Recent developments in the type IIB matrix model

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"Workshop on Noncommutative Field Theory and Gravity"

Corfu, Greece

1. Introduction

Particle physics		Cosmology
Standard Model	successful phenomenological models	Inflation and Big Bang
discovery of SM-like Higgs particle at LHC	experiments and observations	CMB (WMAP, PLANCK) structure formation of galaxies nucleosynthesis
What is the origin of Higgs particle ? SM fermions ? (3 generations) the SM gauge group ?	missing fundamental understanding	What is the origin of inflaton , its potential, its initial condition ? dark matter, dark energy ?
hierarchy problem	serious naturalness problems	cosmological constant problem

All these problems may be solved by superstring theory !

string phenomenology

string cosmology

The most fundamental issue in superstring theory is

space-time dimensionality

(10-dimensional space-time is required for consistency.)

conventional approach:

- compactify extra dimensions.

infinitely many **perturbative** vacua

with various
- space-time dimensionality
- gauge symmetry
- matter (#generations)

- D-branes (representing certain nonperturbative effects)

enriched both string phenomenology and string cosmology

intersecting D-brane models
D-brane inflation models
flux compactification …

Too many models, and no predictive power !!!

However, we should not forget that
all these perspectives are obtained from
mostly perturbative studies of superstring theory

(including at most the nonpertubative effects
represented by the existence of D-branes.)

A totally new perspective might appear
if one studies superstring theory
in a complete nonperturbative framework.

c.f.) lattice gauge theory in the case of QCD

confinement of quarks, hadron mass spectrum, etc.

(Can never be understood from perturbation theory !!!)

type IIB matrix model (Ishibashi, Kawai, Kitazawa, Tsuchiya, 1996)

nonperturbative formulation of superstring theory
<u>based on type IIB superstring theory in 10d</u>

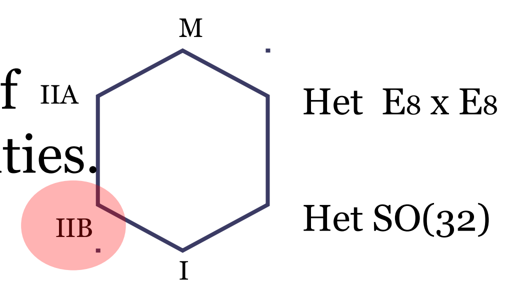
- The connection to perturbative formulations can be seen manifestly by considering 10d type IIB superstring theory in 10d.

  worldsheet action,
  light-cone string field Hamiltonian, etc.

- A natural extension of the "one-matrix model", which is established as a nonperturbative formulation of non-critical strings

  regarding Feynman diagrams in matrix models as string worldsheets.

- It is expected to be a nonperturbative formulation of <u>the unique theory</u> underlying the web of string dualities.



(Other types of superstring theory can be represented as perturbative vacua of type IIB matrix model)

type IIB matrix model

SO(9,1) symmetry

$$S_b = -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$

$$S_f = -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

$N \times N$ Hermitian matrices

A_μ $(\mu = 0, \cdots, 9)$ Lorentz vector

Ψ_α $(\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

Lorentzian metric $\eta = \mathrm{diag}(-1, 1, \cdots, 1)$
is used to raise and lower indices

Wick rotation $(A_0 = -iA_{10}\ , \quad \Gamma^0 = i\Gamma_{10})$

⇒ Euclidean matrix model SO(10) symmetry

Lorenzian v.s. Euclidean

There was a reason why no one dared to study the Lorentzian model for 15 years!

$$S_{\rm b} \propto {\rm tr}\,(F_{\mu\nu}F^{\mu\nu}) = -2\,\underbrace{{\rm tr}\,(F_{0i})^2 + {\rm tr}\,(F_{ij})^2}_{\text{opposite sign!}}$$

$$F_{\mu\nu} = -i[A_\mu, A_\nu]$$

opposite sign!

extremely unstable system.

Once one Euclideanizes it,

$$A_0 = -iA_{10}$$

$$S_{\rm b} \propto {\rm tr}\,(F_{\mu\nu})^2$$

positive definite!

flat direction ($[A_\mu, A_\nu] \sim 0$) is lifted up due to quantum effects.

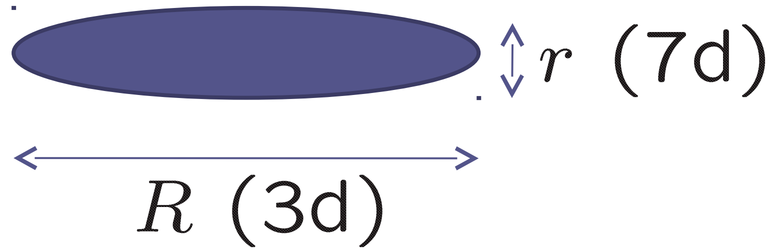
Aoki-Iso-Kawai-Kitazawa-Tada ('99)

Euclidean model is well defined without the need for cutoffs.

Krauth-Nicolai-Staudacher ('98), Austing-Wheater ('01)

Recent developments

- Euclidean model



SSB : SO(10)$\rightarrow$SO(3)

$$\frac{R}{r} \sim 5$$

➢ Interpretation of these results is unclear, though.
➢ Wick rotation is **not** justifiable unlike in ordinary QFT!

- Lorentzian model

➢ Can be made well-defined by introducing IR cutoffs and removing them in the large-*N* limit.
➢ Real-time evolution can be extracted from matrix configurations.
➢ Expanding 3d out of 9d (SSB), inflation, big bang,…
➢ A natural solution to the cosmological constant problem.
➢ Realization of the Standard Model (Tsuchiya's talk)

Lorentzian version of type IIB matrix model is indeed the correct nonperturbative formulation of superstring theory, which describes our Universe.

Plan of the talk

1. Introduction
2. Euclidean type IIB matrix model
3. Lorentzian type IIB matrix model
4. Expanding 3d out of 9d
5. Exponential/power-law expansion
6. Time-evolution at much later times
7. Summary and future prospects

2. Euclidean type IIB matrix model

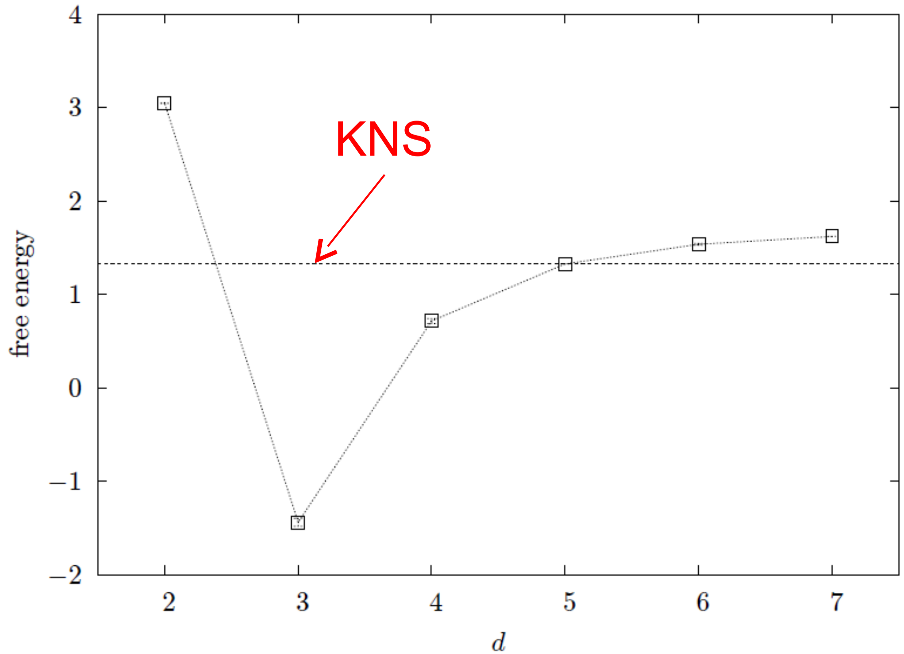
J.N.-Okubo-Sugino, JHEP1110(2011)135, arXiv:1108.1293

Aoyama-J.N.-Okubo, Prog.Theor.Phys. 125 (2011) 537, arXiv:1007.0883

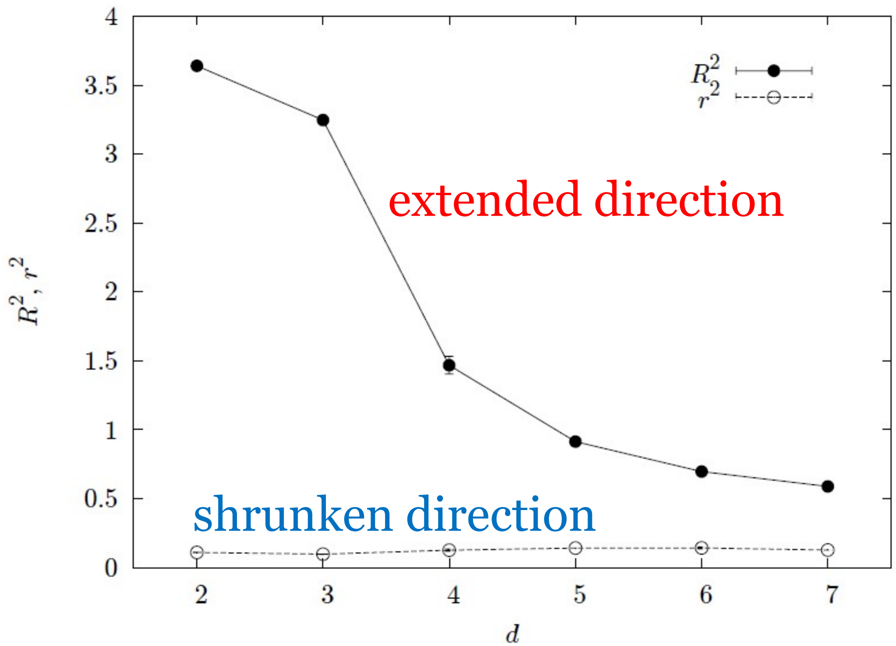
Anagnostopoulos -Azuma-J.N., arXiv:1306.6135

Most recent results based on the Gaussian expansion method

J.N.-Okubo-Sugino, JHEP1110(2011)135, arXiv:1108.1293



d=3 gives the minimum free energy



universal shrunken direction

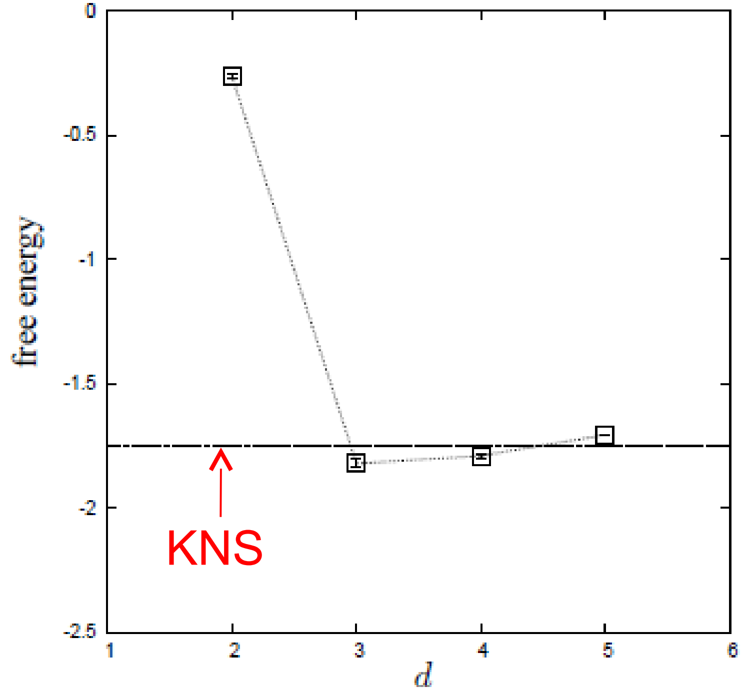
SO(10) $\xrightarrow{\text{SSB}}$ SO(3)

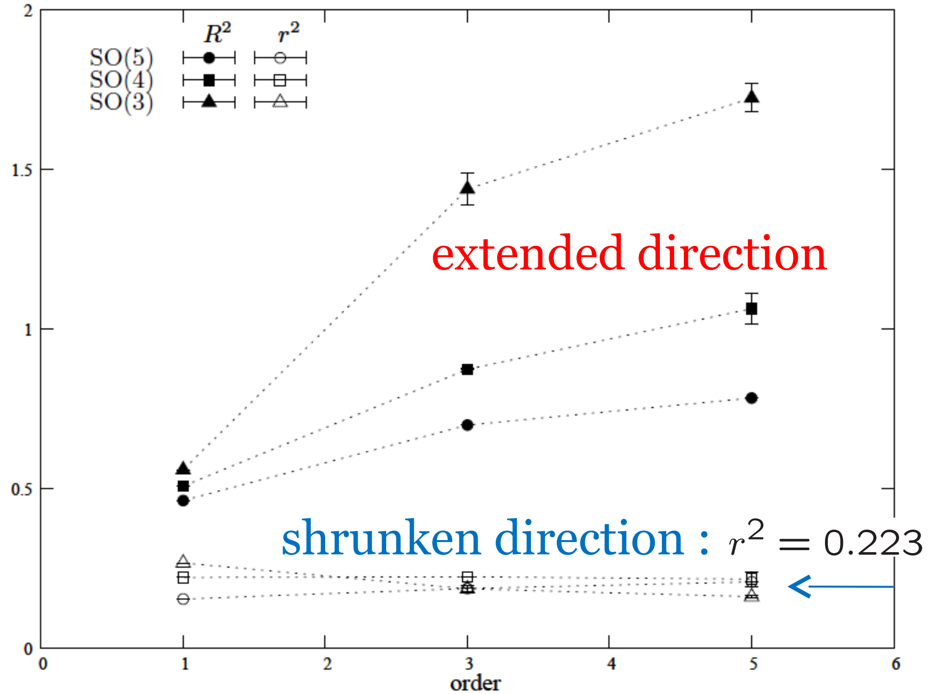
Space-time has finite extent in all directions.

This is an interesting dynamical property of the Euclideanized type IIB matrix model.
However, its physical meaning is unclear…

Results for 6d SUSY model (Gaussian expansion method)

Aoyama-J.N.-Okubo, Prog.Theor.Phys. 125 (2011) 537, arXiv:1007.0883





d=3 gives the minimum free energy

universal shrunken direction

$SO(6) \xrightarrow{SSB} SO(3)$

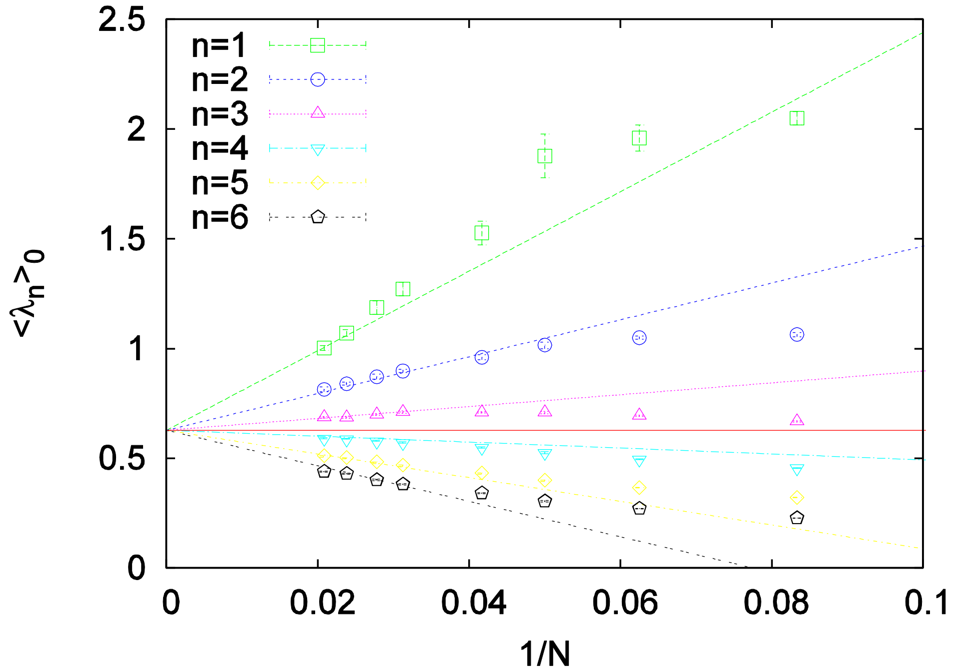
The mechanism for the SSB is demonstrated by Monte Carlo studies in this case. "phase of the fermion determinant"

No SSB if the phase is omitted

Anagnostopoulos –Azuma-J.N., arXiv:1306.6135

$$T_{\mu\nu} = \frac{1}{N}\mathrm{Tr}(A_\mu A_\nu) \qquad 6 \times 6 \text{ real symmetric}$$

eigenvalues : $\lambda_1 > \lambda_2 > \cdots > \lambda_6$

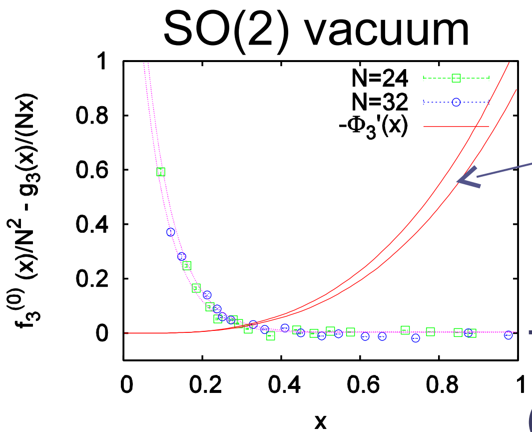


$\ell^2 \sim 0.627$

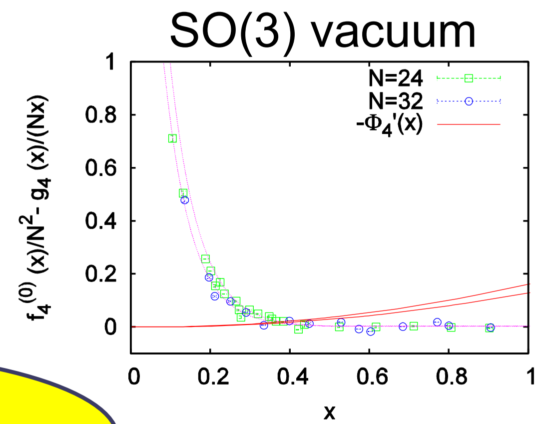
All the eigenvalues converge to the same value at large *N* !

Effects of the phase : universal shrunken direction

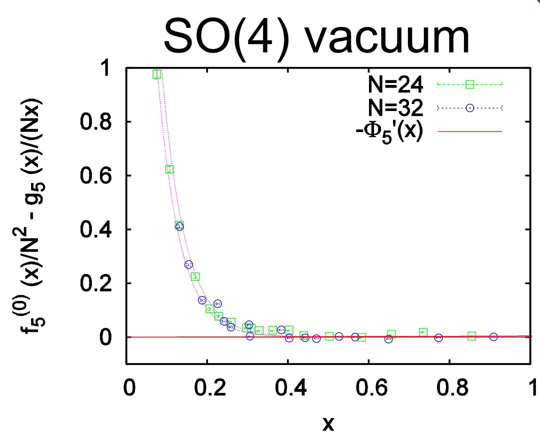
Anagnostopoulos -Azuma-J.N., arXiv:1306.6135



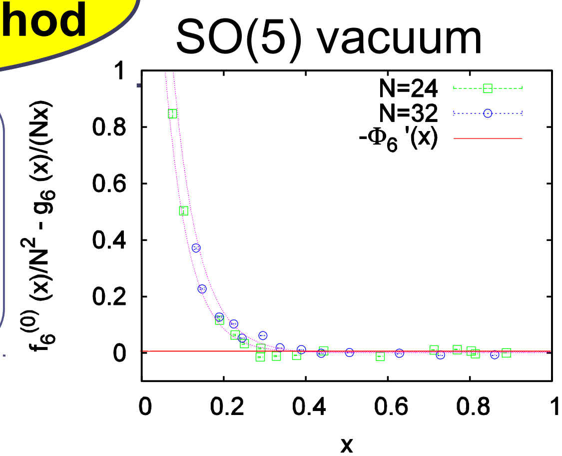
effects of the phase
of fermion determinant



analysis based on
the factorization method



Anagnostopoulos –J.N.
Phys.Rev. D66 (2002) 106008
hep-th/0108041



The extent of the shrunken direction $x \sim 0.35$ for all vacua.

Consistent with the GEM calculations !

c.f.) $x = \frac{r^2}{\ell^2} \sim \frac{0.223}{0.627} \sim 0.355$

After all, the problem was in the Euclideanization ?

- In QFT, it can be fully justified as an analytic continuation.
  (That's why we can use lattice gauge theory.)

- On the other hand, it is subtle in gravitating theory.
  (although it might be OK at the classical level…)

  - Quantum gravity based on dynamical triangulation (Ambjorn et al. 2005)
    (Problems with Euclidean gravity may be overcome in Lorentzian gravity.)

  - Coleman's worm hole scenario for the cosmological constant problem
    (A physical interpretation is possible only by considering the Lorentzian version instead of the original Euclidean version.) Okada-Kawai (2011)

- Euclidean theory is useless for studying the real time dynamics such as the expanding Universe.

3. Lorentzian type IIB matrix model

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601
[arXiv:1108.1540]

Definition of Lorentzian type IIB matrix model

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601 [arXiv:1108.1540]

partition function

$$Z = \int dA\, d\Psi\, e^{iS} = \int dA\, e^{iS_{\mathrm{b}}} \mathrm{Pf}\mathcal{M}(A)$$

This seems to be natural from the connection to the worldsheet theory.

$$S = \int d^2\xi \sqrt{g} \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu, \Psi\} \right)$$

$$\xi_0 \equiv -i\xi_2$$

(The worldsheet coordinates should also be Wick-rotated.)

Regularization and the large-N limit

Unlike the Euclidean model,
the Lorentzian model is NOT well defined as it is.

- The extent in temporal and spatial directions should be made finite.
  (by introducing cutoffs)

$$\frac{1}{N}\mathrm{tr}\,(A_0)^2 \le \kappa L^2$$
$$\frac{1}{N}\mathrm{tr}\,(A_i)^2 \le L^2$$

In what follows, we set $L = 1$ without loss of generality.

- It turned out that these two cutoffs can be removed in the large-N limit.
  (highly nontrivial dynamical property)

- Both SO(9,1) symmetry and supersymmetry are broken explicitly by the cutoffs.
  The effect of this explicit breaking is expected to disappear in the large-N limit. （needs to be verified.)

Sign problem can be avoided !

$$Z = \int dA\, d\Psi\, e^{iS} = \int dA\, \underbrace{e^{iS_{\mathrm{b}}}}_{(2)}\, \underbrace{\boxed{\mathrm{Pf}\mathcal{M}(A)}}_{(1)}$$

The two possible sources of the problem.

(1) Pfaffian coming from integrating out fermions

$\mathrm{Pf}\mathcal{M}(A) \in \mathbf{R}$

The configurations with positive Pfaffian become dominant at large *N*.

In Euclidean model, $\mathrm{Pf}\mathcal{M}(A) \in \mathbf{C}$

This complex phase induces the SSB of SO(10) symmetry.

J.N.-Vernizzi ('00), Anagnostopoulos-J.N.('02)

Sign problem can be avoided ! (Con'd)

(2) What shall we do with $e^{iS_{\text{b}}}$

The same problem occurs in QFT
in Minkowski space
(Studying real-time dynamics in QFT
is a notoriously difficult problem.)

$$Z = \int dA\, e^{iS_{\text{b}}} \mathrm{Pf}\mathcal{M}(A)$$

$$A_\mu = \rho\, \tilde{A}_\mu$$

First, do $\int d\rho$

$$\left.\begin{array}{l} S_{\text{b}} \propto \mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu]) \propto \rho^4 \\ \mathrm{Pf}\mathcal{M}(A) \propto \rho^{16(N^2-1)} \end{array}\right\}$$

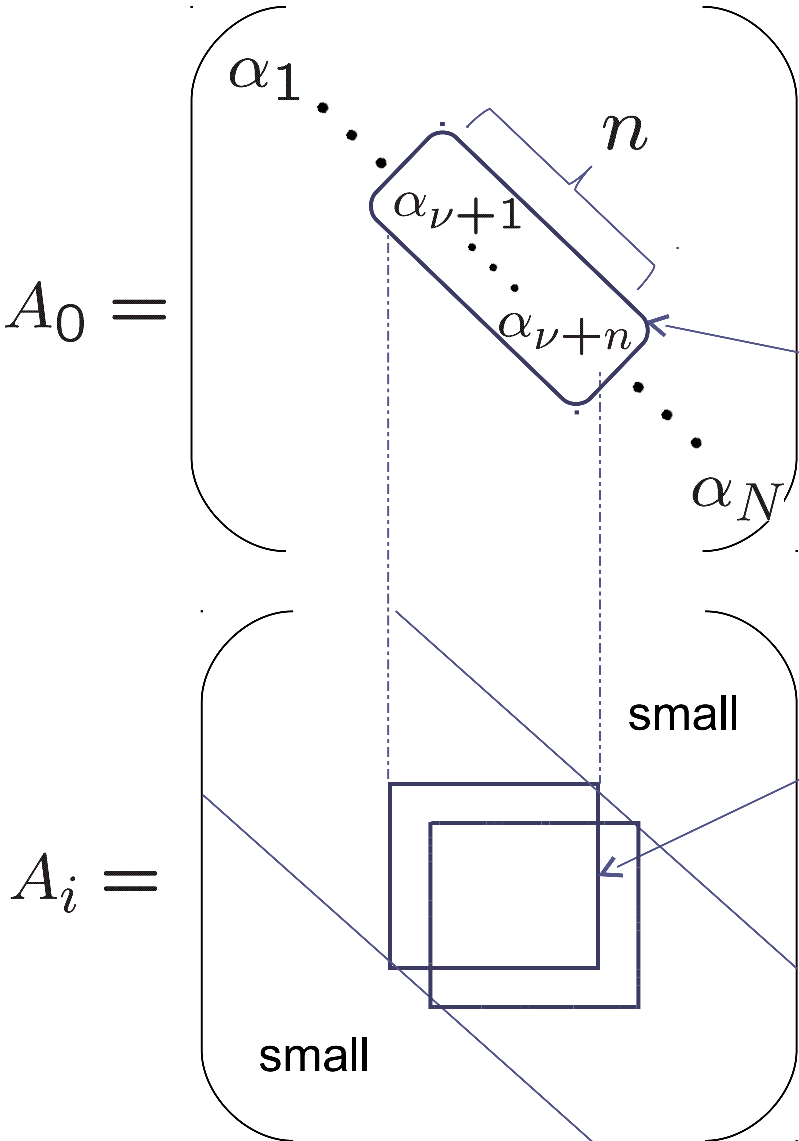
homogenous in A_μ

$$Z \propto \int d\tilde{A}\, \delta(S_{\text{b}}(\tilde{A}))\, \mathrm{Pf}\mathcal{M}(\tilde{A})$$

4. Expanding 3d out of 9d

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601
[arXiv:1108.1540]

How to extract time-evolution



diagonalize A_0

$\alpha_1 < \cdots < \alpha_N$

SU(N) transformation

definition of time "t"

$$t = \frac{1}{n}\sum_{i=1}^{n} \alpha_{\nu+i}$$

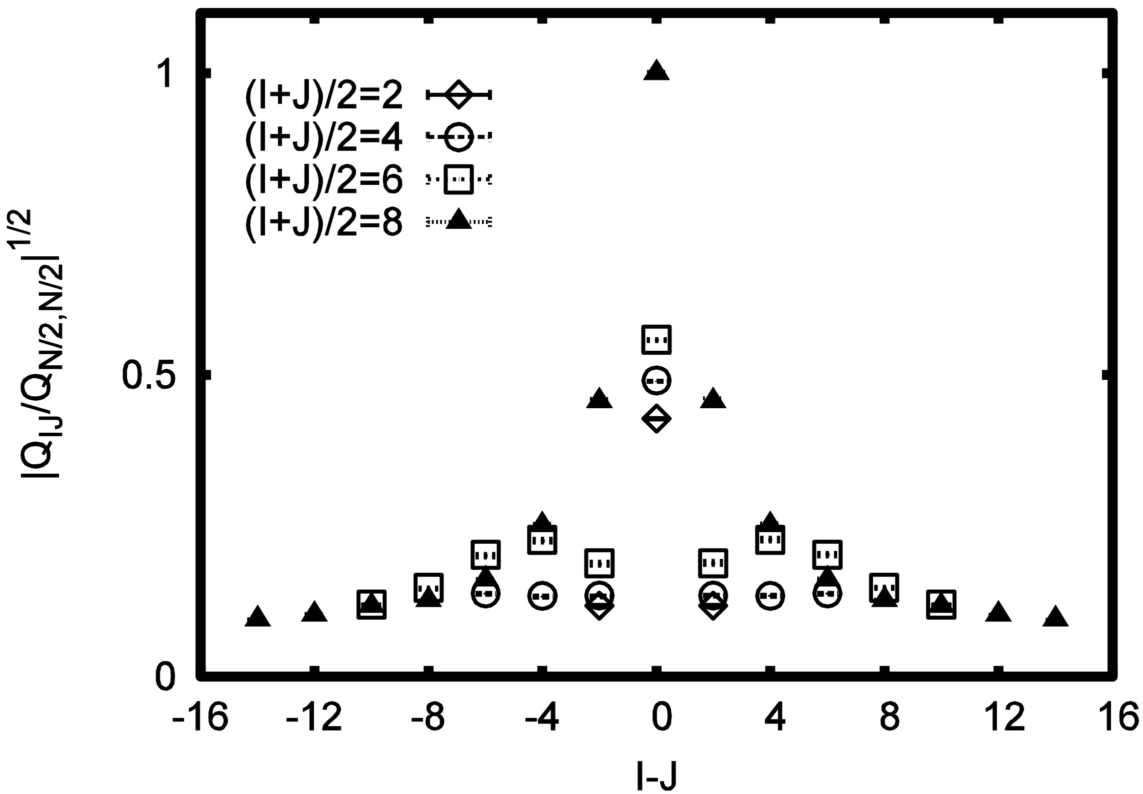
The state of the universe $\bar{A}_i(t)$ at time t

A_i has a band diagonal structure

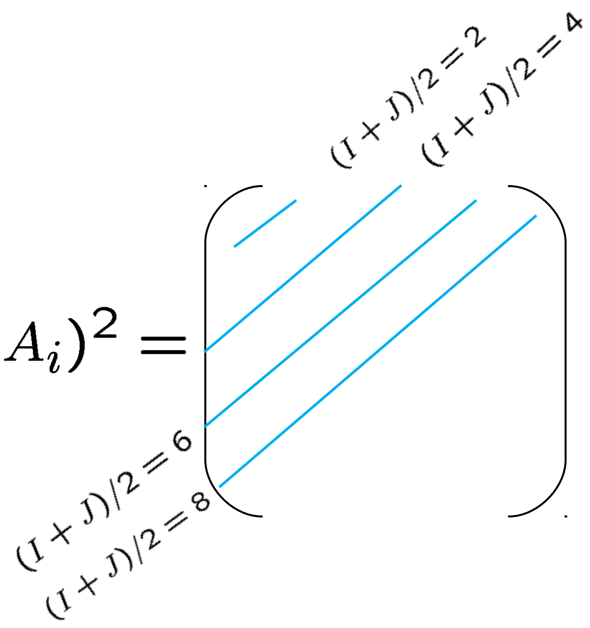
non-trivial dynamical property

Band-diagonal structure

$N = 16$



$(A_i)^2 =$



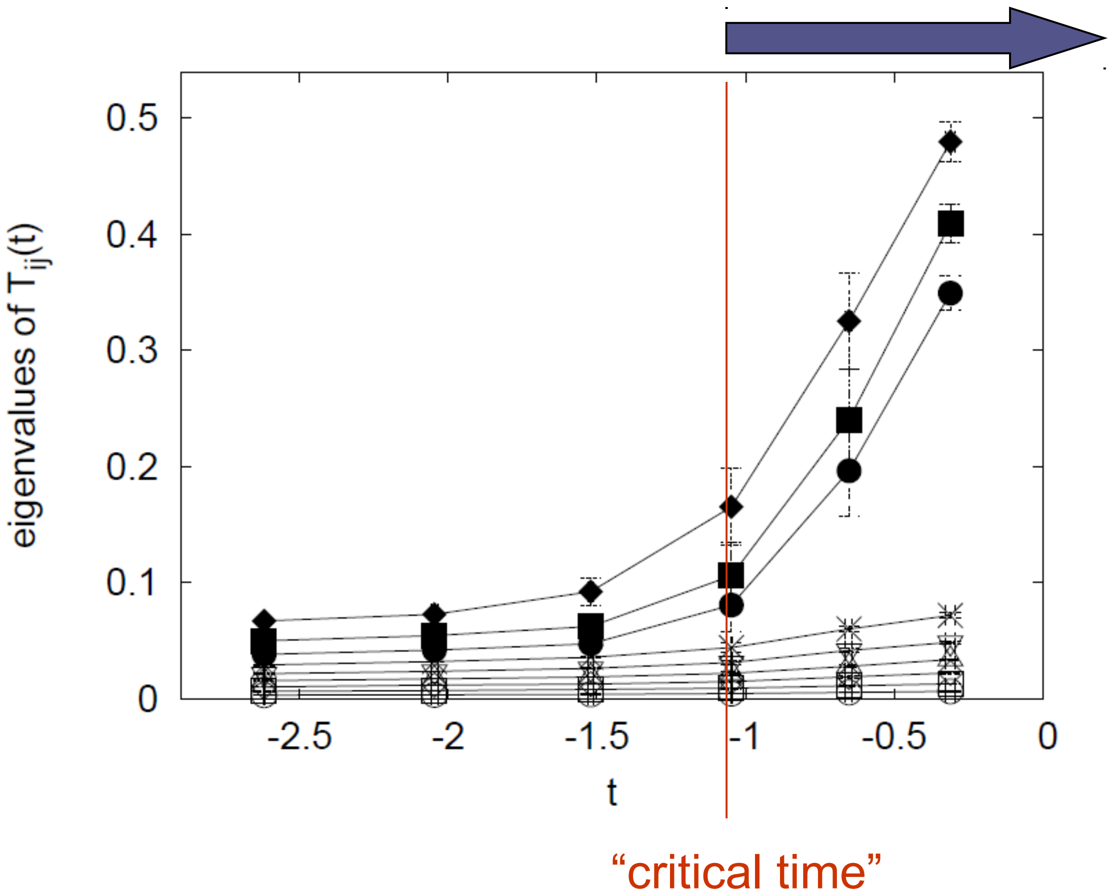
We choose $n = 4$ as the block size

Spontaneous breaking of SO(9)

Kim-J.N.-Tsuchiya, PRL 108 (2012) 011601

$$T_{ij}(t) = \frac{1}{n} \mathrm{tr} \{ \bar{A}_i(t) \bar{A}_j(t) \}$$

$$\mathrm{SO}(9) \overset{\mathrm{SSB}}{\rightarrow} \mathrm{SO}(3)$$



$$N = 16 \ , \quad \kappa = 4.0$$

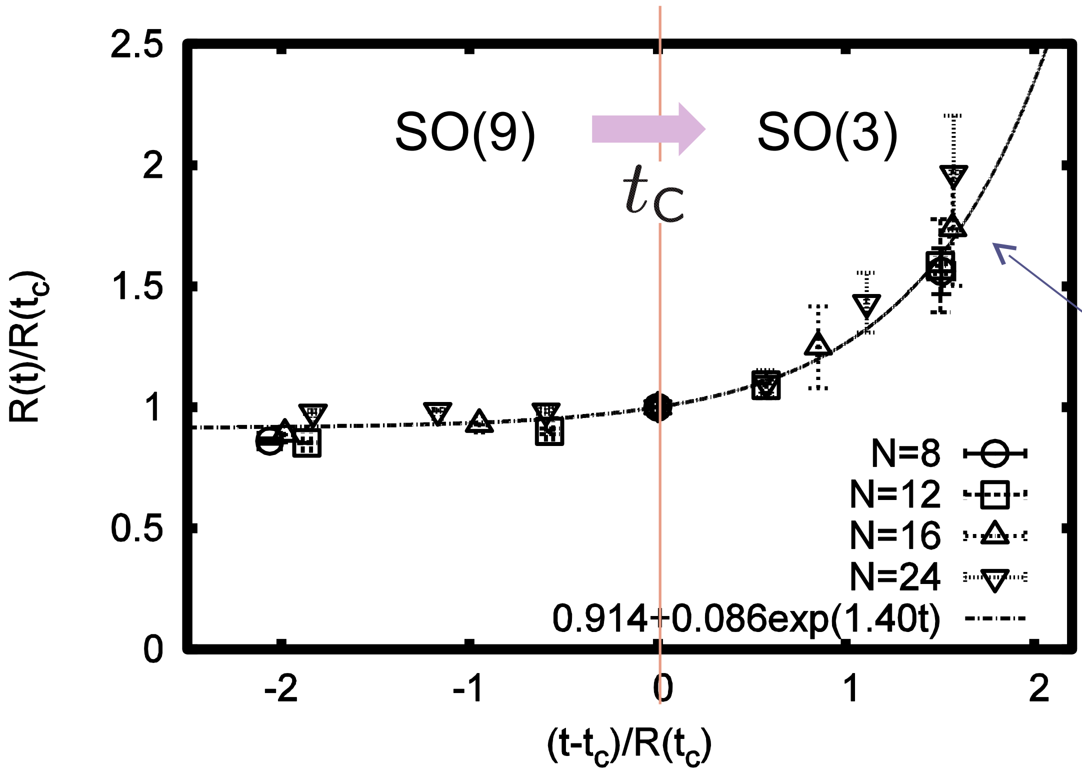
5. Exponetial/power-law expansion

Ito-Kim-Koizuka-J.N.-Tsuchiya, in prep.
Ito-Kim-J.N.-Tsuchiya, work in progress

Exponential expansion

Ito-Kim-J.N.-Tsuchiya, work in progress

$$R(t)^2 \equiv \frac{1}{n} \mathrm{tr}\, \bar{A}_i(t)^2$$



fitted well to

$$f(x) = a + (1-a)e^{bx}$$

Exponential expansion

Inflation

Effects of fermionic action

$$\begin{aligned} S_{\mathrm{f}} &= \mathrm{tr}(\bar{\Psi}_\alpha (\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]) \\ &= \boxed{\mathrm{tr}(\bar{\Psi}_\alpha (\Gamma^0)_{\alpha\beta}[A_0, \Psi_\beta]}) + \boxed{\mathrm{tr}(\bar{\Psi}_\alpha (\Gamma^i)_{\alpha\beta}[A_i, \Psi_\beta])} \end{aligned}$$

dominant term
at early times

dominant term
at late times

keep only the first term

simplified model at early times

simplified model at late time

$$\boxed{\mathrm{Pf}\mathcal{M}(A) \simeq \Delta^{d-1} = \prod_{i<j} (\alpha_i - \alpha_j)^{2(d-1)}}$$

$$\boxed{\mathrm{Pf}\mathcal{M}(A) \simeq 1}$$

repulsive force between eigenvalues of A_0

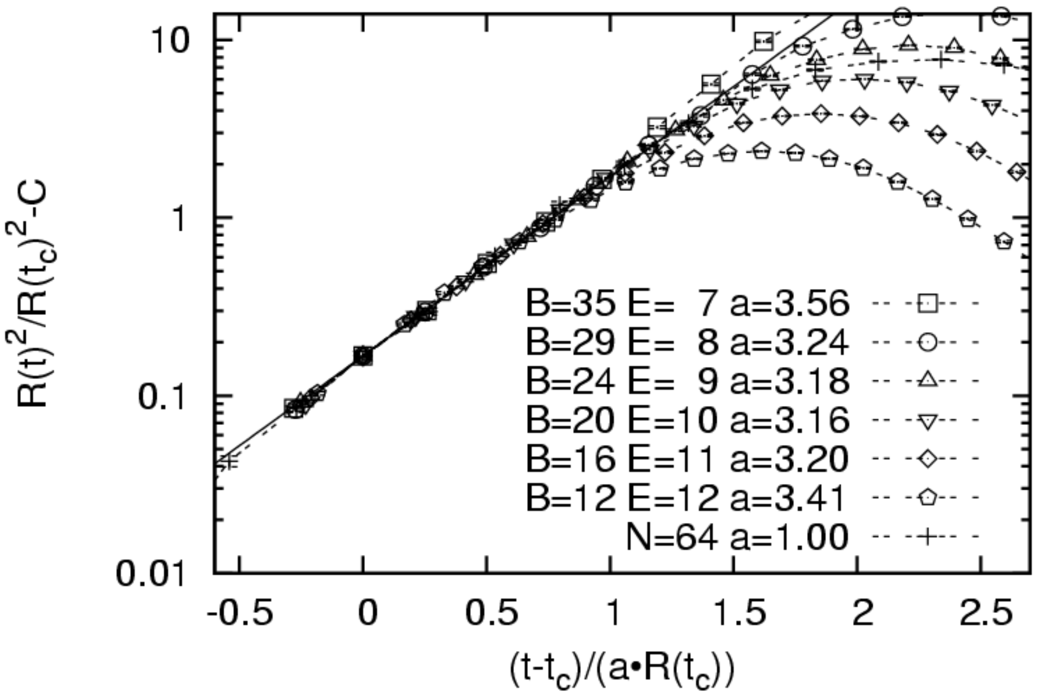
quench fermions

Exponential expansion at early times

Ito-Kim-Koizuka-J.N.-Tsuchiya, in prep.

- simplified model at early times

$$\boxed{\mathrm{Pf}\mathcal{M}(A) \simeq \Delta^{d-1} = \prod_{i<j}(\alpha_i - \alpha_j)^{2(d-1)}}$$



exponential expansion

The first term is important for exponential expansion.

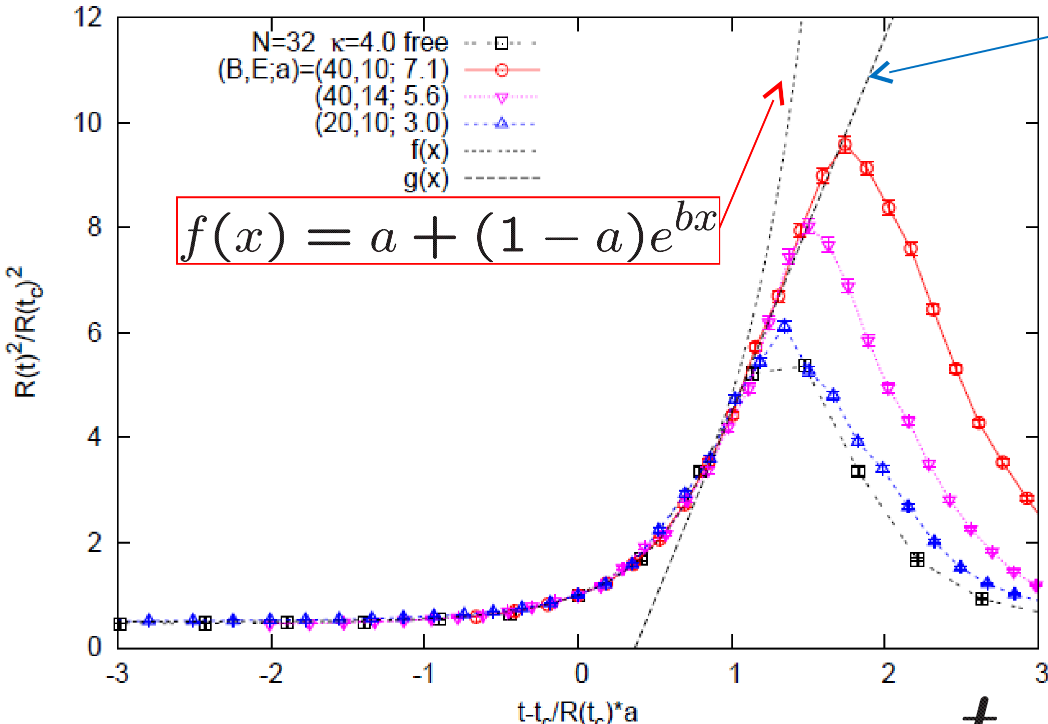
Power-law expansion at late times

Ito-Kim-J.N.-Tsuchiya, work in progress

- simplified model at late times

$$\boxed{\mathrm{Pf}\mathcal{M}(A) \simeq 1}$$

R^2



t

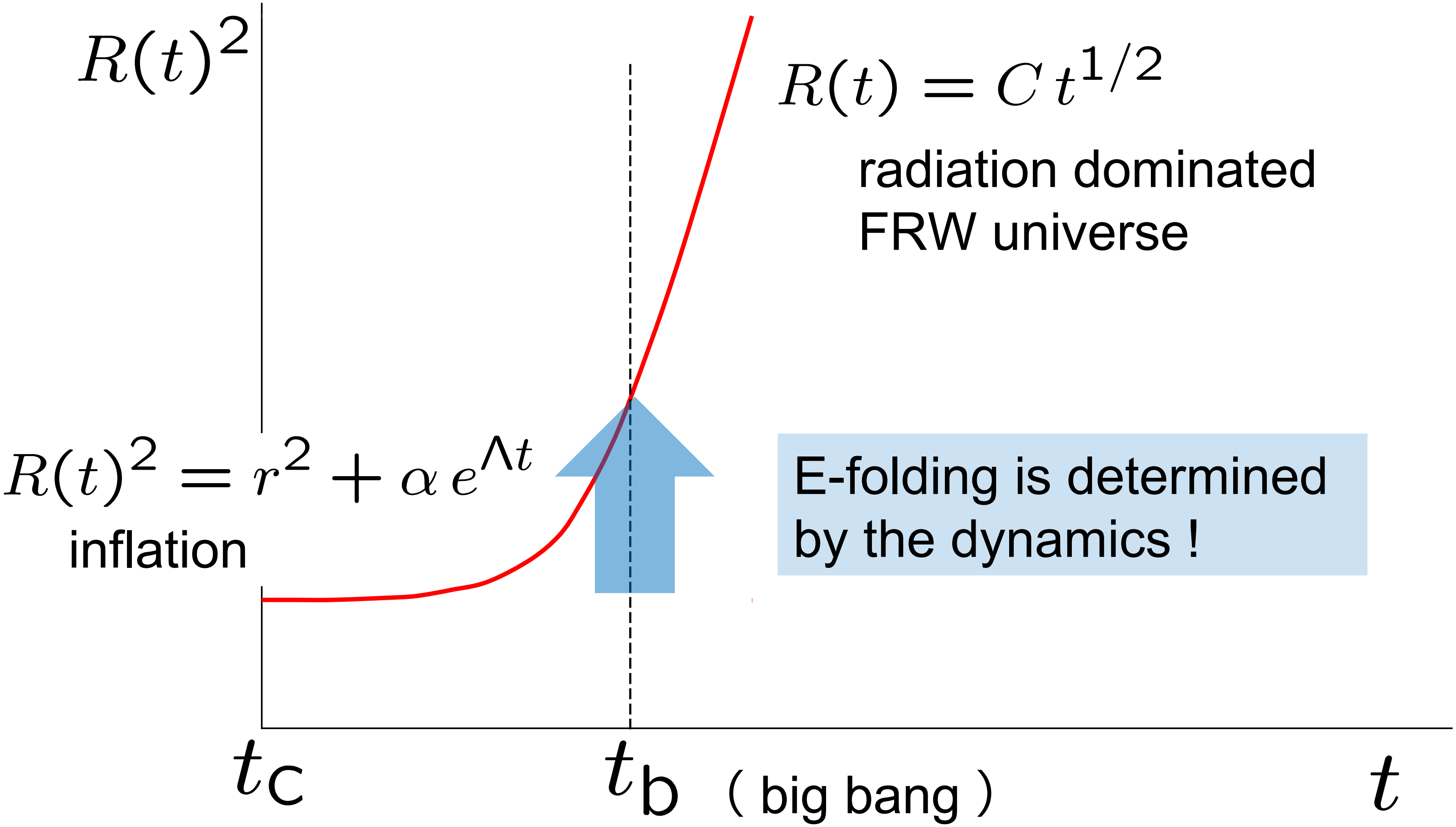
$$g(x) = ax + b$$

$R^2 \sim t \Rightarrow R \sim t^{1/2}$

$t^{1/2}$ behavior

Radiation dominated FRW universe

Expected scenario for the full Lorentzian IIB matrix model



6. Time-evolution at much later times

S.-W. Kim, J. N. and A.Tsuchiya, Phys. Rev. D86 (2012) 027901 [arXiv:1110.4803]
S.-W. Kim, J. N. and A.Tsuchiya, JHEP 10 (2012) 147 [arXiv:1208.0711]

Time-evolution at much later times

$$Z = \int dA\, d\Psi\, e^{iS} = \int dA\, e^{iS_{\mathrm{b}}} \mathrm{Pf}\mathcal{M}(A)$$

The cosmic expansion makes each term in the action larger at much later times.

⇒ Classical approximation becomes valid.

- There are infinitely many classical solutions. (Landscape)
- There is a simple solution representing a (3+1)D expanding universe, which naturally solves the cosmological constant problem.
- Since the weight for each solution is well-defined, one should be able to determine the unique solution that dominates at late times.
- By studying the fluctuation around the solution, one should be able to derive the effective QFT below the Planck scale.

J. N. and A.Tsuchiya, PTEP 2013 (2013) 043B03 [arXiv:1208.4910]

General prescription for solving EOM

- variational function $(i=1,\cdots,9)$

$$\tilde{S}=\mathrm{tr}\left(-\frac{1}{4}[A_\mu,A_\nu][A^\mu,A^\nu]+\boxed{\frac{\tilde{\lambda}}{2}(A_0^2-\kappa L^2)}-\boxed{\frac{\lambda}{2}(A_i^2-L^2)}\right)$$

- classical equations of motion

$$-[A_0,[A_0,A_i]]+[A_j,[A_j,A_i]]-\lambda A_i=0$$
$$[A_j,[A_j,A_0]]-\tilde{\lambda}A_0=0$$

- commutation relations

$$[A_i,A_j]=iC_{ij}$$
$$[A_i,C_{jk}]=iD_{ijk}$$
$$[A_0,A_i]=iE_i$$
$$[A_0,E_i]=iF_i$$
$$[A_i,E_j]=iG_{ij}\quad\cdots$$

Eq. of motion & Jacobi identity

$\Downarrow$

Lie algebra

$\Downarrow$

Unitary representation $\Leftrightarrow$ classical solution

Can be made finite-dimensional by imposing simplifying Ansatz.

An example of SO(4) symmetric solution ($R \times S^3$ space-time)

EOM $$\frac{\delta}{\delta A_\mu}\left(-\frac{1}{4}\mathrm{tr}([A_\mu, A_\nu]^2) - \frac{\lambda}{2}\mathrm{tr}(A_i)^2 + \frac{\tilde{\lambda}}{2}\mathrm{tr}(A_0)^2\right) = 0$$

$$\boxed{\begin{aligned} &A_0 = bT_0 \otimes 1_k \\ &A_i = \alpha b T_1 \otimes M_i \quad (i = 1 \sim 4) \\ &A_5 \sim A_9 = 0 \end{aligned}}$$

$SL(2, R)$ algebra

$$[T_0, T_1] = iT_2 \quad [T_0, T_2] = -iT_1 \quad [T_1, T_2] = -iT_0$$

$$M_i = \mathrm{diag}(n_i^{(1)}, \cdots, n_i^{(k)}) \quad (i = 1, \ldots, 4)$$

$|n^{(I)}| = 1 \quad (I = 1, \cdots, k)$ uniformly distributed on a unit S^3

$$\lambda = -b^2, \quad \tilde{\lambda} = -\alpha^2 b^2$$

Space-space is commutative.

An example of SO(4) symmetric solution ($R \times S^3$ space-time) cont'd

primary unitary series

$$(T_0)_{mn} = n\delta_{mn}$$

$$(T_1)_{mn} = -\frac{i}{2}\left(n - i\rho + \frac{1}{2}\right)\delta_{m,n+1} + \frac{i}{2}\left(n + i\rho - \frac{1}{2}\right)\delta_{m,n-1}$$

Block size can be taken to be n=3.

the extent of space

$$R(n) = \sqrt{\frac{1}{3}\mathrm{tr}(\bar{A}_i(n)^2)} = \frac{\alpha b}{\sqrt{3}}\sqrt{n^2 + \rho^2 + \frac{1}{4}} \longrightarrow \boxed{R(t) = \frac{\alpha}{\sqrt{3}}\sqrt{t^2 + t_0^2}}$$

space-time noncommutativity

$$\frac{-\frac{1}{3}\mathrm{tr}([\bar{A}_0(n), \bar{A}_1(n)]^2)}{\frac{1}{3}\mathrm{tr}(\bar{A}_0(n))^2 \frac{1}{3}\mathrm{tr}(\bar{A}_1(n))^2} = \frac{1}{n^2 + \frac{2}{3}} \xrightarrow[n \to \infty]{} 0$$

commutative space-time !

cont. lim. $b \to 0$ with $t = nb, \quad \rho b = t_0$

$$\lambda = -b^2 \to 0$$
$$\tilde{\lambda} = -\alpha^2 b^2 \to 0$$

consistent!

An example of SO(4) symmetric solution ($R \times S^3$ space-time) cont'd

$$R(t) = \frac{\alpha}{\sqrt{3}}\sqrt{t^2 + t_0^2} \equiv a(t)$$

$$\Rightarrow \quad H = \frac{\dot{a}}{a} \sim a^{-\frac{3}{2}(1+w)} \qquad w = -\frac{1}{3}\left(\frac{2t_0^2}{t^2} + 1\right)$$

This part can be identified as a viable late-time behavior.

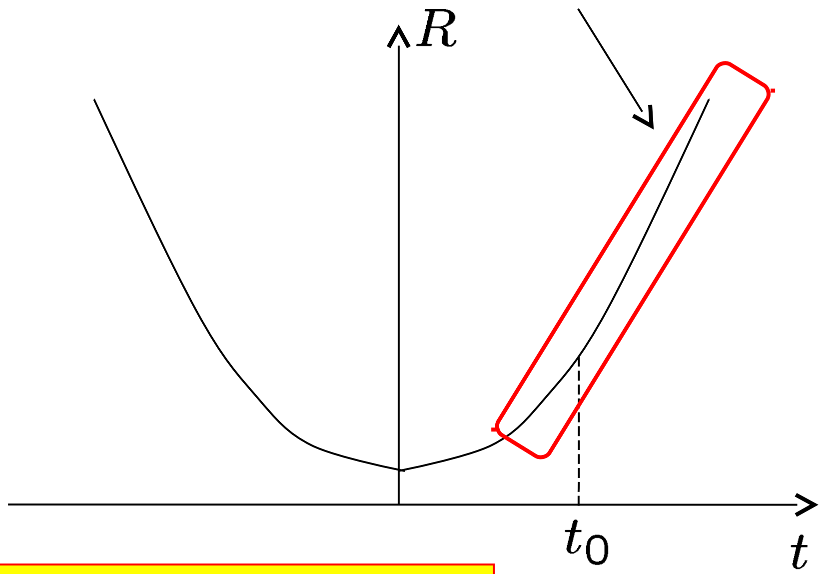
$t = t_0 \longrightarrow w = -1$

cosmological const. $\sim (1/t_0)^4$

explains the accelerated expansion at present time

$t \to \infty \longrightarrow w = -\frac{1}{3}$

Cosmological constant disappears in the far future.



A natural solution to the cosmological constant problem.

7. Summary and future prospects

Summary

type IIB matrix model (1996)

A nonperturbative formulation of superstring theory
based on type IIB theory in 10d.

The problems with the Euclidean model have become clear.

Lorentzian model : untouched until recently because of its instability
Monte Carlo simulation has revealed its surprising properties.

- A well-defined theory can be obtained by introducing cutoffs and removing them in the large-N limit.
- The notion of "time evolution" emerges dynamically

  When we diagonal A_0 ,
  A_i $(i = 1, \cdots, 9)$ has band-diagonal structure.

- After some "critical time", the space undergoes the SSB of SO(9), and only 3 directions start to expand.
  - Exponential expansion observed (Inflation, no initial condition problem.)
  - Power-law $t^{1/2}$) expansion observed in a simplified model for later times.
  - Classical analysis is valid for much later times.

A natural solution to the cosmological coonstant problem suggested.

Future prospects

- Observe directly the transition from the exponential expansion to the power-law expansion by Monte Carlo simulation.
- Does the transition to commutative space-time (suggested by a classical solution) occur at the same time ?
- Can we calculate the density fluctuation to be compared with CMB ?
- Can we read off the effective QFT below the Planck scale from fluctuations around a classical solution ?
- Does Standard Model appear at low energy ? (Tsuchiya's talk)

Various fundamental questions in particle physics and cosmology :

the mechanism of inflation, the initial value problem,
the cosmological constant problem,
the hierarchy problem, dark matter, dark energy, baryogenesis,
the origin of the Higgs field, the number of generations etc..

It should be possible to understand all these problems in a unified manner by using the nonperturbative formulation of superstring theory.

Backup slides

Previous works in the Euclidean matrix model

(A model with SO(10) rotational symmetry instead of SO(9,1) Lorentz symmetry)

Dynamical generation of 4d space-time ?

SSB of SO(10) rotational symmetry

- perturbative expansion around diagonal configurations, branched-polymer picture
  Aoki-Iso-Kawai-Kitazawa-Tada(1999)

- The effect of complex phase of the fermion determinant (Pfaffian)
  J.N.-Vernizzi (2000)

- Monte Carlo simulation
  Ambjorn-Anagnostopoulos-Bietenholz-Hotta-J.N.(2000)
  Anagnostopoulos-J.N.(2002)

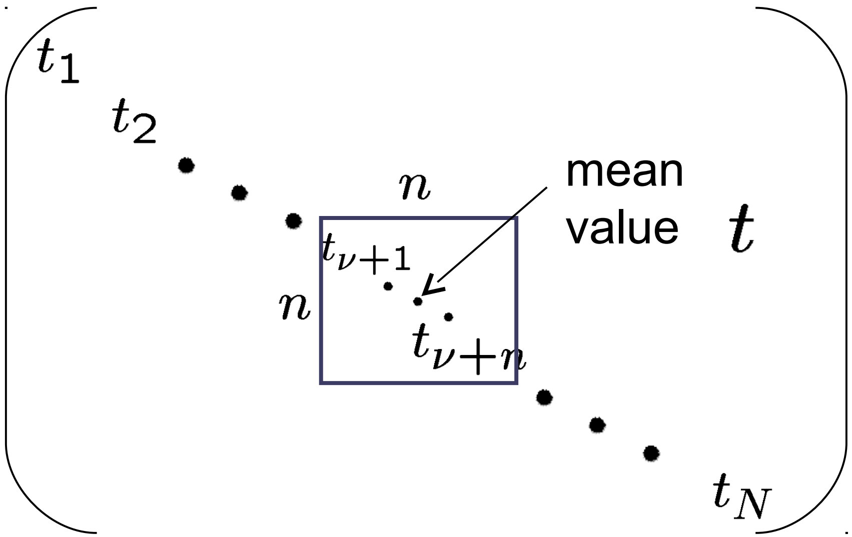
- Gaussian expansion method
  J.N.-Sugino
(2002)、 Kawai-Kawamoto-Kuroki-Matsuo-Shinohara(2002)

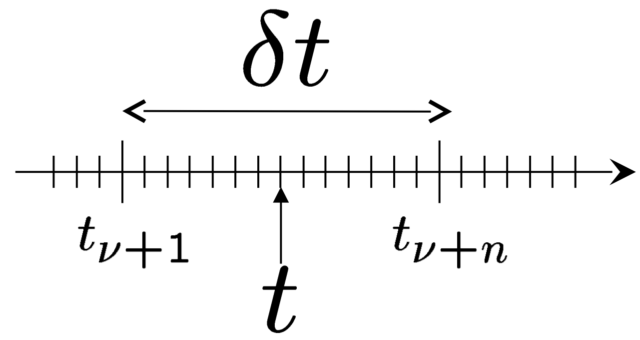
$S^2 \times S^2$

- fuzzy

Emeregence of the notion of “time-evolution”

$A_0 =$

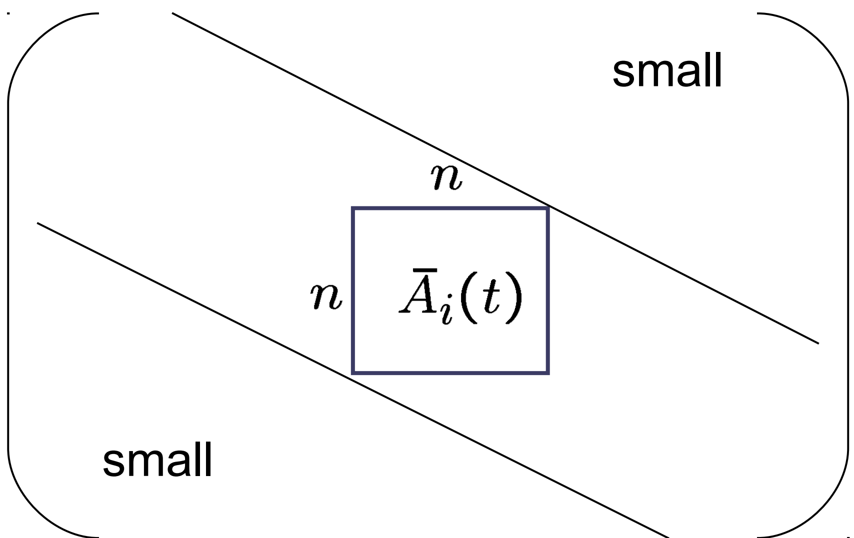




$$\nu = 0, 1, \cdots, N - n$$

$$t = \frac{1}{n} \sum_{a=1}^{n} t_{\nu+a}$$

$A_i =$



band-diagonal structure

$\bar{A}_i(t)$ represents the state at the time t

The emergence of "time"

<u>Supersymmetry plays a crucial role!</u>

Calculate the effective action for

$$A_0 = \mathrm{diag}(\alpha_1, \cdots, \alpha_N)$$

at one loop.

$$\left(\Delta = \prod_{i<j} (\alpha_i - \alpha_j)^2 \right)$$

$A_i \ (i = 1, \cdots, d)$ contributes Δ^{-d}

$\Psi_\alpha \ (\alpha = 1, \cdots, p)$ contributes $\Delta^{p/2}$

Contribution from
van der Monde determinant Δ

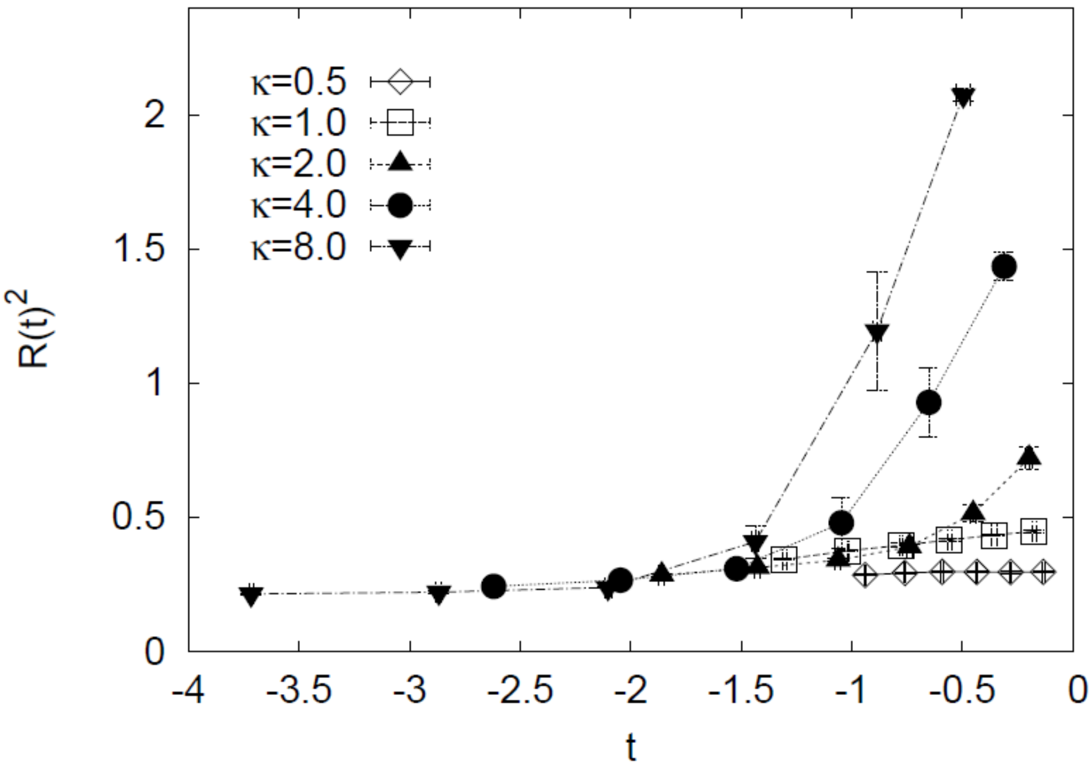
Altogether, $\Delta^{-d+p/2+1}$

Zero, in a supersymmetric model !
(e.g., $d = 9$, $p = 16$)

Attractive force between the eigenvalues in the bosonic model, cancelled in supersymmetric models.

The time-evolution of the extent of space

$$R(t)^2 \equiv \frac{1}{n}\mathrm{tr}\,\bar{A}_i(t)^2$$



$N = 16\ , \quad n = 4$

symmetric under $t \to -t$

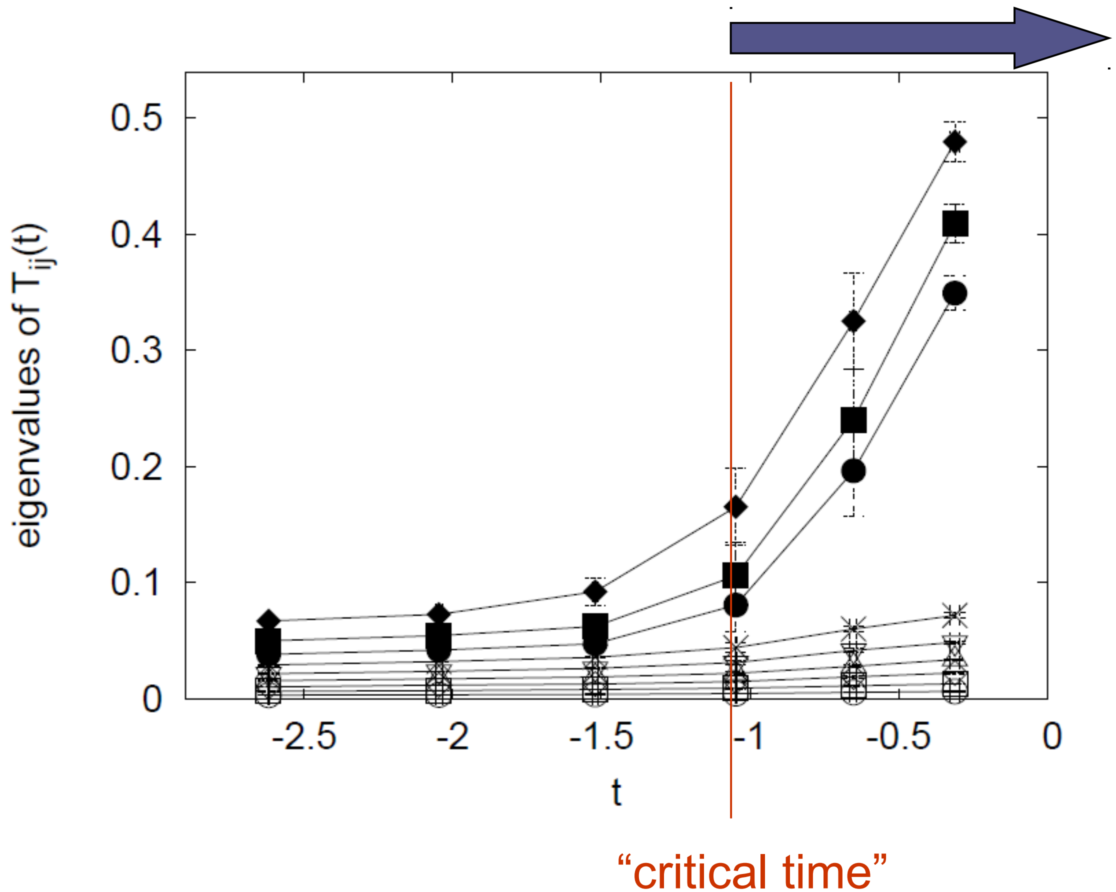
We only show the region $t < 0$

SSB of SO(9) rotational symmetry

$$T_{ij}(t) = \frac{1}{n} \mathrm{tr} \left\{ \bar{A}_i(t) \bar{A}_j(t) \right\}$$

$$\mathrm{SO}(9) \xrightarrow{\mathrm{SSB}} \mathrm{SO}(3)$$

$$N = 16 \ , \qquad \kappa = 4.0$$



What can we expect by studying the time-evolution at later times

- What is seen by Monte Carlo simulation so far is: the birth of our Universe
- What has been thought to be the most difficult from the bottom-up point of view, can be studied first.
  This is a typical situation in a top-down approach !
- We need to study the time-evolution at later times in order to see the Universe as we know it now!

  - Does inflation and the Big Bang occurs ?
    (First-principles description based on superstring theory, instead of just a phenomenological description using "inflaton"; comparison with CMB etc..
  - How does the commutative space-time appear ?
  - What kind of massless fields appear on it ?
  - accelerated expansion of the present Universe (dark energy), understanding the cosmological constant problem
  - prediction for the end of the Universe (Big Crunch or Big Rip or…)

Ansatz

extra dimension is small $\Longrightarrow$ $A_i = 0 \quad \text{for} \quad i > d$

(compared with Planck scale)

commutative space $\Longrightarrow$ $[A_i, A_j] = 0$

$$[\boxed{A_i}, A_j] = i\cancel{C_{ij}}$$

$$[A_i, C_{jk}] = i\cancel{D_{ijk}}$$

$$[\boxed{A_0}, A_i] = i\boxed{E_i}$$

$$[A_0, E_i] = iF_i \quad \Longleftarrow \quad -[A_0, [A_0, A_i]] + [A_j, [A_j, A_i]] - \lambda A_i = 0$$

$$\boxed{F_i = \lambda A_i}$$

$$[A_i, E_j] = iG_{ij} \quad \cdots$$

$$G_{ij} = \boxed{M_{ij}} + \cancel{N_{ij}} + \frac{1}{d}\delta_{ij}H \quad \Longleftarrow \quad [A_j, [A_j, A_0]] - \tilde{\lambda} A_0 = 0$$

$$\boxed{H = \tilde{\lambda} A_0}$$

$$[A_0, [A_i, A_j]] + [A_i, [A_j, A_0]] + [A_j, [A_0, A_i]] = 0$$

Simplification

$$M_{ij} = 0 \ \text{ for } \ i \neq j$$

$$M_i \equiv M_{ii} \qquad \sum_{i=1}^{d} M_i = 0$$

$$\Downarrow$$

Lie algebra

$$
\boxed{
\begin{aligned}
&[A_i, A_j] = 0 \ , \quad [A_0, A_i] = iE_i \ , \quad [A_0, E_i] = i\lambda A_i \ , \\
&[E_i, E_j] = 0 \ , \quad [A_i, E_j] = i\delta_{ij}\left(\frac{\tilde{\lambda}}{d}A_0 + M_i\right) \ , \quad [A_0, M_i] = 0 \ , \\
&[A_i, M_j] = i\frac{\tilde{\lambda}}{d}(1 - d\delta_{ij})E_i \ , \quad [E_i, M_j] = i\frac{\lambda\tilde{\lambda}}{d}(1 - d\delta_{ij})A_i \ , \quad [M_i, M_j] = 0
\end{aligned}
}
$$

e.g.)

$$d = 2, \quad \lambda > 0, \quad \tilde{\lambda} > 0 \quad \Rightarrow \quad SO(2,2)$$

d=1 case

$$[A_0, A_1] = iE \ , \quad [A_0, E] = i\lambda A_1 \ , \quad [A_1, E] = i\tilde{\lambda} A_0$$

SO(9) rotation $\Rightarrow$ $r_i A_1 \ (i = 1, \cdots, 9)$ with $r_i^2 = 1$

Take a direct sum

$$A'_0 = A_0 \otimes \mathbb{1}_K$$
$$A'_i = A_1 \otimes \mathrm{diag}(r_i^{(1)}, r_i^{(2)}, \cdots, r_i^{(K)})$$
$$\text{where} \quad r_i^{(m)2} = 1 \ \ (m = 1, \cdots, K)$$

$r^{(m)}$ distributed on a unit S^3

$\Rightarrow$ (3+1)D space-time $\sim$ $R\times S^3$

A complete classification of d=1 solutions has been done.
Below we only discuss a physically interesting solution.

SL(2,R) solution

- SL(2,R) solution

$$[A_0, A_1] = iE \ , \quad [A_0, E] = i\lambda A_1 \ , \quad [A_1, E] = i\tilde{\lambda} A_0$$

$$A_0 = aT_2 \ , \quad A_1 = bT_0 \ , \quad E = cT_1$$
$$\lambda = a^2 \ , \quad \tilde{\lambda} = b^2 \ , \quad ab = c$$

$$\boxed{[T_0, T_1] = iT_2 \ , \quad [T_2, T_0] = iT_1 \ , \quad [T_1, T_2] = -iT_0}$$

- realization of the SL(2,R) algebra on $\{e^{in\theta}; n \in Z\}$

$$\mathcal{T}_0 = i\frac{d}{d\theta} + \epsilon$$
$$\mathcal{T}_1 = \frac{i}{2}\left[(\tau + \epsilon)e^{i\theta} + (\tau - \epsilon)e^{-i\theta} - 2\sin\theta\frac{d}{d\theta}\right]$$
$$\mathcal{T}_2 = \frac{1}{2}\left[-(\tau + \epsilon)e^{i\theta} + (\tau - \epsilon)e^{-i\theta} - 2i\cos\theta\frac{d}{d\theta}\right]$$

Space-time structure in SL(2,R) solution

- primary unitary series representation

$$
\begin{aligned}
(T_0)_{mn} &= n\delta_{mn} \\
(T_1)_{mn} &= -\frac{i}{2}(n - i\rho + \frac{1}{2})\delta_{m,n+1} + \frac{i}{2}(n + i\rho - \frac{1}{2})\delta_{m,n-1} \\
(T_2)_{mn} &= -\frac{1}{2}(n - i\rho + \frac{1}{2})\delta_{m,n+1} - \frac{1}{2}(n + i\rho - \frac{1}{2})\delta_{m,n-1}
\end{aligned}
$$

tri-diagonal

Space-time noncommutativity disappears in the continuum limit.

$$
\bar{A}_0(n) = a \begin{pmatrix} n-1 & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n+1 \end{pmatrix} \otimes 1\!\!1_K
$$

$$
\bar{A}_1(n) = \frac{ib}{2} \begin{pmatrix} 0 & n + i\rho - \frac{1}{2} & 0 \\ -n + i\rho + \frac{1}{2} & 0 & n + i\rho + \frac{1}{2} \\ 0 & -n + i\rho - \frac{1}{2} & 0 \end{pmatrix} \otimes \mathrm{diag}(r_i^{(1)}, \cdots, r_i^{(K)})
$$

Cosmological implication of SL(2,R) solution

- the extent of space

$$R(n) \equiv \sqrt{\frac{1}{3K}\mathrm{tr}(\bar{A}_1(n))^2} = \sqrt{\frac{b^2}{3}\left(n^2+\rho^2+\frac{1}{4}\right)}$$

$$\longrightarrow \quad \boxed{R(t) = \sqrt{\frac{\alpha^2}{3}(t^2+t_0^2)}}$$

$$t = na, \quad t_0 = \rho a$$

cont. lim. $\quad a \to 0, \quad n \to \infty, \quad \rho \to \infty$

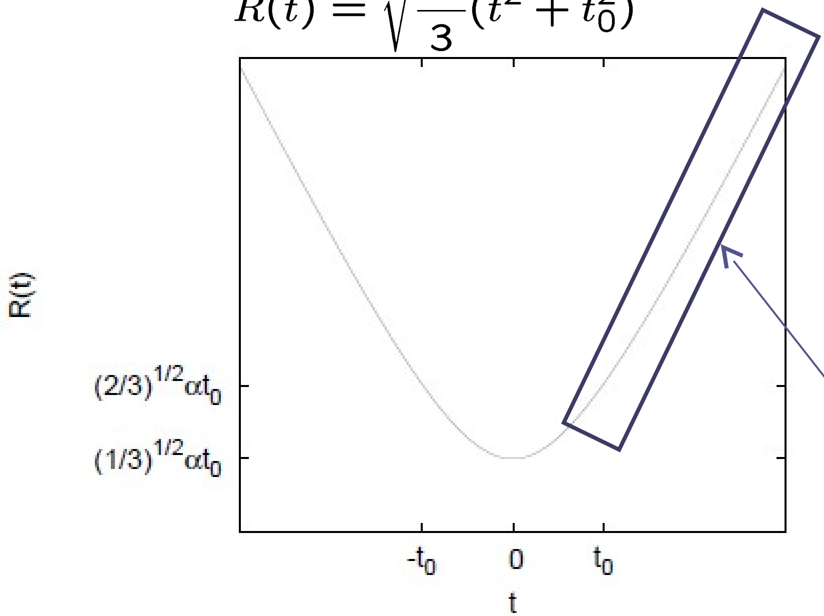
- Hubble constant and the w parameter

$$H(t) = \frac{\dot{R}(t)}{R(t)} = c\,R(t)^{-\frac{3}{2}(1+w)}$$

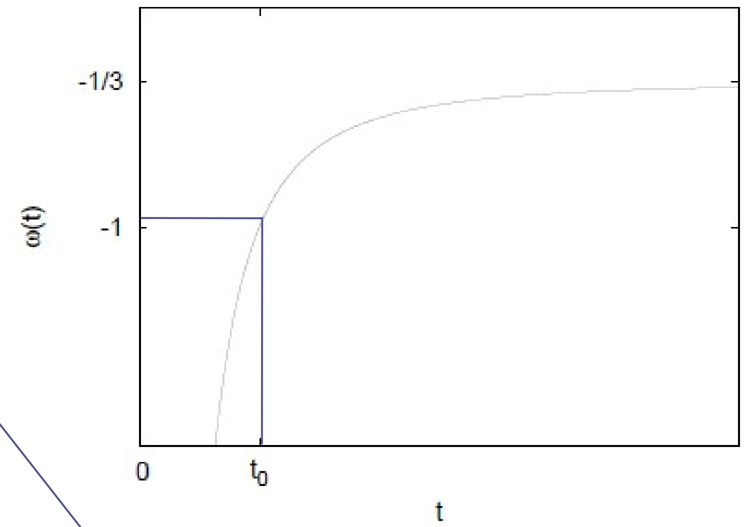
$$\begin{cases} w = \frac{1}{3} & \text{radiation dominant} \\ w = 0 & \text{matter dominant} \\ w = -1 & \text{cosmological constant} \end{cases}$$

Cosmological implication of SL(2,R) solution (cont'd)

$$R(t) = \sqrt{\frac{\alpha^2}{3}(t^2 + t_0^2)}$$



$$w = -\frac{2t_0^2}{3t^2} - \frac{1}{3}$$



t_0 is identified with the present time.

This part is considered to give the late-time behavior of the matrix model

$\Rightarrow$ $w = -1$ present accelerated expansion

cosmological const. $\sim (1/t_0)^4$ a solution to the cosmological constant problem

$w \to -\dfrac{1}{3}$ for $t \to \infty$ Cosmological constant disappears in the future.

Seiberg's rapporteur talk (2005)
at the 23rd Solvay Conference in Physics

hep-th/0601234

"Emergent Spacetime"

Understanding how time emerges will undoubtedly shed new light on some of the most important questions in theoretical physics including the origin of the Universe.

Indeed in the Lorentzian matrix model,
not only space but also time emerges,
and the origin of the Universe seems to be clarified.

The significance of the unique determination of the space-time dimensionality

It strongly suggests that
superstring theory has a unique nonperturbative vacuum.
By studying the time-evolution further, one should be able to see the emergence of commutative space-time and massless fields propagating on it.

It is conceivable that the SM can be derived uniquely.

This amounts to “proving” the superstring theory.

It is sufficient to identify the classical configuration which dominates at late times by studying the time-evolution at sufficiently late times.

Independently of this, it is important to study classical solutions and to study the fluctuations around them.

Does chiral fermions appear ?
Is SUSY preserved ?

The key lies in the structure in the extra dimensions.