NNLO corrections to the decay $B \to D\pi$

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DFG FOR 1873

Main motivation for NNLO correction

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- Test of QCD factorization

$$\Gamma(\bar{B}_0 \to D^+\pi^-) = \frac{G_F^2 (m_B^2 - m_D^2)^2 |\vec{q}|}{16\pi m_B^2} |V_{ud}^* V_{cb}| \, |a_1(D\pi)|^2 \, f_\pi^2 \, F_0^2(m_\pi^2)$$

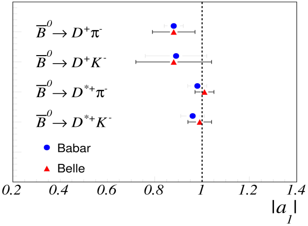
Perturbative correction to the decay $\bar{B}_0 \to D^+\pi^-$

$$a_1(\bar{B}_0 \to D^+\pi^-)_{exp} = (0.88 \pm 0.09)$$

PDG 2012, Belle 2011

$$a_1(\bar{B}_0 \to D^+\pi^-)_{NLO,HQL} = (1.055^{+0.019}_{-0.017})$$

BBNS, 2000

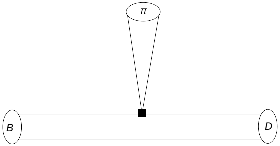


Fleischer et al, 2011

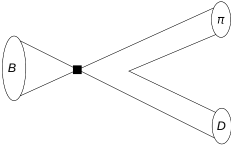
Universal raditive correction $a_1(D^{(*)+}L^-)$, $L = \{\pi, \rho, K\}$?

- NLO correction rather small:
  - color suppression
  - small Wilson coefficient

QCD factorization



Tree topology



Weak annihilation, power suppressed

m_c heavy

m_c/m_b fixed as $m_b \to \infty$

Tree topology can be factorized in the heavy quark limit:

$$\langle D^+\pi^-|\mathcal{Q}_i|\bar{B}_0\rangle = \sum_j F_j^{B\to D}(m_\pi^2)\int_0^1 du\,T_{ij}(u)\phi_\pi(u) + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

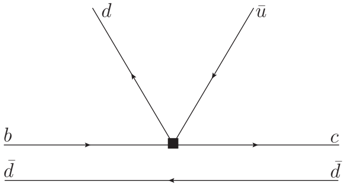
BBNS, 2000

$F_j^{B\to D}$: $B \to D$ form factor
ϕ_π : light-cone distribution amplitude of the pion
} soft part, non-perturbative

T_{ij} : hard scattering kernel   hard part, perturbative

$$T_{ij} = T_{ij}^0 + \alpha_s T_{ij}^1 + \alpha_s^2 T_{ij}^2 + O(\alpha_s^3)$$

Tree topology

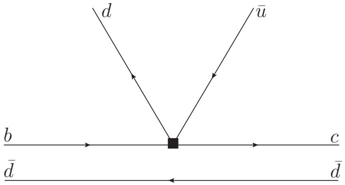


Effective Hamiltonian:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} \left(C_0 \mathcal{Q}_0 + C_8 \mathcal{Q}_8 \right)$$

$$\left.\begin{aligned} \mathcal{Q}_0 &= \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b \\ \mathcal{Q}_8 &= \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b \end{aligned}\right\}$$ Chetyrkin-Misiak-Münz basis

Tree topology



Neglect spectator quark in the following

Effective Hamiltonian:

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NLO hard scattering kernel

NLO diagrams:

NLO result

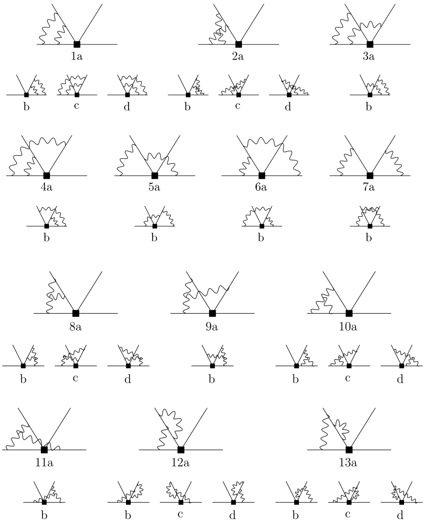
$$T_0(u,z) = 1 + O(\alpha_s^2)$$

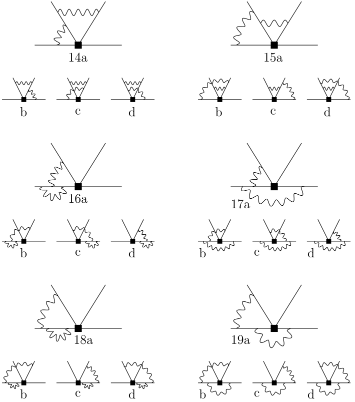
$$T_8(u,z) = \frac{\alpha_s}{(4\pi)} \frac{C_F}{2N_C} \left[-6 ln \frac{\mu^2}{m_b^2} - 11 + F(u,z) \right] + O(\alpha_s^2)$$

$z = m_c / m_b$

u momentum fraction

agrees with *BBNS 2000*





ca. 70 diagrams

Dimensional regularization in $d = 4 - 2\epsilon$ dimensions

Renormalization of strong coupling α_s in $\overline{\text{MS}}$ scheme

Calculation divided in two parts:

(1) Strings of Dirac matrices: reduced to set of operators by inhouse mathematica (MMA) routine

(2) 2-loop integrals: Laporta algorithm (MMA package: FIRE) Laporta, Remiddi 1996 Smirnov, 2008

reduces huge number of complicated 2-loop integrals to a few rather simple master integrals

Calculation methods for unknown master integrals

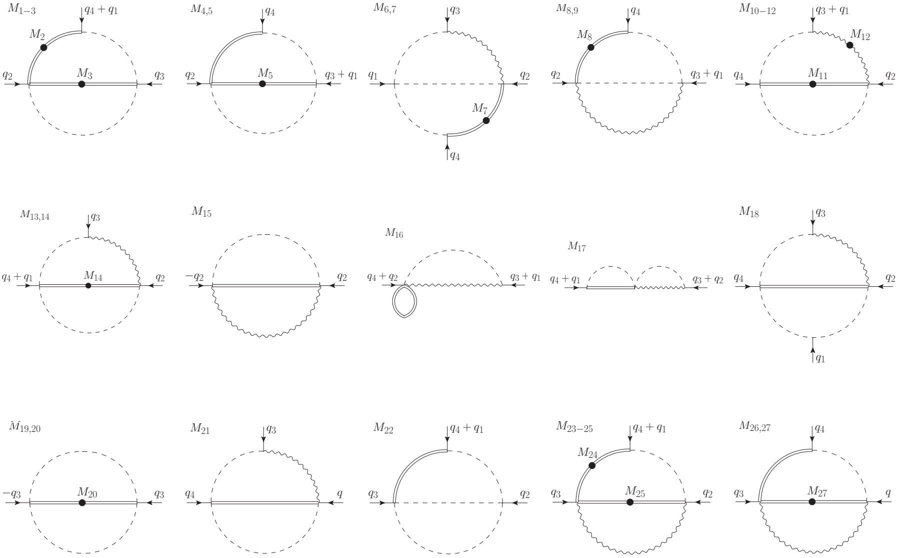
(a) Feynman parameters (MMA package: HypExp) Huber, Maitre, 2005

(b) Mellin Barnes representation (MMA packages: Ambre, MB) Gluza et al, 2007 Czakon, 2006

(c) Differential equation Kotikov, 1991, Caffo et al, 1998, Argeri, Mastrolia 2007

We found: 27 unknown master integrals

Master Integrals



Conclusion and outlook

- Importance of precision calculations to test the SM
- Test model-independent framework of QCD factorization
- Calculation of $BR(\bar{B}_d^0 \to D^+\pi^-)$ and estimation of uncertainties
- Check whether calculation is applicable to other decays like $\bar{B}_d^0 \to D^{*+}\pi^-$ or $\bar{B}_d^0 \to D^+\rho^-$