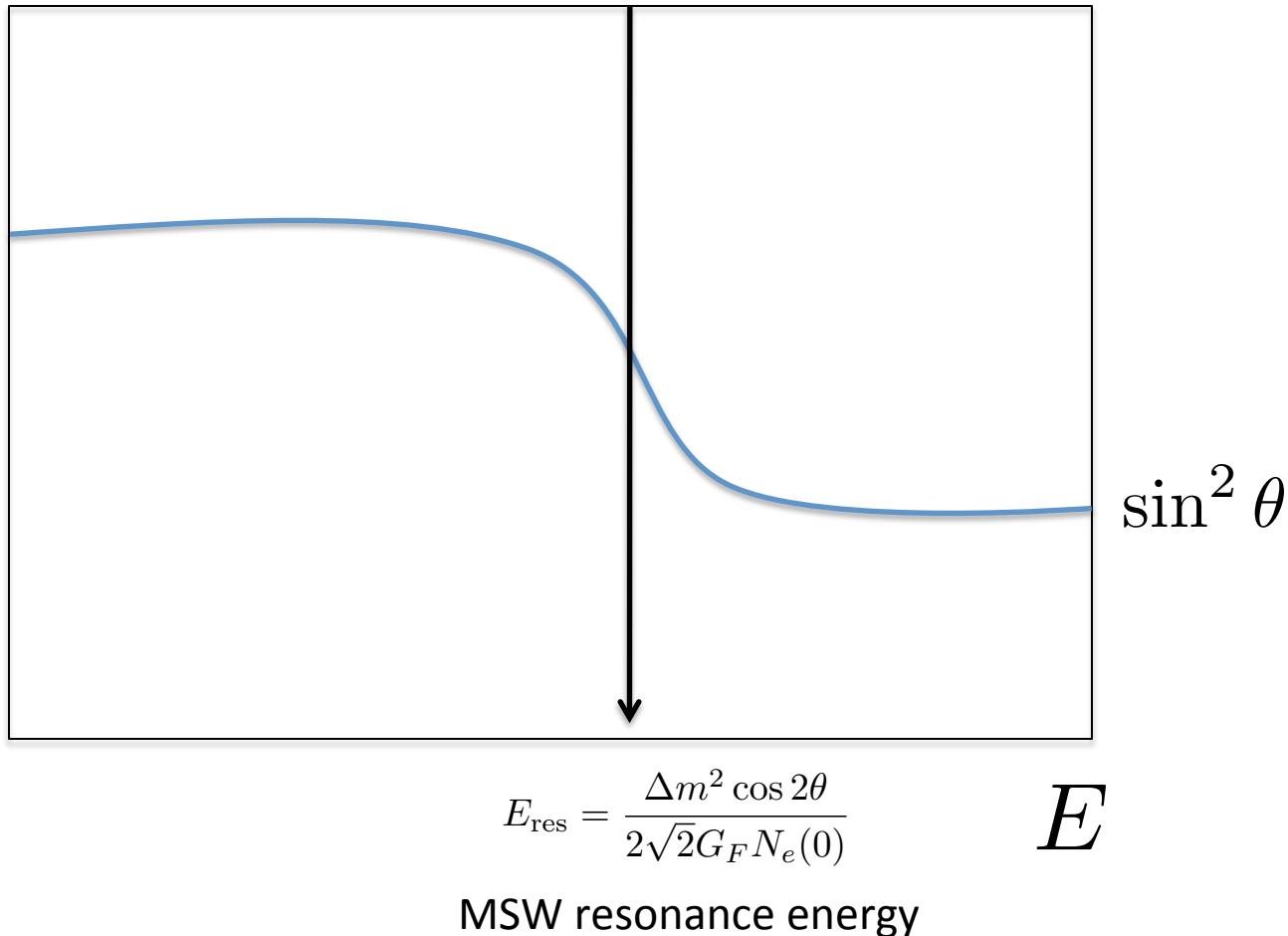


$$P(\nu_e \rightarrow \nu_e) \simeq 1 - \frac{1}{2} \sin^2 2\theta \quad P(\nu_e \rightarrow \nu_e) \simeq \sin^2 \theta$$

Solar neutrinos

$$P(\nu_e \rightarrow \nu_e)$$

$$1 - \frac{1}{2} \sin^2 2\theta$$



In most physical situations: piece-wise constant matter or adiabatic approx. good enough

NEUTRINO PHYSICS

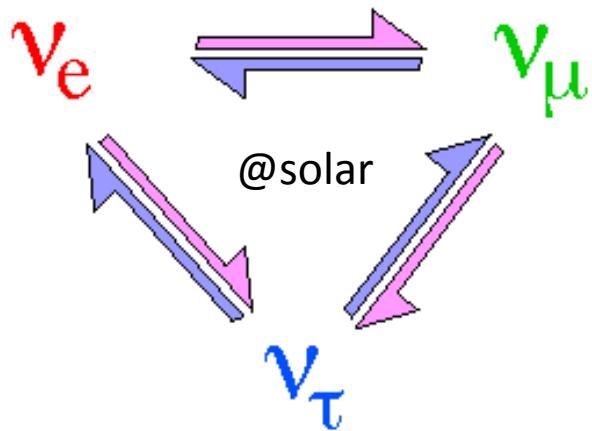
Lecture I :

- A bit of history: neutrinos in the Standard Model
- Neutrino masses and mixing : Majorana versus Dirac
- Neutrino oscillations in vacuum and in matter

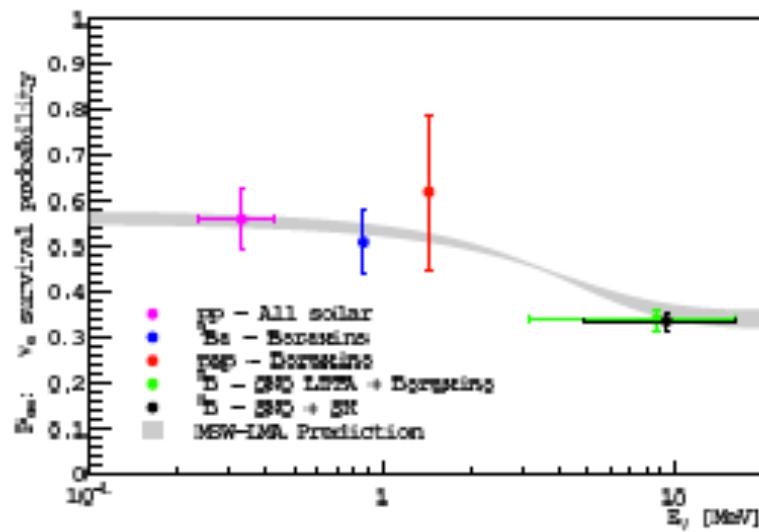
Lecture II:

- The standard 3 ν scenario and a few outliers...
- Prospects in neutrino physics

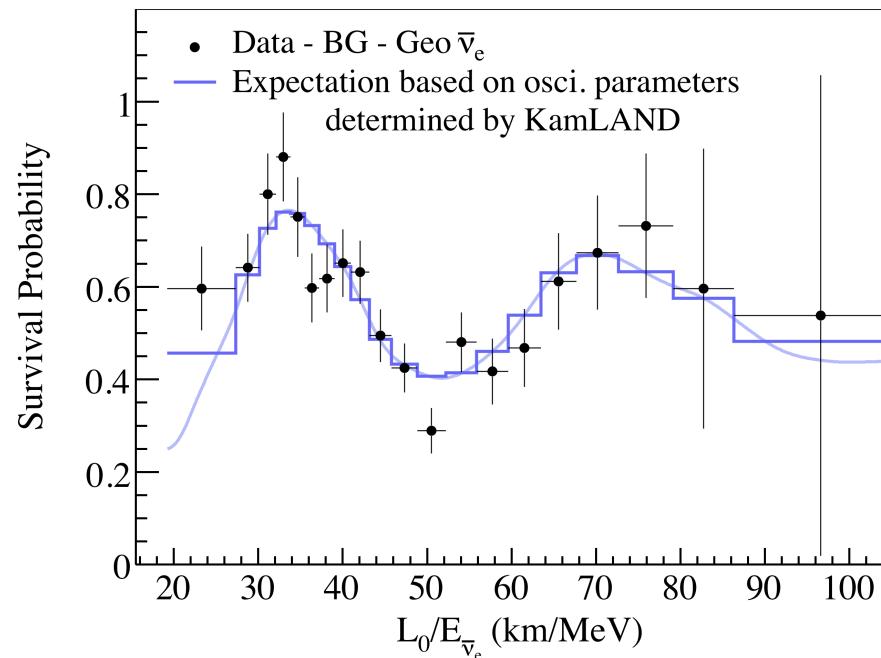
Solar oscillation of ν_e



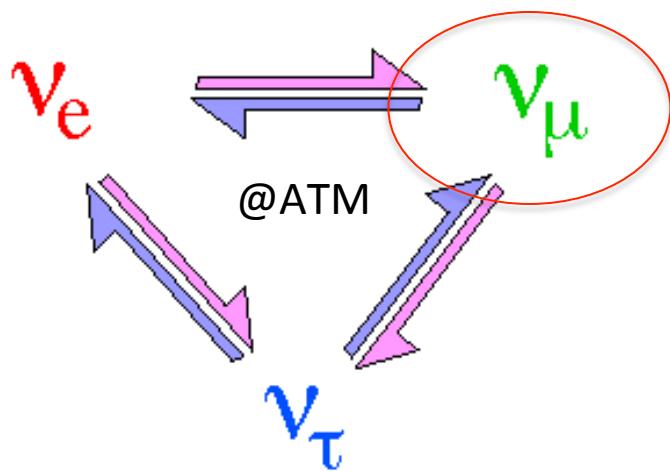
MSW conversion in Sun



$$|\Delta m^2| \sim \frac{\mathcal{O}(MeV)}{\mathcal{O}(100km)}$$



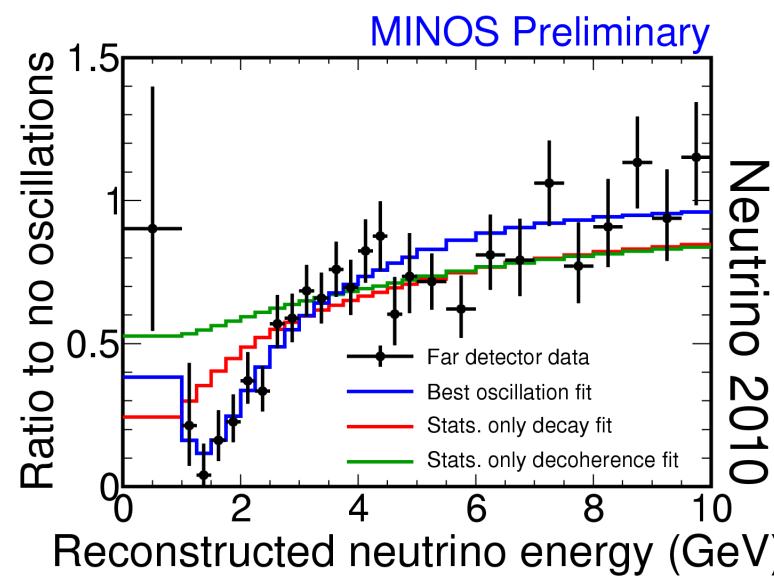
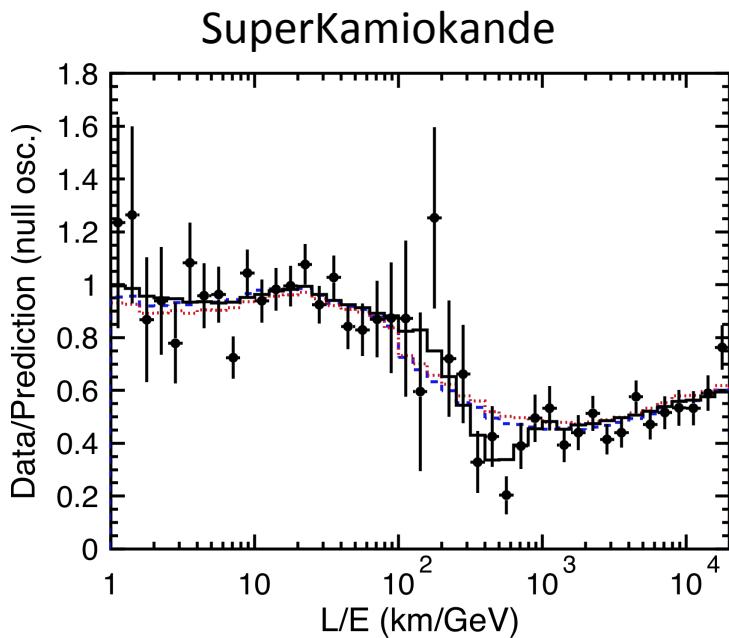
Atmospheric Oscillation of ν_μ



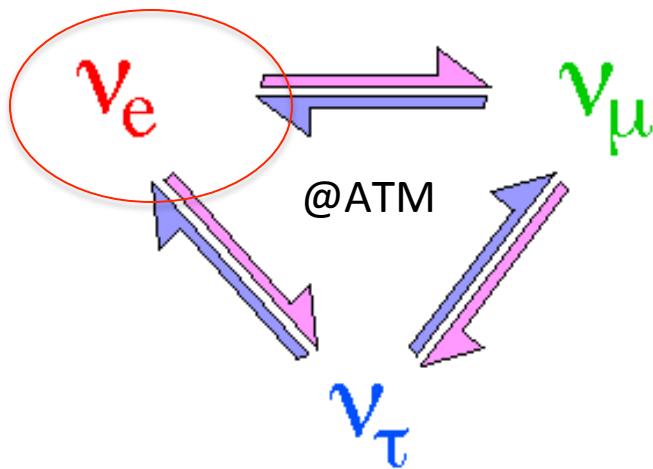
$$|\Delta m^2| \sim$$

$$\frac{\mathcal{O}(GeV)}{\mathcal{O}(1000km)}$$

$$\sim \frac{\mathcal{O}(MeV)}{\mathcal{O}(1km)}$$



Atmospheric Oscillation of ν_e

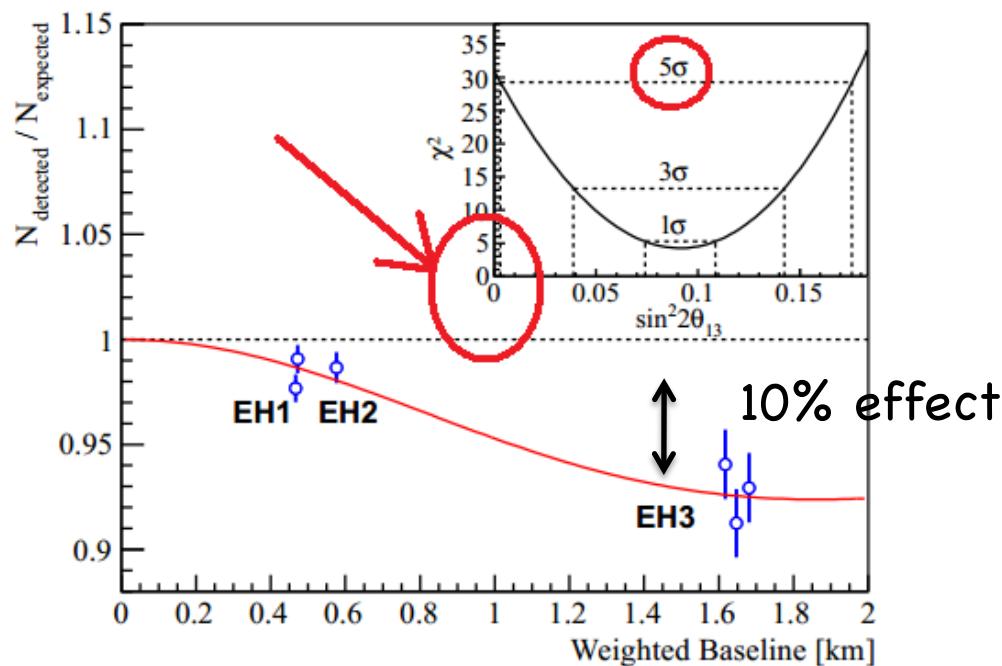


2012

T2K, Double Chooz
Daya Bay, RENO

$$|\Delta m^2| \sim \frac{\mathcal{O}(GeV)}{\mathcal{O}(1000km)}$$

$$\sim \frac{\mathcal{O}(MeV)}{\mathcal{O}(1km)}$$



Standard 3ν scenario

$$\Delta m_{23}^2 = m_3^2 - m_2^2 \equiv \Delta m_{atm}^2$$

$$\Delta m_{12}^2 = m_2^2 - m_1^2 \equiv \Delta m_{sol}^2$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V_{MNS}(\theta_{12}, \theta_{13}, \theta_{23}, \delta) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Solar and atmospheric osc. decouple as 2x2 mixing phenomena:

- hierarchy $\frac{|\Delta m_{atm}^2|}{|\Delta m_{sol}^2|} > 10$
- small θ_{13}

$$E_\nu/L \sim \Delta m^2_{23} \gg \Delta m^2_{12}$$

Chooz

$$P(\nu_e \rightarrow \nu_\mu) = s^2_{23} \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2_{23}}{4E} L \right)$$

$$P(\nu_e \rightarrow \nu_\tau) = c^2_{23} \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2_{23}}{4E} L \right)$$

$$P(\nu_\mu \rightarrow \nu_\tau) = c^4_{13} \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m^2_{23}}{4E} L \right)$$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2_{23}}{4E} L \right) \approx 1 \xrightarrow{\theta_{13}=0}$$

$$E_\nu/L \sim \Delta m_{23}^2 \gg \Delta m_{12}^2$$

Chooz

$$P(\nu_e \rightarrow \nu_\mu) = s_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{23}^2}{4E} L \right) \approx 0$$

$$P(\nu_e \rightarrow \nu_\tau) = c_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{23}^2}{4E} L \right) \approx 0$$

$$P(\nu_\mu \rightarrow \nu_\tau) = c_{13}^4 \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m_{23}^2}{4E} L \right)$$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{23}^2}{4E} L \right) \approx 1$$

Experiments in the atmospheric are described approximately by 2x2 mixing with

$$(\Delta m_{23}^2, \theta_{23}) = (\Delta m_{atm}^2, \theta_{atm})$$

$$E_\nu / L \sim \Delta m_{12}^2 \ll \Delta m_{23}^2$$

$$P(\nu_e \rightarrow \nu_e) = P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq c_{13}^4 \left(1 - \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{12}^2}{4E} L \right) \right) + s_{13}^4$$

Experiments in the solar range are described approximately by 2x2 mixing with

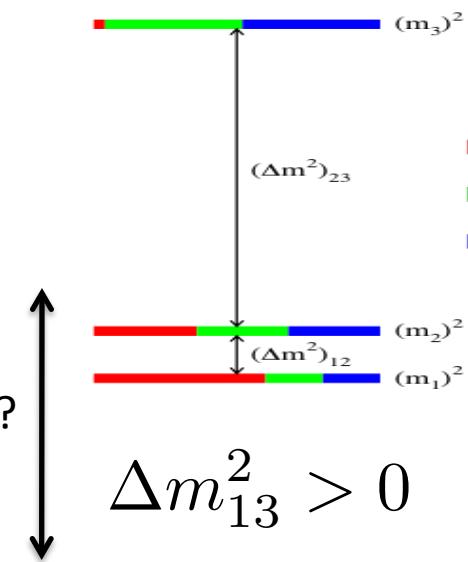
$$(\Delta m_{12}^2, \theta_{12}) = (\Delta m_{\text{sol}}^2, \theta_{\text{sol}})$$

The measurement of $\theta_{13} \sim 9^\circ$ implies that corrections to these approximations are sizeable and need to be included in all analyses

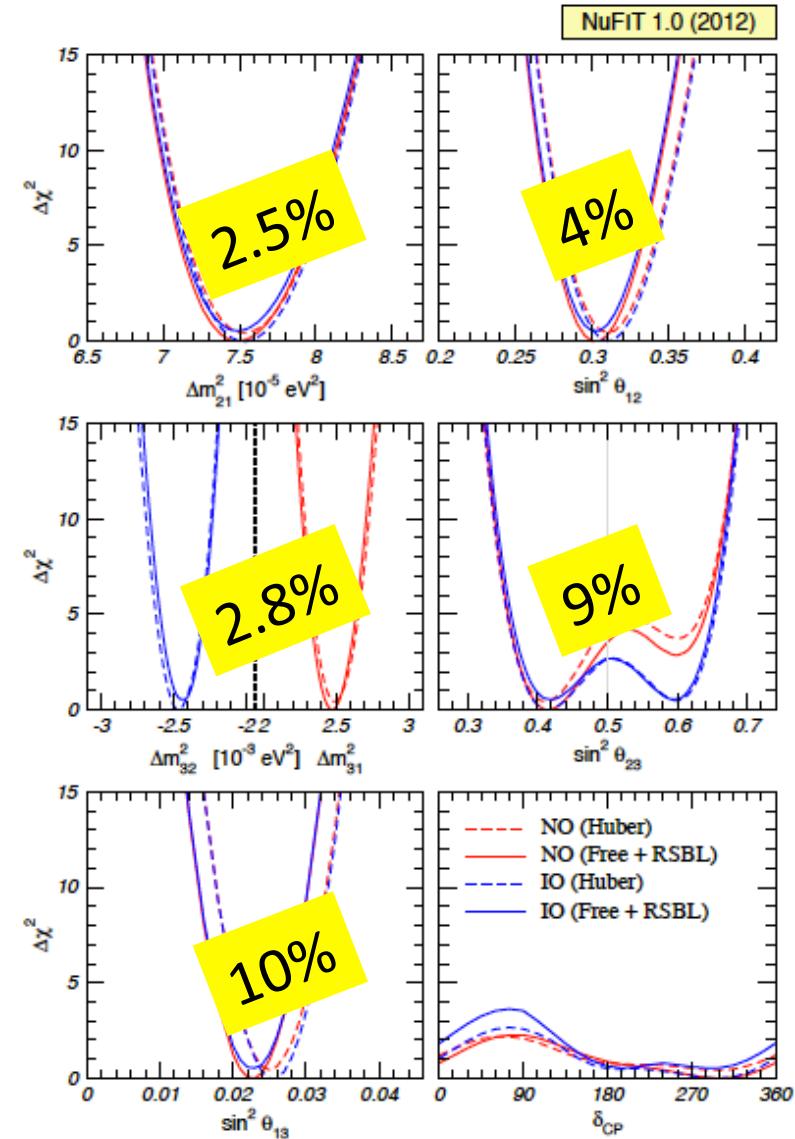
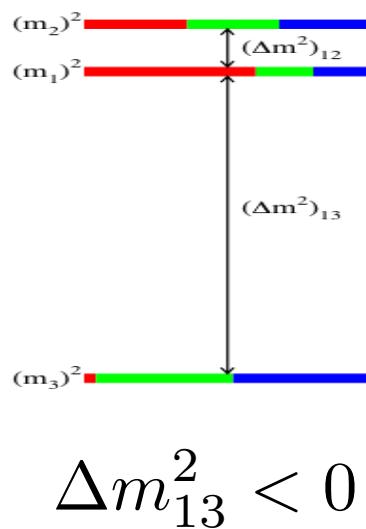
Standard 3ν scenario

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \dots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

normal hierarchy

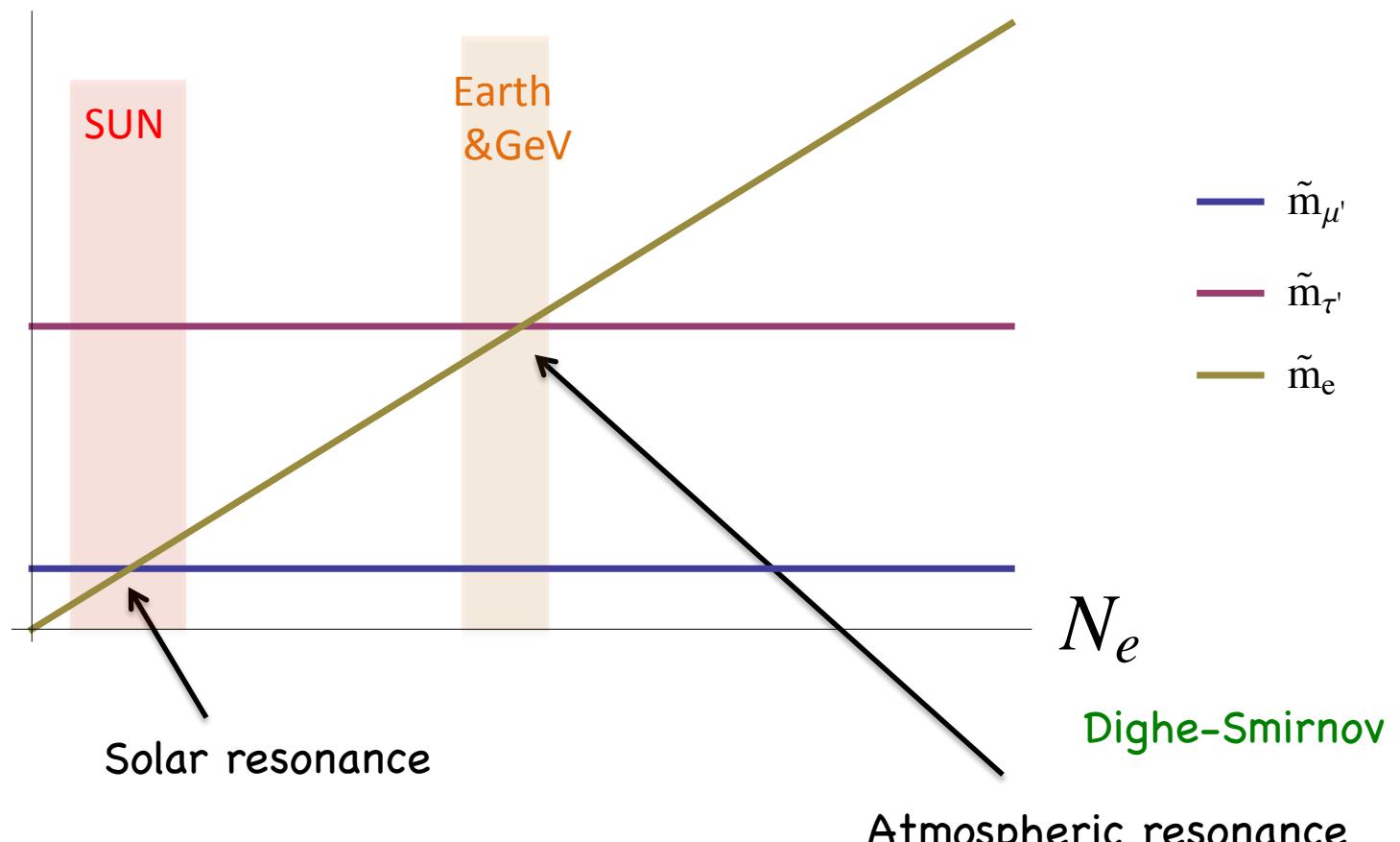


inverted hierarchy



3ν in matter if NH

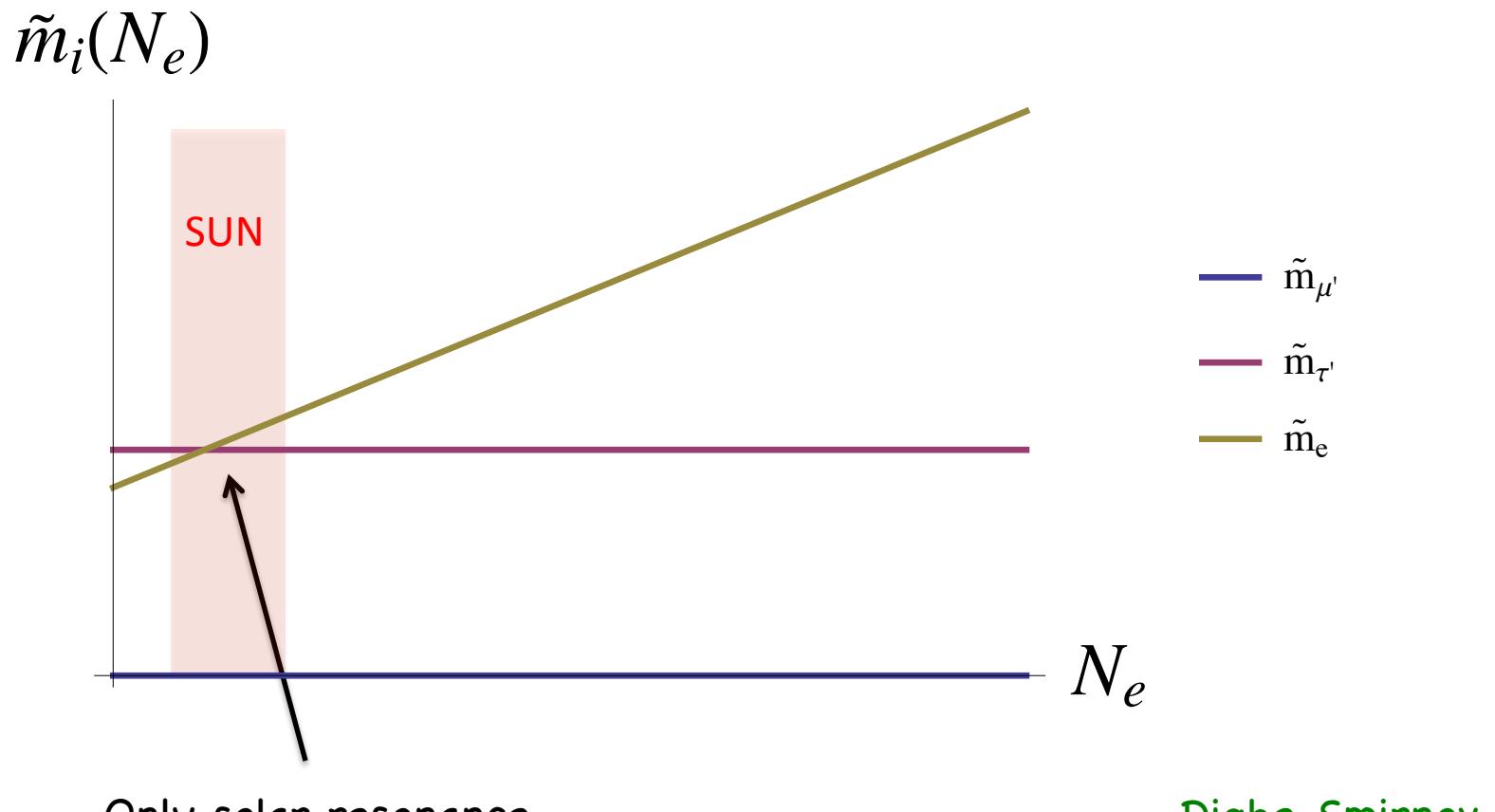
$\tilde{m}_i(N_e)$



$$\Delta m_{12}^2 \cos 2\theta_{12} = 2\sqrt{2}G_F E N_e$$

$$\Delta m_{23}^2 \cos 2\theta_{13} = \pm 2\sqrt{2}G_F E N_e$$

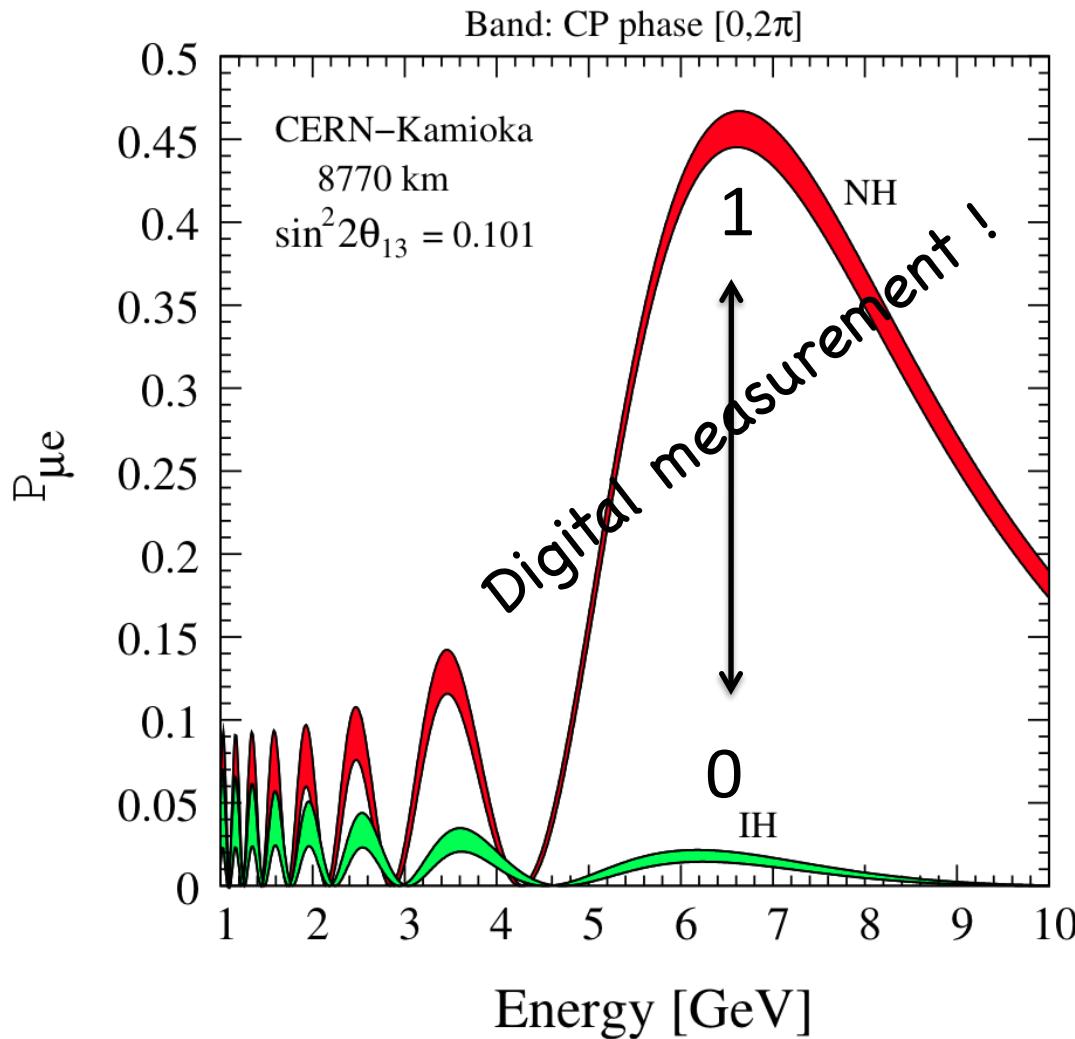
3ν in matter if IH



$$\Delta m_{12}^2 \cos 2\theta_{12} = 2\sqrt{2}G_F E N_e$$

Opposite situation for anti-neutrinos

Hierarchy through MSW



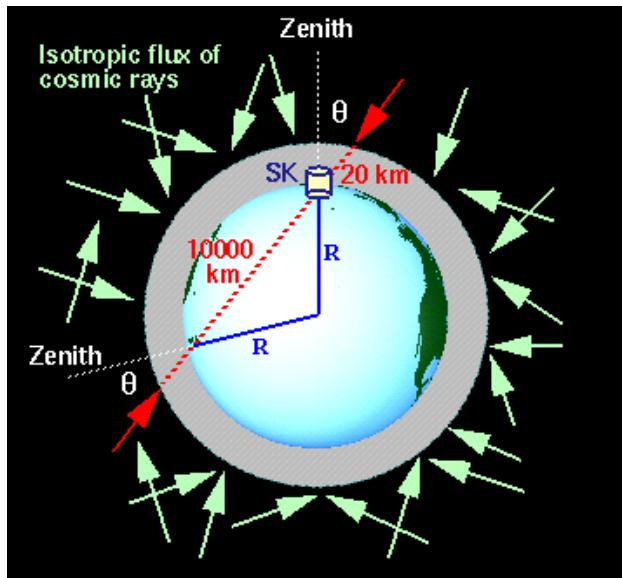
Spectacular MSW effect at $O(6\text{GeV})$ and very long baselines: no need for spectral info nor two channels

$$E_{\text{res}} \equiv \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2\sqrt{2}G_F n_e},$$

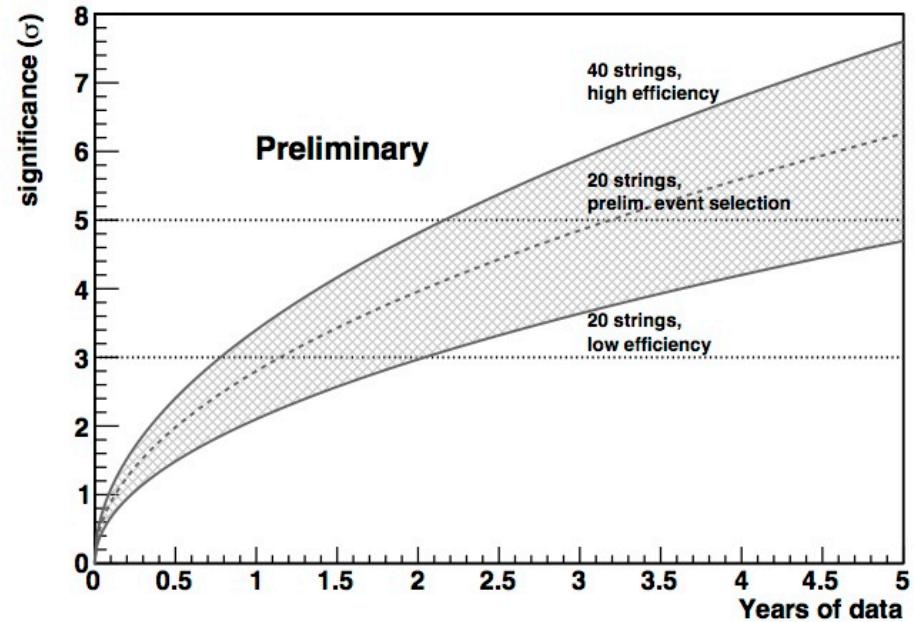
$$n_e(L)L|_{L_{\max}} = \frac{\pi}{\sqrt{2}G_F \tan 2\theta_{13}}$$

Hierarchy from atmospherics ?

$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$



PINGU@ South Pole



Atmospheric data contain the golden signal but not so easy to dig...

Outliers: LSND anomaly

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$\nu_\mu \rightarrow \nu_e$ DIF $(28 \pm 6/10 \pm 2)$

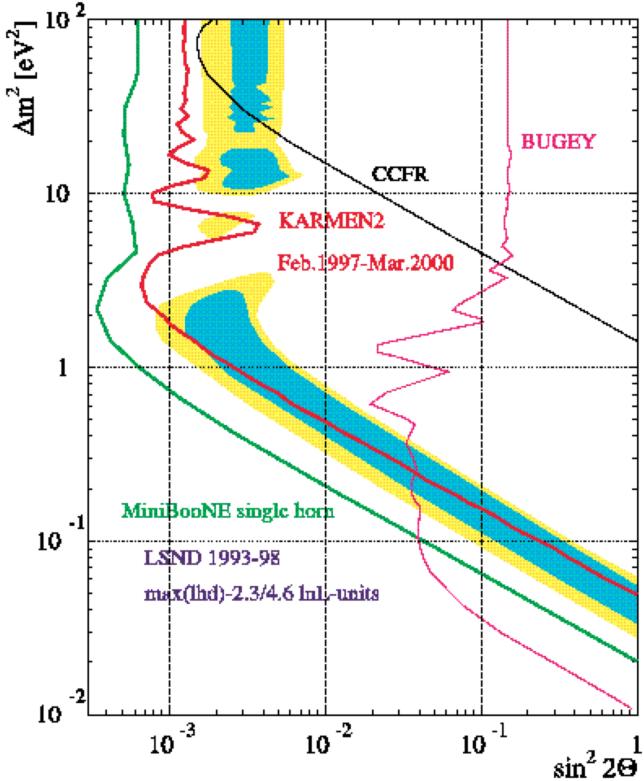
$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ DAR $(64 \pm 18/12 \pm 3)$

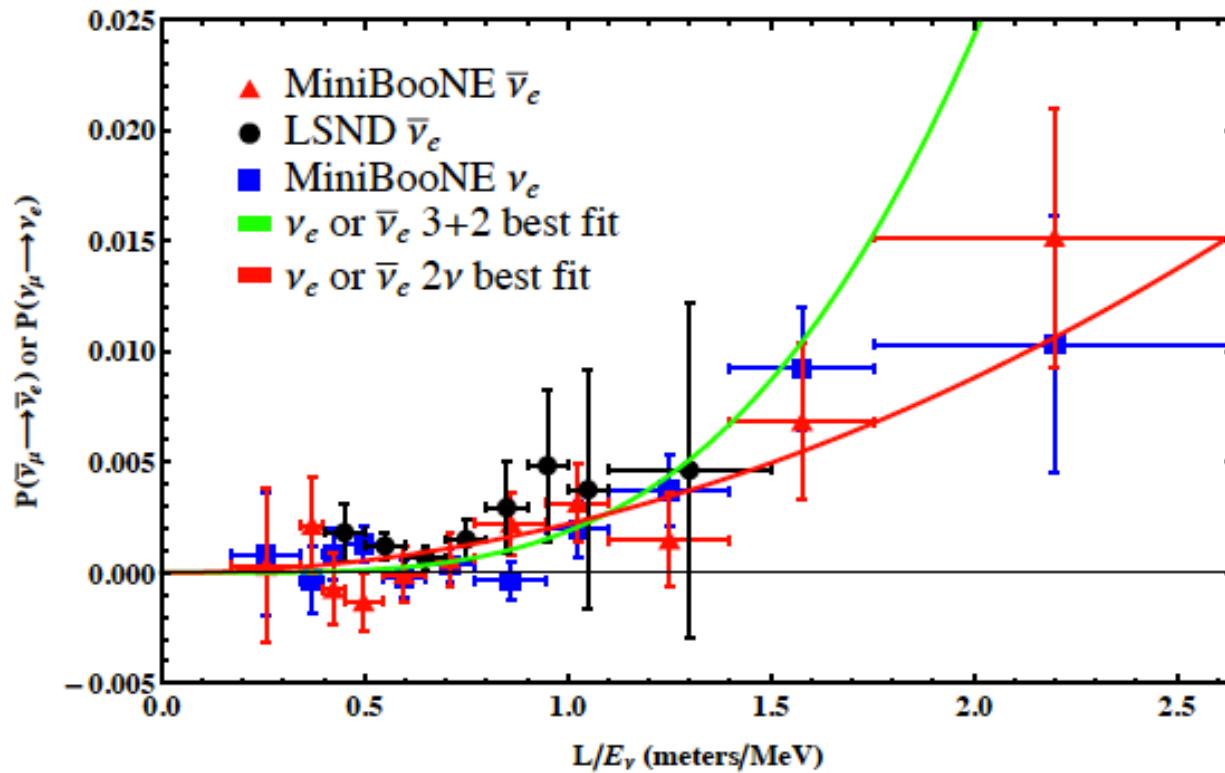
Appearance signal with very different

$$|\Delta m^2| \gg |\Delta m_{atm}^2|$$

LSND vs KARMEN

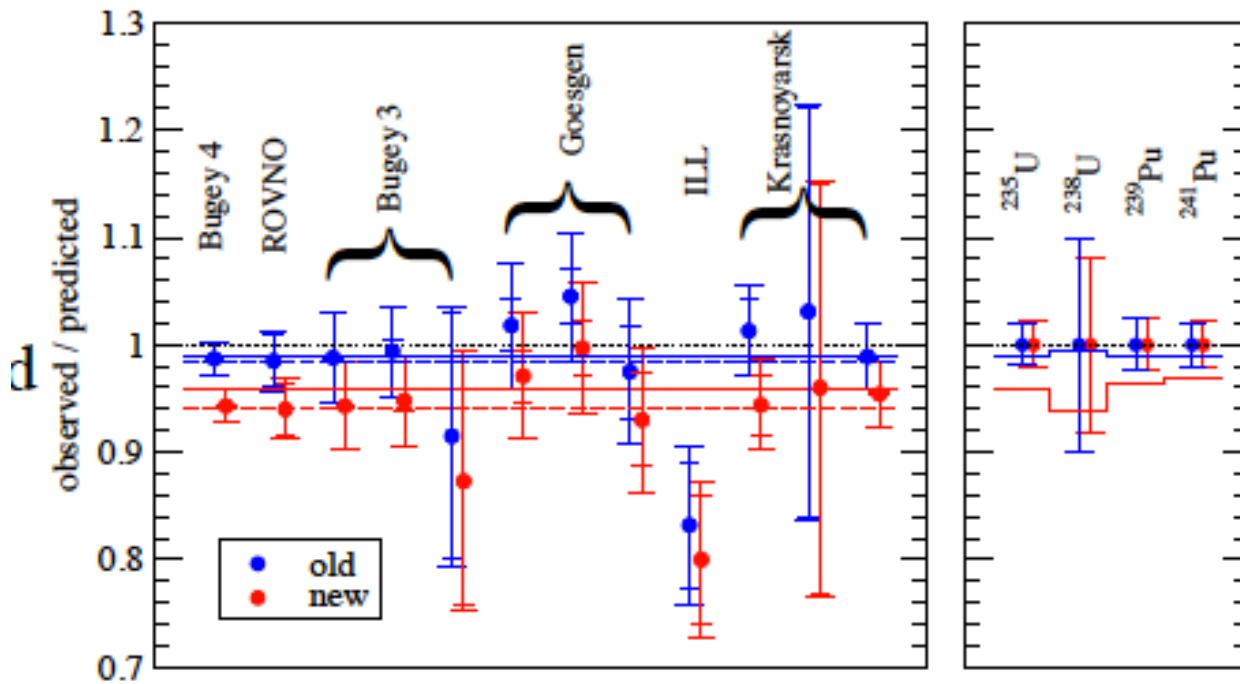


MINIBOONE



Extremely confusing situation!

Outliers: reactor anomaly



T. A. Mueller et al; P. Huber

Recent re-evaluation of reactor fluxes found to be 3% underestimated

+Gallium anomaly...

3+1, 3+2 neutrino mixing models

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = U_{4 \times 4} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \\ \nu'_s \end{pmatrix} = U_{5 \times 5} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \end{pmatrix}$$

Conrad et al; Kopp, et al; Giunti et al

New mass splitting:

$$|\Delta m^2| \sim \frac{\mathcal{O}(MeV)}{\mathcal{O}(1 - 10m)} \sim \frac{\mathcal{O}(1GeV)}{\mathcal{O}(1 - 10km)}$$

Significant improvement over 3ν scenario, but tension appearance/disappearance remains

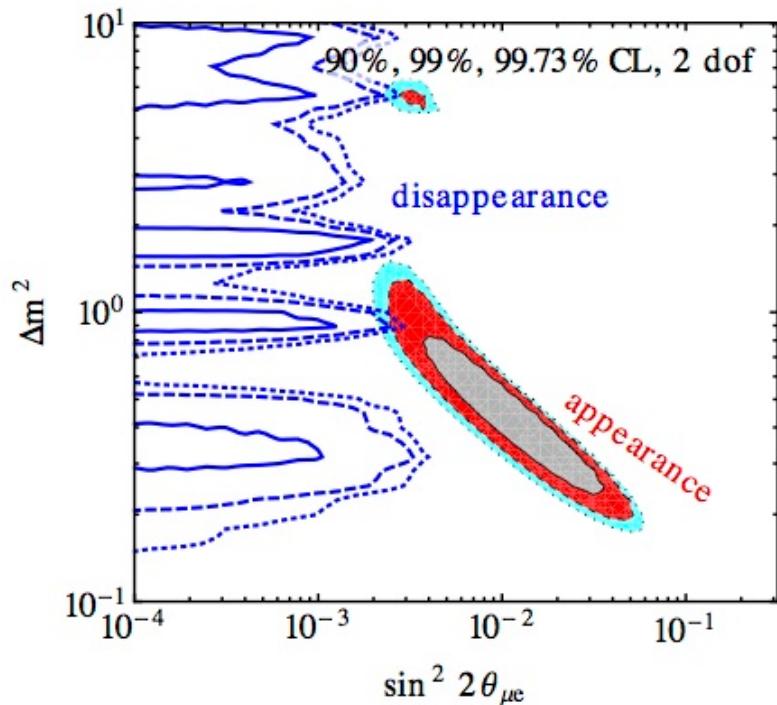
$$P(\nu_e \rightarrow \nu_\mu) = O(|U_{ei}|^2 |U_{\mu i}|^2)$$



$$P(\nu_e \rightarrow \nu_e) = O(|U_{ei}|^2)$$



$$P(\nu_\mu \rightarrow \nu_\mu) = O(|U_{\mu i}|^2)$$

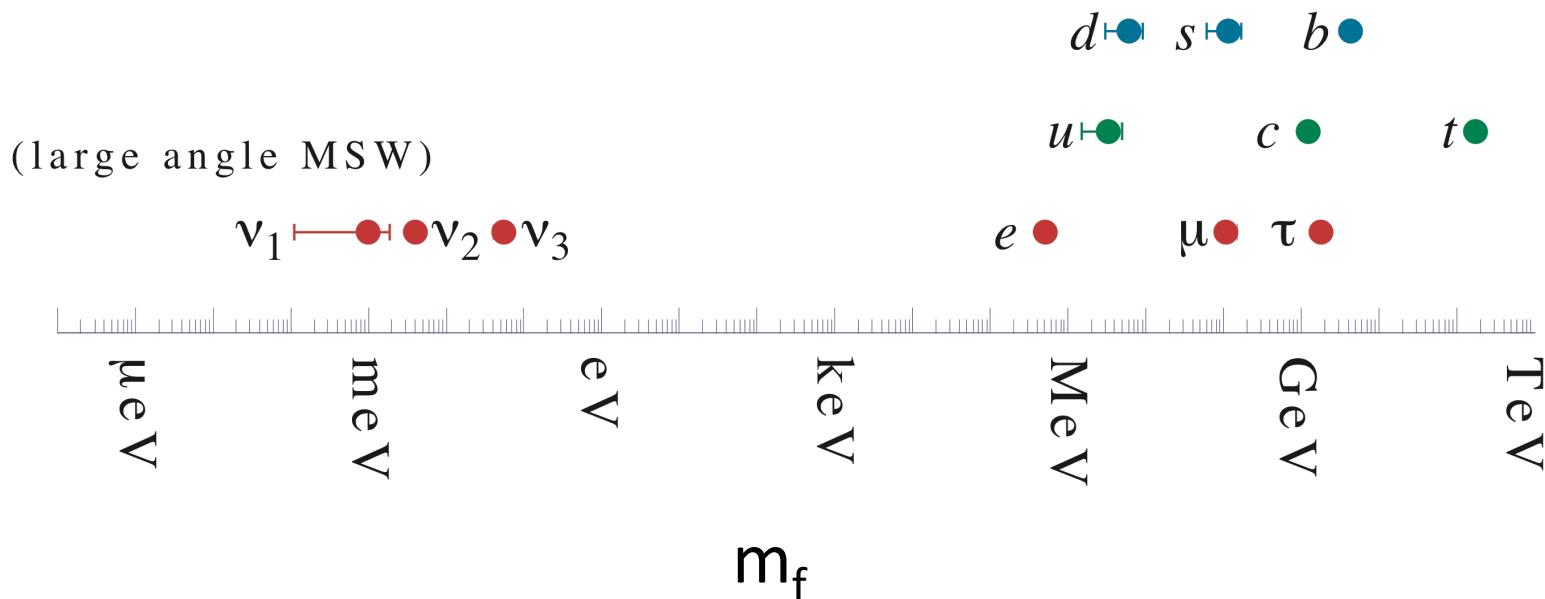


Kopp et al

Strong tension remains:
a convincing signal would be to find it in all the three...

Why are neutrinos so much lighter ?

Neutral vs charged hierarchy ?



Why so different mixing ?

CKM

$$|V|_{\text{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.0065 & (3.51 \pm 0.15) \times 10^{-3} \\ 0.2252 \pm 0.00065 & 0.97344 \pm 0.00016 & (41.2^{+1.1}_{-5}) \times 10^{-3} \\ (8.67^{+0.29}_{-0.31}) \times 10^{-3} & (40.4^{+1.1}_{-0.5}) \times 10^{-3} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

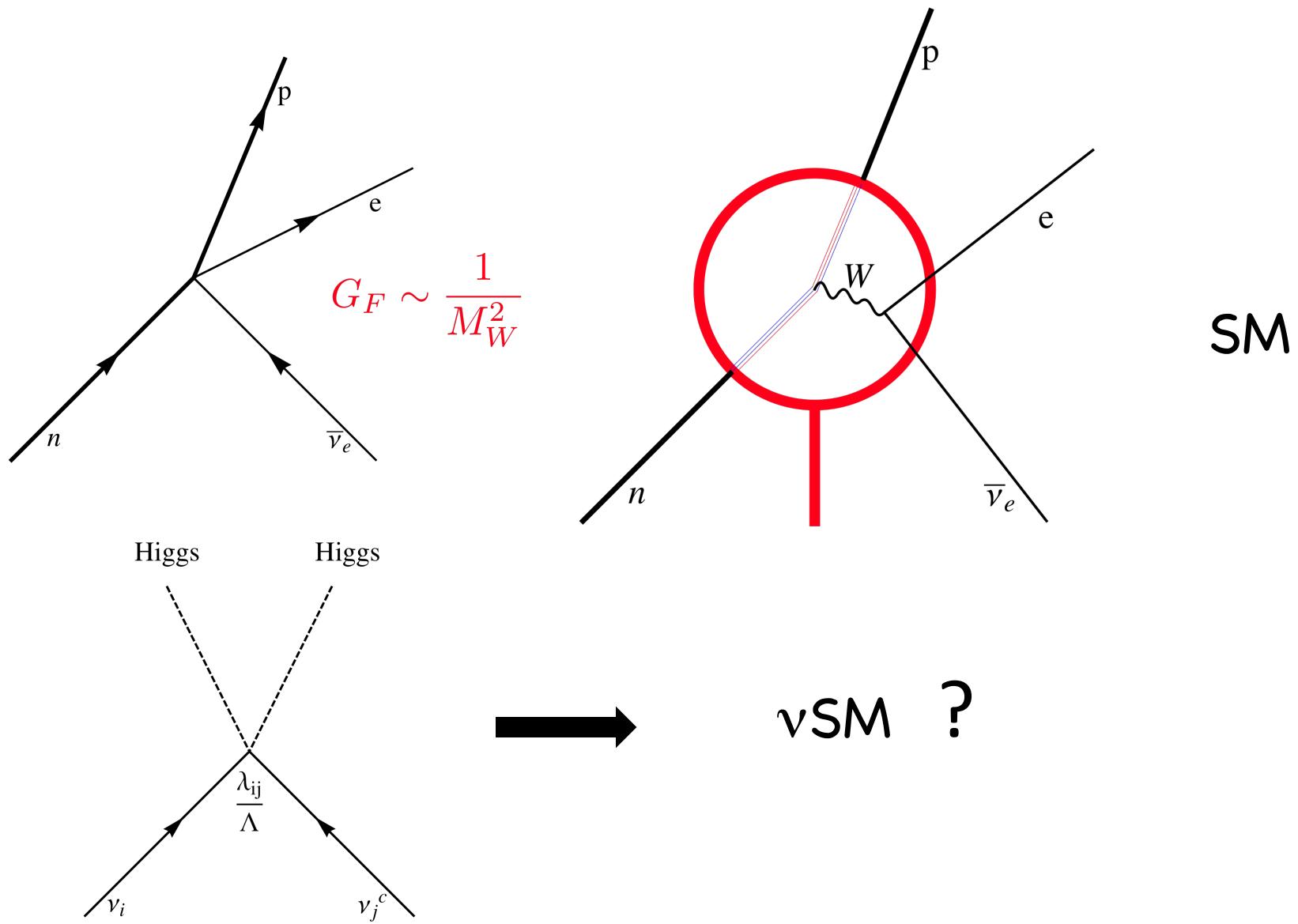
PMNS

3σ

$$|U| = \begin{pmatrix} 0.795 \rightarrow 0.846 & 0.513 \rightarrow 0.585 & 0.126 \rightarrow 0, 178 \\ 0.205 \rightarrow 0.543 & 0.416 \rightarrow 0.730 & 0.579 \rightarrow 0.808 \\ 0.215 \rightarrow 0.548 & 0.409 \rightarrow 0.725 & 0.567 \rightarrow 0.800 \end{pmatrix}$$

Gonzalez-Garcia, et al 1209.3023

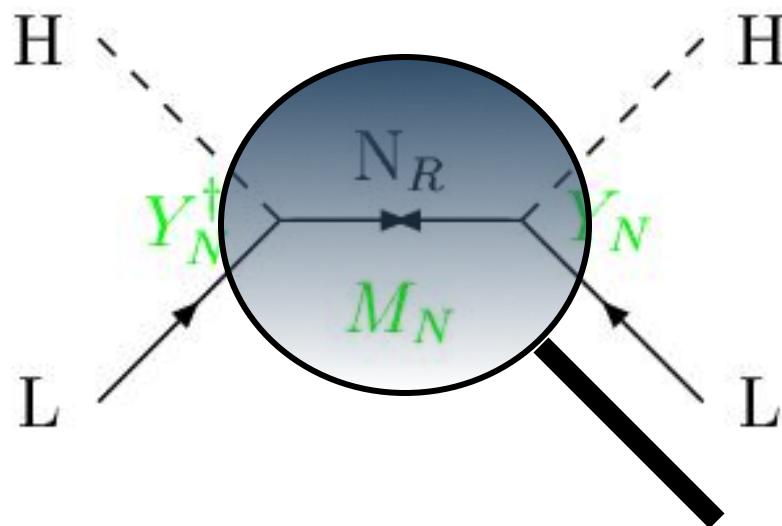
Neutrinos have tiny masses \rightarrow a new physics scale, what ?



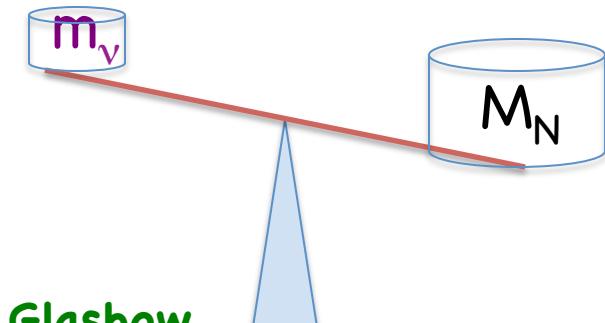
How does the v scale relates to the EW scale ?

Example: Type I seesaw model (interchange heavy singlet fermions)

$$\mathcal{L} = \mathcal{L}_{SM} - \sum^{n_R} \bar{l}_L^\alpha Y^{\alpha i} \tilde{\Phi} \nu_R^i - \sum^{n_R} \frac{1}{2} \bar{\nu}_R^{ic} M_N^{ij} \nu_R^j + h.c.$$



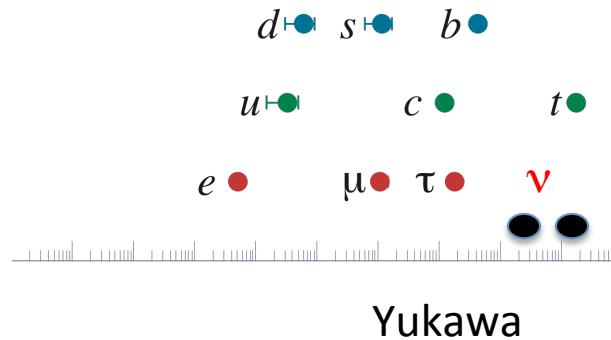
$$\frac{\lambda}{\Lambda} \equiv Y_N^T \frac{1}{M_N} Y_N$$



Minkowski; Gell-Mann, Ramond Slansky; Yanagida, Glashow...

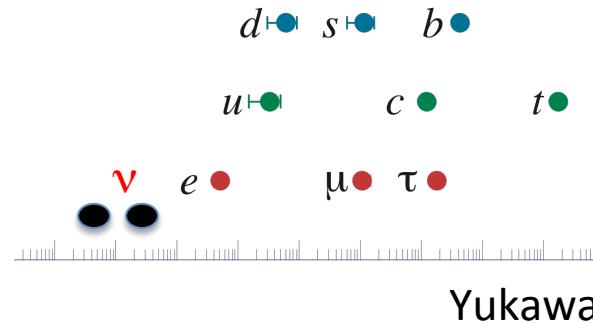
Charged/neutral hierarchy in seesaw (I)

$M_N = \text{GUT}$



Yukawa

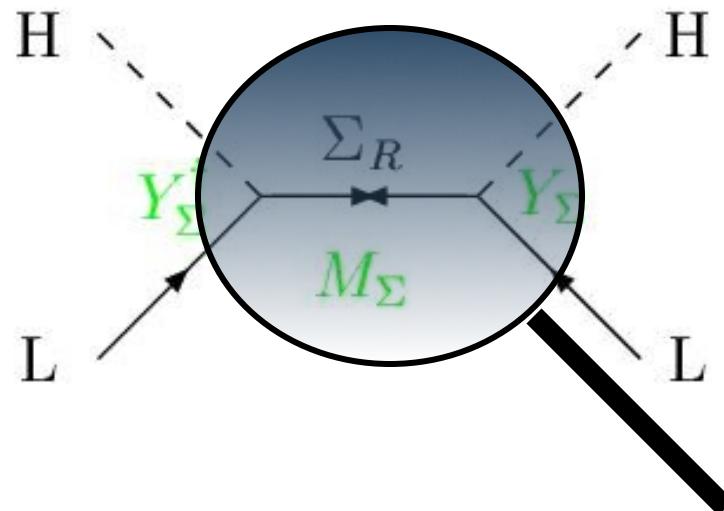
$M_N = \text{TeV}$



Yukawa

New physics scale

Type III see-saw: interchange a heavy triplet fermion

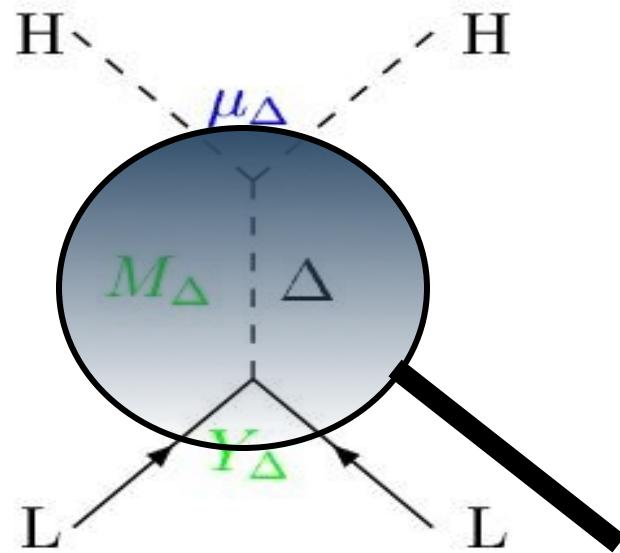


$$m_\nu = \frac{\alpha v^2}{\Lambda} \equiv Y_\Sigma^T \frac{v^2}{M_\Sigma} Y_\Sigma$$

Foot et al; Ma; Bajc, Senjanovic...

New physics scale

Type II see-saw: interchange a heavy triplet scalar



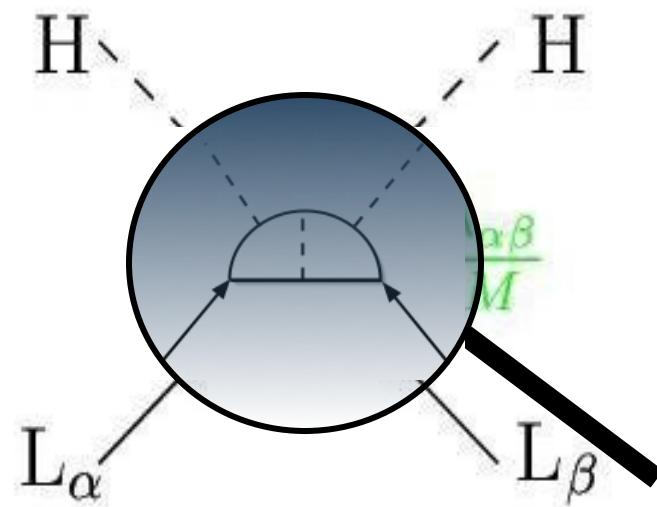
$$m_\nu = \frac{\alpha v^2}{\Lambda} \equiv Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Konetschny, Kummer; Cheng, Li; Lazarides, Shafi, Wetterich ...

New physics scale

Also from loops !

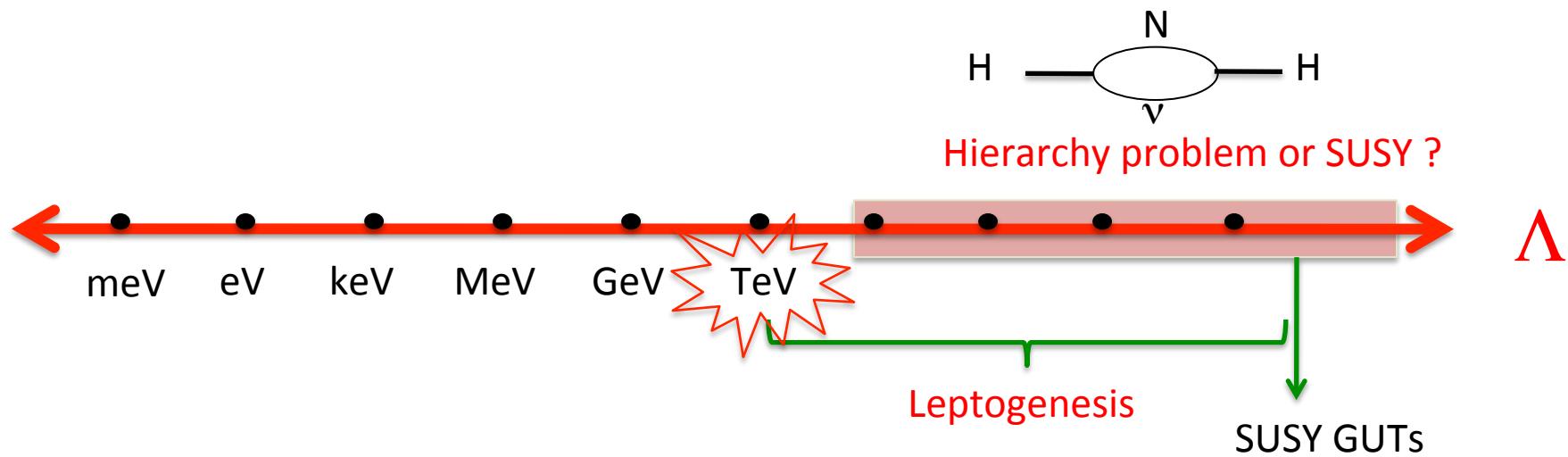
Zee-Babu



$$m_\nu \sim \mathcal{O} \left(\frac{1}{(16\pi^2)^2} \times \frac{\mu m_l^2}{M^2} \right)$$

Pinning down the New physics scale

The new scale is stable under radiative corrections due to Lepton Number Symmetry but the EW is not!



Robust predictions of high (and not so high) scale seesaw models:

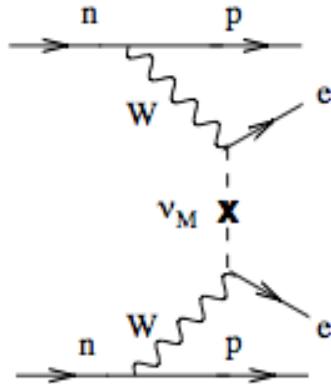
there is **neutrinoless double beta** decay at some level ($\Lambda > 100\text{MeV}$)

a matter-antimatter asymmetry if there is **CP violation** in the lepton sector !

there are other states out there at scale Λ !

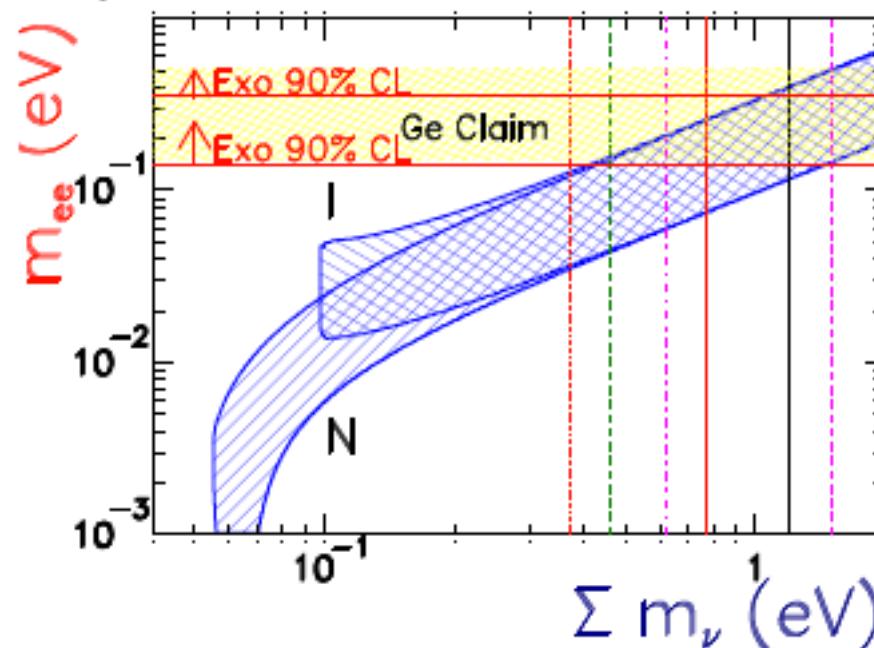
Lepton Number violation: Majorana nature

Plethora of experiments with different techniques/systematics: EXO, KAMLAND-ZEN, GERDA, CUORE, NEXT, SuperNEMO, LUCIFER...



$$m_{\beta\beta} \equiv |m_{ee}|$$

$$\Sigma \equiv \sum_i m_i$$



Vissani 2002; Pascoli et al 2005; (Fogli et al (04))

Update Maltoni, Schwetz,Salvado, MCGG (95%)

$$|m_{ee}| = |c_{13}^2(m_1 c_{12}^2 + m_2 e^{i\alpha} s_{12}^2) + m_3 e^{i\beta} s_{13}^2|$$

Leptonic CP violation (in vacuum)

$$E_\nu/L \sim \Delta m_{23}^2 \gg \Delta m_{12}^2$$

$$\begin{aligned} P_{\nu_e \nu_\mu (\bar{\nu}_e \bar{\nu}_\mu)} &= s_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta_{23} L}{2} \right) \equiv P^{atmos} \\ &+ c_{23}^2 \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta_{12} L}{2} \right) \equiv P^{solar} \\ + \tilde{J} &\quad \cos \left(\pm \delta - \frac{\Delta_{23} L}{2} \right) \frac{\Delta_{12} L}{2} \sin \left(\frac{\Delta_{23} L}{2} \right) \equiv P^{inter} \end{aligned}$$

$$\tilde{J} \equiv c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \qquad \Delta_{ij} \equiv \frac{m_j^2 - m_i^2}{2E}$$

Leptonic CP violation (in vacuum)

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Golden Channel in matter

In matter:

$$\begin{aligned}
 P_{\nu_e \bar{\nu}_e} &= s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{13}}{B_\pm} \right)^2 \sin^2 \left(\frac{B_\pm L}{2} \right) \\
 &\quad + c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A} \right)^2 \sin^2 \left(\frac{AL}{2} \right) \\
 &\quad + \tilde{J} \frac{\Delta_{12}}{A} \sin \left(\frac{AL}{2} \right) \frac{\Delta_{13}}{B_\pm} \sin \left(\frac{B_\pm L}{2} \right) \cos \left(\pm\delta - \frac{\Delta_{13} L}{2} \right)
 \end{aligned}$$

Octant dependence

Hierarchy dependence

$$\tilde{J} \equiv c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \quad B_\pm \equiv \sqrt{2}G_F n_e \pm \Delta_{13}$$

Cervera et al; Freund et al

Parameter degeneracies (eg. neutrino hierarchy, octant) compromise δ sensitivity

Burguet et al; Minakata, Nunokawa; Barger, et al

Golden Channel in matter

In matter:

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 P_{\nu_e \bar{\nu}_e} &= s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{13}}{B_\pm} \right)^2 \sin^2 \left(\frac{B_\pm L}{2} \right) \\
 &\quad + c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A} \right)^2 \sin^2 \left(\frac{AL}{2} \right) \\
 &\quad + \tilde{J} \frac{\Delta_{12}}{A} \sin \left(\frac{AL}{2} \right) \frac{\Delta_{13}}{B_\pm} \sin \left(\frac{B_\pm L}{2} \right) \cos \left(\pm\delta - \frac{\Delta_{13} L}{2} \right)
 \end{aligned}$$

Octant dependence

Hierarchy dependence

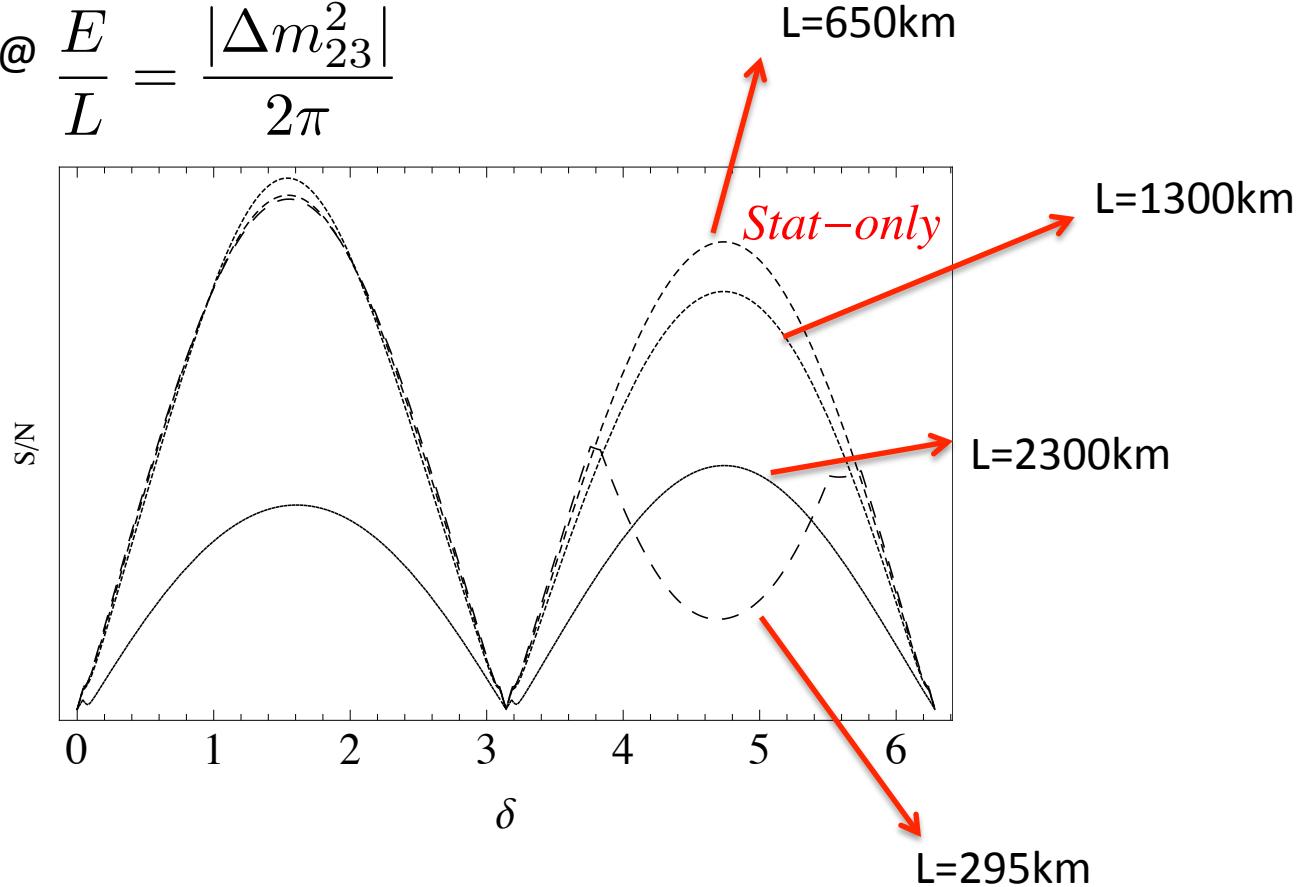
$$\tilde{J} \equiv c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \quad B_\pm \equiv \sqrt{2}G_F n_e \pm \Delta_{13}$$

Cervera et al

Parameter degeneracies (eg. neutrino hierarchy, octant) compromise δ sensitivity

Burguet et al; Minakata, Nunokawa; Barger, et al

$$@ \frac{E}{L} = \frac{|\Delta m_{23}^2|}{2\pi}$$



Naive scaling of S/N assuming statistical errors dominate ...
But systematics could change this conclusion...

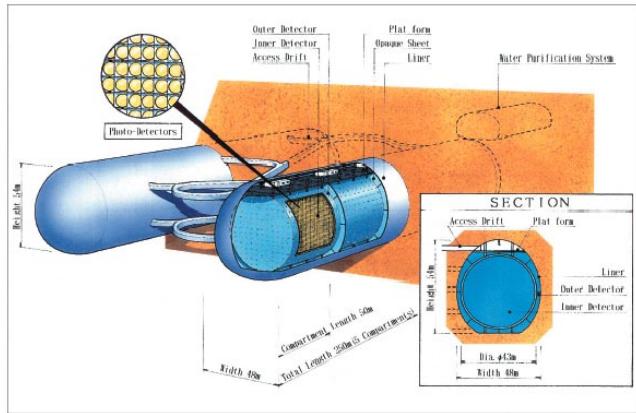
To maximize sensitivity to CP violation don't go too far

Hierarchy + CP in one go...

Three concrete superbeam proposals (to be ready in 10y)

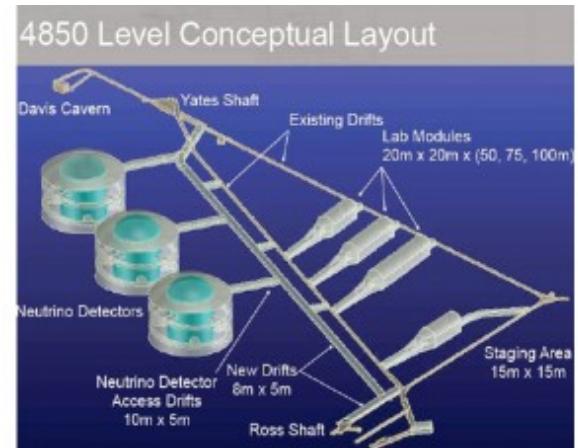
$p \rightarrow \text{Target} \rightarrow K, \pi$ $\nu_\mu, \% \nu_e$

HyperK (Japan)



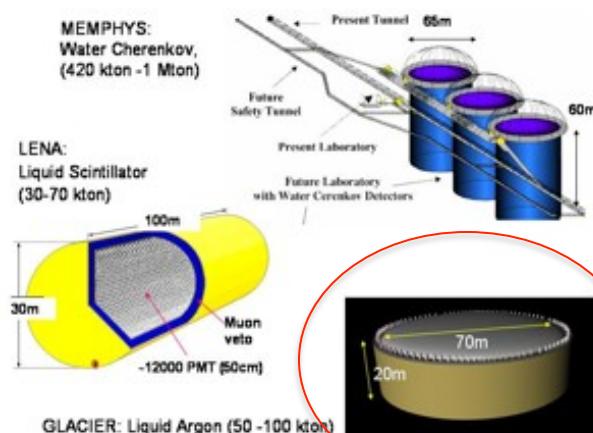
750MW, 560kton WC, Tokai-Kamioka (295km)

LBNE (USA)



800MW, 10kton-> 35kton LAr, Fermilab-Homestake(1300km)

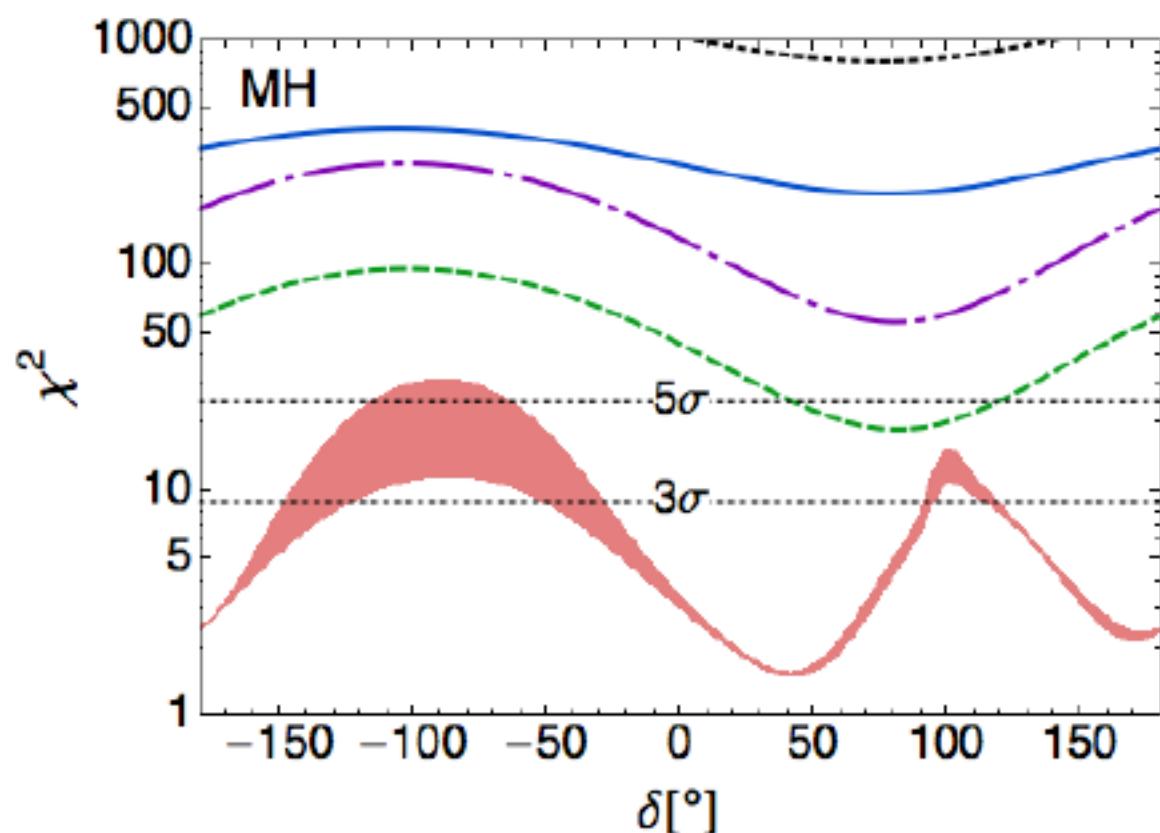
LBNO (Europe)



800MW, 20kton-> 100kton LAr, CERN-Pyhäsalmi (2300km)

In 20 years from now with conventional beams...

---- LBNO-100kt LBNO-20kt
— LBNE-34kt -·- LBNE-10kt
■ T2HK

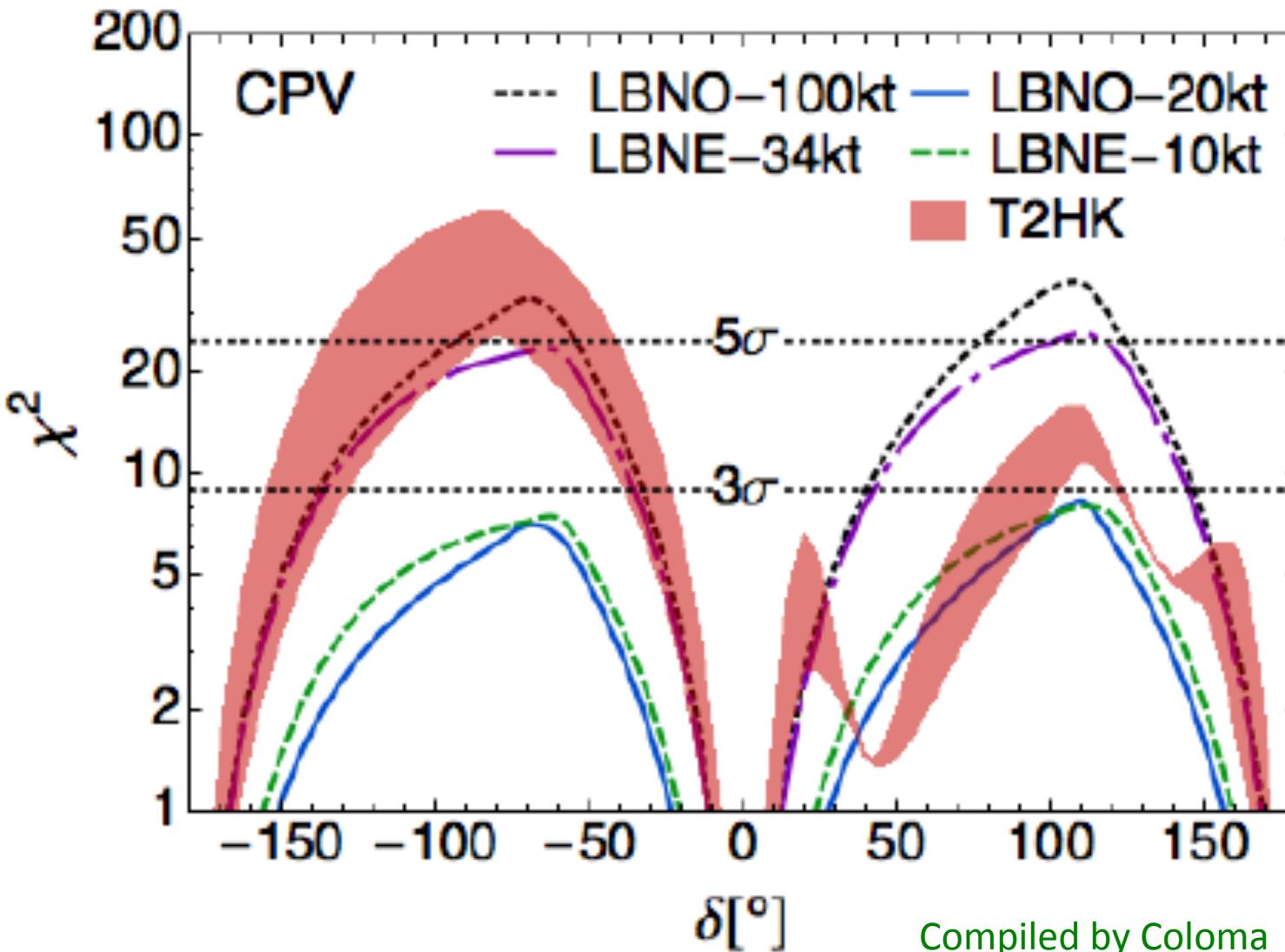


Compiled by Coloma

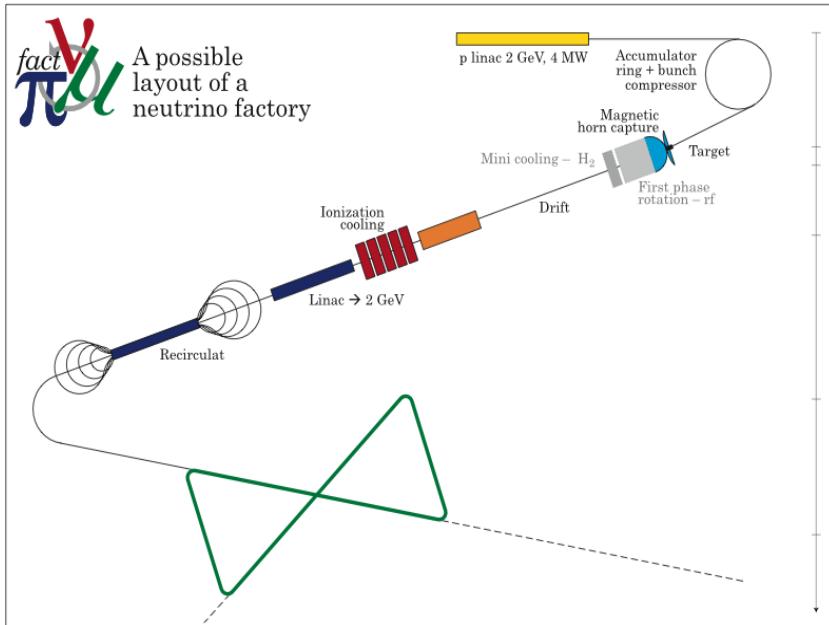
O(10kton) LAr can do the job easily

In 20 years from now with conventional beams...

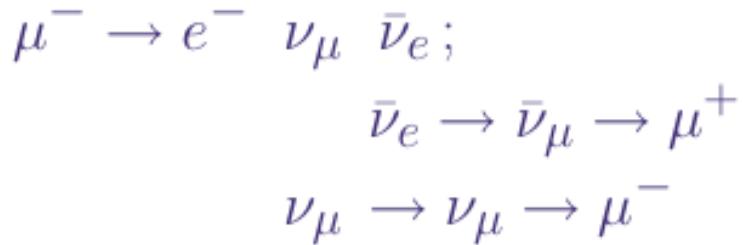
Hierarchy unknown



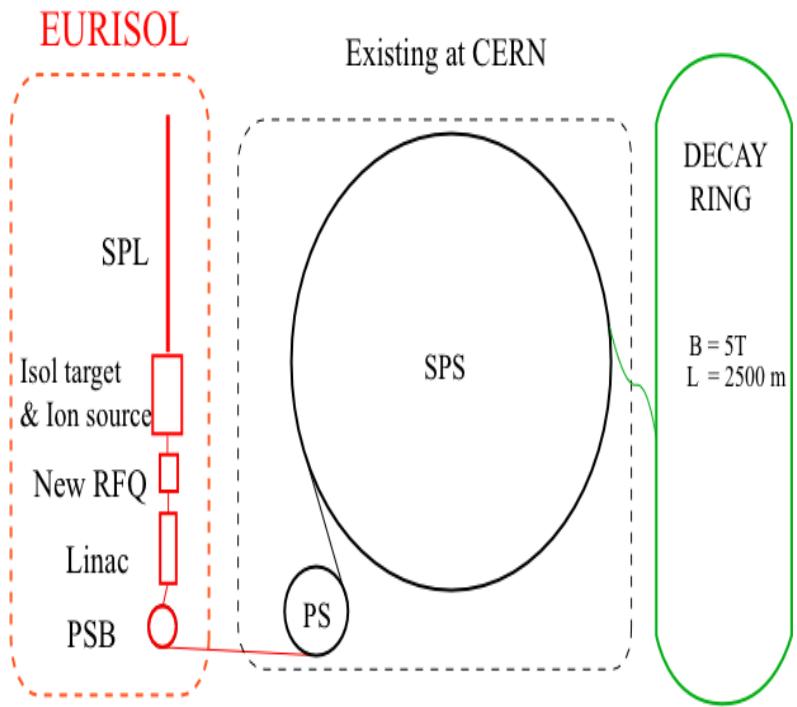
Neutrino factory



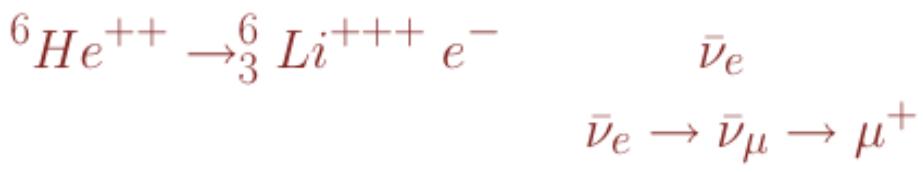
From μ decays



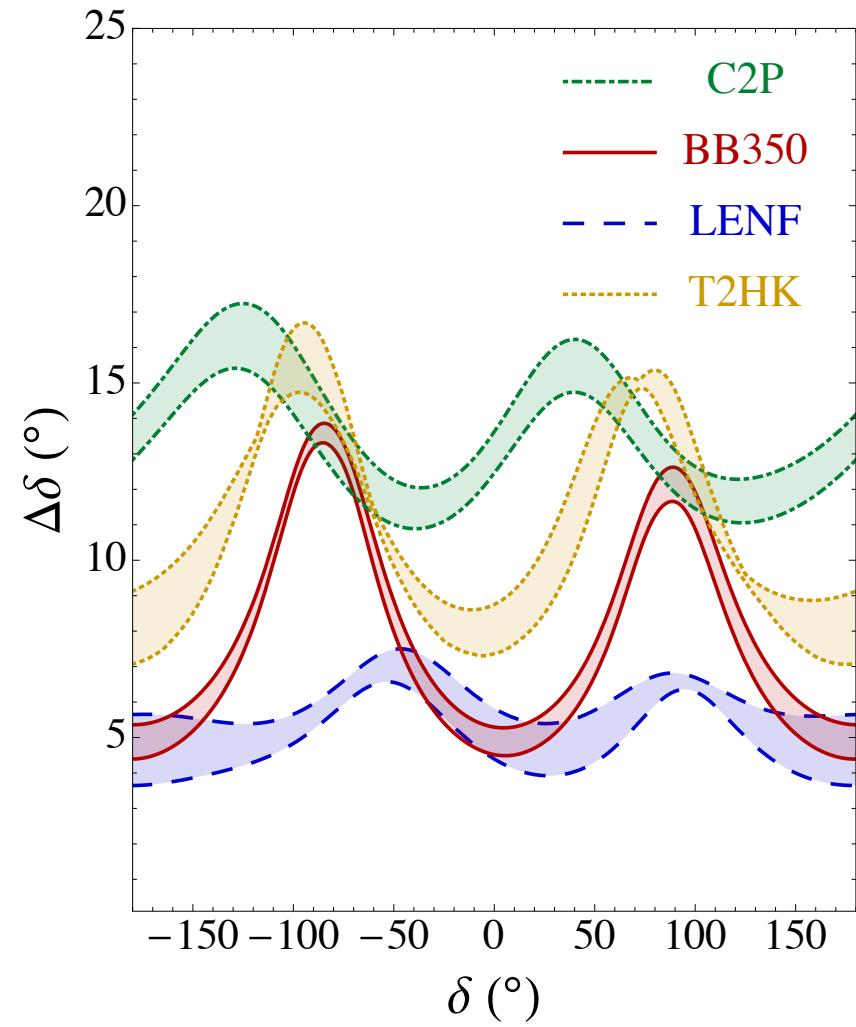
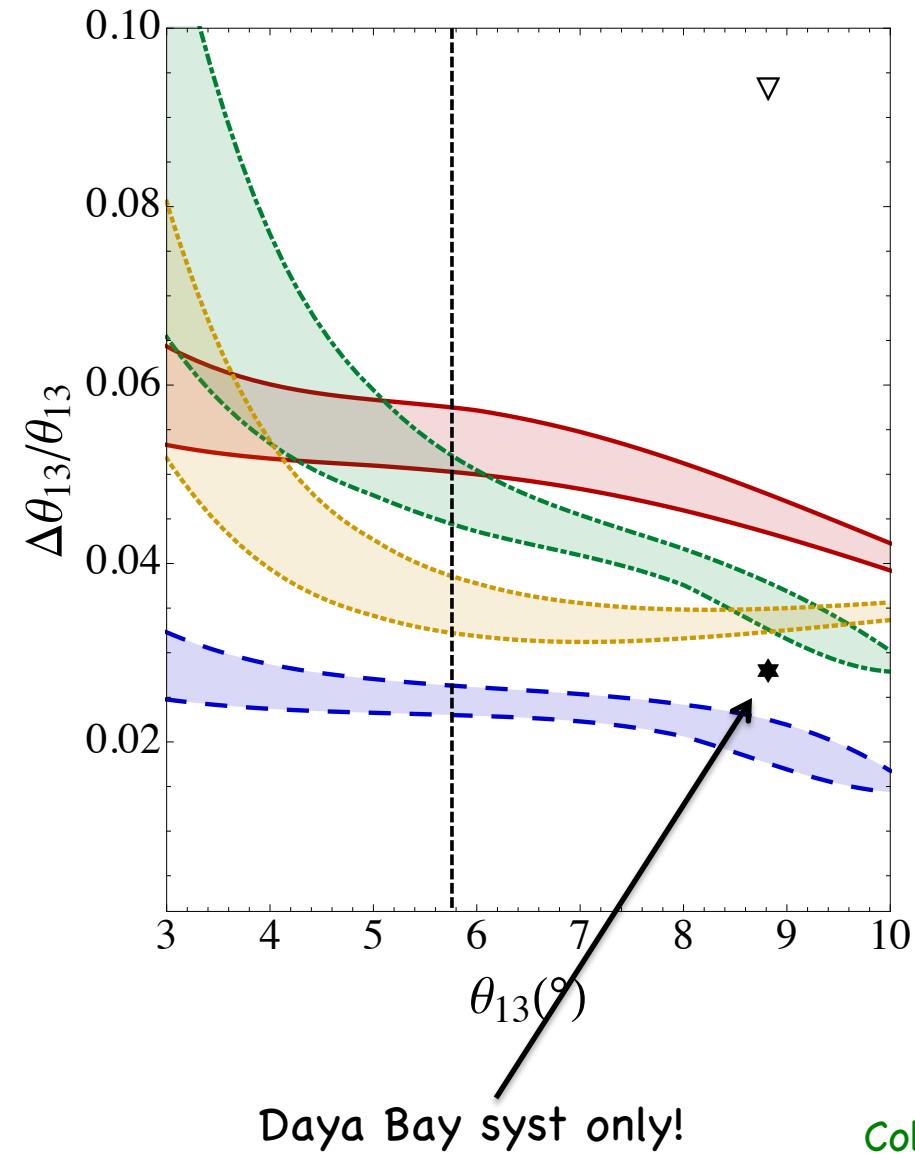
β beam



From radioactive ions



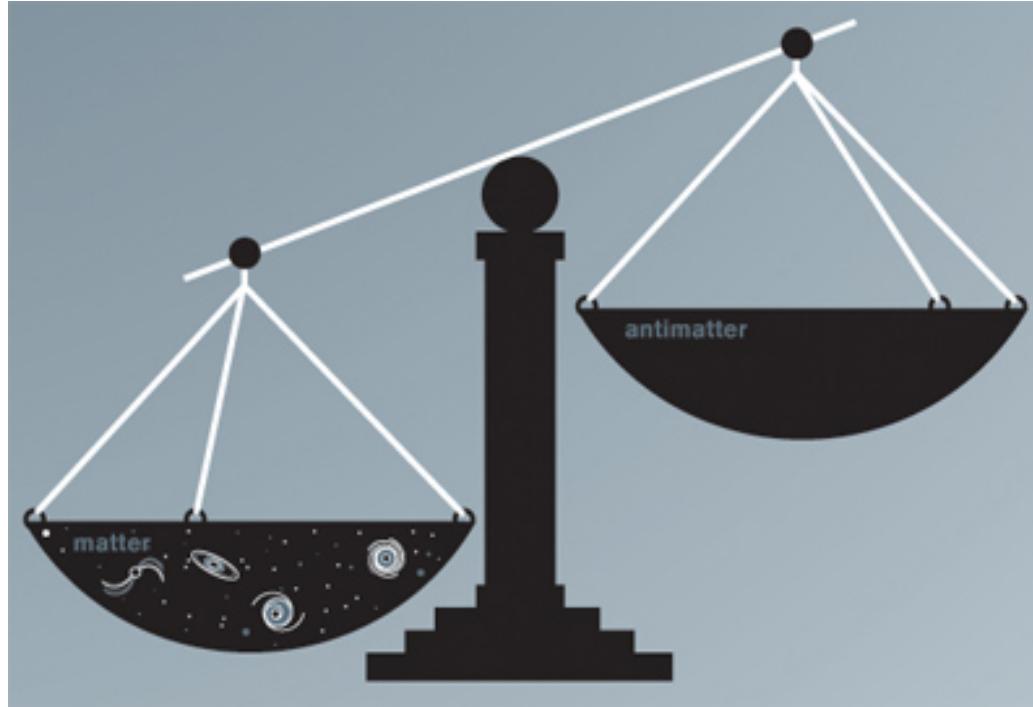
With better beams in XX years...



Coloma, et al 1203.5651

Baryon asymmetry

The Universe seems to be made of matter



WMAP

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.21(16) \times 10^{-10}$$

Baryon asymmetry

Can it arise from a symmetric initial condition with same matter & antimatter ?

Sakharov's necessary conditions for baryogenesis

- ✓ Baryon number violation (B+L violated in the Standard Model)
- ✓ C and CP violation (both violated in the SM)
- ✓ Deviation from thermal equilibrium (at least once: electroweak phase transition)

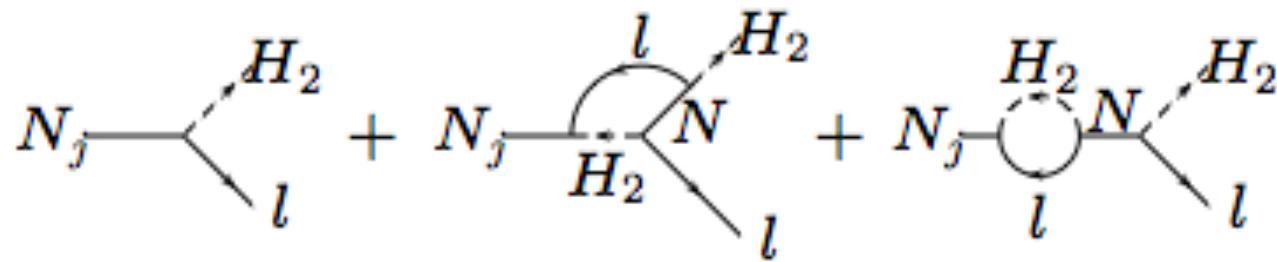
It does not seem to work in the SM with massless neutrinos ...

CP violation in quark sector far to small, EW phase transition too weak...

L, C and CP violation

New sources of CP violation and L violation in the neutrino sector can induce CP asymmetries in decays of heavy Majorana ν

Fukuyita, Yanagida



$$\epsilon_1 = \frac{\Gamma(N \rightarrow \Phi l) - \Gamma(N \rightarrow \Phi \bar{l})}{\Gamma(N \rightarrow \Phi l) + \Gamma(N \rightarrow \Phi \bar{l})}$$

Generic and robust feature of see-saw models

Lepton asymmetry

$$M_{2,3} \gg M_1$$

$$Y_B = 4 \times 10^{-3} \quad \overbrace{\epsilon_1}^{\text{CP-asym eff. factor}} \quad \overbrace{\kappa}$$

$$\epsilon_1 = -\frac{3}{16\pi} \sum_i \frac{Im[(\lambda_\nu^\dagger \lambda_\nu)_{i1}^2]}{(\lambda^\dagger \lambda)_{11}} \frac{M_1}{M_i} \quad \longleftrightarrow \quad m_\nu = \lambda_\nu^T \frac{1}{M} \lambda_\nu$$

Different combinations

Even if we know the neutrino mass we cannot predict the asymmetry accurately...

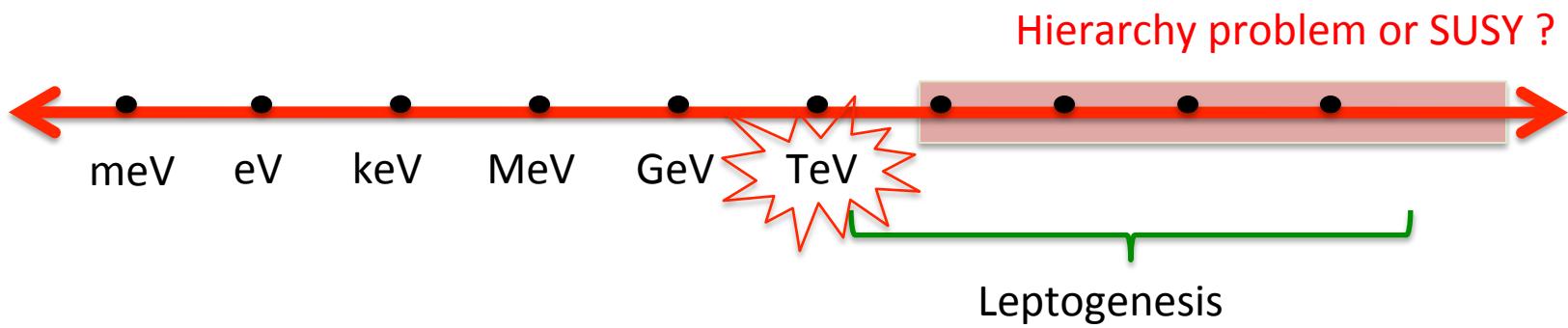
Counting physical parameters in lepton mixing (seesaw I)

physical parameters = # parameters in Yukawas
- # parameters in field redefinitions
+ # parameters of field redefinitions of exact symmetries

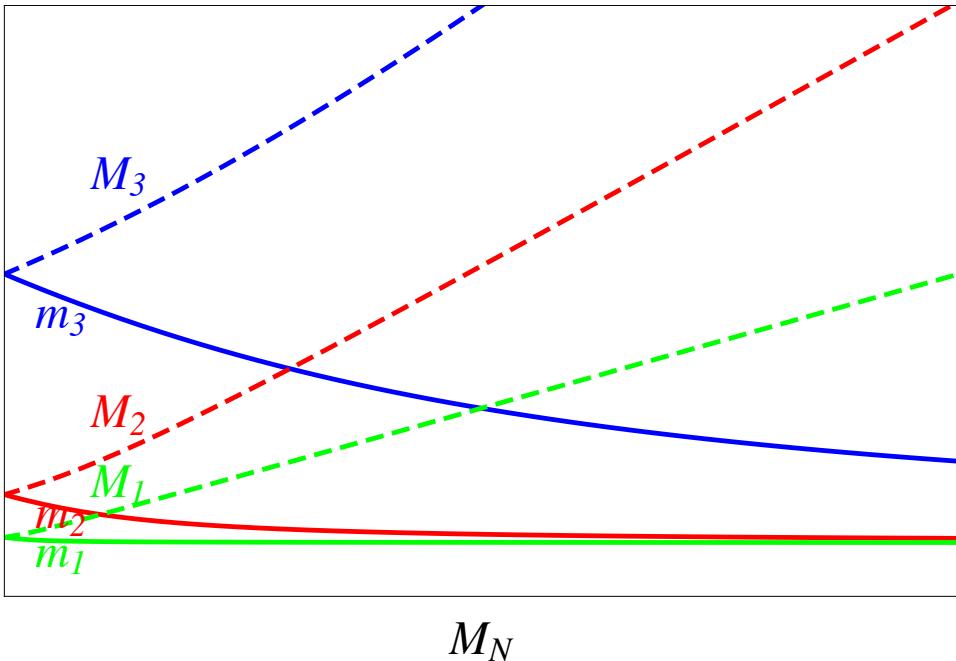
	Yukawas	Field. Red.	Symmetries	Physical
	Y_v, Y_l, M	$U_L(n) \times U_{lR}(n) \times U_{vR}(n)$	0	
Moduli	$2n^2 + n(n+1)/2$	$3n(n-1)/2$	0	$3n + n(n-1)$
Phases	$2n^2 + n(n+1)/2$	$3n(n+1)/2$	0	$n(n-1)$

For $n=3$: 9 masses, 6 angles, 6 phases

Other states out there...



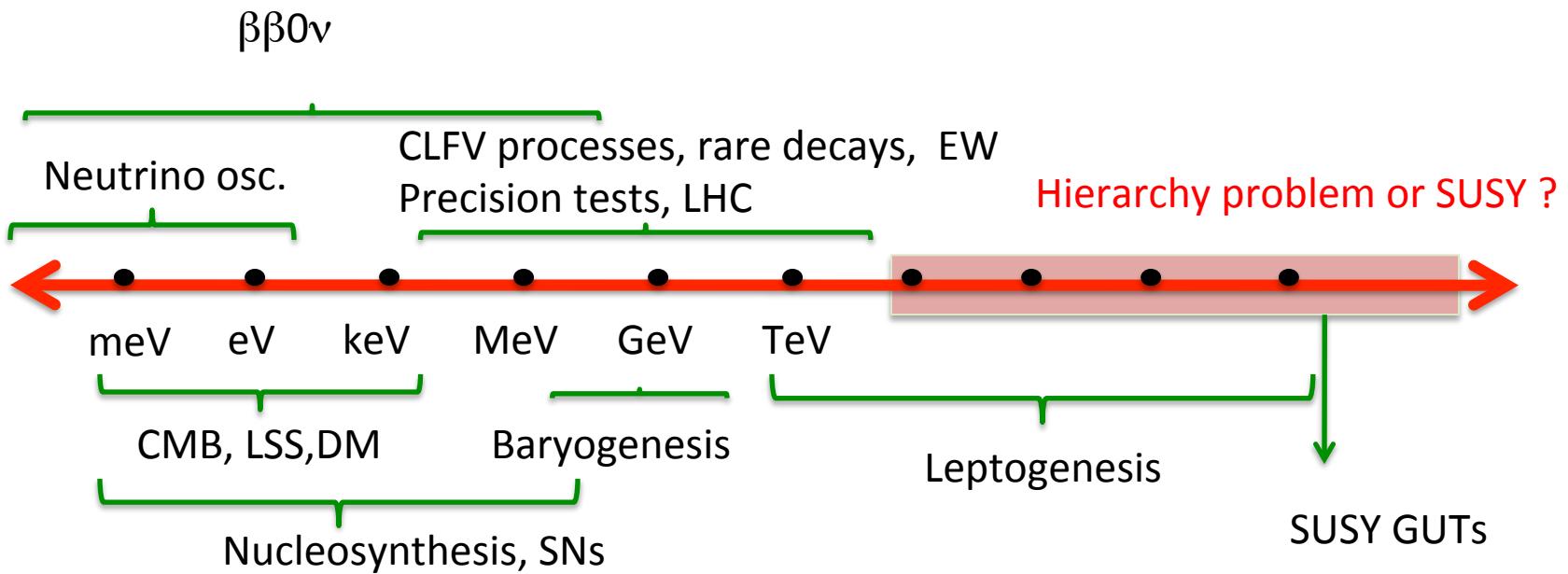
Low scale Type I seesaw models



$$\theta_{hl} \sim \frac{Yv}{M_N} \sim \sqrt{m_\nu/M_N}$$

- kinematically allowed (the lower the mass the better)
- they mix significantly with the rest of the SM (the lower the mass the better)

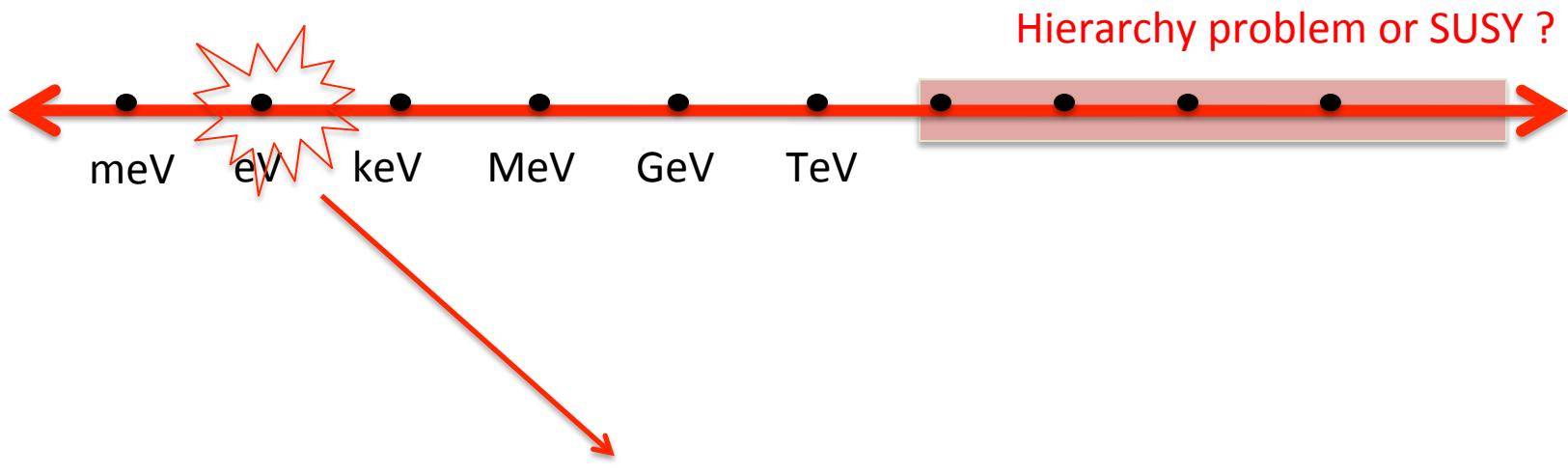
Other states out there ?



Light Sterile Neutrinos White Paper, Abazajian et al arXiv: 1204.5379 and refs. therein

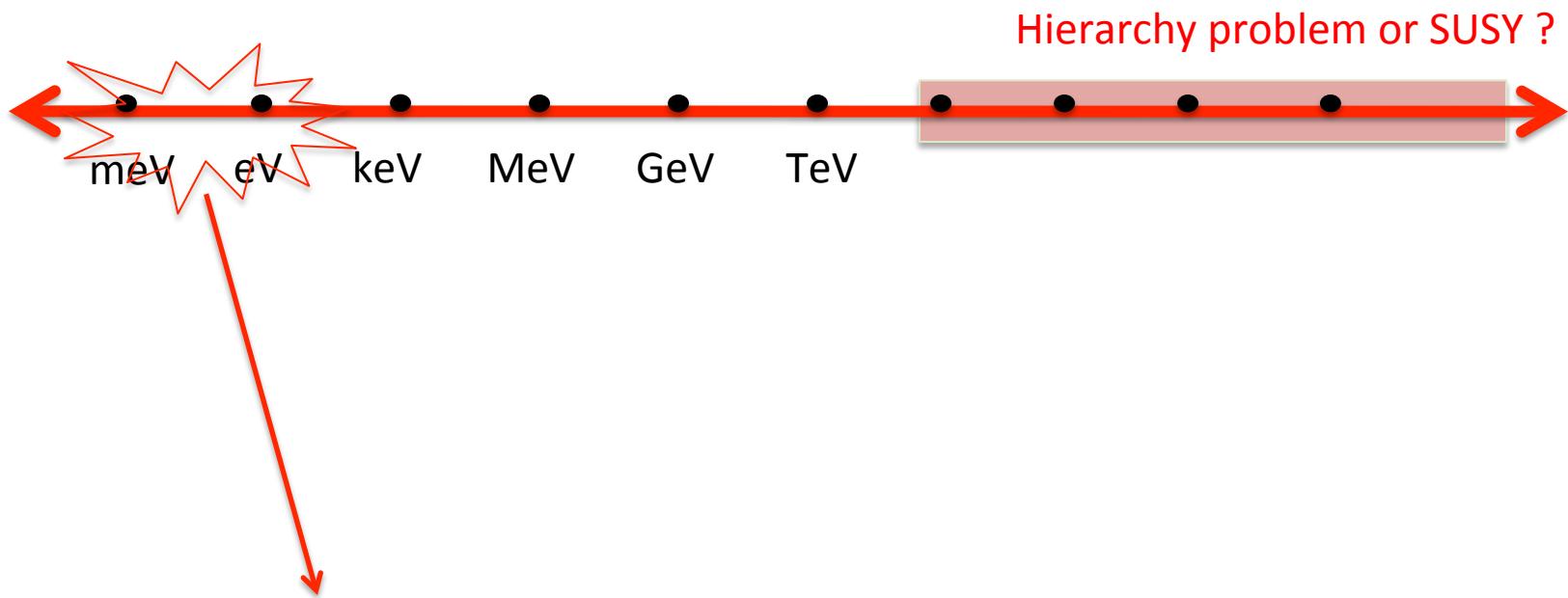
The measurement of any of these additional observables would give complementary information to that in neutrino masses, making the models much more predictive ...

Other states out there ?



Neutrino anomalies: LSND...

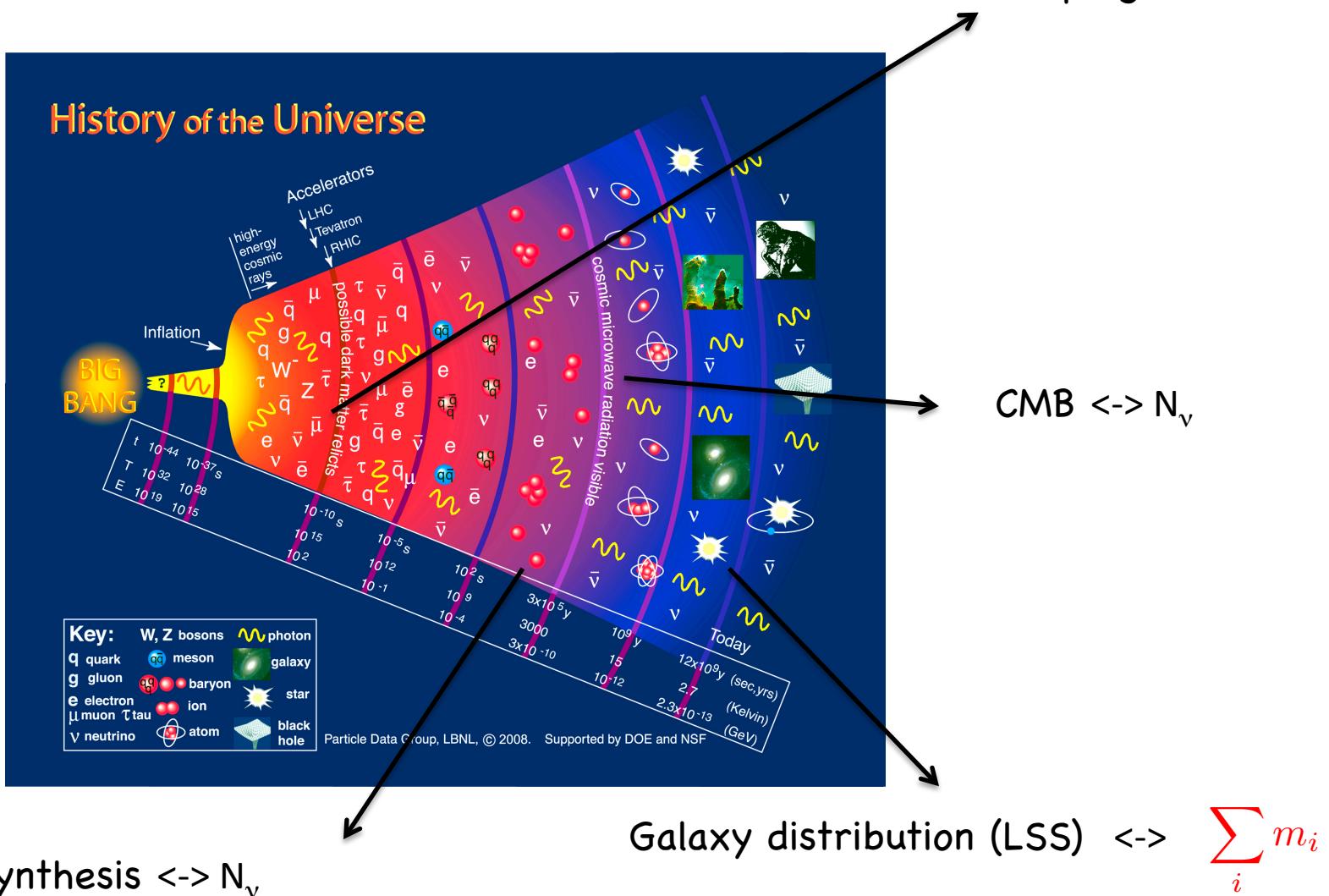
Other states out there ?



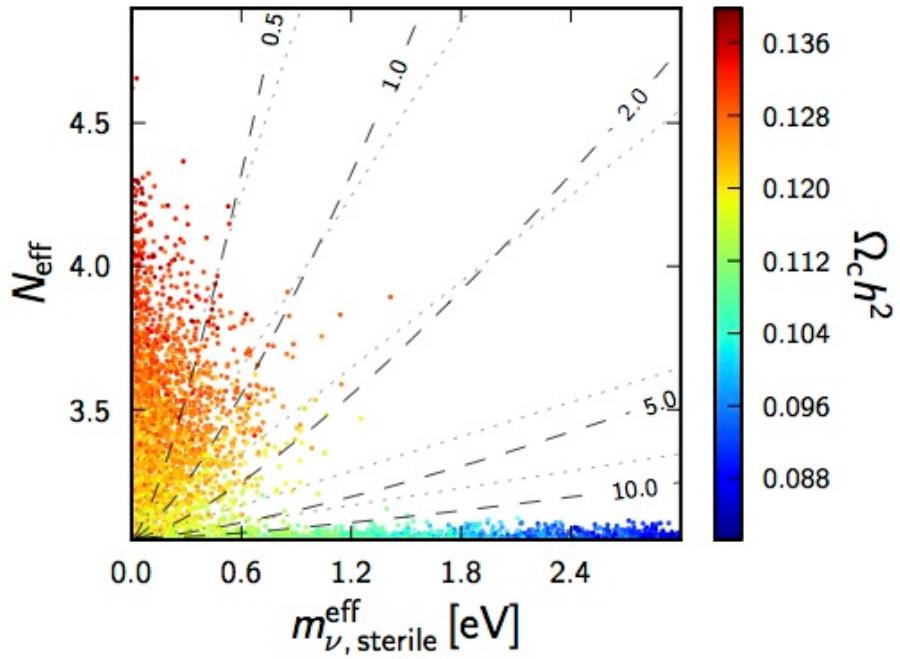
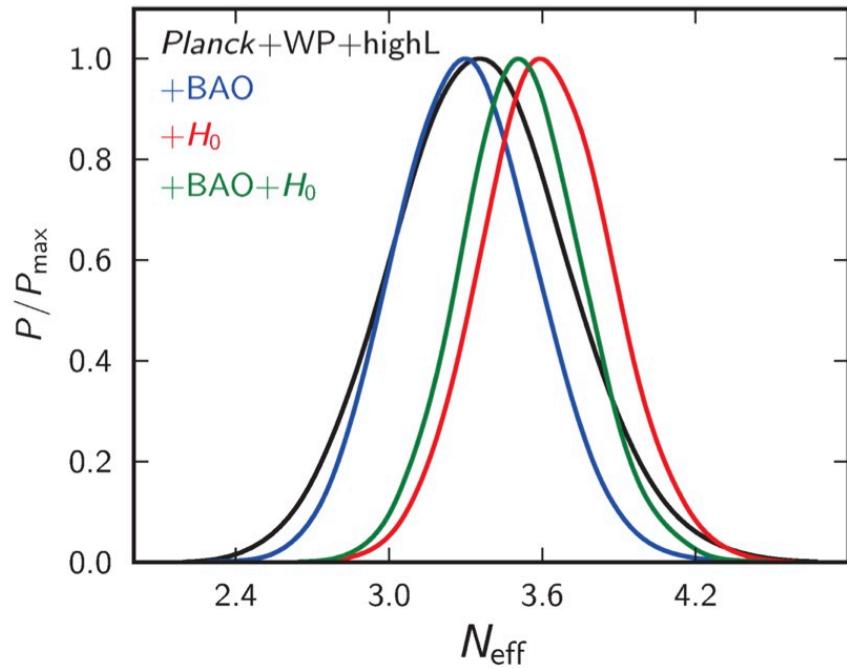
Cosmo anomalies ?

Cosmological neutrinos

they have left traces in the history of the Universe



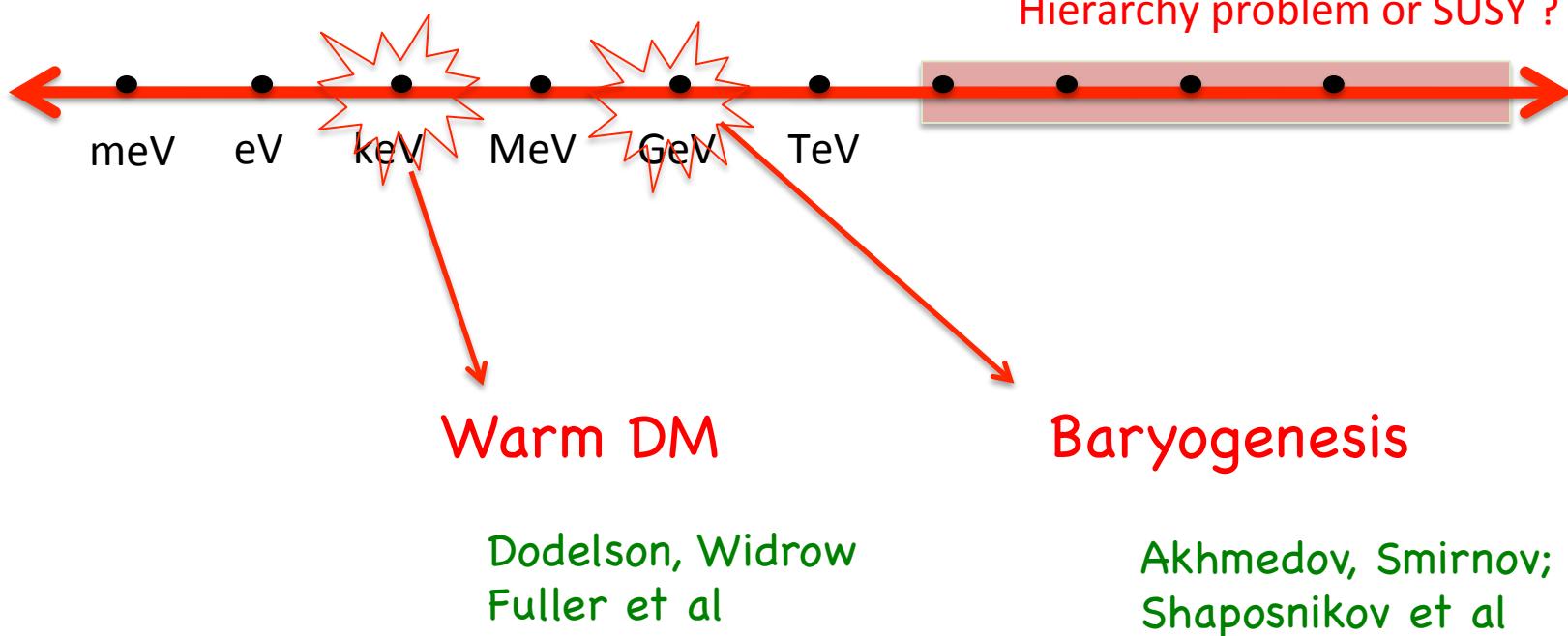
Extra relativistic species welcome...



Ade et al 2013

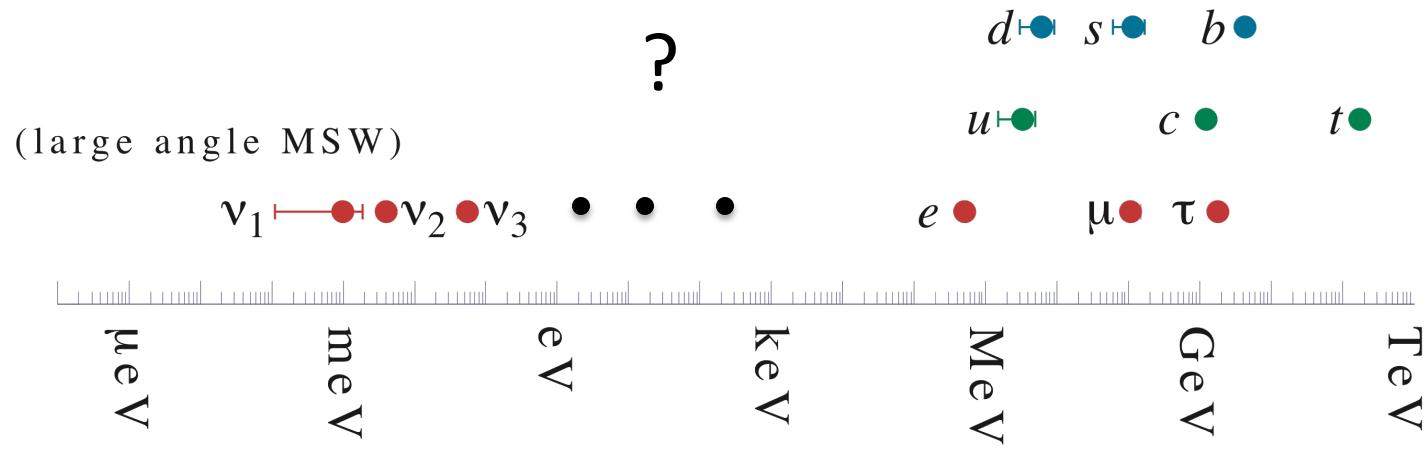
but not too heavy or not too thermal...

Other states out there ?



Even though there are typically more parameters than those in the neutrino mass, there are strong correlations...

Low-scale models...what about the hierarchy ?



Maybe no gap... but still small Yukawa's...

Other states out there: other constraints ?

Stringent constraints from peak and decay searches, unitarity, EW...

Direct production at LHC of heavy states ? Keung, Senjanovic;...

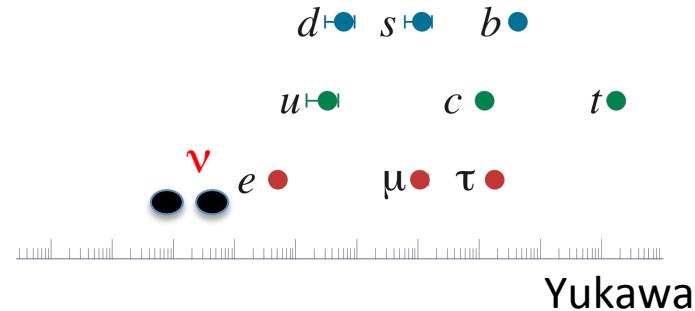
Generically it is needed

- Gauge interactions of extra fields for large enough production
(ex. type II and type III or type I + W' , Z')
- Flavour effects unsuppressed by small Yukawas: approximate $U(1)_L$

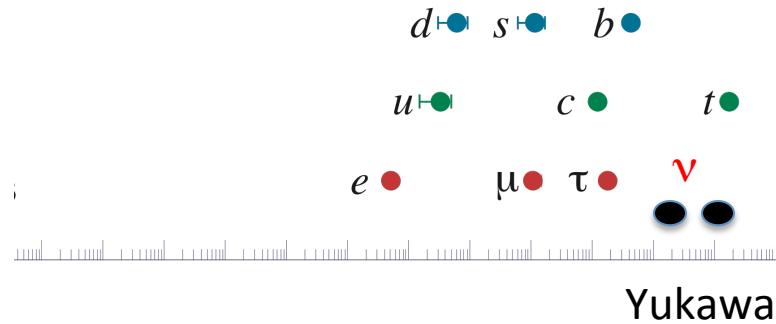
$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{\alpha}{\Lambda_{LN}} O_i^{d=5} + \sum_i \frac{\beta_i}{\Lambda_{LFV}^2} O_i^{d=6} + \dots$$
$$\Lambda_{LN} \gg \Lambda_{LFV}$$

Charged/neutral hierarchy in seesaw

$\Lambda = \text{TeV}$



$\Lambda \leq \text{TeV} + \text{aprox. } U(1)_L$



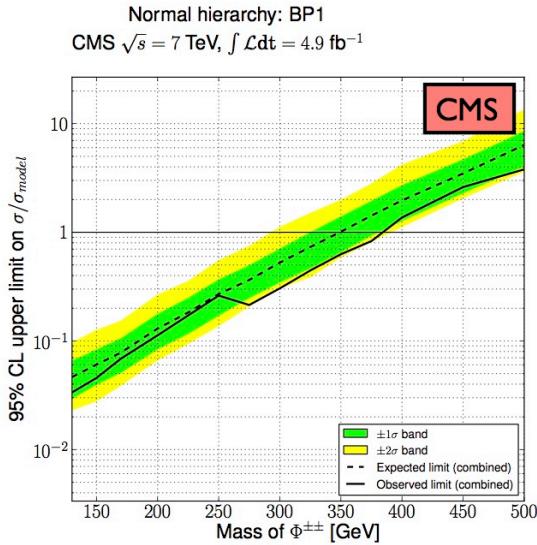
Eg: Inverse seesaw/direct seesaw type I&III or type II

Wyler, Wolfenstein; Mohapatra, Valle;
Branco, Grimus, Lavoura, Malinsky, Romao,

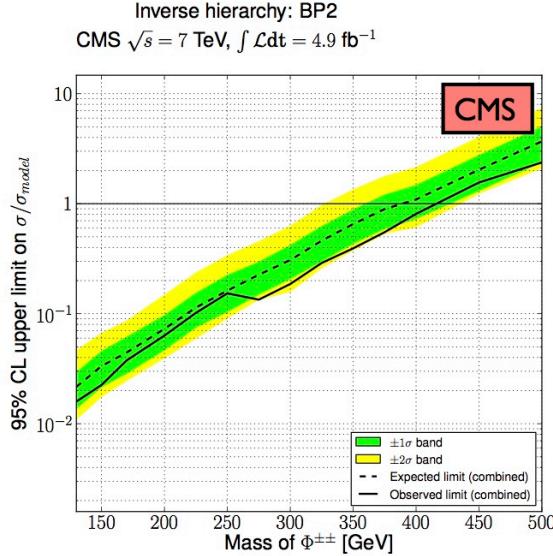
...

$$pp \rightarrow H^{++} H^{-} \rightarrow |^{+}|^{+}|^{-}|^{-}$$

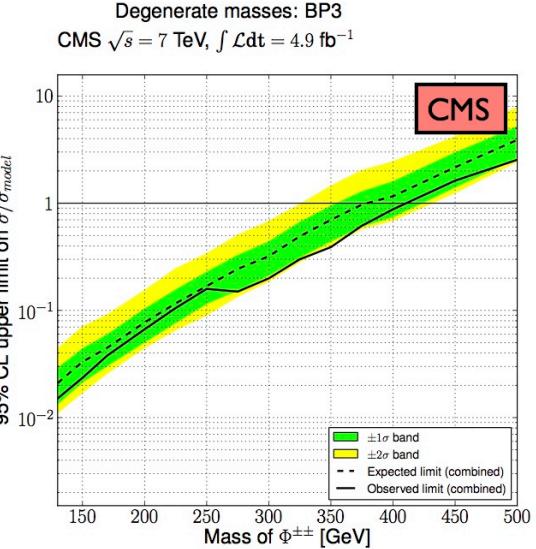
Normal hierarchy



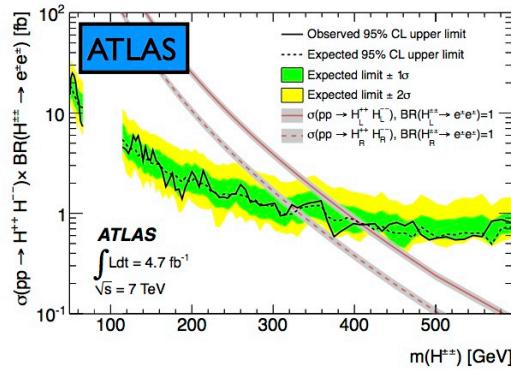
Inverted hierarchy



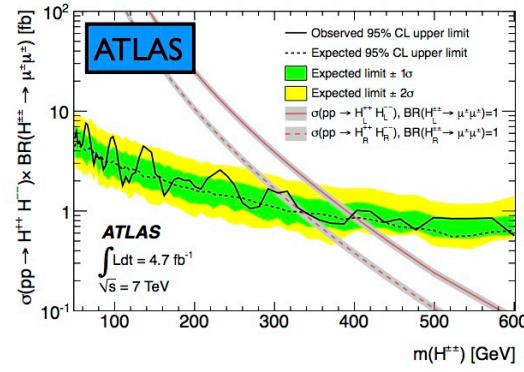
Degenerate v



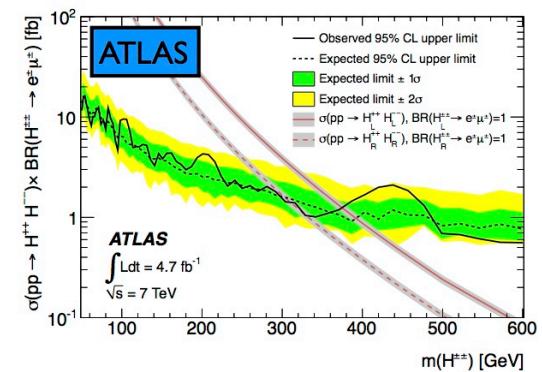
$\text{Br}(ee)=1$



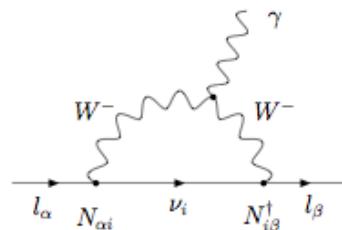
$\text{Br}(\mu\mu)=1$



$\text{Br}(e\mu)=1$

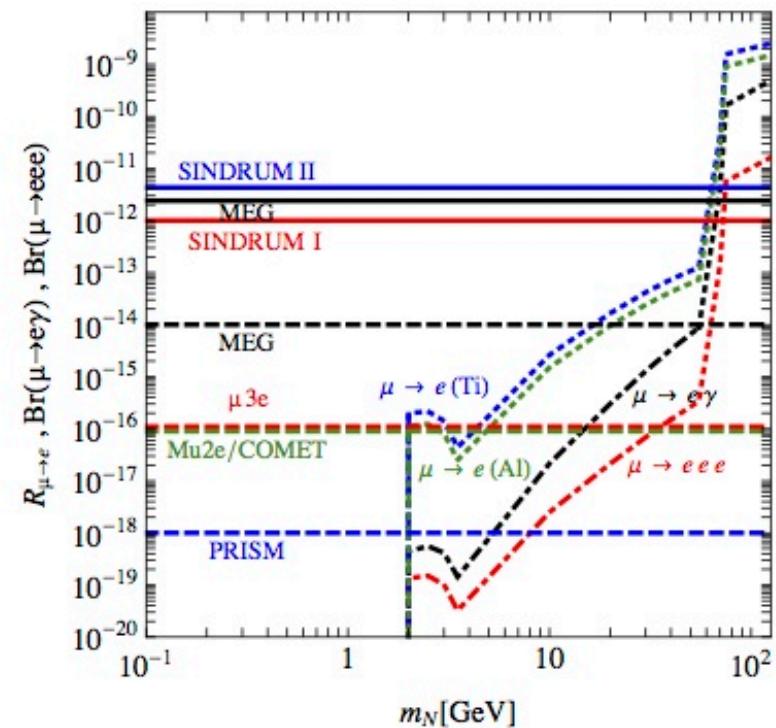
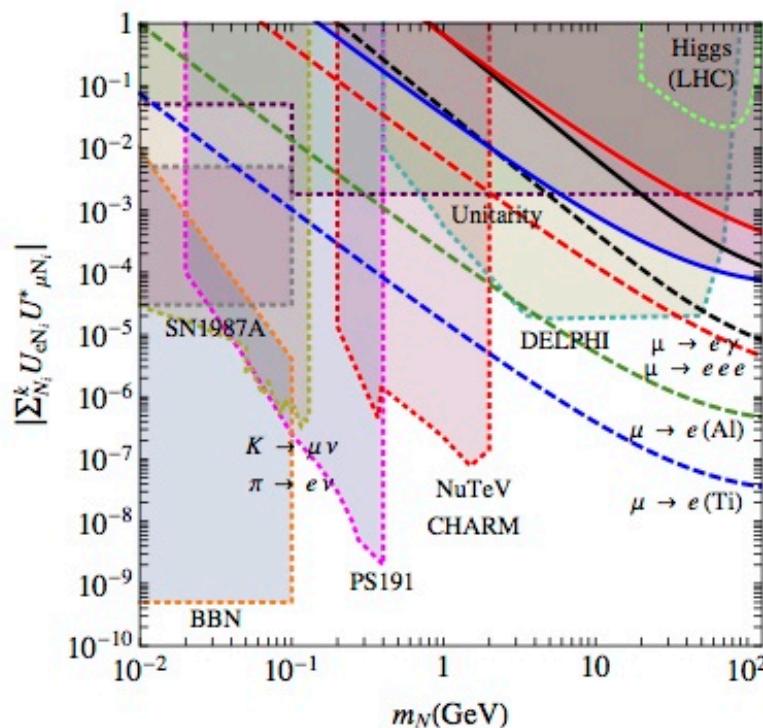


Rich phenomenology of low-scale models with U(1)



$\mu \rightarrow e \gamma$ $\mu \rightarrow eee$ $\mu \rightarrow e$ conversion

Petcov and many others



recent analysis Alonso et al 2012

Detecting such a signal would be a major breakthrough in clarifying the origin of neutrino masses

Conclusions

What the SM does not explain...

Gauge Couplings	EWSB	Quark flavour	Lepton flavour (minimal)
$g_{SU(3)}, g_{SU(2)}, g_{U(1)}$	m_h, v	$\theta_{12}, \theta_{23}, \theta_{13}, \phi$ m_u, m_c, m_t m_d, m_s, m_b	$\theta_{12}, \theta_{23}, \theta_{13}, \delta$ m_1, m_2, m_3 $, m_e, m_\mu, m_\tau$

A more fundamental theory BSM should explain the origin of these parameters

- We still don't know what the vSM is
- Neutrinos add at least as many parameters as quarks to the puzzle, but with features that might hint to a new physics scale

- The existence of a new physics scale in vSM whether related or not to the EW scale would have clear implications for the hierarchy problem and EWSB
- The observation of neutrinoless double beta decay would be the discovery of such a new physics scale
- Predicting the matter-antimatter asymmetry in the Universe would be a major achievement of the vSM

Two key ingredients: Leptonic CP violation
Lepton number violation

- Mass Hierarchy essential for reconstructing the underlying model of neutrino masses & predictions for other observables

- Sterile neutrinos are not particularly exotic but a **generic** prediction of seesaw models

It is of extreme importance to clarify neutrino anomalies and establish if there are other light sterile states since

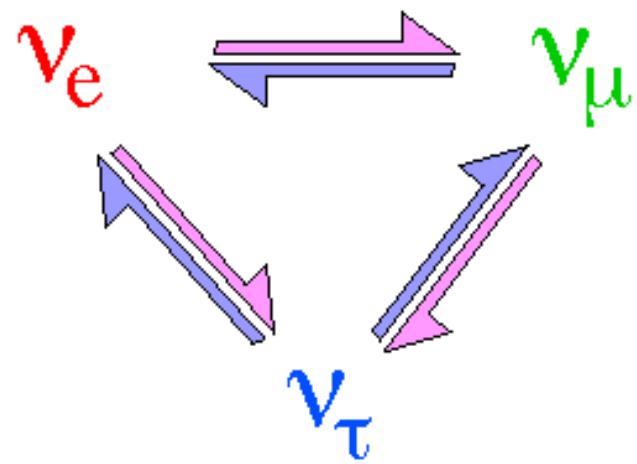
Our predictions/constraints on

- 1) matter-antimatter asymmetry
- 2) large-scale structure
- 3) nucleosynthesis
- 4) supernova explosions
- 5) the dark matter content of the Universe
- 6) rate of neutrinoless double beta decay

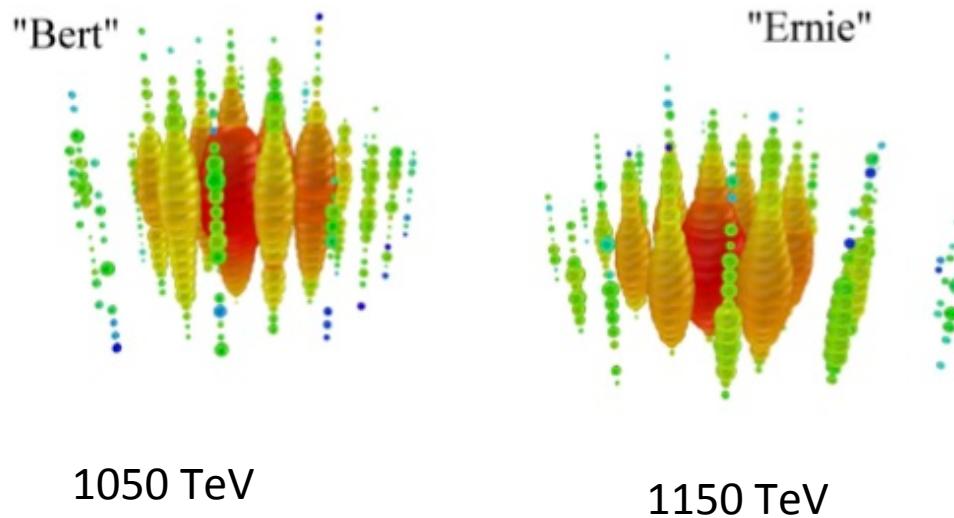
....

would depend on it !

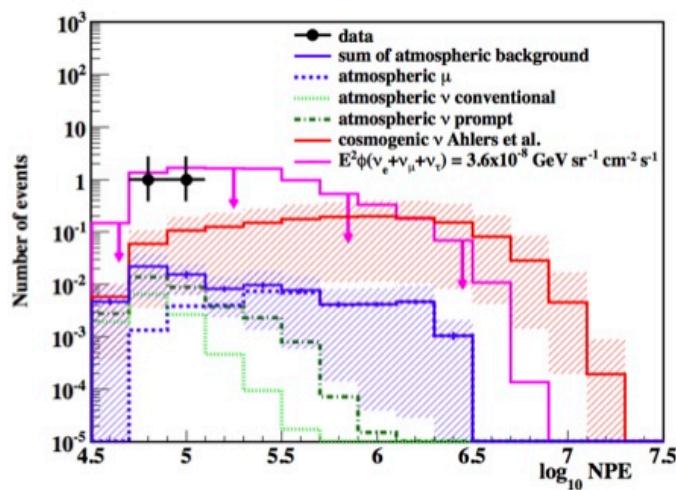
These elusive pieces of reality have brought many surprises, maybe they will continue with their tradition...



Some unexpected PeV ν events...

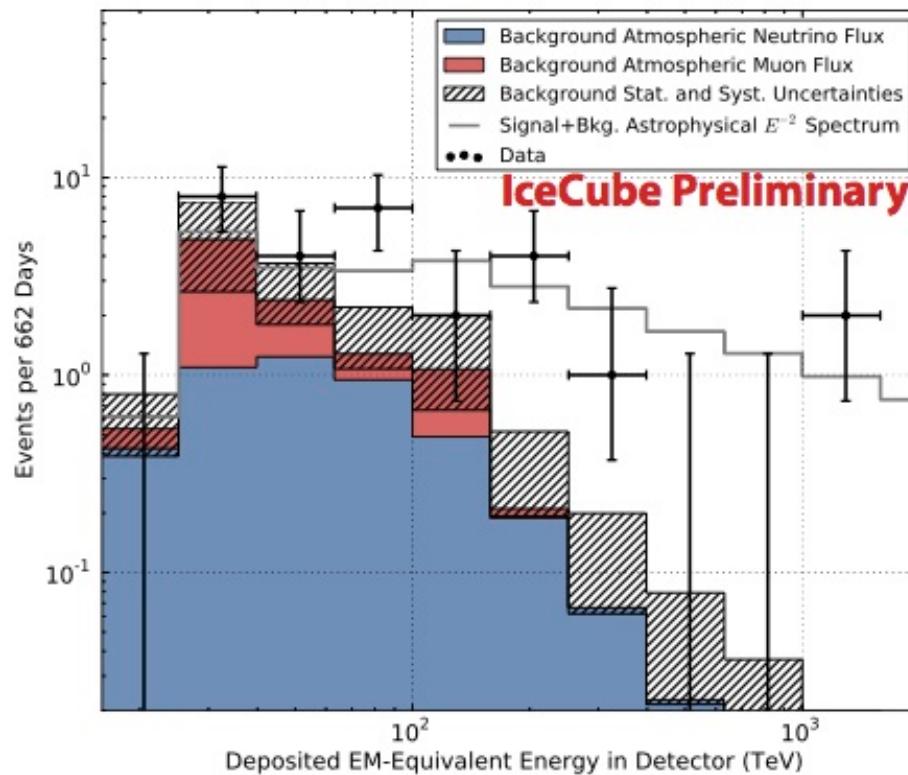


Icecube col. 1304.5356



- ▶ Too low in energy for GZK
- ▶ Seems too high in energy for atmospheric

Some more with new analysis...

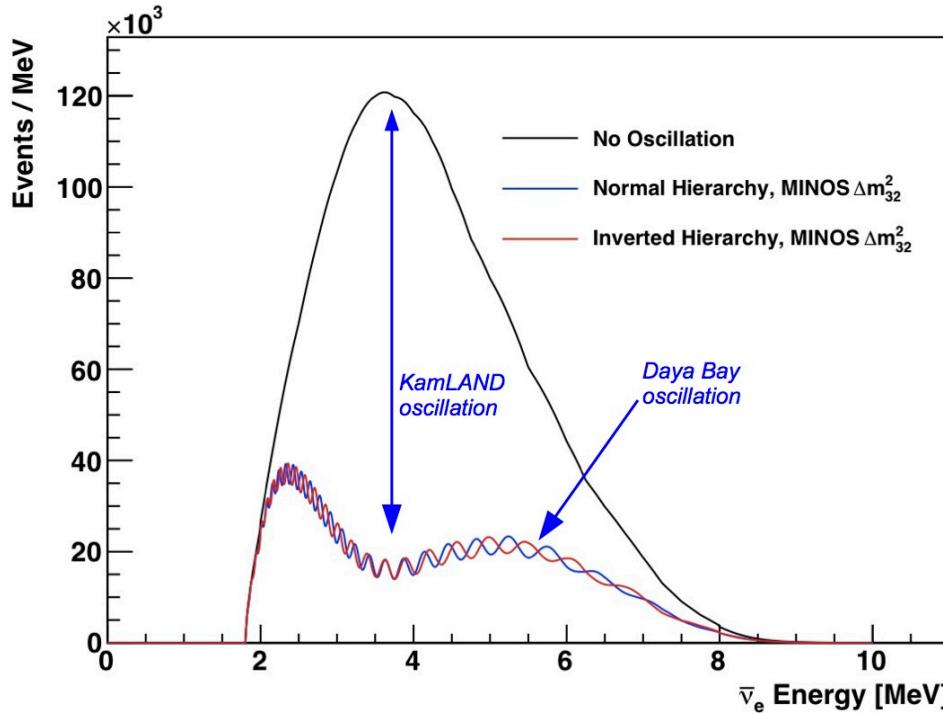


28 events: 21 cascades, 7 muons

Icecube recent talk

Hierarchy from reactor ν's

Petcov, Piai; Choubey et al; Learned et al



$$P_{\nu_e \nu_e} = 1 - c_{13}^2 \sin^2 2\theta_{12} \sin^2 \Delta_{21} - 2s_{13}^2 c_{13}^2 \left(1 - \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}} \cos(2\Delta_{32} \pm \phi) \right)$$

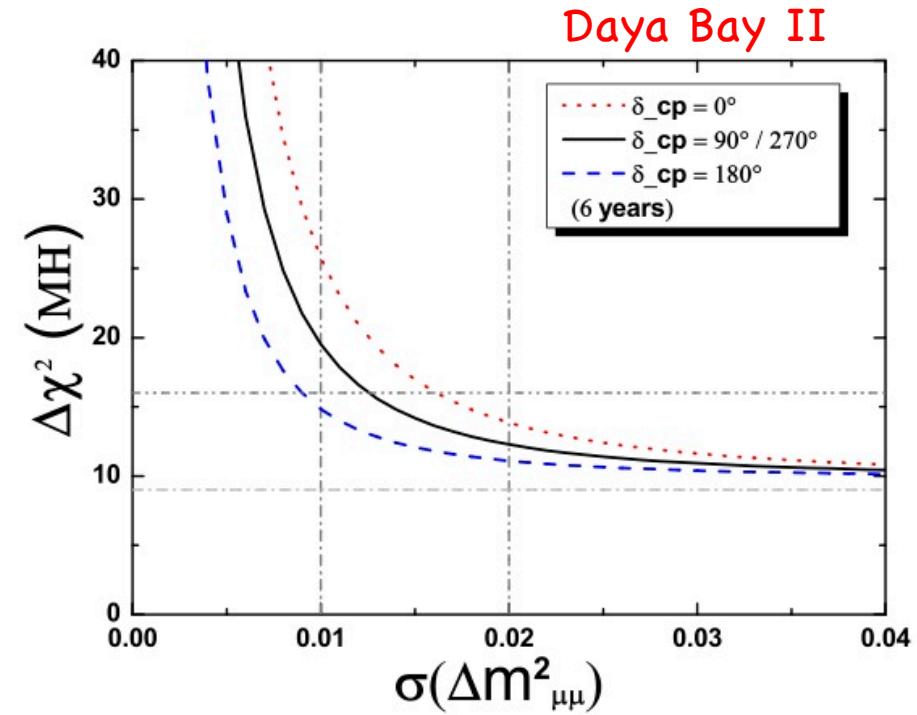
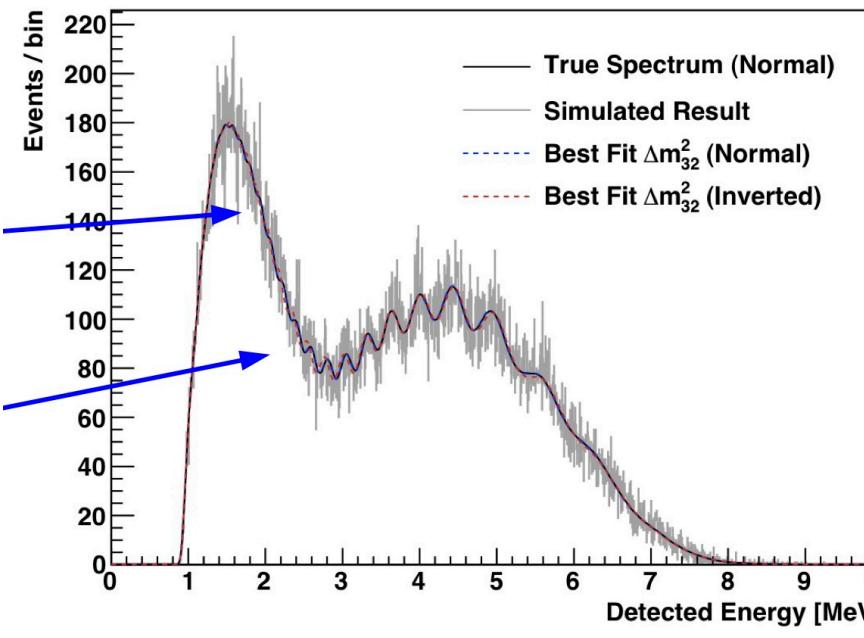
$$\sin \phi = \frac{c_{12}^2 \sin 2\Delta_{21}}{\sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}}} \quad \phi \simeq 0.12 \times 10^{-3} eV^2 \quad \pm \text{NH (IH)}$$

Hierarchy in reactors

Extremely challenging

large mass 20kton
3% energy resolution (present 6.5%)
<1% linearity in energy scale,
error on $|\Delta m_{23}^2|$

Qian, et al 1208.1551



Li et al 1303.6733

Cosmological neutrinos @nucleosynthesis

Before LEP, the best constraint on N_ν came from nucleosynthesis:

$$\nu_e \ n \leftrightarrow p \ e^-$$

$$e^+ \ n \leftrightarrow p \ \bar{\nu}_e$$

$$\langle \sigma_\nu N_\nu v \rangle \sim H(T_\nu) \sim \sqrt{g^*} \frac{T_\nu^2}{M_{\text{Planck}}} \rightarrow T_\nu \sim \left(\frac{\sqrt{g^*}}{G_F^2 M_{\text{Planck}}} \right)^{1/3} \sim \mathcal{O}(MeV)$$

At this temperature the ratio p/n gets fixed and determines the abundance of light elements:

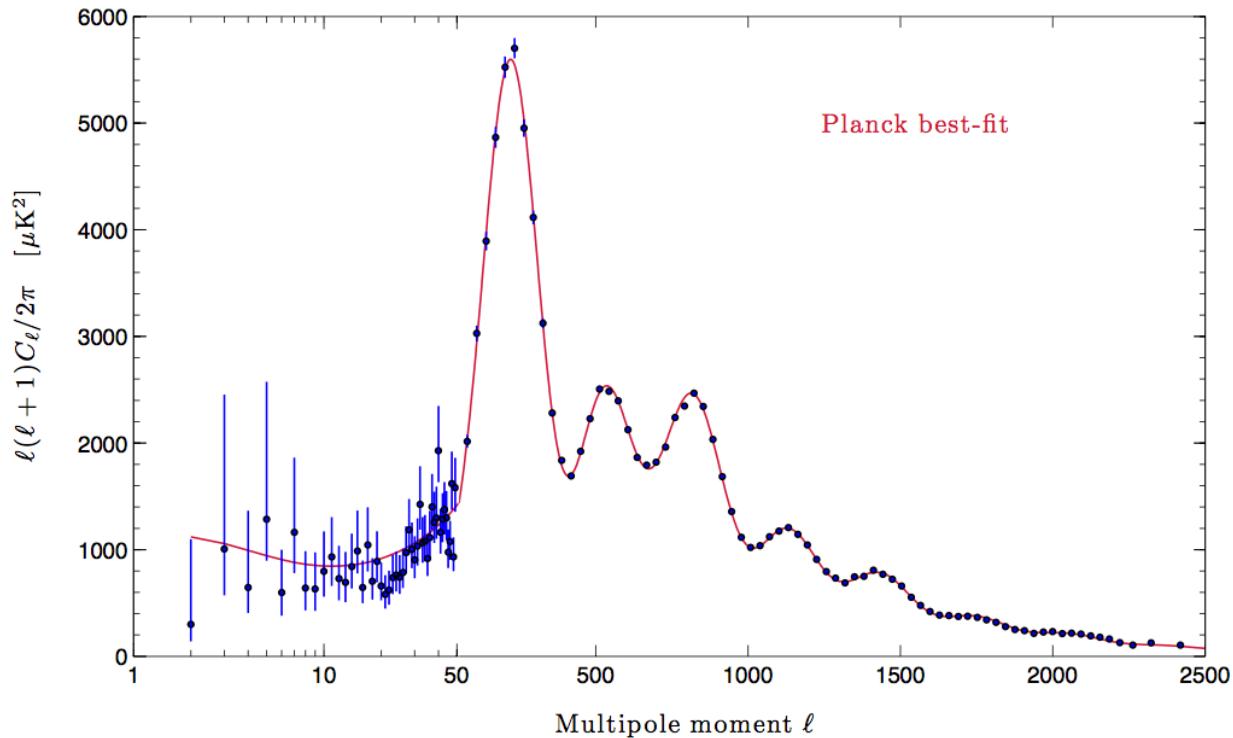
$$\frac{N_n}{N_p} = \exp \left(\frac{m_p - m_n}{T_\nu} \right) \simeq \frac{1}{6} \quad Y_{^4\text{He}} = \frac{\text{Mass of } ^4\text{He}}{\text{Total Mass}} = \frac{2N_n}{N_p + N_n}$$

Recent analysis: $N_s = 0.68^{+0.80}_{-0.70}$

Izotov, Thuan

Cosmological neutrinos @ CMB

$$T_{CMB} \sim 0.3 \text{ eV}$$



Anisotropies sensitive to the total energy density in relativistic particles ($m_\nu \ll T_{CMB}$):

$$\rho_\nu \sim \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_\nu^{\text{CMB}}$$

Massive neutrinos & LS Structure

Neutrino distribution gets frozen at BBN ($T \sim \text{MeV}$), therefore if thermal at BBN

$$N_\nu \simeq N_{\bar{\nu}} \simeq \frac{4}{11} T_\gamma^3$$

$$\Omega_\nu = \frac{\sum_i m_i}{93.5 \text{ eV}} h^{-2} < \Omega_m \rightarrow \sum_i m_i \leq 11.2 \text{ eV}$$

Gershtein, Zeldovich

But such neutrinos cannot be DM: free-streaming would distort the large scale structure power spectrum...

Sarkar's Lectures