Axion Physics

FROM STRONG CP TO AXIONS OUT OF DISCRETE SYMMETRIES

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1. Introduction

2. Axions and the strong CP problem

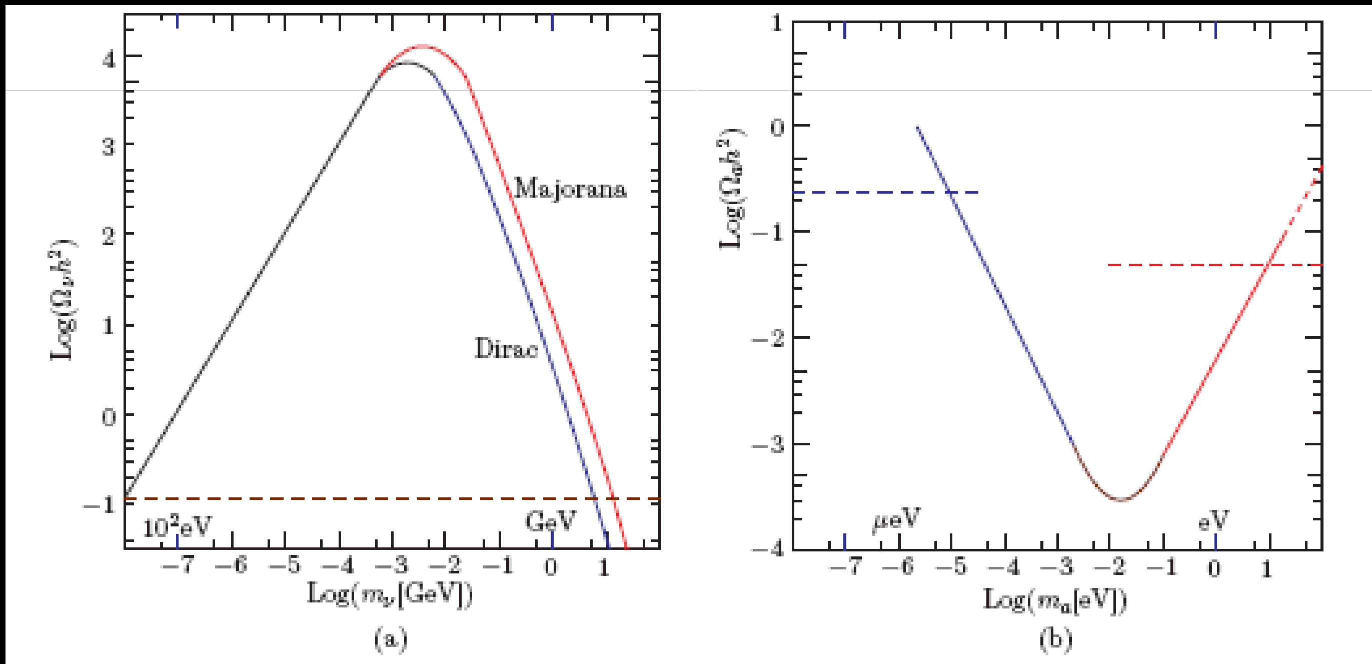
3. Axions and SUSY

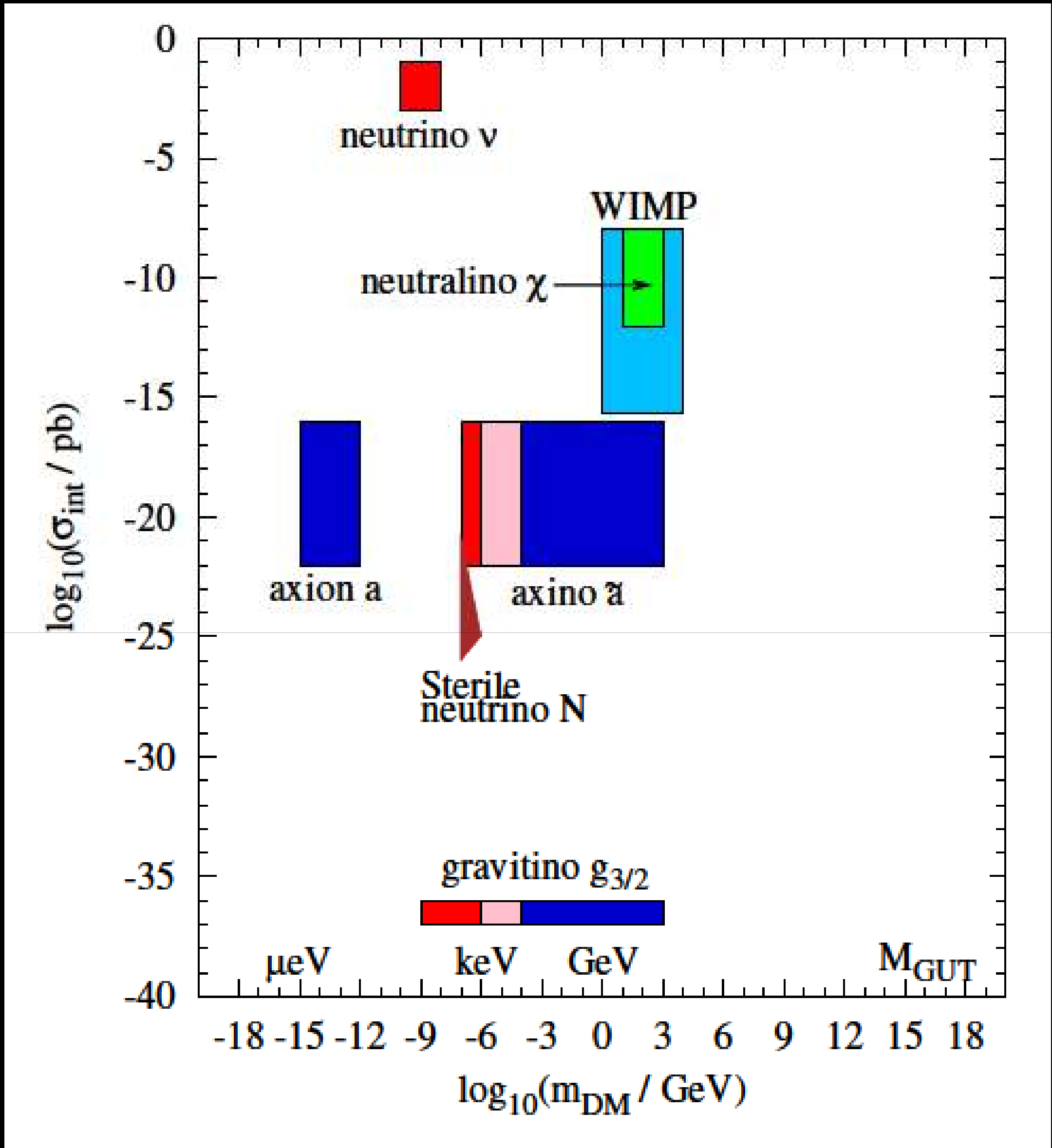
4. Discrete symmetries and axions : Axions from string

1. Introduction

The axion cosmic energy density has the opposite cosmological behavior from that of WIMP.

It is because it is the bosonic collective motion.

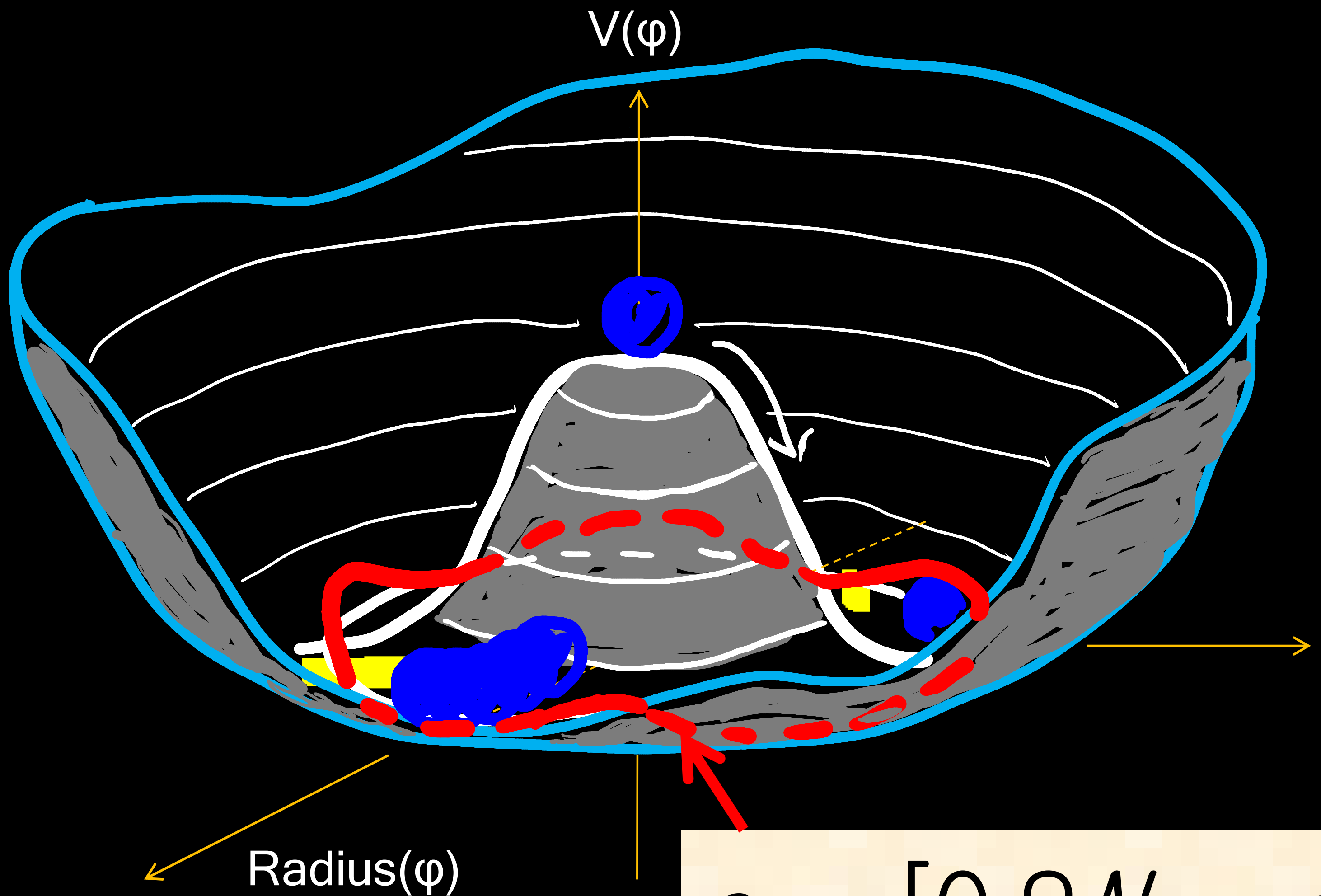




A rough sketch of WIMP masses and cross sections.

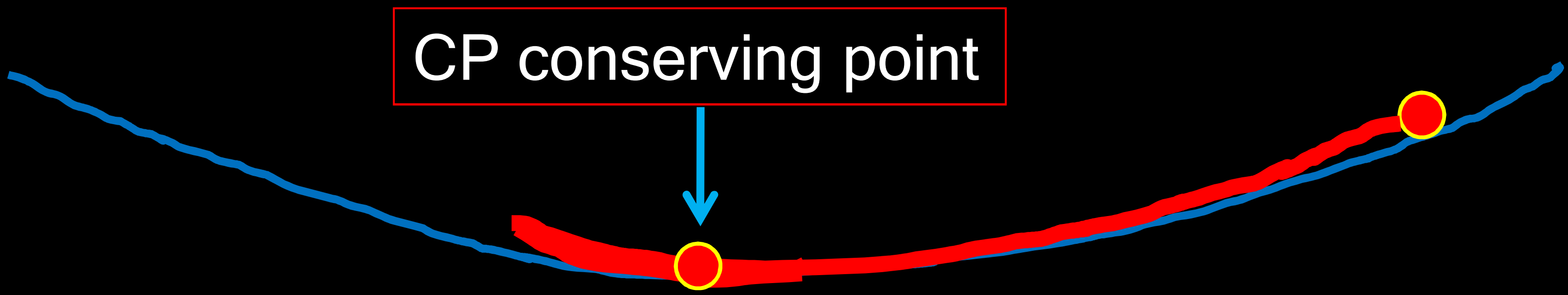
[Choi-Kim-Roszkowski, 2013]

2. Axions and Strong CP



$$a = [0, 2N_{DW} f_a]$$

It is very flat if the axion decay constant is large,



In the evolving universe, at some temperature, say T_1, *a* starts to roll down to end at the CP conserving point sufficiently closely. This analysis constrains the axion decay constant (upper bound) and the initial VEV of *a* at T_1.

The Lagrangian is invariant under changing $\theta \to \theta - 2\boldsymbol{\alpha}$.
But θ becomes dynamical and the $\theta = a/F_a$ potential becomes

$$V = \frac{Z f_\pi^2 m_\pi^2}{(1+Z)^2}\left(1 - \cos\frac{a}{F_a}\right), \qquad Z = \frac{m_u}{m_d}$$

The true vacuum chooses $\theta = a/F_a$ at

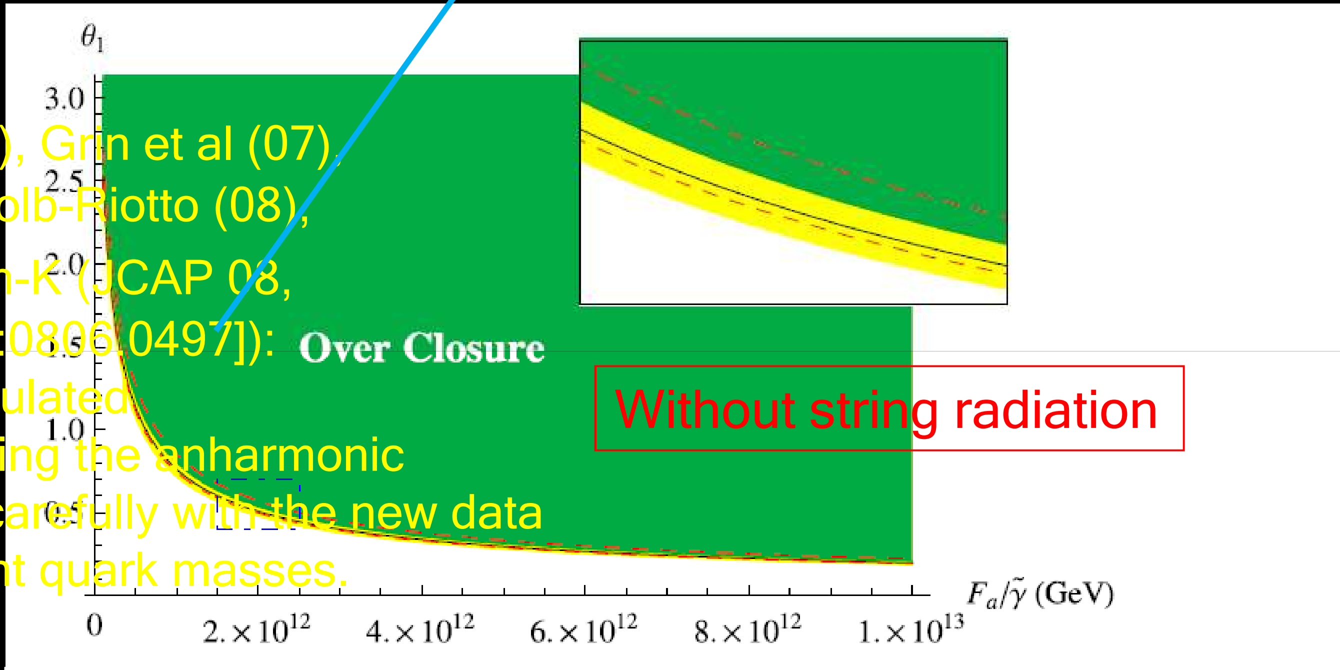
$$\boxed{\theta = 0}$$

A recent calculation of the cosmic axion density is,

$$10^9 \text{ GeV} < F_a < \{10^{12} \text{ GeV ?}\}$$

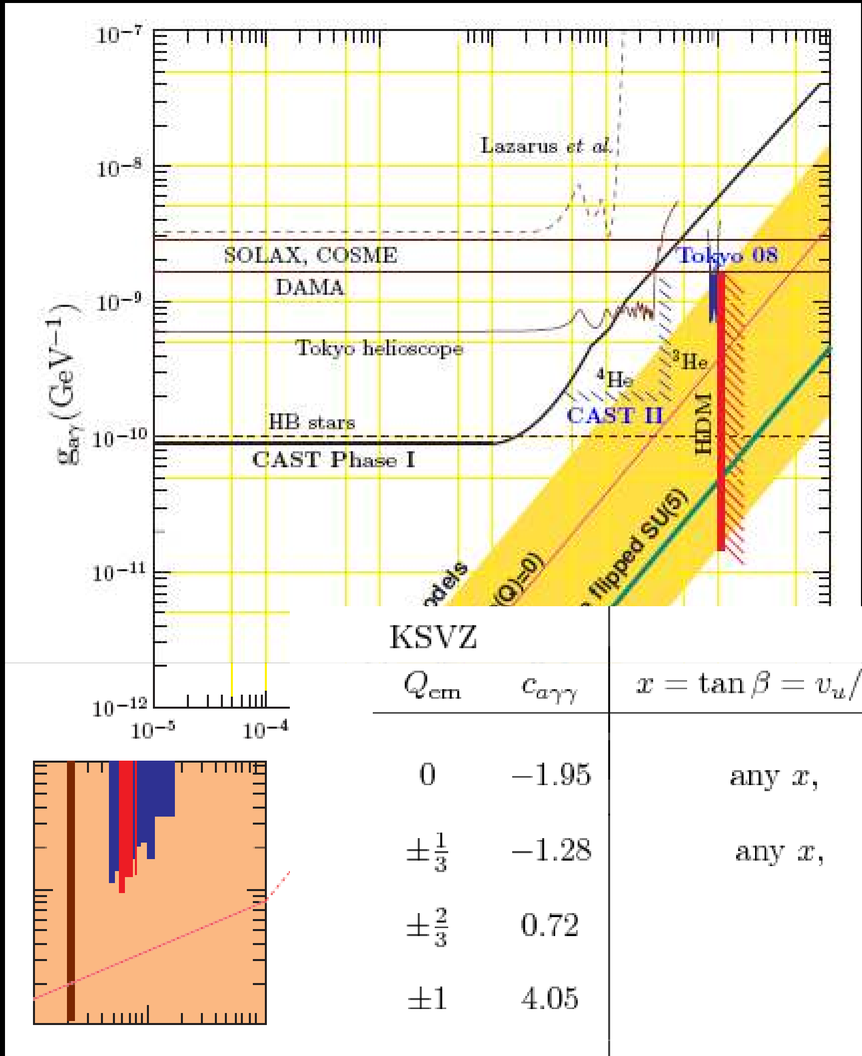
Turner (86), Grin et al (07), Giudice-Kolb-Riotto (08),

Bae-Huh-K (JCAP 08, [arXiv:0806.0497]): recalculated including the anharmonic term carefully with the new data on light quark masses.



It is the basi
argument fo

Figure 9. The bound from overclosure of the universe. The yellow band shows the error bars of Λ and two red dashed lines are the limits of the allowed current quark masses. The anharmonic effect is taken into account, including the initial correction factor of equation (15). Here, the entropy production ratio γ is absorbed into the bracket of F_a: $\tilde{\gamma} = \gamma^{(n+4)/(n+6)} \simeq \gamma^{0.84}$.



KSVZ		DFSZ		
$Q_{\rm em}$	$c_{a\gamma\gamma}$	$x = \tan\beta = v_u/v_d$,	same Higgs for (q^c, e) masses,	$c_{a\gamma\gamma}$
0	-1.95	any x,	(d^c, e)	0.72
$\pm\frac{1}{3}$	-1.28	any x,	(u^c, e)	-1.28
$\pm\frac{2}{3}$	0.72			
± 1	4.05			
(m, m)	-0.28			

Many lab. searches were made, and we hope the axion be discovered .

Strong CP

Axion is a Goldstone boson arising when the PQ global symmetry is spontaneously broken.
The simple form dictates that its interaction is only through the anomaly term(hadronic axion), etc. The axion models have the spontaneous symmetry breaking scale F and the axion decay constant F_a which are related by $F=N_{DW} F_a$.

But, gravity is not friendly with the global symmetries due to wormholes and black holes. Discrete gauge symmetries are not plagued by gravity. [Krauss-Wilczek; Barr-Seckel; Kamionkowski-March-Russel; Holman et al]

History: The Peccei-Quinn-Weinberg-Wilczek axion is ruled out early in one year [Peccei, 1978]. The PQ symmetry can be incorporated by heavy quarks, using a singlet Higgs field [KSVZ axion]

$$L = \overline{Q}_L Q_R S - V(S, H_u, H_d) + \cdots$$

Here, Higgs doublets are neutral under PQ. If they are not neutral, then it is not necessary to introduce heavy quarks [DFSZ axion]. In any case, the axion is the phase of the SM singlet *S,* if the VEV of *S* is much above the electroweak scale.

Because F_a can be in the intermediate scale, axions can live up to now (m<24 eV) and constitute DM of the Universe.

Recent review: Kim-Carosi, RMP 82, 557 (2010) arXiv:0807.3125

Historically, Peccei-Quinn tried to mimick the symmetry $\theta \to \theta - 2\alpha$, by the full electroweak theory. They found such a symmetry if H_u is coupled to up-type quarks and H_d couples to down-type quarks,

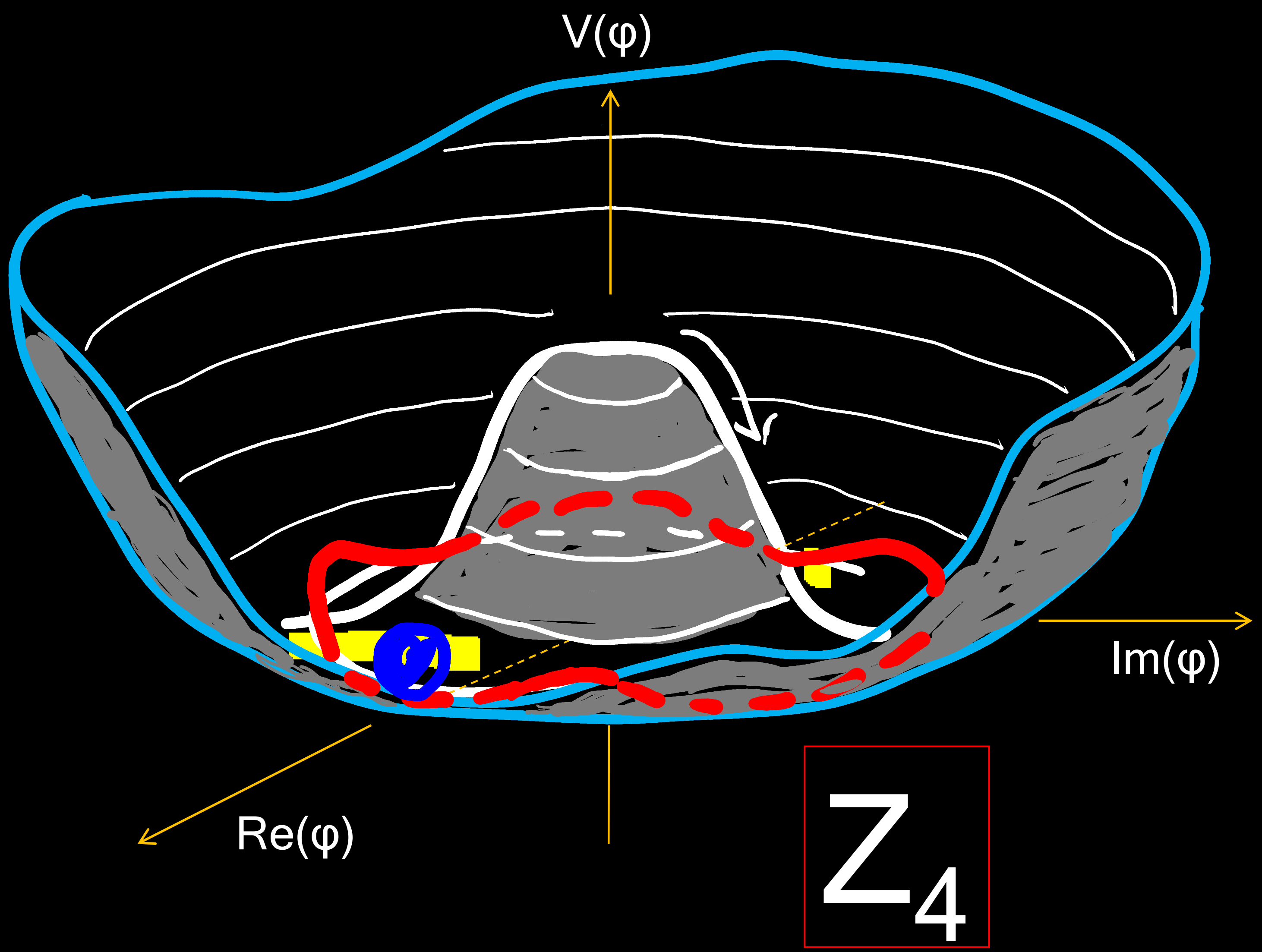
$$L = \bar{q}_L u_R H_u + \bar{q}_L d_R H_d - V(H_u, H_d) + \cdots$$

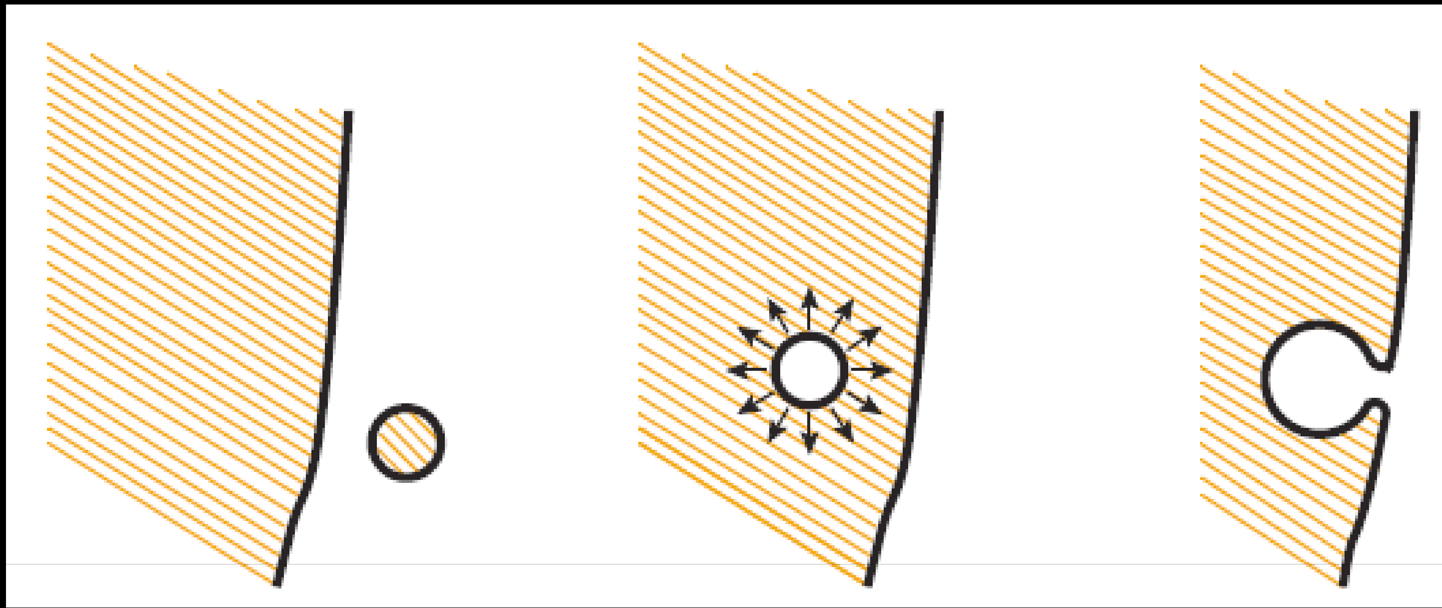
$$q \to e^{i\gamma_5 \alpha} q, \{H_u, H_d\} \to e^{i\beta}\{H_u, H_d\}$$
$$\to \int (-H_u e^{i\beta} \bar{u} e^{i\gamma_5 \alpha} u - H_d e^{i\beta} \bar{d} e^{i\gamma_5 \alpha} d + \frac{\theta - 2\alpha}{32\pi^2} G\tilde{G})$$

Eq. β=α achieves the same thing as the m=0 case.

The axion is created at $T=F_a$, but the universe • ($<a>$)does not roll until $3H=m_a$ ($T=0.92$ GeV [Bae-Huh-Kim]). From then on, the classical field $<a>$ starts to oscillate.
Harmonic oscillator motion:
$m_a^2\, F_a^2$ = energy density = m_a x *number density* = like CDM.
See, Bae-Huh-Kim, arXiv:0806.0497 [JCAP09 (2009) 005]

$$10^9 \text{ GeV} < F_a < 10^{12} \text{ GeV},$$


V(φ)
Im(φ)
Re(φ)
Z4

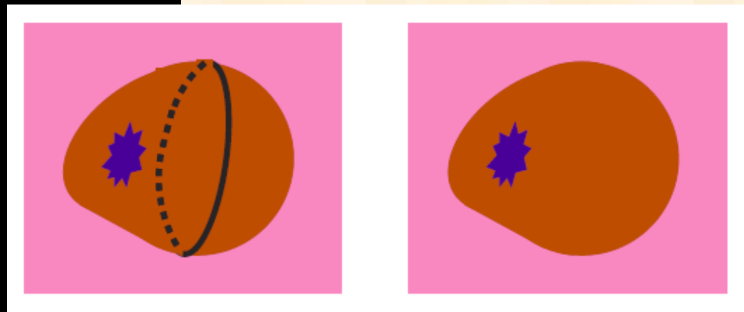


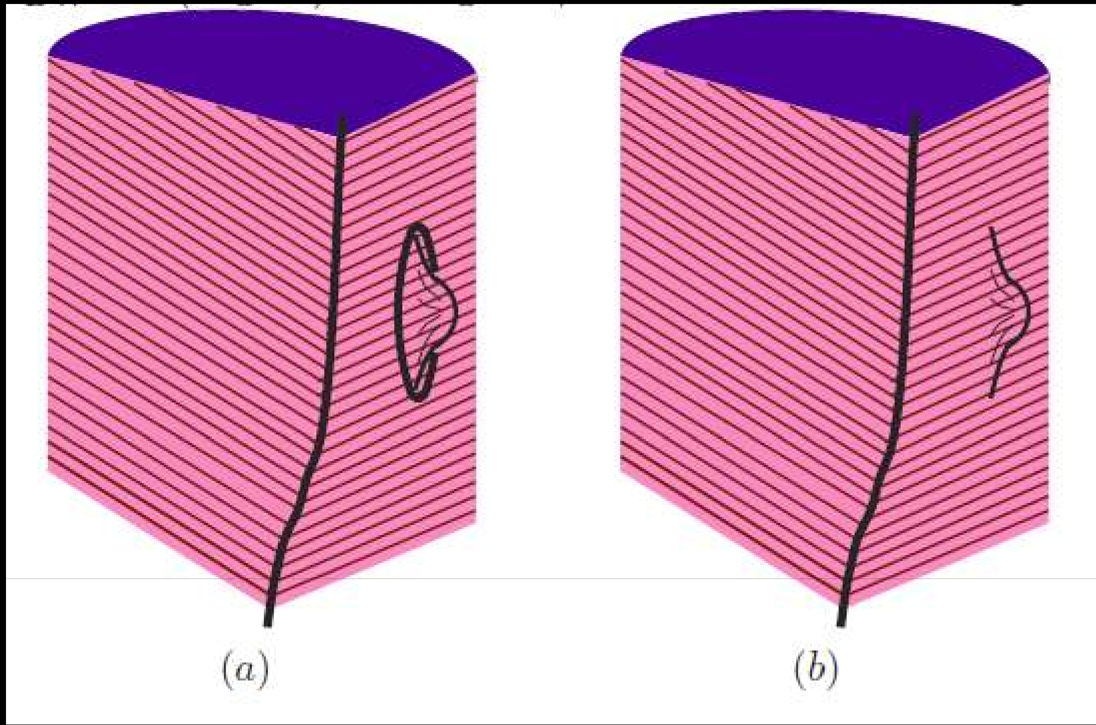
For N=1, the horizon scale DWs can be efficiently washed out. [Vilenkin-Everett, PRL 48, 1867 (1982); Barr-Choi-Kim, NPB 283, 591 (1987)]

The discrete symmetries can result from spontaneous breaking of continuous symmetries, but it is not absolutely necessary. DS as a subgroup of U(1) is useful and relevant for the QCD axion.

$$Z_N: \quad \psi \to e^{i\alpha}\psi$$
$$\Phi \to e^{iN\alpha}\Phi$$

Z_2 :





So, it is not likely that the domain wall of the horizon scale is washed out for N≥2. [Sikivie, PRL 48, 1156 (1982)]

3. Axions and SUSY

Nilles-Raby, Frere-Gerard, Tamvakis-Wyler,
Kim-Masiero-Nanopoulos, Rajagopal-Turner-Wilczek,
Chun-Kim-Nilles, Chun-Lukas, Nieves, Covi-Kim-Roszkowski,
Covi-Kim-Kim-Roszkowski, Covi-Roszkowski-Small,
Covi-Roszkowski-Ruis de Austri, Small, Brandenburg-Steffen,
Strumia, Covi-Kim, Freitas-Steffen-Tajuddin-Wyler,
Chun-Kim-Kohri-Lyth, Kim-Park-Stewart,
Baer-Dermisiek-Rajaopalan-Summy, Baer-Box-Summy,
Baer-Kraml-Lessa-Sekmen ,
Kim-Seo, Choi-Kim-Kim, Covi-Kim, Covi-Choi-Kim-Roszkowski,
Bae-Choi-Im, Goto-Yamaguchi, Moxhay-Yamamoto, Chang-Kim,
Roszkowski-Seto, , Choi-Kim-Lee-Seto, Berger-Covi-Kraml-Palorini,
Covi-Hasenkamp-Pokorski-Roberts, Pospelov,

....

Axino introduction by SUSY extension

axion : implies a superpartner

gravitino

$T_R < 10^9$ GeV(old), or 10^7 GeV(recent)

Ellis-Kim-Nanopoulos   Khori-Kawasaki-Moroi

axino

$T_R <$ multi TeV

Covi et al.

relatively small reheating temperature.

So, the reheating temperature is the inescapable parameter in axino cosmology. Also, R-parity is better to be introduced:

T_{reheat}

R-parity or effective R-parity

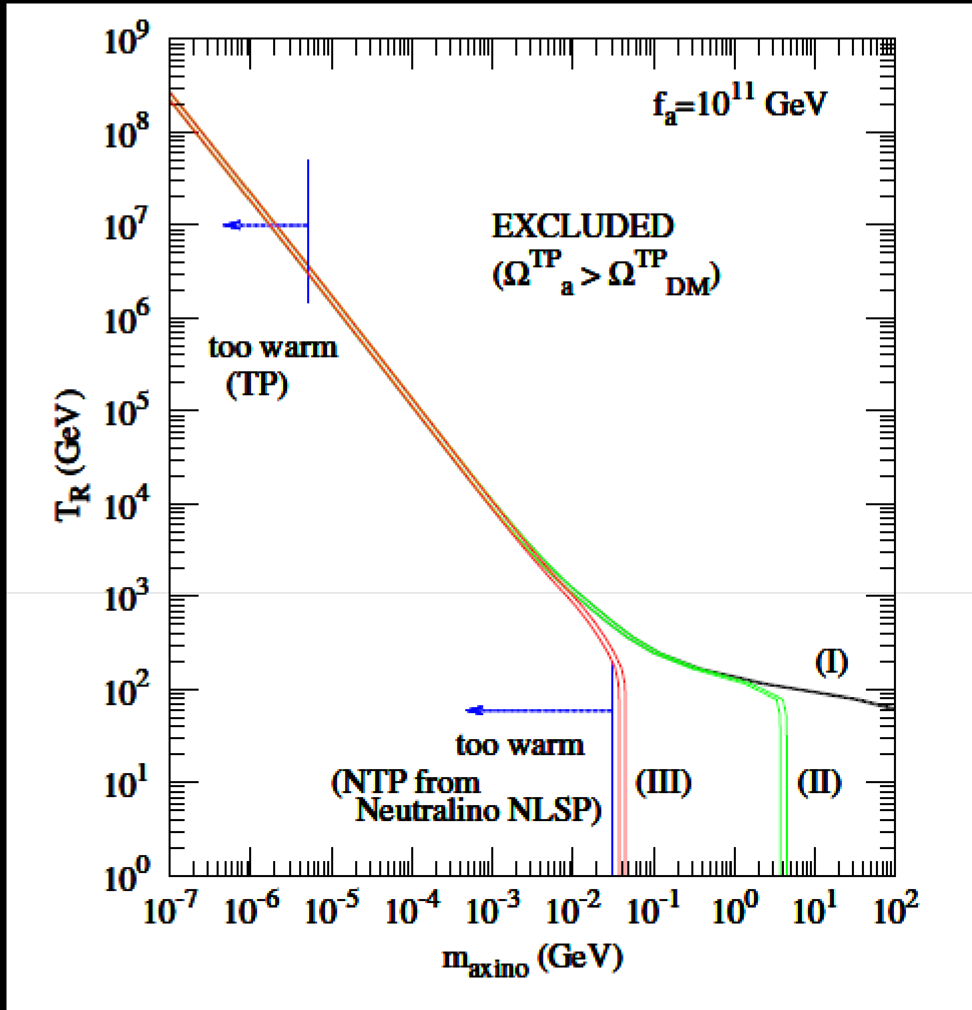
For axino to be LSP, it must be lighter than the lightest neutralino. The axino mass is of prime importance. The conclusion is that there is no theoretical upper bound on the axino mass.

eV axinos can be HDM (80s) [Kim-Masiero-Nanopoulos]

KeV axinos can be warm DM in SBB (90s) [Rajagopal-Turner-Wilczek]

GeV axinos can be CDM (00s) [Covi-H. B. Kim-K-Roszkowski]

TeV axino (decaying) to DM [K-Y Choi-Kim-Lee-Seto(2008); Huh- Kim, PRD 80, 075012 (2009)]



Choi et al: 1108.2282

In this figure, NTP axinos can be CDM for relatively low reheating temperature < 0.5 TeV, in the region 10 MeV < m(axino) < m(neutralino)

The region corresponding to the MSSM models with $\Omega_\chi h^2 < 10^4$ is allowed: a small axino mass renders the possibility of axino closing the universe or just 30 % of the energy density due to the suppression $\frac{m_{\tilde{a}}}{m_\chi}$

NTP axino as CDM possibility

If all SUSY mass parameters are below 1 TeV, then $\Omega\chi\, h^2 < 100$ but a sufficient axino energy density may not result for

$$m_{\tilde{a}} > 1\,\mathrm{GeV} \text{ and } T_{rh} > 1\,\mathrm{TeV}$$

But, what is the axino mass?

Or, what is really the axino?

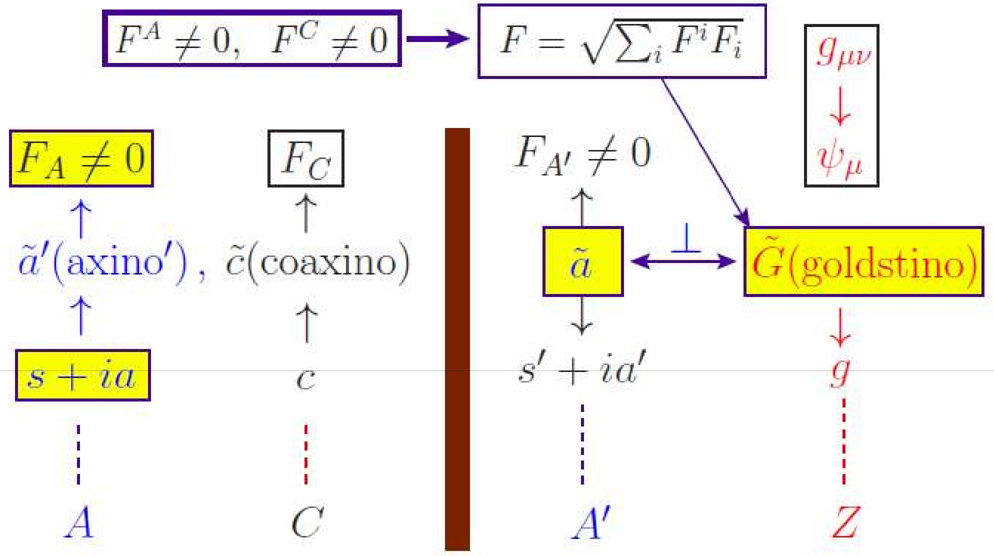


FIG. 1: The axion (blue) and goldstino (red) multiplets. F_A can be zero also. The axion direction a is defined by the PQ symmetry and the goldstino ($\tilde{G}$) and axino ($\tilde{a}$) directions are defined by the fermion mass eigenvalues. The primed fields are not mass eigenstates.

It is related to the PQ symmetry breaking and the SUSY breaking [Kim-Seo, NPB 864, 296 arXiv:1204.5495].

For example, the axino study so far performed from

$$
\begin{aligned}
\mathcal{L}^{\rm eff}_{\tilde a} = & \, i\frac{\alpha_s}{16\pi f_a}\bar{\tilde a}\gamma_5[\gamma^\mu,\gamma^\nu]\tilde g^b G^b_{\mu\nu} + \frac{\alpha_s}{4\pi f_a}\bar{\tilde a}\tilde g^a \sum_{\tilde q} g_s \tilde q^* T^a \tilde q \\
& + i\frac{\alpha_2 C_{aWW}}{16\pi f_a}\bar{\tilde a}\gamma_5[\gamma^\mu,\gamma^\nu]\tilde W^b W^b_{\mu\nu} \\
& + \frac{\alpha_2}{4\pi f_a}\bar{\tilde a}\tilde W^a \sum_{\tilde f_D} g_2 \tilde f_D^* T^a \tilde f_D \\
& + i\frac{\alpha_Y C_{aYY}}{16\pi f_a}\bar{\tilde a}\gamma_5[\gamma^\mu,\gamma^\nu]\tilde Y Y_{\mu\nu} \\
& + \frac{\alpha_Y}{4\pi f_a}\bar{\tilde a}\tilde Y \sum_{\tilde f} g_Y \tilde f^* Q_Y \tilde f,
\end{aligned}
$$

does not include the axino mass.

One must introduce the PQ symmetry and SUSY. The first example is: Kim, PLB136, 78 (1984)

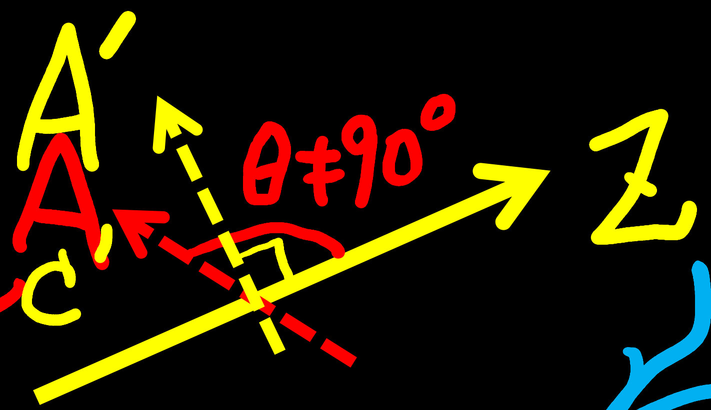
$$W = \sum_{i=1}^{n_I} Z_i(S\overline{S} - f_i^2), \; n_I \geq 2.$$

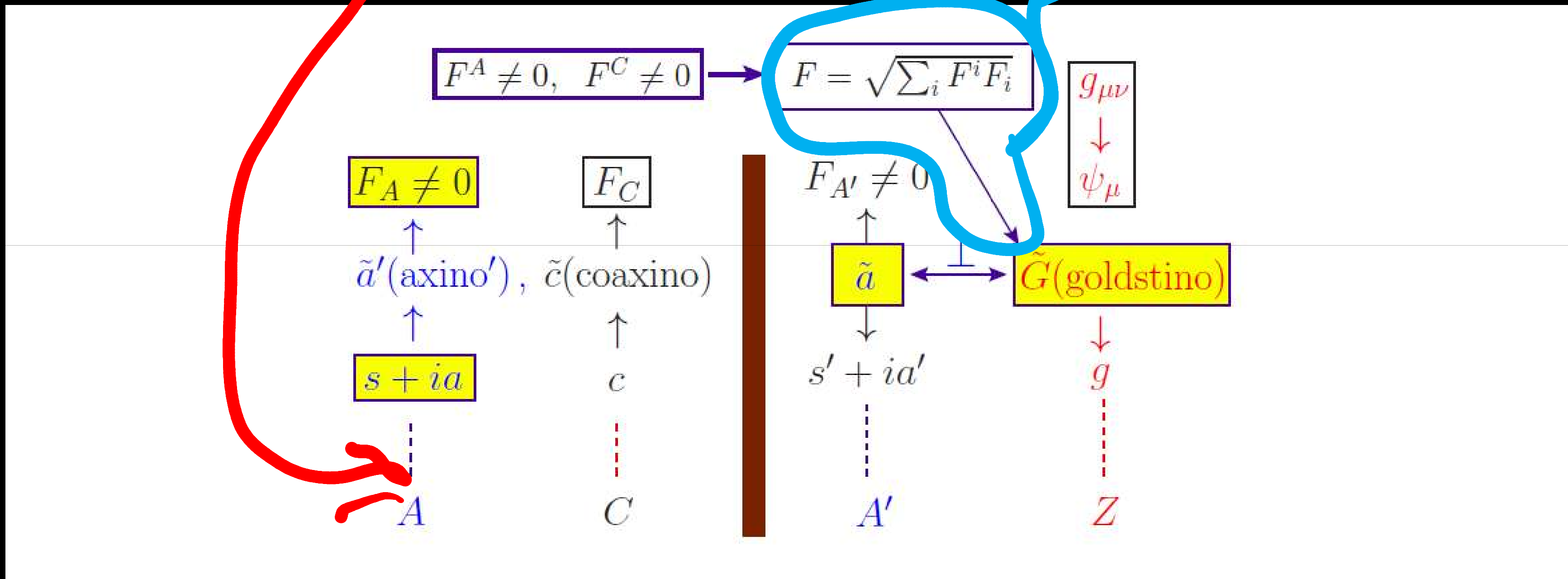
PQ symmetry breaking → axion

SUSY breaking → goldstino

One lucid but mostly unaware aspect on axino is that its definition must be given as a mass eigenstate. Axino is connected to two kinds of symmetry breaking, the PQ global symmetry breaking and the SUSY breaking. In general, these two breakings are not orthogonal to each other.

This happens if
F-term of A is nonzero,
which is generally
expected.





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One notable corollary of this is that the axino CDM study in most references is an over-estimation. I.e. T_{reheat} can be a bit larger than the previous study. For

$$m_{\tilde{a}} < m_{3/2}$$

an over-estimation was given if the axion multiplet A obtains an F-term.

4. Discrete symmetries and axions : Axions from string

Wormholes: Giddings-Strominger, Gilbert

Discrete gauge symmetry: Krauss-Wilczek

Gravity does not like PQ symmetries:

Barr-Seckel, Kamionkowski-March-Russell,
Holman-Hsu-Kephart-Kolb-Watkins-Widrow

Therefore, go for a discrete symmetry with an approximate PQ symmetry.

★ Discrete symmetries seem required, though they often seem “ad hoc”

★ Bottom-up approach:

fermion flavor mixing problem,

proton decay→ e.g. R-parity with SUSY

★ Discrete symmetries in QFT and string theory:

Ibanez-Ross, Strominger-Witten, Banks-Dine,
Acharya-Kane-Kumar-Lu-Zheng,
Kobayashi-Nilles-Raby,
Lee-Raby-Ross-Ratz-Schieren-SchmidtHolberg-Vaudrev.,
Kappl-Petersen-Raby-Ratz-Schrien-Vaudrevange,
Chen-Ratz-Staudt-Vaudrevange,
Fischer-Ratz-Torrado-Vaudrevange,

. . . .

★ Abelian discrete symmetries Z_N and Z_{nR} , from string theory, example in Z(12-I) orbifold : [JEK, 1308.0344]

Kobayashi-Nilles-Raby,
Lee-Raby-Ross-Ratz-Schieren-SchmidtHolberg-Vaudrev.,
Kappl-Petersen-Raby-Ratz-Schrien-Vaudrevange,
Chen-Ratz-Staudt-Vaudrevange,
Fischer-Ratz-Torrado-Vaudrevange,

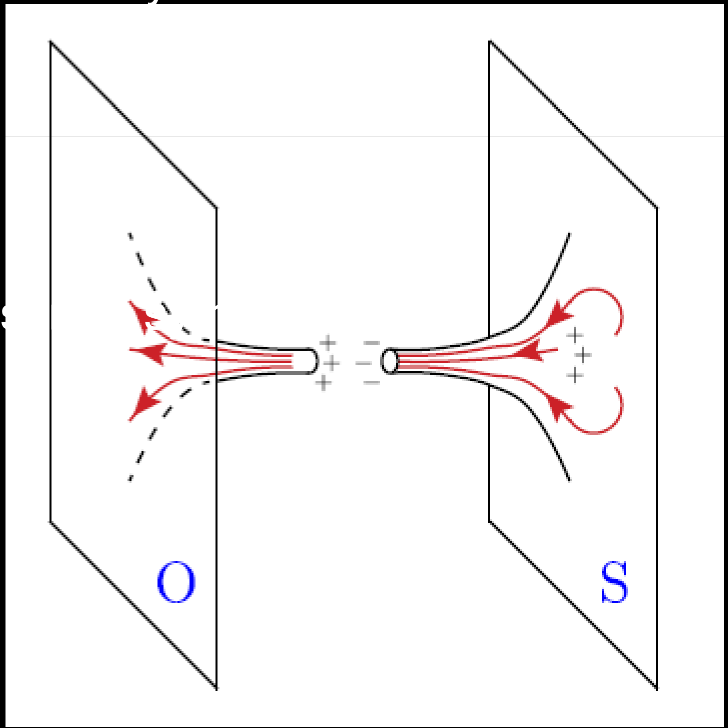
. . . .

How quantum gravity effects allow interactions below MP?

★ Quantum gravity effects occurring at the Planck scale connect the observable universe O to the shadow world S via the Planck size wormholes. Probability of connecting through the Planck scale size wormholes is O(1). We can think of two possibilities of discrete symmetries from string compactification, resulting from the Planck size scale wormholes:

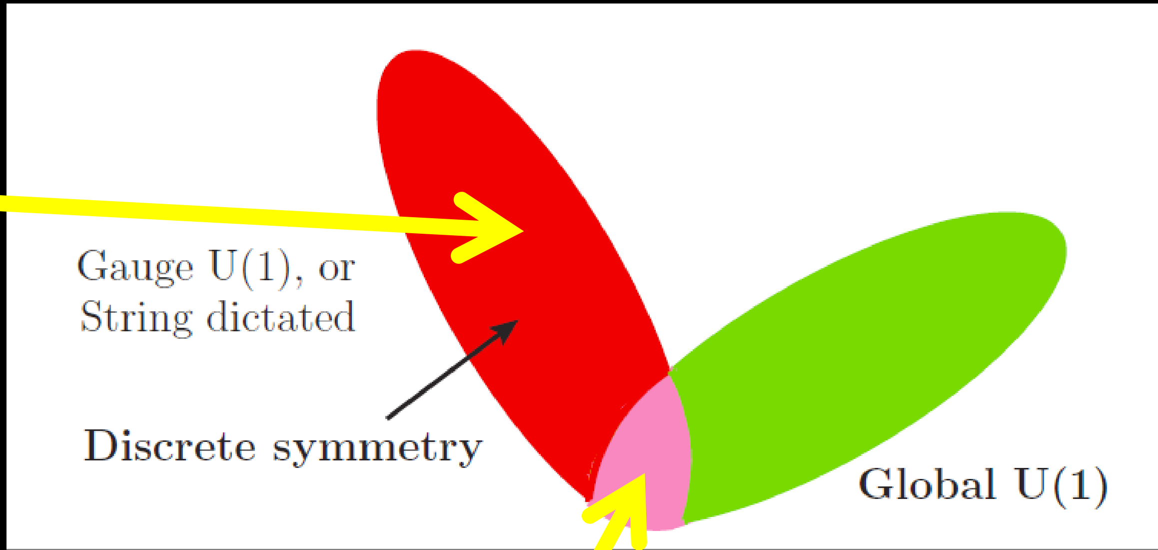
(1) the discrete symmetry is a part of a gauge symmetry.

(2) The string selection rules directly give the di



Same thing?

The global symmetry violating terms.



A few low order W's are respected by discrete symmetry.

If some matter fields arise at the same fixed point, then there must be the maximal discrete symmetry among the fields of that fixed point. So, can it be considered “ad hoc”?

$$T_2 \otimes T_2 \otimes T_2$$

Triangle
Cushion:SU(3)

Z_3

3 families
by LHC data?

T_4 of Z_{12}

Z_{12} on 4-torus

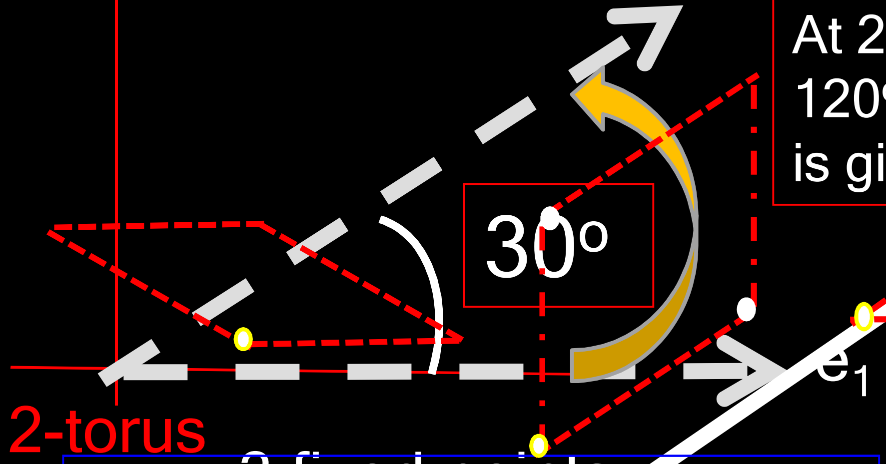
SO(8) lattic

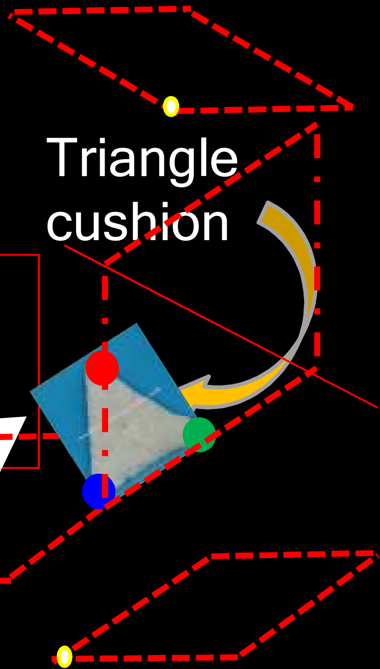
$$\phi = \begin{pmatrix} \frac{5}{12} & \frac{4}{12} & \frac{1}{12} \end{pmatrix}$$

On SO(8) lattice, 30° rotation gives only origin as FP

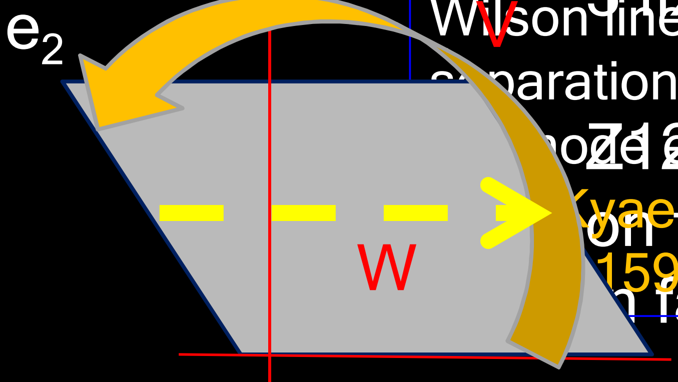
At 2nd torus, 120° rotation is given: Z_3

Triangle cushion





Z_3 on 2-torus



3 fixed points

Wilson line shift introduces the separation between fixed points.

Z12 is simpler than Z3 on fixed point analysis.

…mode effects studied

[Kyae: PRD77, 106008 … 1596]]

In fact, Z(12-I) is the simplest.

$$\phi = \begin{pmatrix} \frac{5}{12} & \frac{4}{12} & \frac{1}{12} \end{pmatrix}$$

30°

Two pairs of Higgs doublets:

Brown-He-Ovrut-Pantev, JHEP 0605, 043 (2006)
Kim-Kim-Kyae, JHEP 0706, 034 (2007),
. . . .

Sector	Weight	Mult.	$SU(5)_X$
T_4^0	$(\underline{1\,0\,0\,0\,0};\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3})\ (0^8)'$	2	$\mathbf{5}_{-2}\ (h_u^{(i)})$
T_4^0	$(\underline{-1\,0\,0\,0\,0};\ \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3})\ (0^8)'$	2	$\overline{\mathbf{5}}_{2}\ (h_d^{(i)})$
T_4^0	$(0^5;\ \frac{-2}{3}\ \frac{-2}{3}\ \frac{-2}{3})\ (0^8)'$	3	$\mathbf{1}_0\ (X_i)$
T_3	$\left(\underline{\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{-1}{2}\ \frac{-1}{2}};\ 0^3\right)\ (0^5;\ \frac{-1}{4}\ \frac{-1}{4}\ \frac{2}{4})'$	1	$\overline{\mathbf{10}}_{-1}\ (\overline{H})$
T_9	$\left(\underline{\frac{1}{2}\ \frac{1}{2}\ \frac{-1}{2}\ \frac{-1}{2}\ \frac{-1}{2}};\ 0^3\right)\ (0^5;\ \frac{1}{4}\ \frac{1}{4}\ \frac{-2}{4})'$	1	$\mathbf{10}_{1}\ (H)$
T_2^0	$(0^5;\ \frac{-1}{3}\ \frac{-1}{3}\ \frac{-1}{3})\ (0^5;\ \frac{-1}{2}\ \frac{1}{2}\ 0)'$	$1+1$	$\mathbf{1}_0$
T_2^0	$(0^5;\ \frac{-1}{3}\ \frac{-1}{3}\ \frac{-1}{3})\ (0^5;\ \frac{1}{2}\ \frac{-1}{2}\ 0)'$	$1+1$	$\mathbf{1}_0$

$\boxed{S_2(L) \times S_2(R)}$

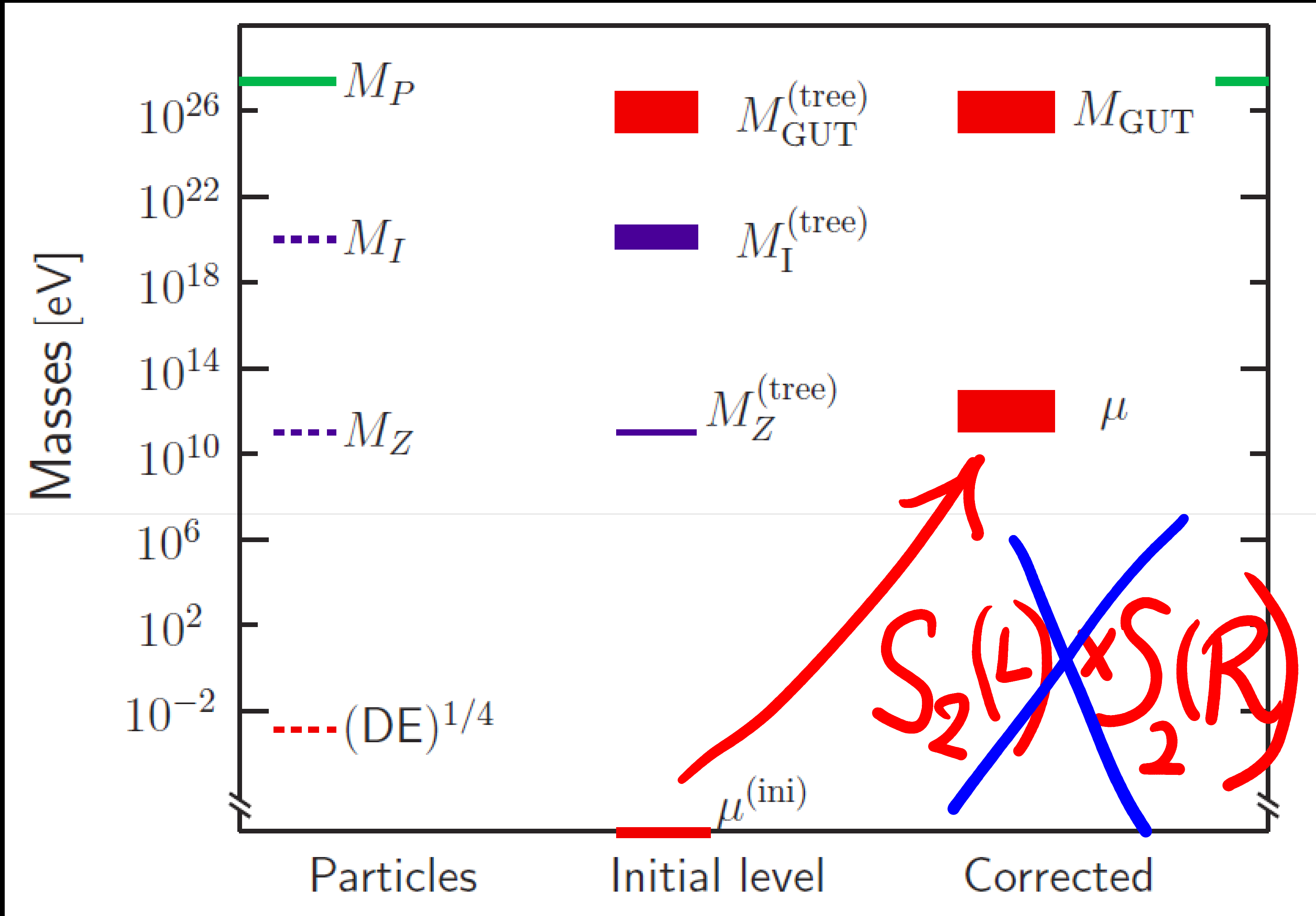
$S_2(L)$ x $S_2(R)$ symmetric fermion masses can arise from

$$\mathcal{L} = -\frac{m}{2}\left(\overline{\Psi}_L^{(1)}\Psi_R^{(1)} + \overline{\Psi}_L^{(1)}\Psi_R^{(2)} + \overline{\Psi}_L^{(2)}\Psi_R^{(1)} + \overline{\Psi}_L^{(2)}\Psi_R^{(2)}\right) + h.c.$$

The fermion mass matrix will be

$$M_0 = \begin{pmatrix} m/2 & m/2 \\ m/2 & m/2 \end{pmatrix} \qquad M = \begin{pmatrix} 0 & 0 \\ 0 & m \end{pmatrix}$$

The first step for the solution of the μ-problem.
[JEK, arXiv:1303.1822].



[JEK, arXiv:1303.1822].

In string theory, matter fields are from

$E_8 \times E_8$ representations. Not from B_{MN} .

Kim-Nilles μ-term arises from

$$W = \frac{\lambda X \bar{X}}{2M_P} H_u H_d$$

Where do X and X-bar belong? Probably, in matter reps.

$$\begin{pmatrix} T_u^r \\ T_u^g \\ T_u^b \\ H_u^+ \\ H_u^0 \\ X_u \\ \cdots \end{pmatrix}^{(i=1,2)}, \quad \begin{pmatrix} T_d^r \\ T_d^g \\ T_d^b \\ H_d^0 \\ H_d^- \\ X_d \\ \cdots \end{pmatrix}^{(i=1,2)}$$

Anyway,
B_{MN} fields: decay constant is very large
$F > 10^{16}$ GeV
[Choi-Kim (1984), Svrcek-Witten(2006)]

How can we break $S_2(L) \times S_2(R)$ symmetry ?

Spontaneously by

$$\langle X^{(1)}\rangle = \langle \overline{X}^{(1)}\rangle = F_a\,, \quad \langle X^{(2)}\rangle = \langle \overline{X}^{(2)}\rangle = 0$$

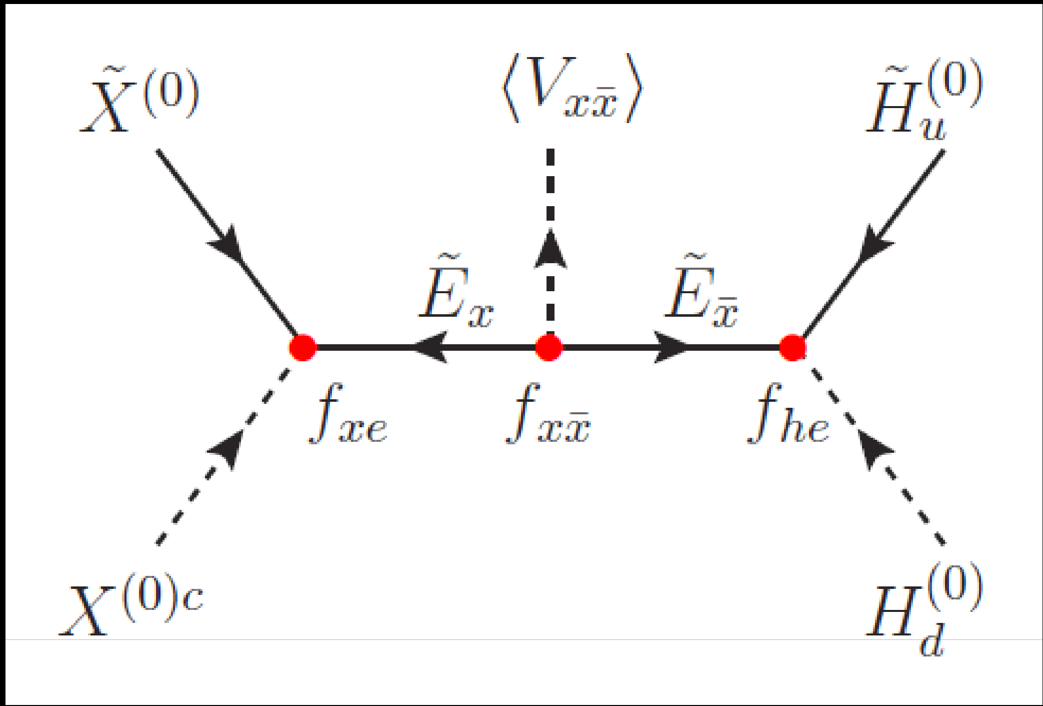
The massless (0) fields, and superheavy (G) fields

$$\frac{\lambda}{M_P} X^{(i)}\overline{X}^{(i)} H_u^{(j)} H_d^{(j)}$$

Nonzero VEV

$$X^{(0),(G)} = \frac{1}{\sqrt{2}}(X^{(1)} \mp X^{(2)})$$

$$H_{u,d}^{(0),(G)} = \frac{1}{\sqrt{2}}(H_{u,d}^{(1)} \mp H_{u,d}^{(2)})$$



This defines the PQ charges of X and H.

$$\frac{\lambda}{2M_P} X^{(0)} X^{(0)c} H_u^{(0)} H_d^{(0)}, \qquad \lambda = f_{xe} f_{he} \frac{M_P}{M_E}$$

$$\langle X^{(0)} \rangle = 10^{12}\ \text{GeV}, \qquad f = O(10^{-2}), \frac{M_P}{M_E} = 10, \text{gives } \lambda = 10^{-3}$$

Then, $\mu = 200$ GeV

$(\lambda / 2M_P)$

1 $X^{(0)} X^{(0)c}$

-1 $H_u^{(0)} H_d^{(0)}$

But PQ $\neq$ exact

We choose (2) of

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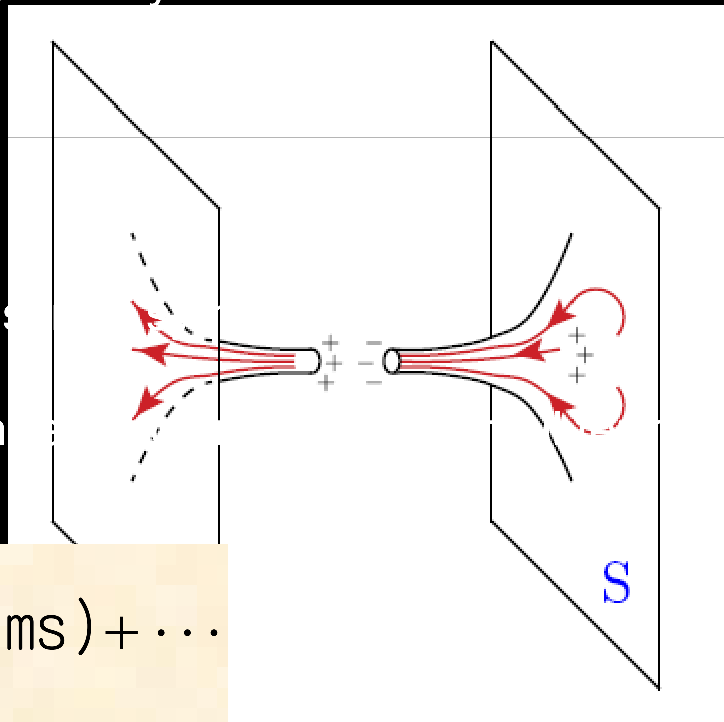
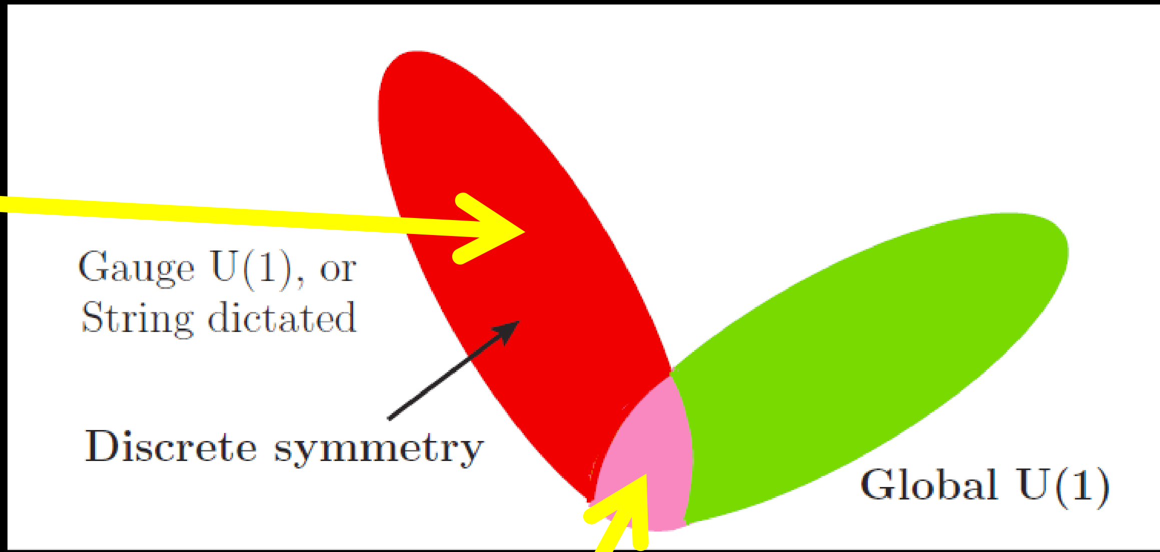


(2) The string selection rules directly give the di

In both cases, an exact discrete symmetry is a
written at the scale M_P.

$$(D=4\ \text{terms}) + \frac{1}{M_P}(D=5\ \text{terms}) + \cdots$$

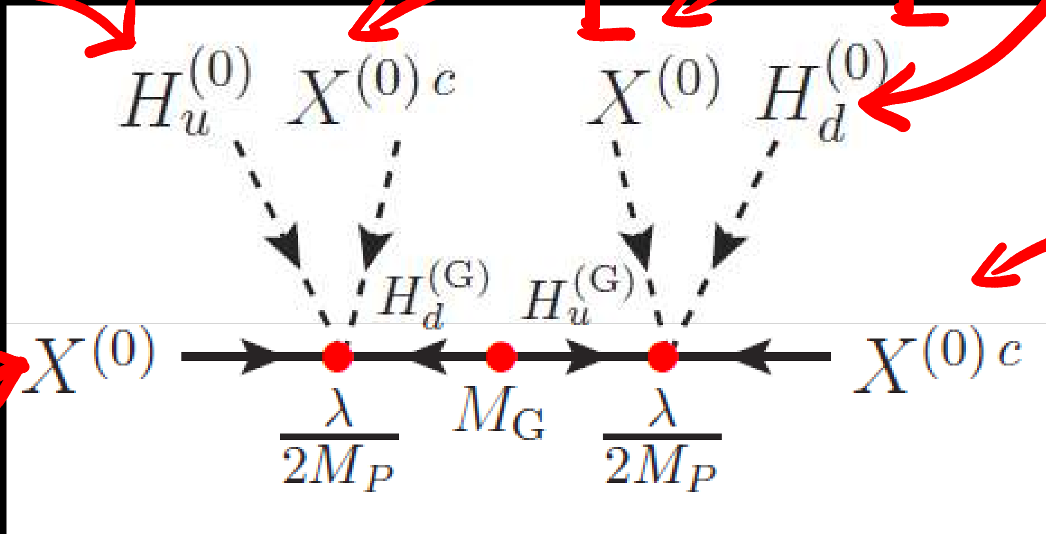
The global symmetry violating terms.



A few low order W's are respected by discrete symmetry.

The PQ breaking diagram is

-1 1 1 -1



1 1

Total 2 units

The A-term

$$m_{3/2}\frac{\lambda^2}{4M_P^2}\left(\frac{1}{M_G}H_uH_d\right)(XX^c)^2=\frac{\lambda^2 m_{3/2}v_uv_dF_a^4}{8M_P^2M_G}\approx\left(\frac{\lambda^2\sin\beta\cos\beta}{8}\right)\frac{v_{\rm ew}^2}{M_G}m_{3/2}\mu^2$$
$$\approx\left(\frac{\lambda^2}{\tan\beta}\right)3\times10^{-6}\left(\frac{m_{3/2}}{\rm TeV}\right)\left(\frac{\mu}{\rm TeV}\right)^2[{\rm GeV}^4].$$

This can be compared with the main axion potential

$$V=\frac{Z f_\pi^2 m_\pi^2}{(1+Z)^2}\left(1-\cos\frac{a}{F_a}\right)+10^{-13}\sin\frac{a}{F_a}[GeV^4]\to|\bar\theta|\approx10^{-9}$$

For gravity mediation, |θ| may be of order

$$\lambda=10^{-3}\to\quad|\bar\theta|\approx10^{-9}$$

★ Gauge mediated SUSY breaking: $m_{3/2}$ is of eV order. So, $|\theta|$ is completely negligible. Then, λ can be larger.

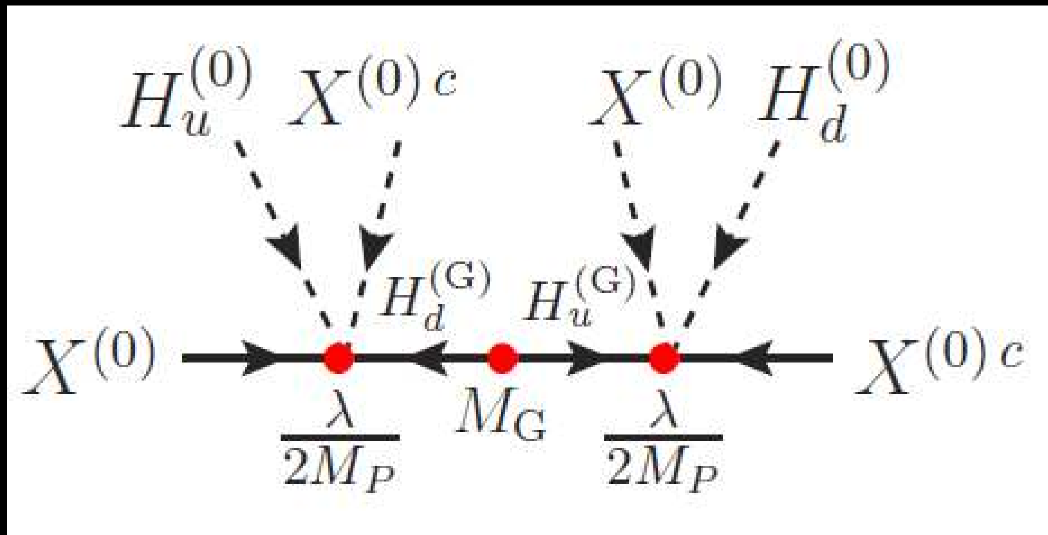
★ With $\lambda = 10^{-3}$, in the gravity mediation,

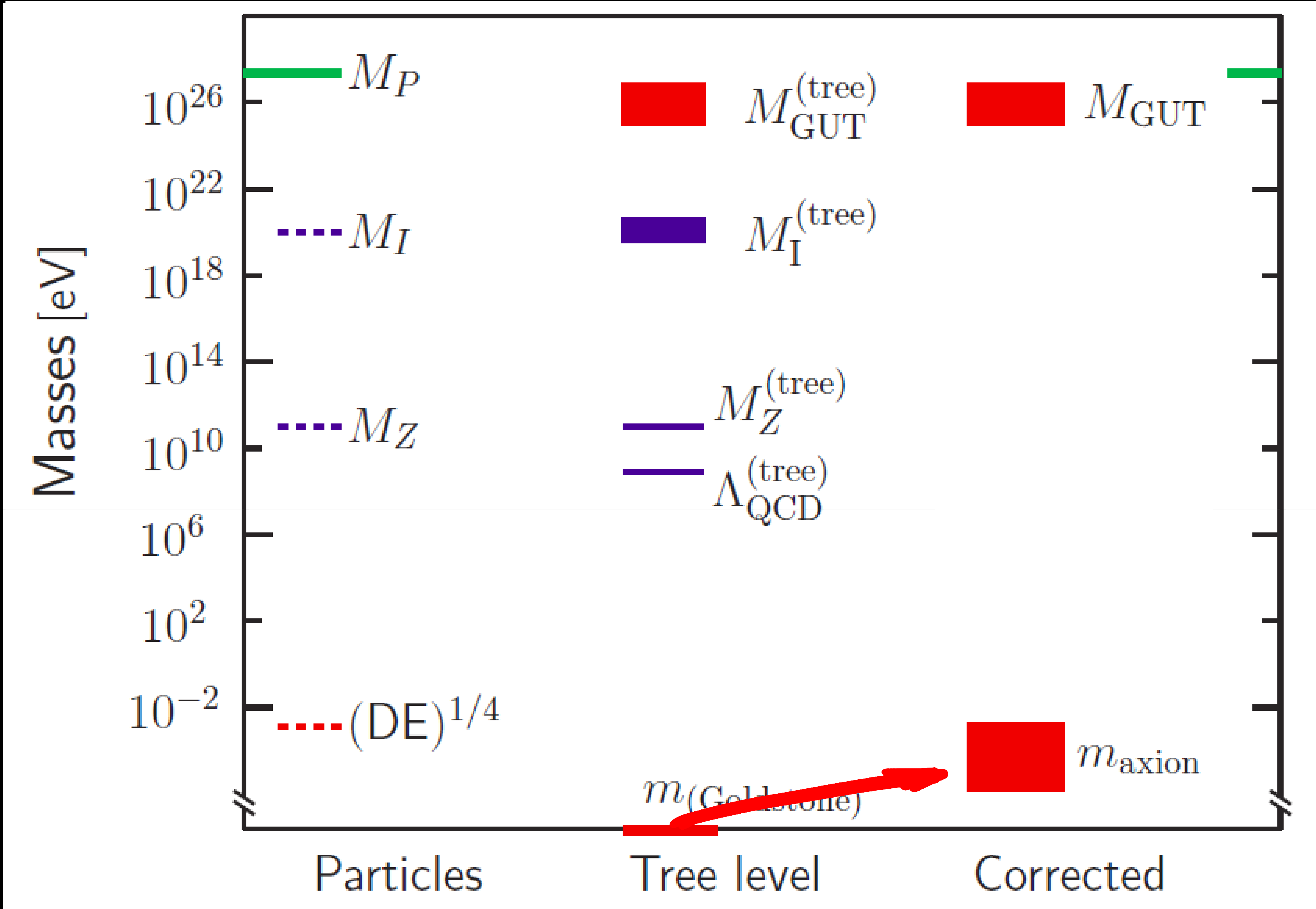
$|\bar{\theta}| \approx 10^{-9}$ is in the experimentally detectable range at pEDM exp.

Still,

$$\bar{\theta} \approx \frac{\mu}{M_G}$$

From the PQ violating





Masses [eV]
10^{26}
10^{22}
10^{18}
10^{14}
10^{10}
10^{6}
10^{2}
10^{-2}
M_P
M_I
M_Z
$(\mathrm{DE})^{1/4}$
$M_{\mathrm{GUT}}^{(\mathrm{tree})}$
$M_{\mathrm{I}}^{(\mathrm{tree})}$
$M_Z^{(\mathrm{tree})}$
$\Lambda_{\mathrm{QCD}}^{(\mathrm{tree})}$
$m_{(\mathrm{Goldstone})}$
M_{GUT}
m_{axion}
Particles
Tree level
Corrected

Other discrete symmetries Z_N and Z_{nR} are also useful:

[JEK, 1308.0344] [Many papers by Kappl, Ratz et al.]

★ Model building methodology is to look for SM singlets. For example, in Z(12-I) model of [Huh-Kim-Kyae, PRD 80, 115012 (2009), 0904.1108]:

$P+4V$	χ	$(N^L)_j$	$\mathcal{P}_4(f_0)$	$SU(5)_X$	$B-L$	P	Z_4, Z_8, Z_{10}, Z_{12}
$(\underline{+----};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	L	0	2	$2\cdot\mathbf{5}_3$	$u^c:-\frac{1}{3}$, $(\nu_e,e)^T:-1$	-1	-1
$(\underline{+++--};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	L	0	2	$2\cdot\overline{\mathbf{10}}_{-1}$	$d^c:-\frac{1}{3}$, $(u,d):\frac{1}{3}$, $\nu^c:+1$	-1	-1
$(+++++;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	L	0	2	$2\cdot\mathbf{1}_{-5}$	$e^+:+1$	-1	-1
$(\underline{1\,0\,0\,0\,0};\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	L	0	2	$2\cdot\mathbf{5}_{-2}$	$H_d:0$	$+2$	$+2$
$(\underline{-1\,0\,0\,0\,0};\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	L	0	2	$2\cdot\overline{\mathbf{5}}_2$	$H_u:0$	$+2$	$+2$

Needed for DM

All the SU(5) × U(1)$_X$ × SU(5)′ × SU(2)′ singlet states of Ref. [22].

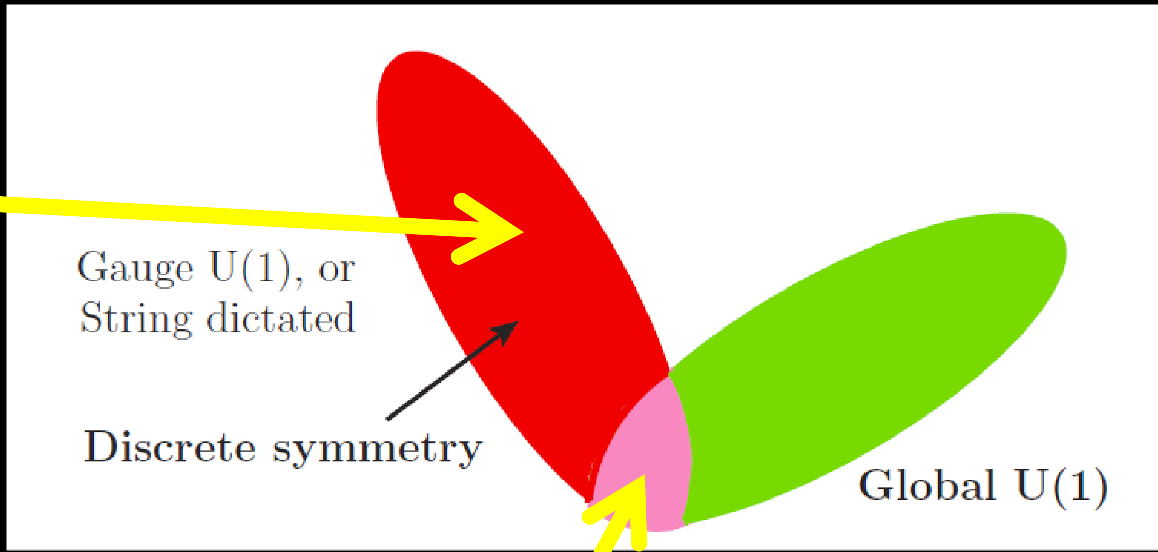
Sectors	Singlet states	χ	$(N^L)_j$	$\mathcal{P}(f_0)$	Label	P	Z_4	Z_8	Z_{10}	Z_{12}
T_4^0	$(0\,0\,0\,0\,0;\,\frac{-2}{3}\,\frac{-2}{3}\,\frac{-2}{3})(0^8)'$	L	0	3	S_1	−4	−4	−4	−4	−4
T_4^0	$(0\,0\,0\,0\,0;\,\frac{-2}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	L	$1_{\bar 1}, 1_2, 1_3$	2, 3, 2	S_2	0	−2	−2	−2	−2
T_4^0	$(0\,0\,0\,0\,0;\,\frac{1}{3}\,\frac{-2}{3}\,\frac{1}{3})(0^8)'$	L	$1_{\bar 1}, 1_2, 1_3$	2, 3, 2	S_3	0	0	0	0	0
T_4^0	$(0\,0\,0\,0\,0;\,\frac{1}{3}\,\frac{1}{3}\,\frac{-2}{3})(0^8)'$	L	$1_{\bar 1}, 1_2, 1_3$	2, 3, 2	S_4	0	+2	+2	+2	+2
T_6	$(0\,0\,0\,0\,0;\,0\,1\,0)(0\,0\,0\,0\,0\,\frac{1}{2}\,\frac{-1}{2}\,0)'$	L	0	2	S_5	+2	+1	−3	−5	−7
T_6	$(0\,0\,0\,0\,0;\,0\,0\,1)(0\,0\,0\,0\,0\,\frac{-1}{2}\,\frac{1}{2}\,0)'$	L	0	2	S_6	+2	+3	+7	+9	+11
T_6	$(0\,0\,0\,0\,0;\,0\,-1\,0)(0\,0\,0\,0\,0\,\frac{-1}{2}\,\frac{1}{2}\,0)'$	L	0	2	S_7	−2	−1	+3	+5	+7
T_6	$(0\,0\,0\,0\,0;\,0\,0\,-1)(0\,0\,0\,0\,0\,\frac{1}{2}\,\frac{-1}{2}\,0)'$	L	0	2	S_8	−2	−3	−7	−9	−11
T_3	$(0\,0\,0\,0\,0;\,\frac{-1}{2}\,\frac{-1}{2}\,\frac{-1}{2})(0\,0\,0\,0\,0\,\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	L	0	1	S_9	−3	−4	−8	−10	−12
T_3	$(0\,0\,0\,0\,0;\,\frac{-1}{2}\,\frac{1}{2}\,\frac{1}{2})(0\,0\,0\,0\,0\,\frac{3}{4}\,\frac{-1}{4}\,\frac{-1}{2})'$	L	0	1	S_{10}	+1	0	−4	−6	−8
T_3	$(0\,0\,0\,0\,0;\,\frac{1}{2}\,\frac{1}{2}\,\frac{-1}{2})(0\,0\,0\,0\,0\,\frac{-1}{4}\,\frac{3}{4}\,\frac{-1}{2})'$	L	0	1	S_{11}	+1	+2	+6	+8	+10
T_3	$(0\,0\,0\,0\,0;\,\frac{1}{2}\,\frac{1}{2}\,\frac{-1}{2})(0\,0\,0\,0\,0\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{1}{2})'$	L	$1_1, 1_3$	2,1	S_{12}	+1	+1	+1	+1	+1
T_9	$(0\,0\,0\,0\,0;\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2})(0\,0\,0\,0\,0\,\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{2})'$	L	0	1	S_{13}	+3	+4	+8	+10	+12
T_9	$(0\,0\,0\,0\,0;\,\frac{1}{2}\,\frac{-1}{2}\,\frac{-1}{2})(0\,0\,0\,0\,0\,\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{2})'$	L	0	2	S_{14}	−1	0	+4	+6	+8
T_9	$(0\,0\,0\,0\,0;\,\frac{-1}{2}\,\frac{-1}{2}\,\frac{1}{2})(0\,0\,0\,0\,0\,\frac{1}{4}\,\frac{-3}{4}\,\frac{1}{2})'$	L	0	2	S_{15}	−1	−2	−6	−8	−10
T_9	$(0\,0\,0\,0\,0;\,\frac{-1}{2}\,\frac{-1}{2}\,\frac{1}{2})(0\,0\,0\,0\,0\,\frac{1}{4}\,\frac{1}{4}\,\frac{-1}{2})'$	L	$1_{\bar 1}, 1_{\bar 3}$	1, 1	S_{16}	−1	−1	−1	−1	−1
T_2^0	$(0\,0\,0\,0\,0;\,\frac{-1}{3}\,\frac{-1}{3}\,\frac{-1}{3})(0\,0\,0\,0\,0\,\frac{-1}{2}\,\frac{1}{2}\,0)'$	L	$2_{\bar 1}, 2_3$	1, 1	S_{17}	−3	−2	+2	+4	+6
T_2^0	$(0\,0\,0\,0\,0;\,\frac{-1}{3}\,\frac{-1}{3}\,\frac{-1}{3})(0\,0\,0\,0\,0\,\frac{1}{2}\,\frac{-1}{2}\,0)'$	L	$2_{\bar 1}, 2_3$	1, 1	S_{18}	−3	−4	−8	−10	−12
T_1^0	$(0\,0\,0\,0\,0;\,\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0\,0\,0\,0\,0\,\frac{-3}{4}\,\frac{1}{4}\,\frac{1}{2})'$	L	3_3	1	S_{19}	−1	0	+4	+6	+8
T_1^0	$(0\,0\,0\,0\,0;\,\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0\,0\,0\,0\,0\,\frac{1}{4}\,\frac{-3}{4}\,\frac{1}{2})'$	L	3_3	1	S_{20}	−1	−2	−6	−8	−10
T_1^0	$(0\,0\,0\,0\,0;\,\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0\,0\,0\,0\,0\,\frac{1}{4}\,\frac{1}{4}\,\frac{-1}{2})'$	L	$\{1_1, 1_3\}, \{2_3, 1_2\}, 6_3$	1, 1, 1	S_{21}	−1	−1	−1	−1	−1
T_7^0	$(0\,0\,0\,0\,0;\,\frac{5}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0\,0\,0\,0\,0\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{1}{2})'$	L	$2_{\bar 1}$	1	S_{22}	+1	+1	+1	+1	+1
T_7^0	$(0\,0\,0\,0\,0;\,\frac{-1}{6}\,\frac{5}{6}\,\frac{-1}{6})(0\,0\,0\,0\,0\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{1}{2})'$	L	$2_{\bar 1}$	1	S_{23}	+1	+1	+1	+1	+1
T_7^0	$(0\,0\,0\,0\,0;\,\frac{-1}{6}\,\frac{-1}{6}\,\frac{5}{6})(0\,0\,0\,0\,0\,\frac{-1}{4}\,\frac{-1}{4}\,\frac{1}{2})'$	L	$2_{\bar 1}$	1	S_{24}	+1	+1	+1	+1	+1

A U(1) gauge symmetry is broken Z_{10R} . The center of SU(5) not taken into account, but it is still useful for making mu=0.

$(\underline{1\,0\,0\,0\,0};\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$ Hd

$(\underline{-1\,0\,0\,0\,0};\,\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$ Hu

The global symmetry violating terms.



These conditions are from gauge symmetry. Next page works here.

Sector	n·Field	Weight	Z_{10}
$T_4^{(0)}$	$2 \cdot H_d$	$(0\,0\,0\,\underline{1\,0}\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3})(0^8)'$	$+2$
$T_4^{(0)}$	$2 \cdot H_u$	$(0\,0\,0\,\underline{-1\,0}\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3})(0^8)'$	$+2$
$T_4^{(0)}$	$2 \cdot q$	$(\underline{\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{-1}{2}}\,\underline{\tfrac{1}{2}\,\tfrac{-1}{2}}\,\tfrac{-1}{6}\,\tfrac{-1}{6}\,\tfrac{-1}{6})(0^8)'$	-1
$T_4^{(0)}$	$2 \cdot d^c$	$(\underline{\tfrac{1}{2}\,\tfrac{-1}{2}\,\tfrac{-1}{2}}\,\tfrac{-1}{2}\,\tfrac{-1}{2}\,\tfrac{-1}{6}\,\tfrac{-1}{6}\,\tfrac{-1}{6})(0^8)'$	-1
T_3	s_9	$(0^5\,\tfrac{-1}{2}\,\tfrac{-1}{2}\,\tfrac{-1}{2})(0^5\,\tfrac{3}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{2})'$	-10
T_9	s_{13}	$(0^5\,\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2})(0^5\,\tfrac{-3}{4}\,\tfrac{1}{4}\,\tfrac{1}{2})'$	$+10$
T_9	$2 \cdot s_{14} \equiv X$	$(0^5\,\tfrac{1}{2}\,\tfrac{-1}{2}\,\tfrac{-1}{2})(0^5\,\tfrac{-3}{4}\,\tfrac{1}{4}\,\tfrac{1}{2})'$	$+8$
T_9	$2 \cdot s_{15} \equiv \overline{X}$	$(0^5\,\tfrac{-1}{2}\,\tfrac{-1}{2}\,\tfrac{1}{2})(0^5\,\tfrac{1}{4}\,\tfrac{-3}{4}\,\tfrac{1}{2})'$	-10

$$Z_{10} = \sum_{i=1}^{8} \alpha_i Q_i.$$

VEV of s_9 and s_{13} :

$U(1) \to Z_{10}$

$$-\frac{1}{2}(\alpha_2+\alpha_3+\alpha_4)+\frac{3}{4}\alpha_6-\frac{1}{4}\alpha_7-\frac{1}{2}\alpha_8=0.$$

For PQ, if Z10 charges of X and X-bar are required To be 0,

$$\frac{1}{2}\alpha_2-\frac{1}{2}(\alpha_3+\alpha_4)-\frac{3}{4}\alpha_6+\frac{1}{4}\alpha_7+\frac{1}{2}\alpha_8=0,$$
$$-\frac{1}{2}(\alpha_2+\alpha_3+\alpha_4)+\frac{1}{4}\alpha_6-\frac{3}{4}\alpha_7+\frac{1}{2}\alpha_8=0.$$

Conclusion

I discussed the axion with related issues.

1. Solutions of the strong CP problem
2. Axions can be detected by cavity experiments: KAC

3. With SUSY extension, O(GeV) axino can be CDM or decaying axino to CDM [Choi-K-Lee-Seto(08)] can produce the needed number of nonthermal neutralinos. In any case, to understand the strong CP with axions in SUSY framework, the axino must be considered in the discussion.

4. Discrete symmetry $S_2(L) \times S_2(R)$ or some discrete subgroup of gauge symmetry may be very useful in relating the μ solution to the QCD axion scale.

END