A non-perturbative view of the Higgs hierarchy problem

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Nikos Irges
National Technical University of Athens

Based on

N.I. and F. Knechtli

Nucl. Phys. B822 (2009) 1.

Phys. Lett. B685 (2010) 86.

N.I., F. Knechtli and K. Yoneyama

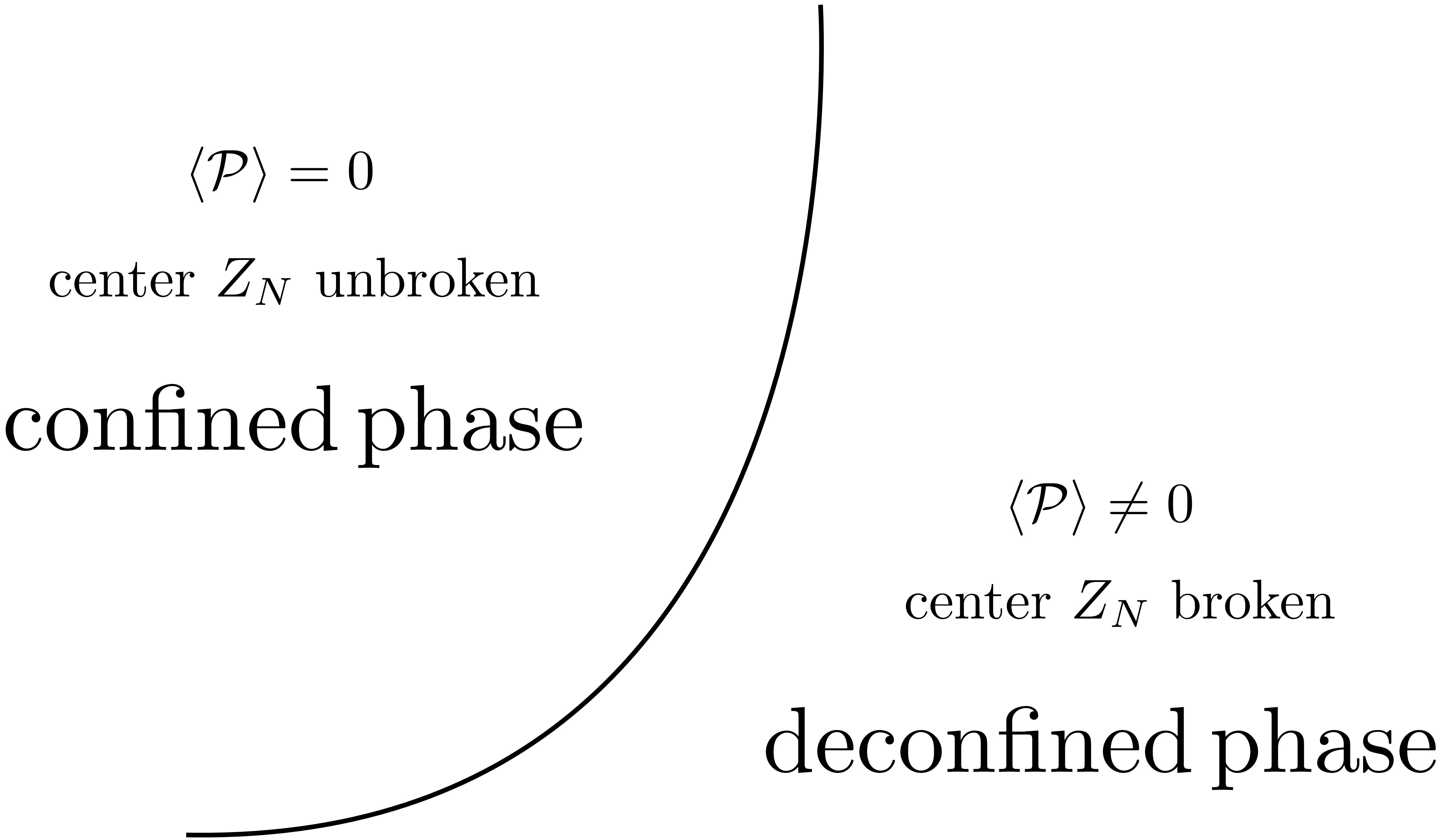
Nucl. Phys. B865 (2012) 541.

Phys. Lett. B722 (2013) 378.

P. Dziennik, N.I., F. Knechtli and K. Yoneyama

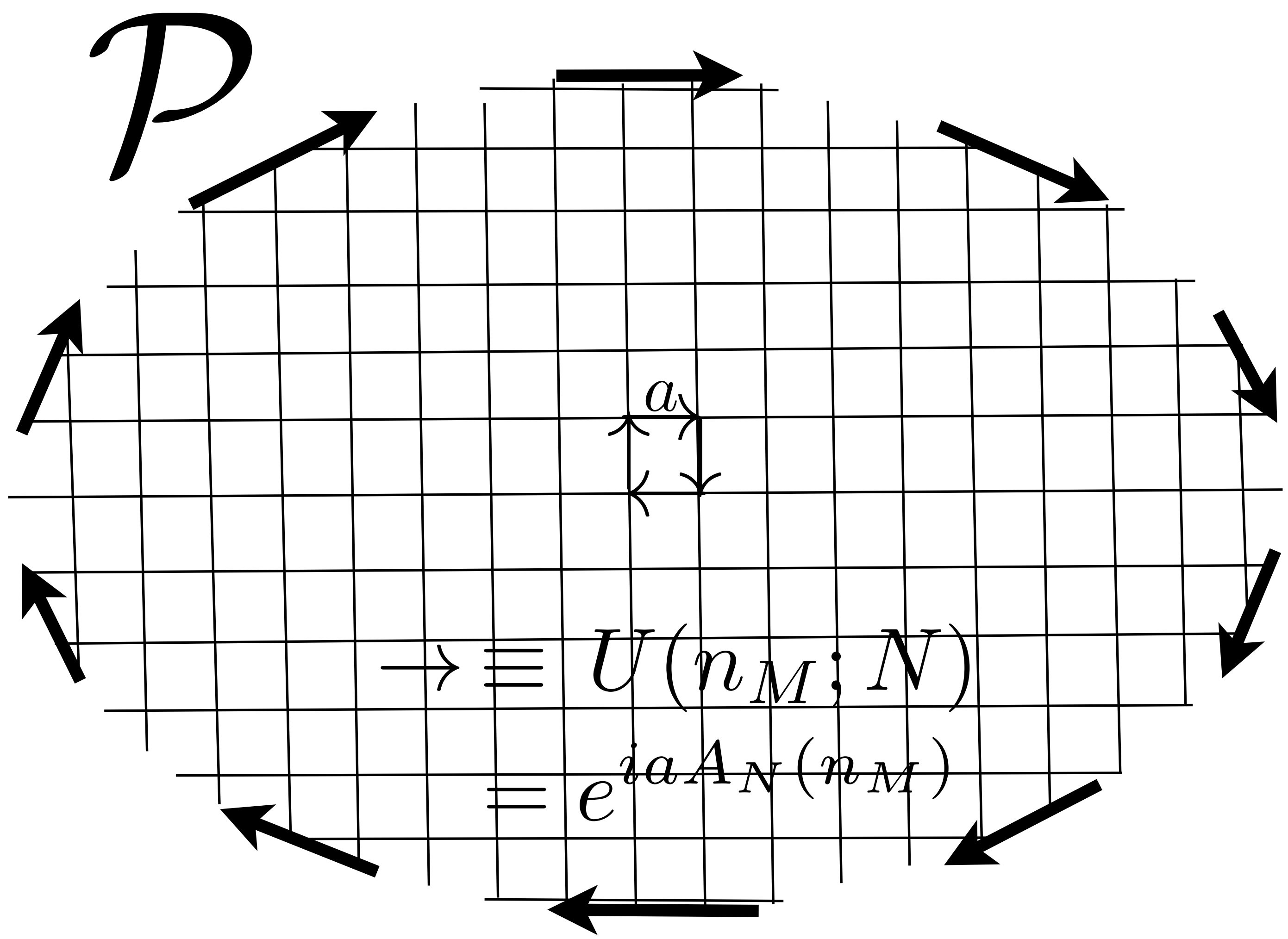
Work in progress

Phases of "usual" SU(N) gauge theories



$N = 2:$ center $= Z_2$

under a center element : $U \to -U$



Action : $S_W[-U] = S_W[U]$

Polyakov Loop : $\mathcal{P}[-U] = -\mathcal{P}[U]$

This is a fully gauge invariant formulation.

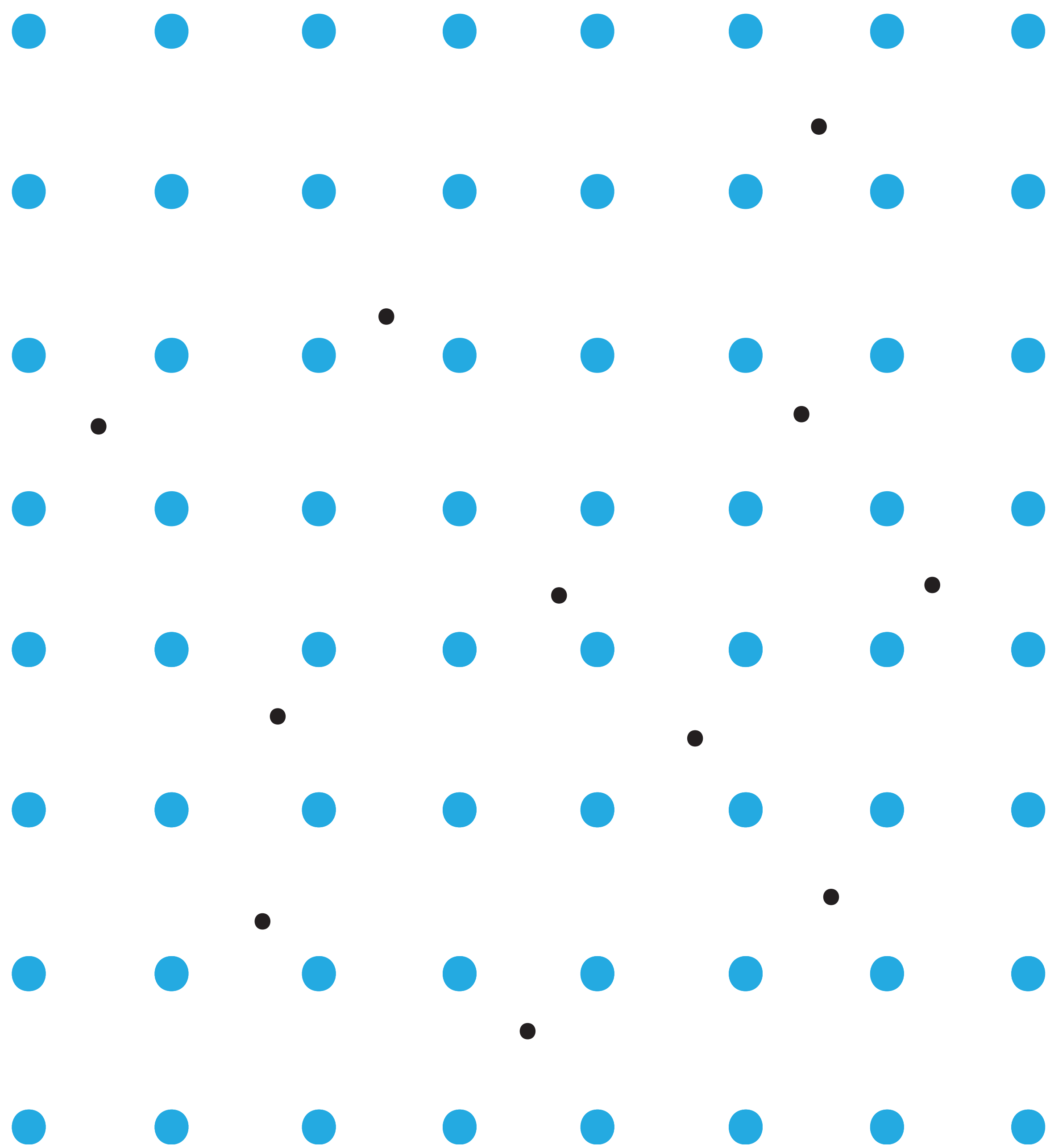
Assume now that somehow we reduce dimensionally the system. The scalar Polyakov Loop (sPL) is a (massive) adjoint "Higgs" scalar in the continuum limit, which takes a vev in the deconfined phase. Does this vev break the gauge symmetry spontaneously? This is a meaningful (i.e. non-empty) question. Recall that in the SM a non-zero Higgs vev implies SSB. But here the non-zero value of the vev is reserved to play the role of the order parameter for the transition from the confined into the deconfined phase. At the same time we know that QCD in the infinitely heavy quark limit, in the deconfined phase, is a gluon plasma (where the gluons are massless). So what is the non-perturbative criterion for SSB? Does the sPL vev contains additional, more refined information about SSB? If it does, we would say that the sPL vev can qualify further the deconfined phase as "Coulomb" or "Higgs".
An example of this is the "non-perturbative Hosotani mechanism" where the sPL vev is recently claimed to contain information about SSB through the fermion matter representation and its boundary conditions (Cossu, D'Elia, 2009). The boundary conditions in this case matter as long as the sPL is made of a finite number of links. Checks of this via Monte Carlo simulations are in progress (see talk by J. Hetrick at the Lattice 2013 conference).

If on the other hand the sPL is blind to SSB, non-perturbatively we need an additional order parameter. This could be the Wilson Loop or a vector PL, in principle neither of them a problem to construct. Now, if we choose (as here) to consider a 5d "Gauge-Higgs Unification" model in order to generate a Higgs field and a dynamical Higgs mechanism, we should be able to also address the Higgs hierarchy problem, given that we live in a world where "the Z is light". It turns out that we are able to do this in the pure gauge theory and only via boundary conditions whose effects remain as the 5th dimension is decompactified.

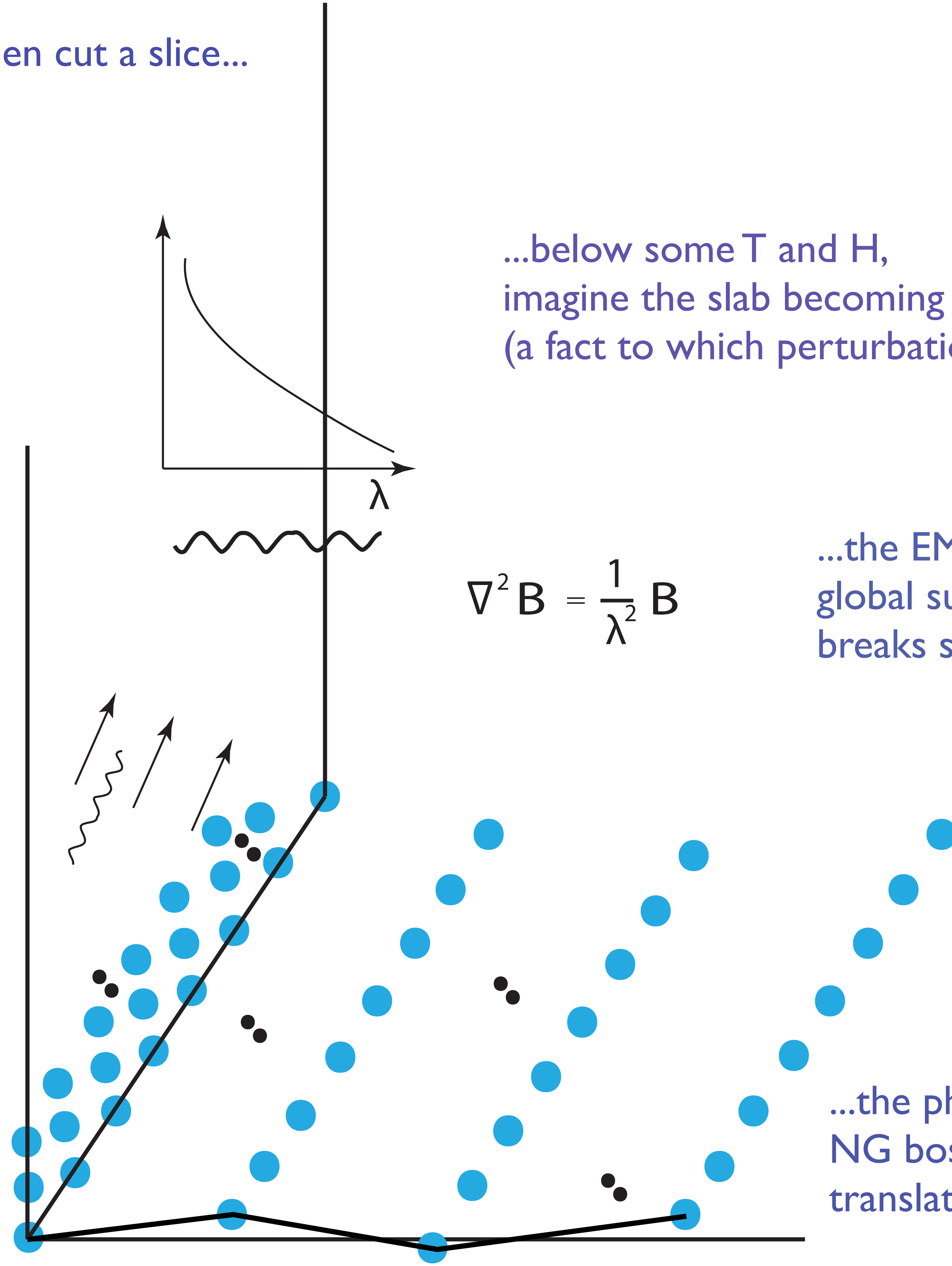
We call these "Non-Perturbative GHU" (NP-GHU) models for short.

So, perturbatively (on the circle or the interval, doesn't matter), the gauge symmetry is not broken by the Coleman-Weinberg-Hosotani mechanism. We know now that this is a consequence of Elitzur's theorem.

Non-perturbatively though a different picture emerges. Consider an infinite, perfectly ordered chrystal:



...and then cut a slice...



...below some T and H,
imagine the slab becoming superconducting...
(a fact to which perturbation theory is blind)

$$\nabla^2 B = \frac{1}{\lambda^2} B$$

...the EM field becomes massive...a global subgroup of the U(1) symmetry breaks spontaneously...

...the phonon can be understood as the NG boson of the spontaneously broken translational invariance...

Lessons:

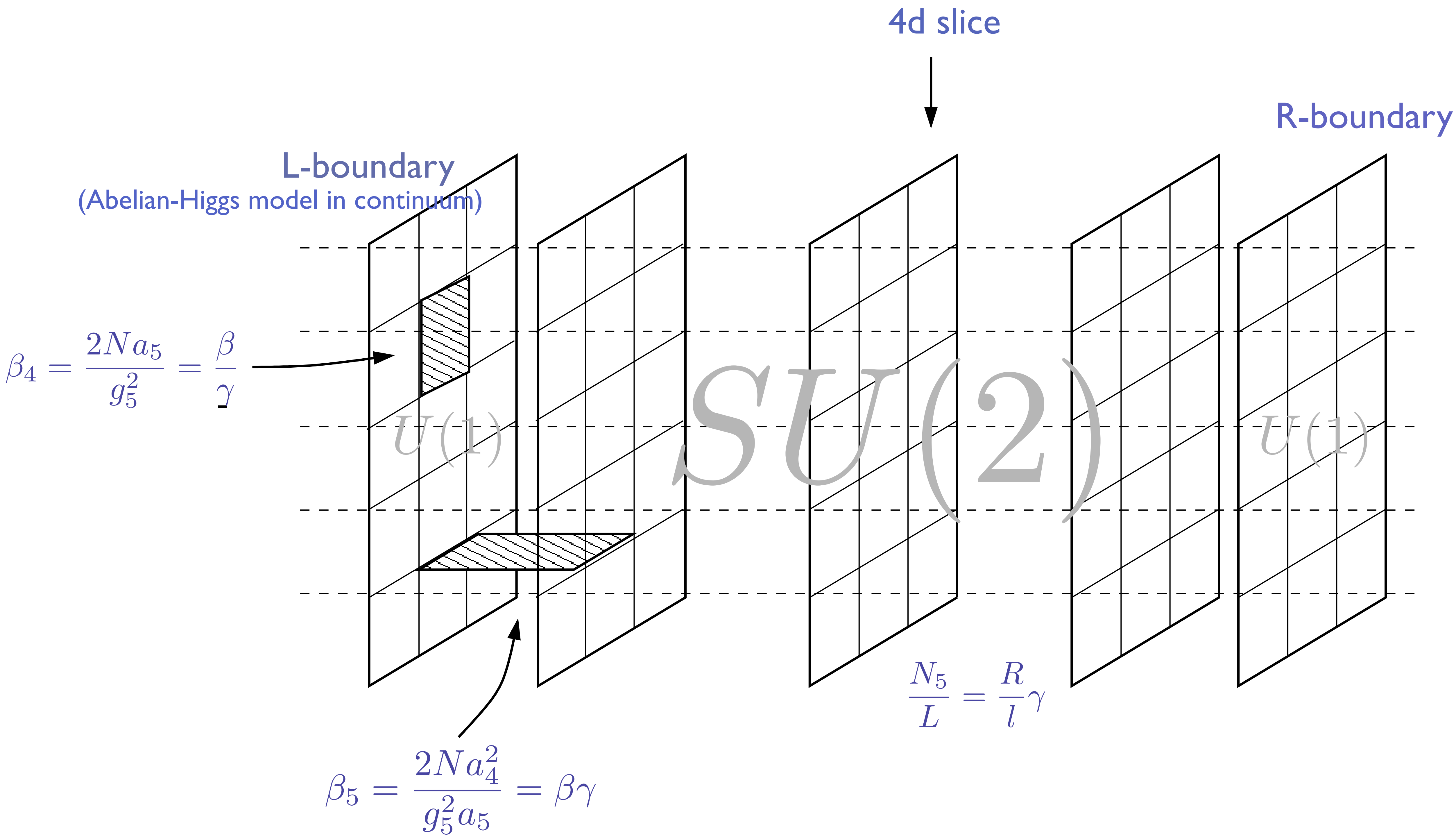
- In order to construct a non-perturbative and microscopic formulation of Spontaneous Symmetry Breaking (SSB), a gauge-invariant lattice helps.

- The necessary lattice deformation generating a phonon can be triggered by breaking translational invariance. We can achieve this with boundaries. In a dimensionally reduced state there will be a Higgs.

- In order to quantify analytically the phonon-Higgs interaction we need a Mean-Field (recall Landau-Ginzburg) expansion. It is not possible to see the results of this interaction in perturbation theory at any order.

- To verify the Mean-Field results Monte Carlo simulations are advised

Prototype NP-GHU models are anisotropic 5d orbifold lattices

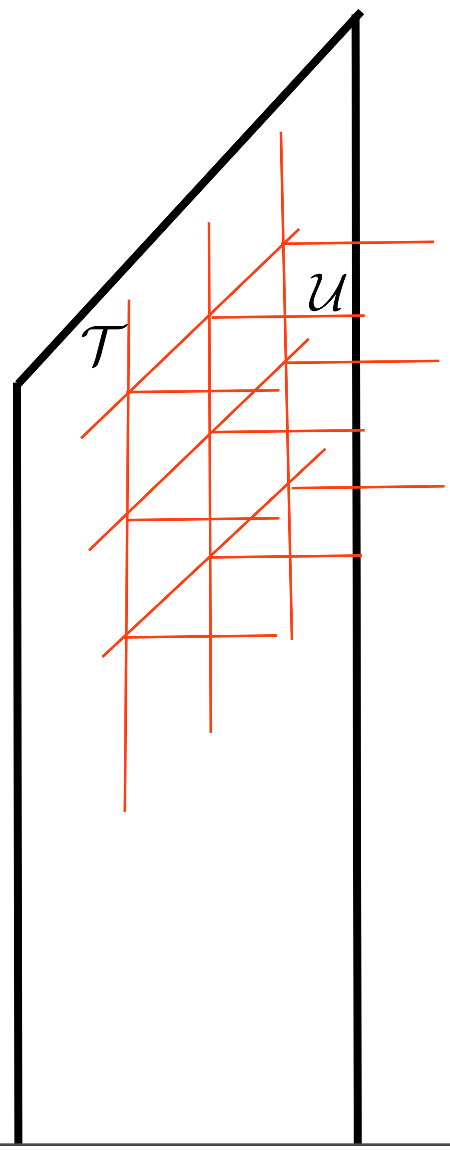


$$\text{Parameters}@L = \infty : \ N_5, \beta(\text{coupling}) \text{ and } \gamma = a_4/a_5(\text{anisotropy parameter})$$

Global symmetries and Elitzur's theorem

$$G \otimes S_L \otimes S_R \otimes U(1)_L \otimes U(1)_R \otimes F \otimes C$$

local lattice gauge invariance



Global symmetries

S_L : left "stick" symmetry

$$\mathcal{T}_\nu(n_\mu) \longrightarrow g_s^{-1}\, \mathcal{T}_\nu(n_\mu)\, g_s$$
$$\mathcal{U}(0) \longrightarrow g_s^{-1}\, \mathcal{U}(0)$$

S_R : right "stick" symmetry

Ishiyama, Murata, So and Takenaga (2010)

$$\mathcal{V}_\nu(n_\mu) \longrightarrow g_s^\dagger\, \mathcal{V}_\nu(n_\mu)\, g_s$$
$$\mathcal{U}(N_5) \longrightarrow \mathcal{U}(N_5)\, g_s$$

$U(1)_L$: left boundary global U(1)

$U(1)_R$: right boundary global U(1)

F : flip around the middle

C : global sign flip

the topologically non-trivial objects: $\langle \mathcal{P} \rangle : \not{S}_L, \ \not{S}_R, \ U(1)_L, \ U(1)_R, \ F \ , C$

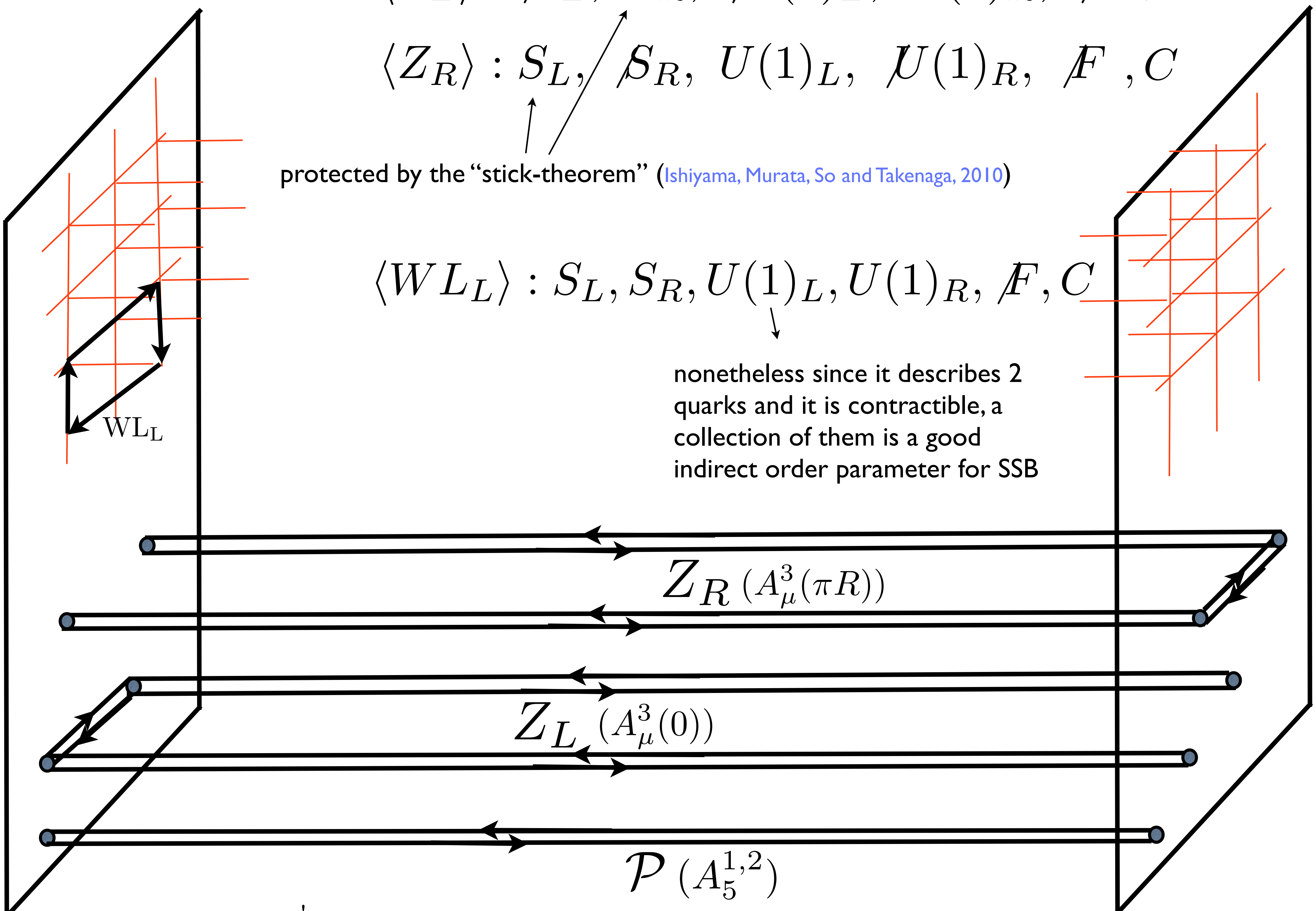
$$\langle Z_L \rangle : \not{S}_L, \ S_R, \ \not{U}(1)_L, \ U(1)_R, \ \not{F} \ , C$$

$$\langle Z_R \rangle : S_L, \ \not{S}_R, \ U(1)_L, \ \not{U}(1)_R, \ \not{F} \ , C$$

protected by the "stick-theorem" (Ishiyama, Murata, So and Takenaga, 2010)

$$\langle WL_L \rangle : S_L, S_R, U(1)_L, U(1)_R, \not{F}, C$$

nonetheless since it describes 2 quarks and it is contractible, a collection of them is a good indirect order parameter for SSB



$$\bullet = g \text{ or } g^\dagger \qquad g = \text{"orbifold projection"} \ \in SU(N)$$

vev "along the algebra"

Elitzur: $A_5 \longrightarrow A_5 + v$ is not allowed when $S_{R(L)}$ is broken !

$\downarrow$

SSB must be realized in a non-local, non-perturbative way (like in superconductors)

$\langle \mathcal{P} \rangle, \langle \{\mathrm{WL_{L,R}}\} \rangle :$ order parameter for confined $\longrightarrow$ deconfined

$\langle Z_L \rangle, \langle \{\mathrm{WL_L}\} \rangle :$ order parameter for $U(1)_L$ SSB

$\langle Z_R \rangle, \langle \{\mathrm{WL_R}\} \rangle :$ order parameter for $U(1)_R$ SSB

The Mean-Field expansion

$$S_G[U] = \frac{1}{2N} \sum_{i=4,5} \sum_p \beta_i w_{p_i} \mathrm{tr}\{1 - U(p_i)\}$$

$$Z = \int DU \int DV \int DH e^{(1/N)\mathrm{Re}[\mathrm{tr}H(U-V)]} e^{-S_G[V]}$$

the link MF effective action

$$Z = \int DV \int DH e^{-Seff[V,H]}, \quad Seff = S_G[V] + u(H) + (1/N)\mathrm{Retr}HV$$

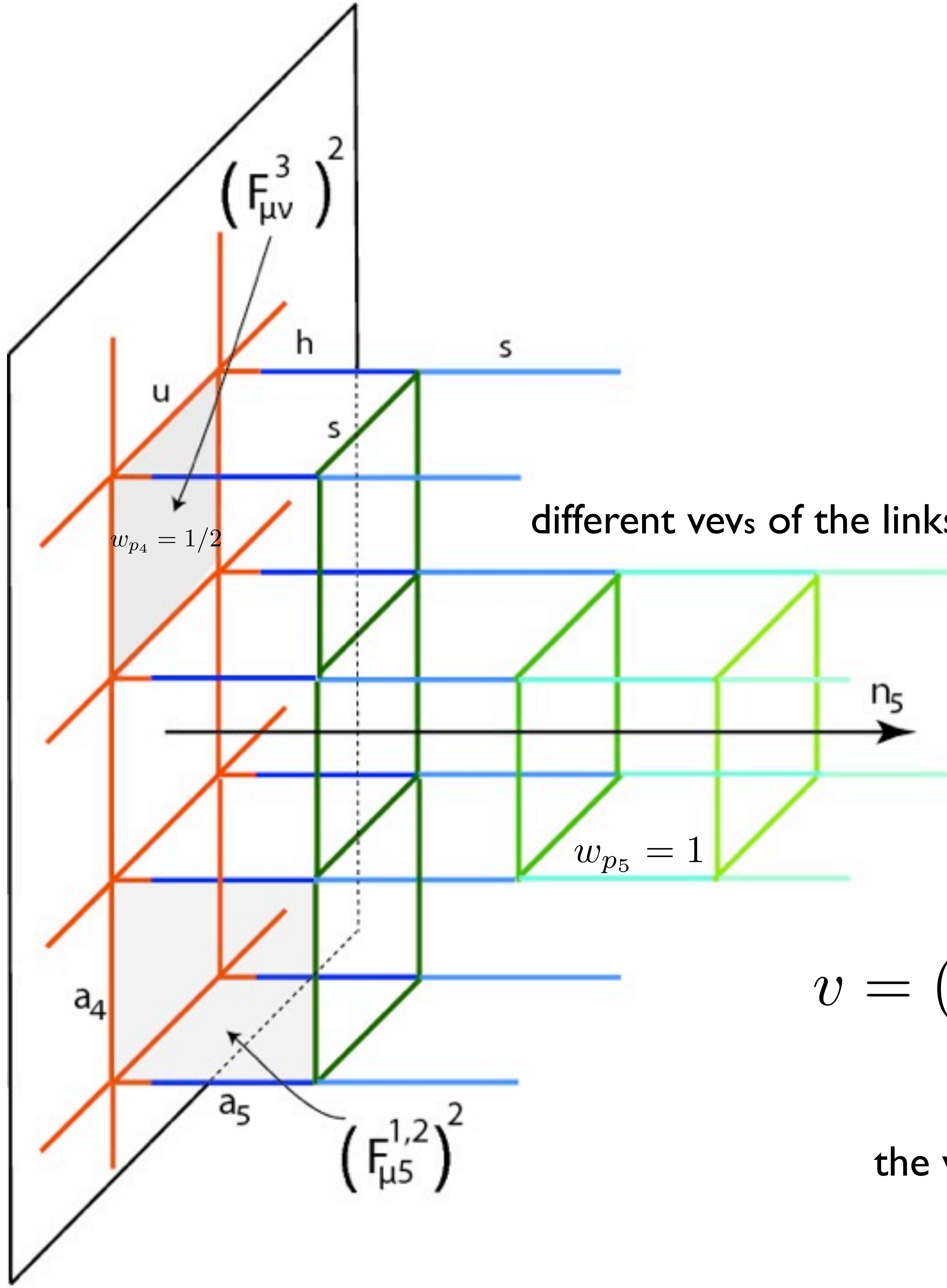
$$e^{-u(H)} = \int DU e^{(1/N)\mathrm{Retr}UH}$$

defines the phase diagram and introduces the phonon

The background is determined by the MF saddle point:

$$\overline{V} = -\frac{\partial u}{\partial H}\Big|_{\overline{H}} \qquad \overline{H} = -\frac{\partial S_G[V]}{\partial V}\Big|_{\overline{V}}$$

These equations are solved iteratively, to yield $\overline{v}_0(n_5)$ and $\overline{v}_{50}(n_5)$

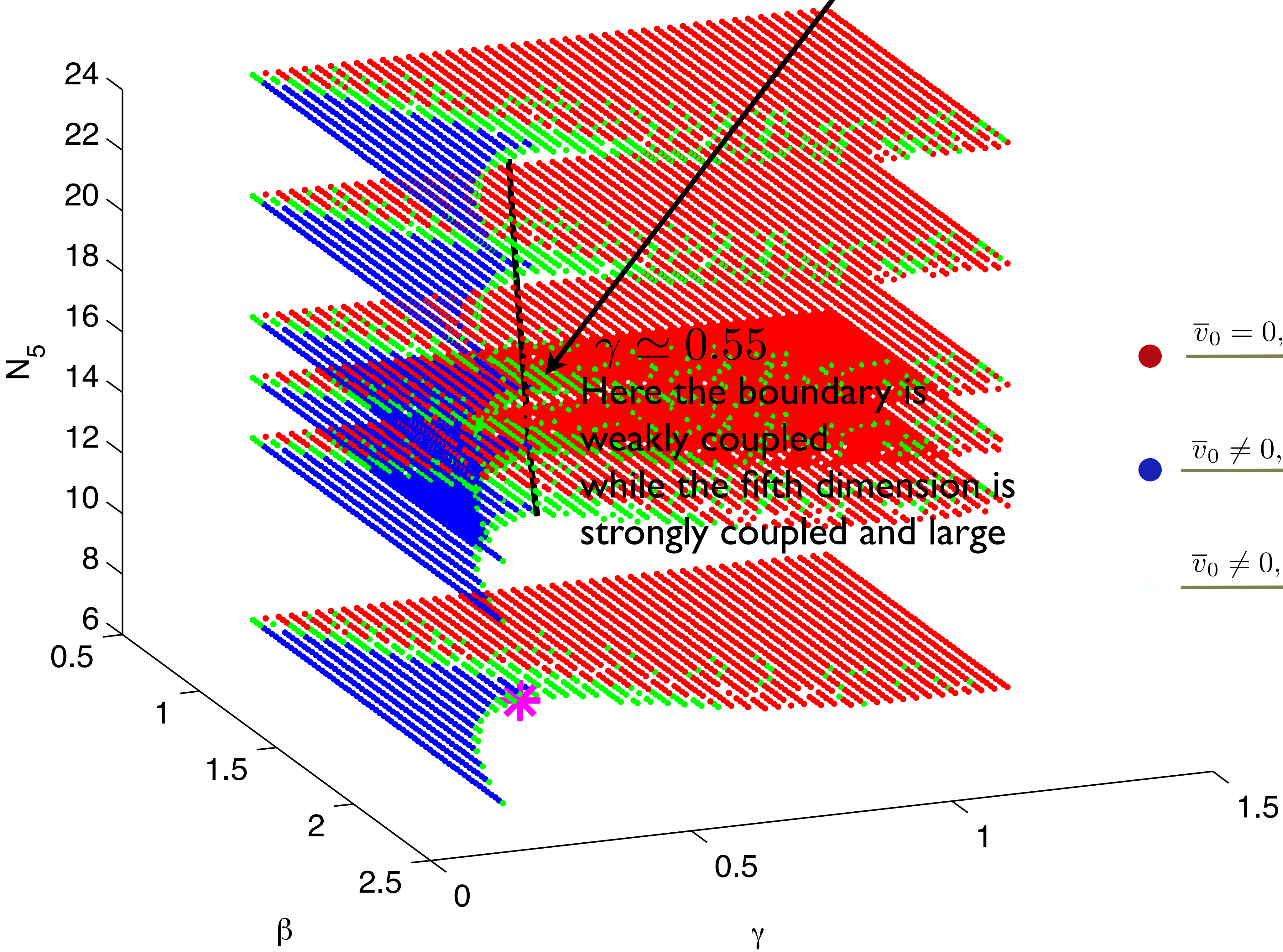


different vevs of the links denoted by different colors

$$v = (\overline{v}_0 + v_0(x))\mathbf{1} + i v^A(x)\sigma^A$$

the vev is not along the algebra!

The mean-field phase diagram and Lines of Constant Physics (LCP)



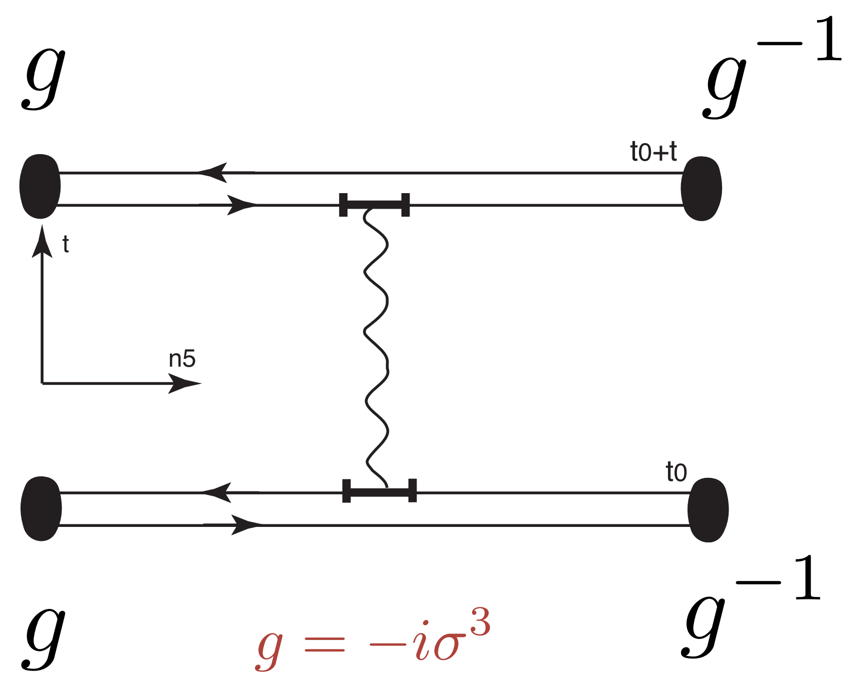
$\overline{v}_0 = 0, \overline{v}_{50} = 0$ → confined phase

$\overline{v}_0 \neq 0, \overline{v}_{50} = 0$ → layered or “Fu & Nielsen” phase

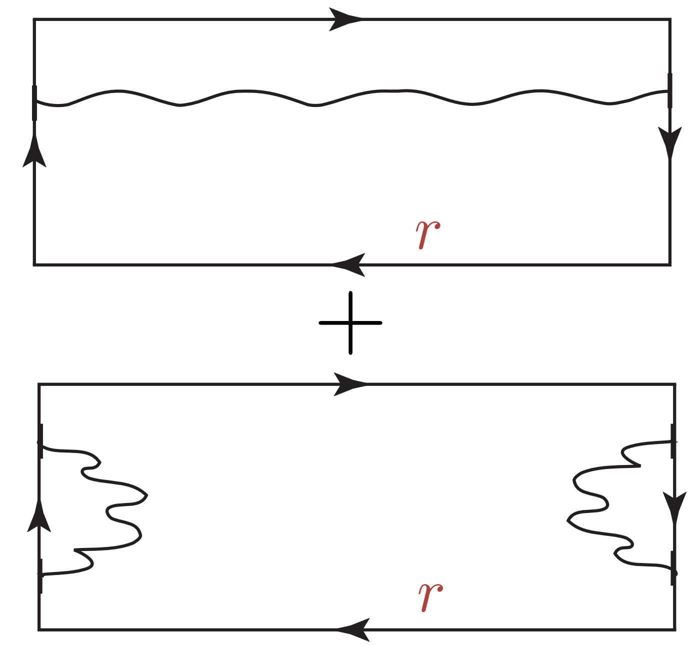
$\overline{v}_0 \neq 0, \overline{v}_{50} \neq 0$ → deconfined phase

Observables in the MF Expansion

the scalar PL or "Higgs":
(order parameter of the deconfined phase; also used to extract the Higgs mass)



the L-boundary WL:
in the MF, we take it as the order parameter for SSB; also used to extract the Z and Z' masses)



$$\mathrm{t} \to \infty : \qquad \mathrm{e}^{-Vt} \simeq < \mathcal{O}_W >$$

$$C(t) =< \mathcal{O}(t_0+t)\mathcal{O}(t_0) > - < \mathcal{O}(t_0+t) >< \mathcal{O}(t_0) >= C^{(0)}(t) + C^{(1)}(t) + \cdots$$

$$C(t) = \sum_\lambda c_\lambda e^{-E_\lambda t}$$

$$E_0 = m_H, \quad E_1 = m_H^*, \cdots$$

$$m \approx \lim_{t\to\infty} \ln \frac{C^{(1)}(t)}{C^{(1)}(t-1)}$$

Dimensional Reduction

(Our) Definition of dimensional reduction to 4d:

- The fit to $V(r) = const. + b\frac{e^{-m_Z r}}{r}$ is possible with $m_Z \neq 0$. All other fits must be excluded. The potential is extracted from boundary WL's.

- The quantities $M_H = a_4 m_H$ and $M_Z = a_4 m_Z < 1$ so that the observables are not dominated by the cut-off.

- $m_H R < 1$ and $\rho_{HZ} = m_H/m_Z > 1$ .The Higgs and the Z are lighter than $1/R$ and the Higgs is heavier than the Z. We will target the value

$$\rho_{HZ} = 1.38$$

LCP

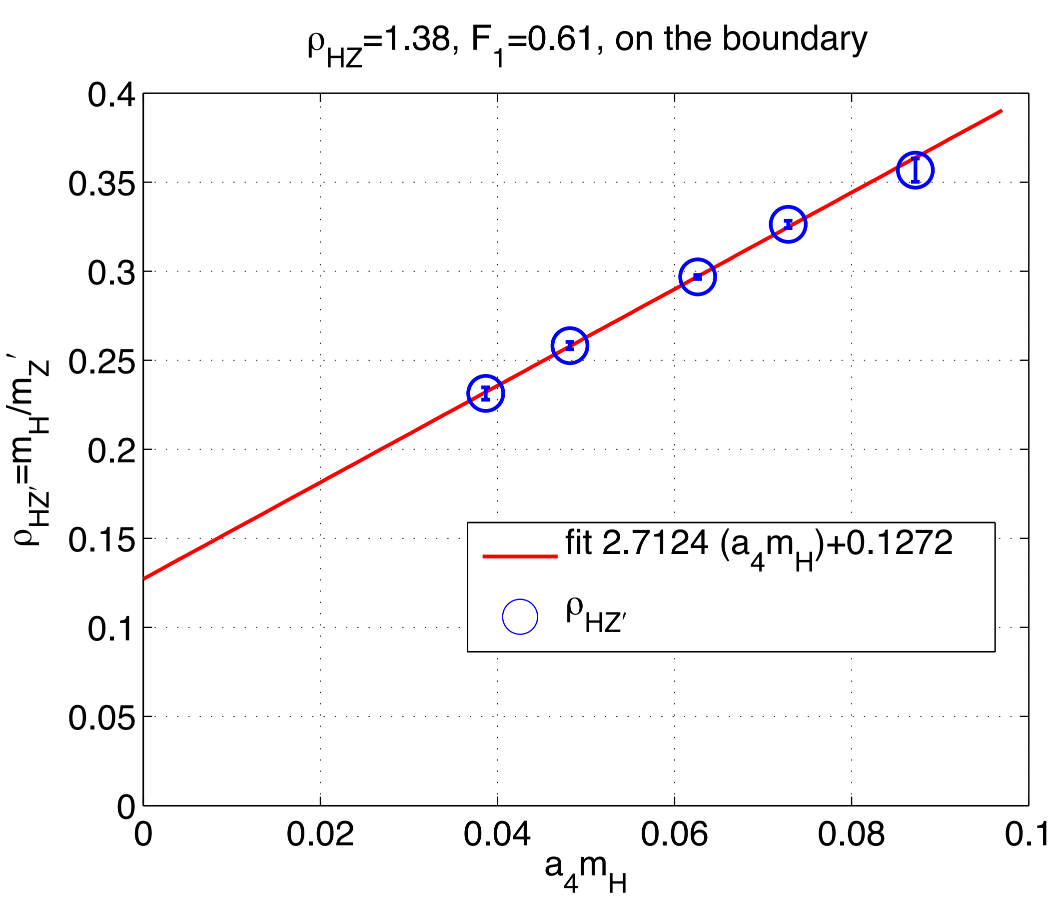
We have 3 observables: $M_H(\beta,\gamma,N_5)$, $M_Z(\beta,\gamma,N_5)$ and $M_{Z'}(\beta,\gamma,N_5)$.

We take $L \longrightarrow \infty$.

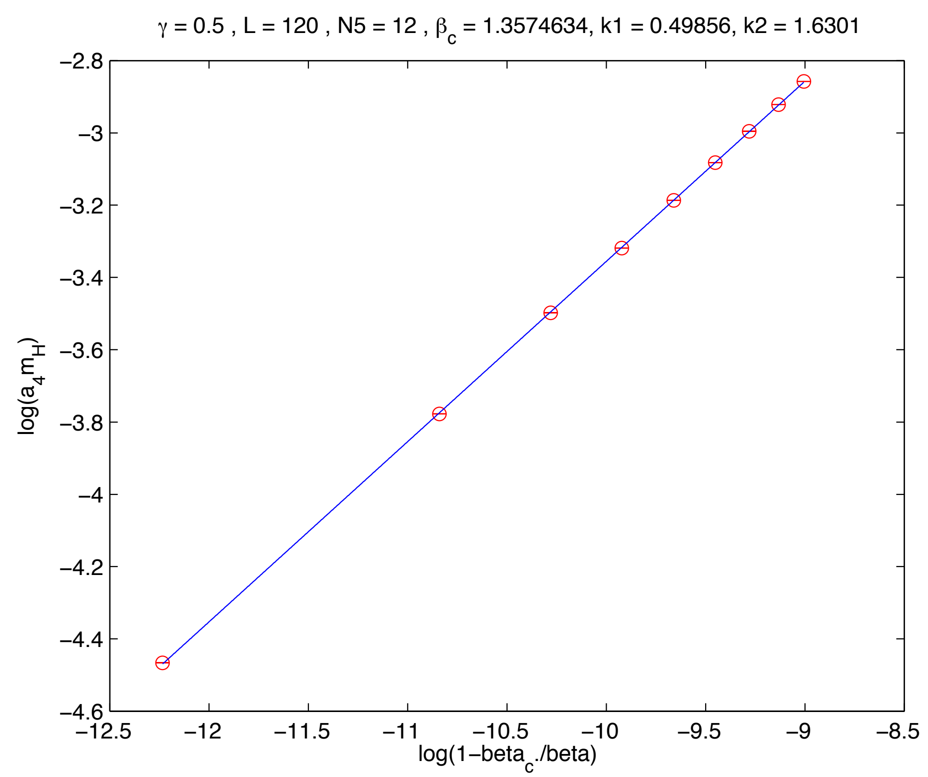
Tuning β and γ we fix $m_H R = 0.61$ and $\rho_{HZ} = 1.38$.

This process defines a line on the phase diagram: $\rho_{HZ'}(N_5)$

The phase transitions seen by the MF are bulk phase transitions. It predicts that the phase transition for $\gamma \lesssim 0.70$ becomes second order. Then, $N_5 \longrightarrow \infty$ is a continuum limit in a finite physical box. Otherwise it is an effective theory with a cut-off which may be a useful theory nevertheless.



the critical exponent:



Observations (after long discussions with Holger)

- Moving by ϵ off the LCP at constant couplings, changes the Higgs mass by $O(\epsilon)$: "the Higgs is insensitive to the cut-off".

- In the case of a first order phase transition, the lattice spacing assumes a minimum value: "there is a physical cut-off". This cut-off is however at most $\approx O(100 \cdot m_H)$. The model is not sensitive to an exponentially large scale like M_{Pl} for example.

- A danger may be that in the case of a second order phase transition the continuum limit may be scale invariant, i.e. the theory could become massless and therefore trivial. However, the phase on the other side of the PT is the Fu & Nielsen phase, which is by definition massive. Thus, masslessness would imply a discontinuous jump which is in contradiction with the PT being second order. Moreover, the lattice generates renormalized masses. It is the lattice spacing that changes as one moves around on the phase diagram.

Summary of the Mean-Field picture:

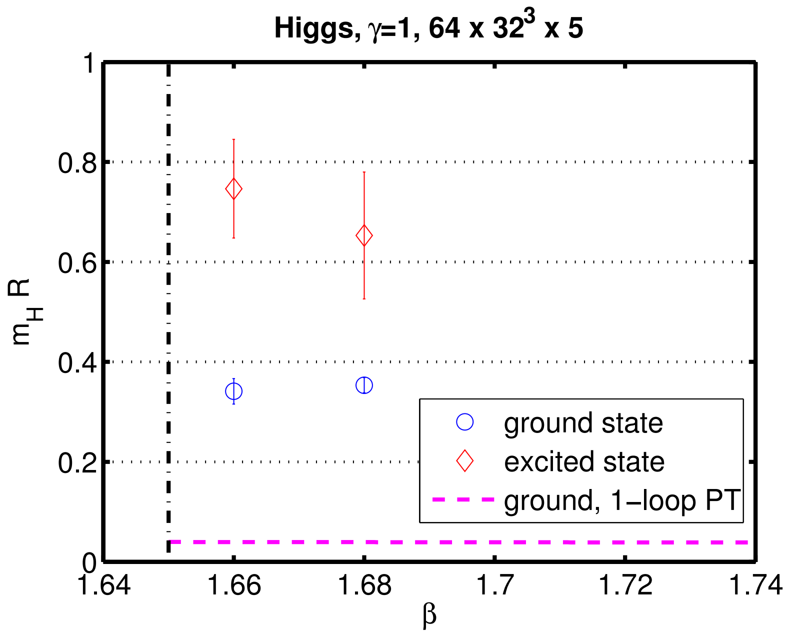
The boundary effective theory of the orbifolded 5d pure SU(2) gauge theory is a 4d Abelian-Higgs model with the U(1) spontaneously broken, with excited states present though in the relatively low energy spectrum (in the TeV regime). SSB is realized due to the broken translational invariance: non-perturbatively the lattice orbifold looks like a relativistic, bosonic superconductor.

That the hierarchy may be protected can be seen by the fact that the system, in a specific regime of its phase diagram, reduces dimensionally and allows the construction of LCP's. If one thinks for a moment:
dimensional reduction + LCP+non-zero masses = a stable Higgs

But is the MF saddle point the dominant one in the path integral? The only way we know how to check this is to look at the full non-perturbative system via Monte Carlo simulations.

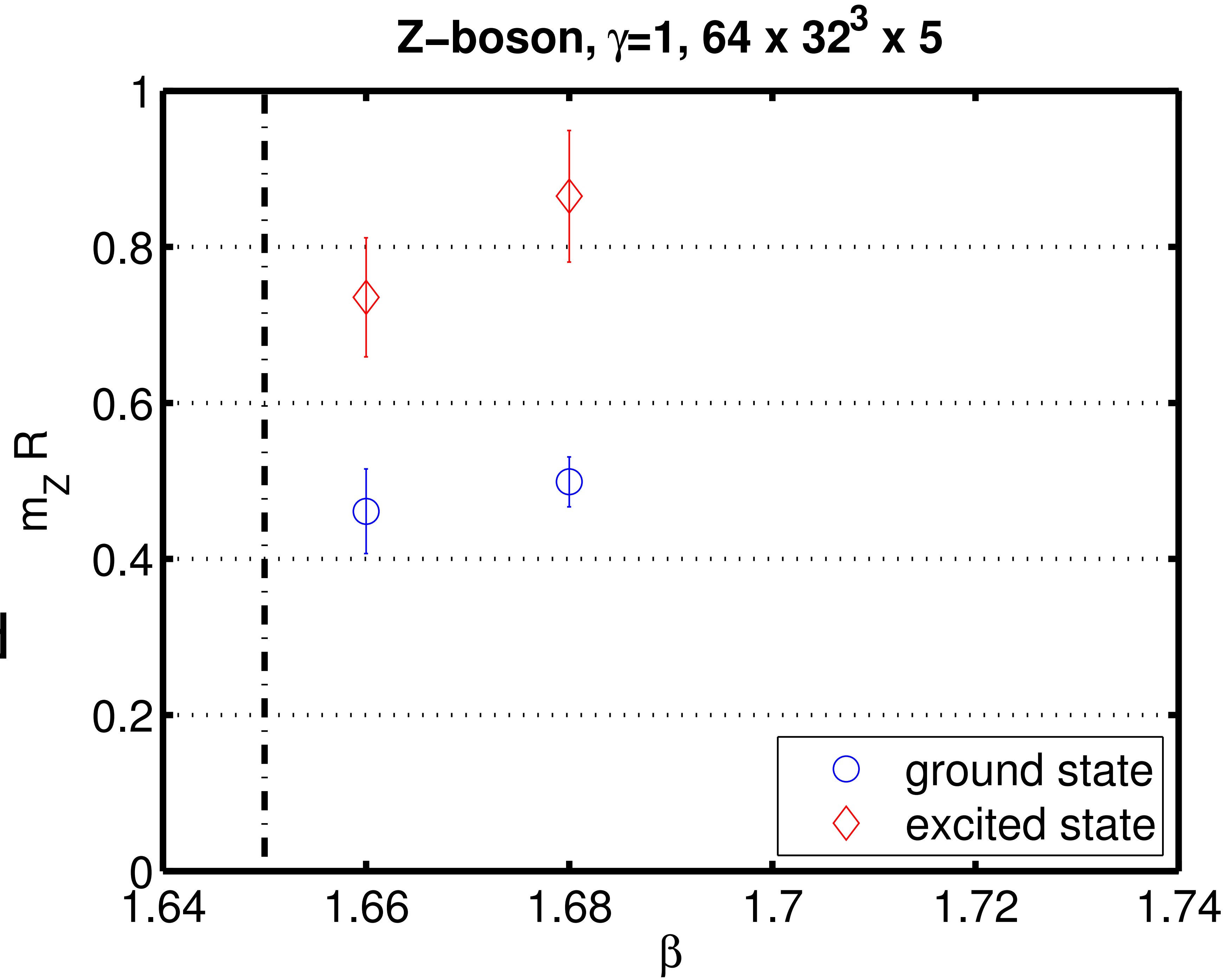
Monte Carlo - isotropic lattice

P. Dziennik, N.I., F. Knechtli and K. Yoneyama



1. SSB is confirmed

2. $m_H/m_Z \lesssim 1$
   as in the MF



the anisotropic regime (where for $\gamma < 1$ the MF suggests that $m_H/m_Z > 1$) is work in progress...

Conclusions and Outlook

- According to the Mean-Field expansion, if SSB in Gauge-Higgs Unification is an exclusively non-perturbative phenomenon, it is due to pure gauge dynamics and is related to the fact that the lattice orbifold is a five-dimensional superconductor. Along LCP's the system looks effectively like a simplified version of the SM Higgs sector. We called this construction "Non-Perturbative GHU".

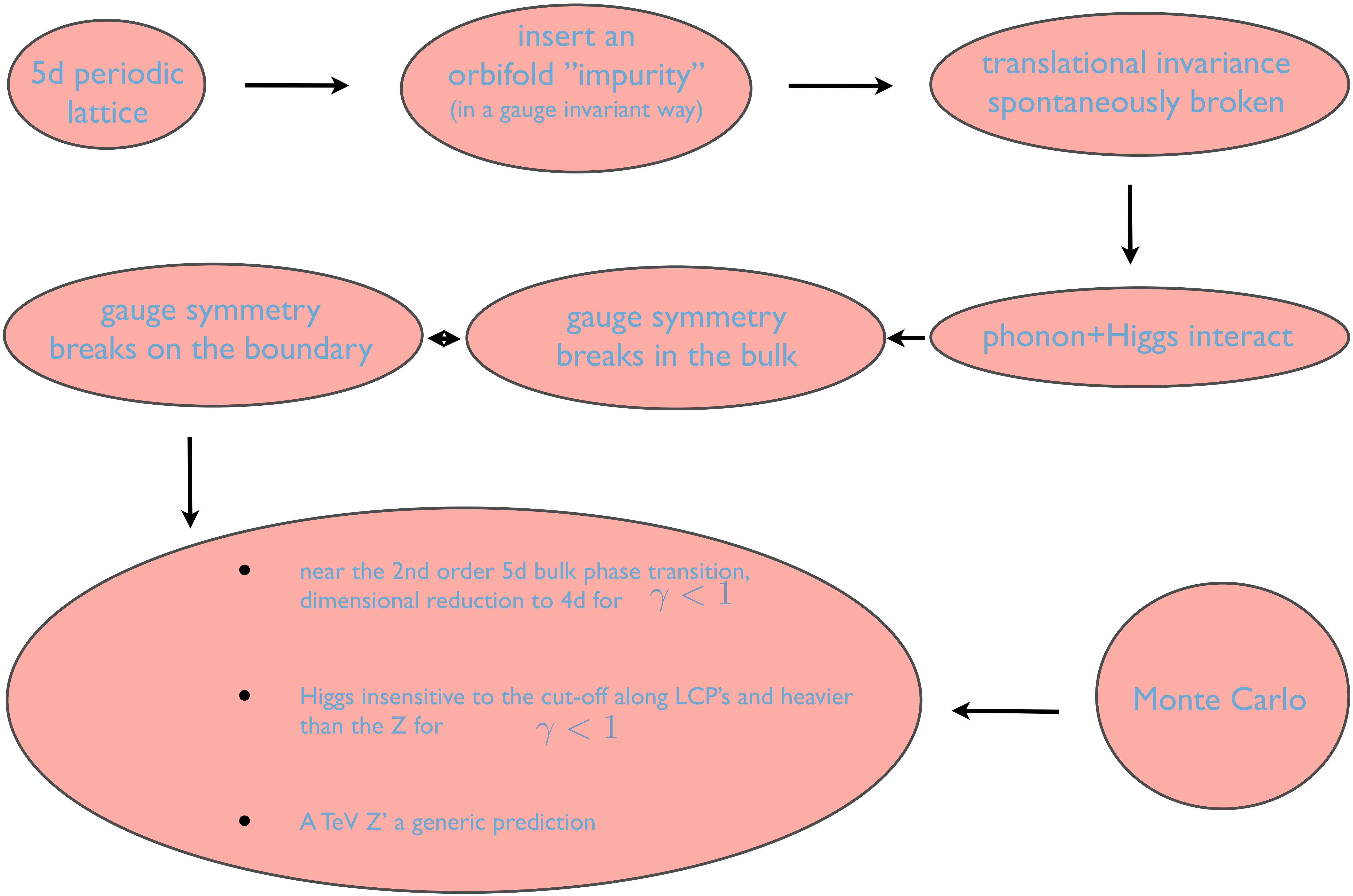
- Monte Carlo simulations verify the spontaneous symmetry breaking; regarding the dependence of the Higgs mass on the cut-off, work is in progress.

- Even though for now we are far from being phenomenologically competitive, later we plan to use larger groups and add fermions to construct more realistic models. But before that it would be really nice to know what is the "Landau functional" H of this superconductor, from which $-\mu H^2 + \lambda H^4$ should emerge...

- And perhaps the most important question (expressed in a cond-mat language): why the mass of the Cooper pair in a superconductor does not receive quantum corrections proportional to some huge scale? And if we understand this, can the argument be applied to the bosonic superconductor?

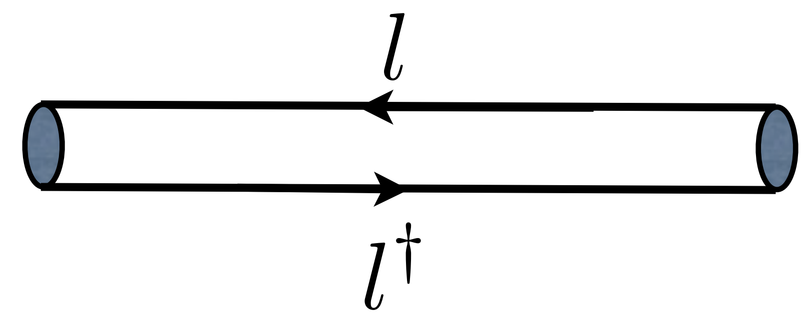
Extra slides

In a picture:

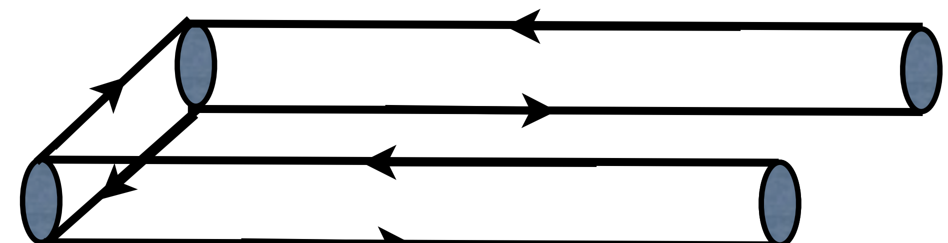


5d periodic lattice
insert an orbifold "impurity" (in a gauge invariant way)
translational invariance spontaneously broken
phonon+Higgs interact
gauge symmetry breaks in the bulk
gauge symmetry breaks on the boundary
near the 2nd order 5d bulk phase transition, dimensional reduction to 4d for $\gamma < 1$
Higgs insensitive to the cut-off along LCP's and heavier than the Z for $\gamma < 1$
A TeV Z' a generic prediction
Monte Carlo

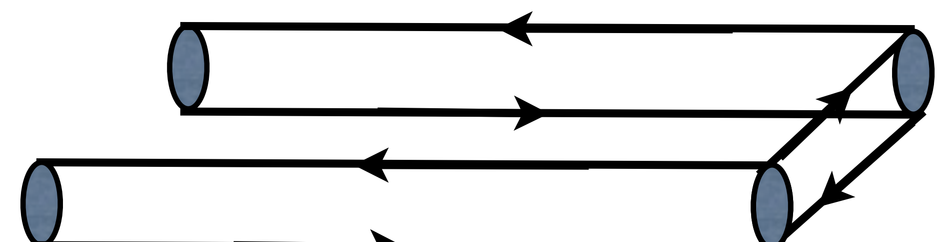
$\mathcal{P} \; =$



$Z_L \; =$



$Z_R \; =$



Fluctuations to first non-trivial order

$$H = \bar{H} + h \qquad V = \bar{V} + v$$

$$Seff = Seff[\bar{V}, \bar{H}] + \frac{1}{2}\left(\left.\frac{\delta^2 Seff}{\delta H^2}\right|_{\overline{V},\overline{H}} h^2 + 2\left.\frac{\delta^2 Seff}{\delta H \delta V}\right|_{\overline{V},\overline{H}} hv + \left.\frac{\delta^2 Seff}{\delta V^2}\right|_{\overline{V},\overline{H}} v^2 \right)$$

$$\left.\frac{\delta^2 Seff}{\delta H^2}\right|_{\overline{V},\overline{H}} h^2 = h_i K_{ij}^{(hh)} h_j = h^T K^{(hh)} h$$

$$\left.\frac{\delta^2 Seff}{\delta V^2}\right|_{\overline{V},\overline{H}} v^2 = v_i K_{ij}^{(vv)} v_j = v^T K^{(vv)} v$$

$$\left.\frac{\delta^2 Seff}{\delta V \delta H}\right|_{\overline{V},\overline{H}} v^2 = v_i K_{ij}^{(vh)} h_j = v^T K^{(vh)} h$$

$$S^{(2)}[v, h] = \frac{1}{2}\left(h^T K^{(hh)} h + 2 v^T K^{(vh)} h + v^T K^{(vv)} v \right)$$

Observables

$$\mathcal{O}[V] = \mathcal{O}[\overline{V}] + \frac{\delta \mathcal{O}}{\delta V}\bigg|_{\overline{V}} v + \frac{1}{2}\frac{\delta^2 \mathcal{O}}{\delta V^2}\bigg|_{\overline{V}} v^2 + \dots$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z}\int Dv \int Dh \left(\mathcal{O}[\overline{V}] + \frac{1}{2}\frac{\delta^2 \mathcal{O}}{\delta V^2}\bigg|_{\overline{V}} v^2 \right) e^{-\left(Seff[\overline{V},\overline{H}] + S^{(2)}[v,h]\right)}$$

$$= \mathcal{O}[\overline{V}] + \frac{1}{2}\frac{\delta^2 \mathcal{O}}{\delta V^2}\bigg|_{\overline{V}} \frac{1}{z}\int Dv \int Dh v^2 e^{-S^{(2)}[v,h]}$$

since $< v_i v_j >= \frac{1}{z}\int Dv \int Dh\, v_i v_j e^{-S^{(2)}[v,h]} = (K^{-1})_{ij}$

we define the propagator $K = -K^{(vh)} K^{(hh)^{-1}} K^{(vh)} + K^{(vv)}$

contains the phonon-Higgs interaction we are after ...and much more...

and finally: $$\boxed{< \mathcal{O} >= \mathcal{O}[\overline{V}] + \frac{1}{2}\mathrm{tr}\left\{ \frac{\delta^2 \mathcal{O}}{\delta V^2}\bigg|_{\overline{V}} K^{-1} \right\}}$$

First order master formula