Phenomenological studies in the matrix models

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Corfu Summer Institute 2013
9/12/2013

§1 Introduction

- **Standard Model of Particle physics**
  - Successful. Agrees well with experiments.
  - Unsatisfactory as a final theory.
    - ∗ No quantum gravity
    - ∗ Too arbitrary

⇑ **String phenomenology**

  - Calabi-Yau, Orbifolds, Intersecting D-branes, etc.

- **String theory**
  - Too many vacua. Landscape.
  - We can not compare them dynamically.
  - **Need an underlying, master theory**

Candidates for nonperturbative formulations of String theory

- String field theories
- Gauge/gravity (AdS/CFT) dualities
- **Matrix Models**

  T. Banks, W. Fischler, S.H. Shenker, L. Susskind, 1997

  N. Ishibashi, H. Kawai, Y. Kitazawa, A. Tsuchiya, 1997

  R. Dijkgraaf, E. P. Verlinde and H. L. Verlinde, 1997

IIB (IKKT) matrix model

H. Kawai's talk

- Action

$$S_{\mathrm{IIBMM}} = -\frac{1}{g_{\mathrm{IIBMM}}^2}\mathrm{Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right) \qquad (1)$$

- $$\int dA\, d\psi\, e^{-S_{\mathrm{IIBMM}}}$$

  $N \to \infty$

  Nonperturbative formulation of String theory

Spacetime structures from IIBMM

J. Nishimura's talk

HA, S. Iso, H. Kawai, Y. Kitazawa, T. Tada, 1998;

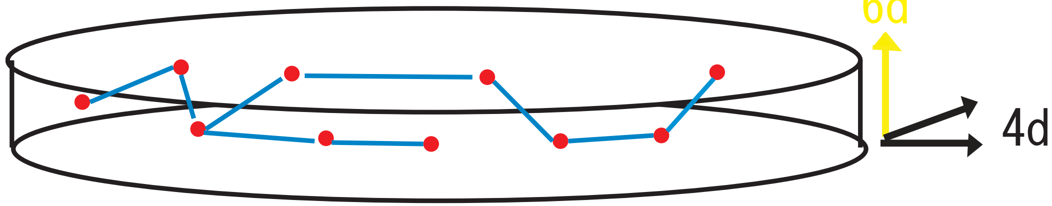
J. Nishimura, J. Ambjorn, K. Anagnostopoulos, T. Azuma, W.Bietenholtz, F. Hofheinz, T. Hotta, T. Okubo, F. Sugino, A. Tsuchiya, G. Vernizzi;

H. Kawai, S. Kawamoto, T. Kuroki, T. Matsuo, S. Shinohara

Lorentzian IIBMM

S. Kim, J. Nishimura, A. Tsuchiya, 2012

- Eigenvalues distribution of $A_\mu \longleftrightarrow$ Spacetime structure



$\Rightarrow$ 4-dimensional spacetime?

Then, next, let's consider matter on the spacetime

SM $\longleftarrow$ MM

Importance of these studies:

1. Such a path may give us a guide for coming close from either side.
   - Study phenomenology beyond the SM
   - Justify/Modify the formulation of MM

2. We can, in principle, analyze dynamics and calculate everything, since MM has **definite action and measure**.
   An advantage that MM has over the ordinary string theories

Mechanism to obtain chiral fermions

- An important ingredient of SM is the chiral fermions.

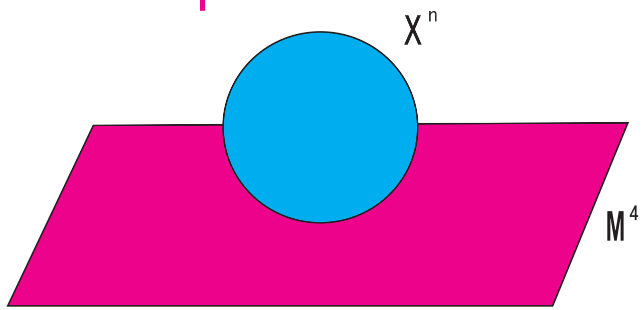
- Nontrivial topology in the extra dimensions

  $\Downarrow$ Index Theorem

  Chiral zero modes in the extra dimensions

  $\Downarrow$ $D_{10} = D_4 + D_6$

  Chiral massless fields on our spacetime



- This is the standard way.
  - Euler characteristics in CY
  - Boundary conditions in orbiforlds
  - Intersection numbers in intersecting D-branes

Chiral fermions and SM from MM

- Orbifolds

  HA, S. Iso, T. Suyama, 2002

  A. Chatzistavrakidis, H. Steinacker, G. Zoupanos, 2010

- Intersecting D-branes — A. Tsuchiya's talk

  A. Chatzistavrakidis, H. Steinacker, G. Zoupanos, 2011

  J. Nishimura, A. Tsuchiya, 2013

  HA, J. Nishimura, A. Tsuchiya, 2013

$\Updownarrow$ via T-duality

- Toroidal compactifications with magnetic fluxes $\sim$ magnetized D-branes

  HA, 2011, 2013, 2013

Outline

$\surd$ §1 Introduction

§2 Formulation of NC tori

§3 Matrix configurations for phenomenological models

§4 Probability distribution over the phenomenological models
(Semiclassical analyses of MM)

§5 Conclusions and Discussions

§2 Formulation of NC tori

Spacetime and matter in matrices

- **Hermitian-matrix formulation**

  HA, N. Ishibashi, S. Iso, H. Kawai, Y. Kitazawa, T. Tada, 2000

  NC background

  $$[p_\mu, p_\nu] = i\theta_{\mu\nu}$$

  Decompose

  $$A_\mu = p_\mu + a_\mu$$

- MM gives NC field theory.

$$\begin{aligned} &\mathrm{tr}\ \left([A_\mu, A_\nu]^2 + \bar{\psi}\Gamma_\mu[A_\mu, \psi]\right) \\ = &\int d^d x \Big((F_{\alpha\beta})^2 + (D_\alpha\phi_i)^2 + [\phi_i, \phi_j]^2 \\ &\quad + \bar{\psi}\Gamma_\alpha D_\alpha\psi + \bar{\psi}\Gamma_i[\phi_i, \psi]\Big)_\star \end{aligned}$$

- **Unitary-matrix formulation**

J. Ambjorn, Y. M. Makeenko, J. Nishimura, R. J. Szabo, 2000

NC background

$$\Gamma_\mu \Gamma_\nu = \mathcal{Z}_{\mu\nu} \Gamma_\nu \Gamma_\mu$$

Decomposition

$$V_\mu = U_\mu \Gamma_\mu$$

- Correspondence between unitary and Hermitian matrices:

$$V_\mu \sim e^{iA_\mu/R} \,, \quad \Gamma_\mu \sim e^{ip_\mu/R} \,, \quad U_\mu \sim e^{ia_\mu/R}$$

- Twisted Eguchi-Kawai model gives NC plaquette action

$$\begin{aligned} & -\mathcal{N}\beta \sum_{\mu\neq\nu} \mathcal{Z}_{\nu\mu} \mathrm{tr}\left(V_\mu V_\nu V_\mu^\dagger V_\nu^\dagger \right) \\ = \ & -\beta \sum_{\mu\neq\nu} \sum_x U_\mu(x) \star U_\nu(x+\epsilon\hat{\mu}) \star U_\mu(x+\epsilon\hat{\nu})^* \star U_\nu(x)^* \end{aligned}$$

- We will use finite-unitary-matrix formulation for NC tori.

- $M^4 \times T^6$ compactification

$$\begin{aligned} M^4 &: \quad A_\mu = x_\mu \otimes \mathbb{1} \\ T^6 &: \quad e^{iA_i/R} \sim \mathbb{1} \otimes V_i \end{aligned}$$

with $\mu = 0, \ldots, 3$ and $i = 4, \ldots, 9$.

- Alternatively, one can sonsider $T^4 \times T^6$ with a huge anisotropy of sizes.

- Topological configurations V_i are given by using the Morita equivalence.

J. Ambjorn, Y. M. Makeenko, J. Nishimura, R. J. Szabo, 2000

R. J. Szabo, 2003

HA, J. Nishimura, Y. Susaki, 2009

Topological configurations V_i in $T^6 = T^2 \times T^2 \times T^2$

$$V_{i+3} = \begin{pmatrix} \Gamma^1_{1,i} \otimes \mathbb{1}_{n^1_2} \otimes \mathbb{1}_{n^1_3} \otimes \mathbb{1}_{p^1} & & & \\ & \Gamma^2_{1,i} \otimes \mathbb{1}_{n^2_2} \otimes \mathbb{1}_{n^2_3} \otimes \mathbb{1}_{p^2} & & \\ & & \ddots & \\ & & & \Gamma^h_{1,i} \otimes \mathbb{1}_{n^h_2} \otimes \mathbb{1}_{n^h_3} \otimes \mathbb{1}_{p^h} \end{pmatrix}$$

$$V_{i+5} = \begin{pmatrix} \mathbb{1}_{n^1_1} \otimes \Gamma^1_{2,i} \otimes \mathbb{1}_{n^1_3} \otimes \mathbb{1}_{p^1} & & & \\ & \mathbb{1}_{n^2_1} \otimes \Gamma^2_{2,i} \otimes \mathbb{1}_{n^2_3} \otimes \mathbb{1}_{p^2} & & \\ & & \ddots & \\ & & & \mathbb{1}_{n^h_1} \otimes \Gamma^h_{2,i} \otimes \mathbb{1}_{n^h_3} \otimes \mathbb{1}_{p^h} \end{pmatrix}$$

$$V_{i+7} = \begin{pmatrix} \mathbb{1}_{n^1_1} \otimes \mathbb{1}_{n^1_2} \otimes \Gamma^1_{3,i} \otimes \mathbb{1}_{p^1} & & & \\ & \mathbb{1}_{n^2_1} \otimes \mathbb{1}_{n^2_2} \otimes \Gamma^2_{3,i} \otimes \mathbb{1}_{p^2} & & \\ & & \ddots & \\ & & & \mathbb{1}_{n^h_1} \otimes \mathbb{1}_{n^h_2} \otimes \Gamma^h_{3,i} \otimes \mathbb{1}_{p^h} \end{pmatrix}$$

with $i = 1, 2$

- $\mathbb{1}_{p^1}, \ldots, \mathbb{1}_{p^h}$ yield the gauge group $\mathrm{U}(p^1) \times \mathrm{U}(p^2) \times \cdots \times \mathrm{U}(p^h)$.

- The unitary matrix $\Gamma^a_{l,i}$ represents NC T^2 with magnetic flux q^a_l.

- $a = 1, \ldots, h$ label the blocks.
  $l = 1, 2, 3$ label lth T^2 in $T^6 = T^2 \times T^2 \times T^2$

- A topological configuration V_i is specified by the integers p^a and q^a_l.

- In fact, V_i are classical solutions for the unitary MM.

The off-diagonal block ψ^{ab} of adjoint fermion

$$\psi = \begin{pmatrix} \psi^{11} & \psi^{12} & \cdots & \psi^{1h} \\ \psi^{21} & \psi^{22} & \cdots & \psi^{2h} \\ \vdots & \vdots & \ddots & \vdots \\ \psi^{h1} & \psi^{h2} & \cdots & \psi^{hh} \end{pmatrix}$$

- Bifundamental rep. $(p^a, \bar{p^b})$ under the gauge group $\mathrm{U}(p^a) \times \mathrm{U}(p^b)$
- Topological charge, i.e., magnetic flux, in T^6:

$$\frac{1}{2}\mathcal{T}r\left[P^{aL}P^{bR}(\gamma + \hat{\gamma})\right] = p^a p^b \prod_{l=1,2,3} (q_l^a - q_l^b)$$

The Dirac index takes the correct values in a GW Dirac operator.

$\Rightarrow$ Generation number on our spacetime

§3 Phenomenological models

- **Fermion content** = SM fermions with 3 generations

- SM gauge group $\subset$ **Gauge group** $\subset$ $\mathrm{U}(8)$

(i) $\mathrm{U}(4) \times \mathrm{U}_L(2) \times \mathrm{U}_R(2)$: Pati-Salam-like model
(ii) $\mathrm{U}_c(3) \times \mathrm{U}(1) \times \mathrm{U}_L(2) \times \mathrm{U}_R(2)$
(iii) $\mathrm{U}(4) \times \mathrm{U}_L(2) \times \mathrm{U}(1)^2$
(iv) $\mathrm{U}_c(3) \times \mathrm{U}(1) \times \mathrm{U}_L(2) \times \mathrm{U}(1)^2$

(i) $\mathrm{U}(4) \times \mathrm{U}_L(2) \times \mathrm{U}_R(2)$ model

Fermion embedding into matrix:

$$\psi = \left(\begin{array}{c|c|c} o & \begin{matrix} q \\ l \end{matrix} & \begin{matrix} u & d \\ \nu & e \end{matrix} \\ \hline & o & o \\ \hline & & o \end{array}\right)$$

q: quark doublets
l: lepton doublets
u,d: quark singlets
ν,e: lepton singlets
o: Vanishing index $\Rightarrow$No massless modes

Flux q_l^a

$$
\begin{aligned}
q_l^{ab} &= q_l^a - q_l^b && \cdots T^2 \text{ flux} \\
q^{ab} &= \prod_{l=1}^{3} q_l^{ab} && \cdots T^6 \text{ flux}
\end{aligned}
$$

T^6 flux must take values

$$
q^{ab} = \begin{pmatrix} 0 & -3 & 3 \\ & 0 & 0 \\ & & 0 \end{pmatrix}
$$

4 solutions for T^2 flux:

$\hat{q}_1^{ab}$	$\hat{q}_2^{ab}$	$\hat{q}_3^{ab}$
$\begin{pmatrix} -1 & 1 \\ & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 \\ & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -3 \\ & -6 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 \\ & -4 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 \\ & -2 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 \\ & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 \\ & 2 \end{pmatrix}$	$\begin{pmatrix} 3 & -1 \\ & -4 \end{pmatrix}$

(ii) $\mathrm{U}_c(3) \times \mathrm{U}(1) \times \mathrm{U}_L(2) \times \mathrm{U}_R(2)$ model

Fermion embedding into matrix:

$$\psi = \left(\begin{array}{c|c|c|c} o & o & q & u\ d \\ \hline & o & l & \nu\ e \\ \hline & & o & o \\ \hline & & & o \end{array}\right)$$

T^6 **flux must take values**

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 \\ & 0 & -3 & 3 \\ & & 0 & 0 \\ & & & 0 \end{pmatrix}$$

4 solutions for T^2 flux:

$\hat{q}_1^{ab}$	$\hat{q}_2^{ab}$	$\hat{q}_3^{ab}$
$\begin{pmatrix} -2 & -1 & -1 \\ & 1 & 1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} -2 & -1 & -1 \\ & 1 & 1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -3 & 3 \\ & -3 & 3 \\ & & 6 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & 1 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 1 & 1 \\ & -1 & -1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 6 & 3 & 3 \\ & -3 & -3 \\ & & 0 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & 1 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 4 & 3 & 3 \\ & -1 & -1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 4 & 1 & 1 \\ & -3 & -3 \\ & & 0 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & 1 \\ & -1 & 1 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 3 & 3 \\ & 1 & 1 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} -2 & 1 & 1 \\ & 3 & 3 \\ & & 0 \end{pmatrix}$

(iii) $\mathrm{U}(4) \times \mathrm{U}_L(2) \times \mathrm{U}(1)^2$ model

Fermion embedding into matrix:

$$\psi = \left(\begin{array}{c|c|c|c} o & \begin{matrix} q \\ l \end{matrix} & \begin{matrix} u \\ \nu \end{matrix} & \begin{matrix} d \\ e \end{matrix} \\ \hline & o & o & o \\ \hline & & o & o \\ \hline & & & o \end{array}\right)$$

T^6 flux must take values

$$q^{ab} = \begin{pmatrix} 0 & -3 & 3 & 3 \\ & 0 & 0 & 0 \\ & & 0 & 0 \\ & & & 0 \end{pmatrix}$$

9 solutions for T^2 flux:

$\hat{q}_1^{ab}$	$\hat{q}_2^{ab}$	$\hat{q}_3^{ab}$
$\begin{pmatrix} -1 & 1 & -1 \\ & 2 & 0 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & -1 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & 3 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & 1 & -1 \\ & 2 & 0 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & -3 \\ & 0 & -6 \\ & & -6 \end{pmatrix}$
$\begin{pmatrix} -1 & 1 & 3 \\ & 2 & 4 \\ & & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & 1 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$
$\begin{pmatrix} -1 & 1 & -3 \\ & 2 & -2 \\ & & -4 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 3 & -1 \\ & 0 & -4 \\ & & -4 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & 3 \\ & 0 & 4 \\ & & 4 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -3 & 1 \\ & -6 & -2 \\ & & 4 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & -3 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -3 & -1 \\ & -6 & -4 \\ & & 2 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & 3 \\ & 0 & 4 \\ & & 4 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 & 1 \\ & -4 & 0 \\ & & 4 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & 1 \\ & -2 & -2 \\ & & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & -1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -3 & 3 \\ & -4 & 2 \\ & & 6 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & -1 \\ & -2 & -4 \\ & & -2 \end{pmatrix}$
$\begin{pmatrix} -1 & -1 & -3 \\ & 0 & -2 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 & 1 \\ & 2 & 0 \\ & & -2 \end{pmatrix}$	$\begin{pmatrix} 3 & -1 & -1 \\ & -4 & -4 \\ & & 0 \end{pmatrix}$

(iv) $\mathrm{U}_c(3) \times \mathrm{U}(1) \times \mathrm{U}_L(2) \times \mathrm{U}(1)^2$ model

Fermion embedding into matrix:

$$\psi = \left(\begin{array}{c|c|c|c|c} o & o & q & u & d \\ \hline & o & l & \nu & e \\ \hline & & o & o & o \\ \hline & & & o & o \\ \hline & & & & o \end{array}\right)$$

T^6 **flux must take values**

$$q^{ab} = \begin{pmatrix} 0 & 0 & -3 & 3 & 3 \\ & 0 & -3 & 3 & 3 \\ & & 0 & 0 & 0 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$$

No solution for T^2 flux

§4 Probability distribution

Semiclassical analyses of MM dynamics

Unitary MM

$$S_b = -\beta \mathcal{N} \sum_{i \neq j} \mathcal{Z}_{ji} \, \mathrm{tr} \, \left(\mathcal{V}_i \, \mathcal{V}_j \, \mathcal{V}_i^\dagger \, \mathcal{V}_j^\dagger \right) + \ldots \tag{2}$$

with

$$\mathcal{Z}_{45} = \exp\left(2\pi i \frac{s_1}{N_1}\right) \,, \quad \mathcal{Z}_{67} = \exp\left(2\pi i \frac{s_2}{N_2}\right) \,, \quad \mathcal{Z}_{89} = \exp\left(2\pi i \frac{s_3}{N_3}\right) \,,$$

Background configurations:

$$\mathcal{V}_i = \mathbb{1} \otimes V_i$$

with

$$V_{3+i} = \begin{pmatrix} \Gamma^1_{1,i} \otimes \mathbb{1}_{n^1_2} \otimes \mathbb{1}_{n^1_3} \otimes \mathbb{1}_{p^1} & & & \\ & \Gamma^2_{1,i} \otimes \mathbb{1}_{n^2_2} \otimes \mathbb{1}_{n^2_3} \otimes \mathbb{1}_{p^2} & & \\ & & \ddots & \\ & & & \Gamma^h_{1,i} \otimes \mathbb{1}_{n^h_2} \otimes \mathbb{1}_{n^h_3} \otimes \mathbb{1}_{p^h} \end{pmatrix}$$

$$V_{5+i} = \begin{pmatrix} \mathbb{1}_{n^1_1} \otimes \Gamma^1_{2,i} \otimes \mathbb{1}_{n^1_3} \otimes \mathbb{1}_{p^1} & & & \\ & \mathbb{1}_{n^2_1} \otimes \Gamma^2_{2,i} \otimes \mathbb{1}_{n^2_3} \otimes \mathbb{1}_{p^2} & & \\ & & \ddots & \\ & & & \mathbb{1}_{n^h_1} \otimes \Gamma^h_{2,i} \otimes \mathbb{1}_{n^h_3} \otimes \mathbb{1}_{p^h} \end{pmatrix}$$

$$V_{7+i} = \begin{pmatrix} \mathbb{1}_{n^1_1} \otimes \mathbb{1}_{n^1_2} \otimes \Gamma^1_{3,i} \otimes \mathbb{1}_{p^1} & & & \\ & \mathbb{1}_{n^2_1} \otimes \mathbb{1}_{n^2_2} \otimes \Gamma^2_{3,i} \otimes \mathbb{1}_{p^2} & & \\ & & \ddots & \\ & & & \mathbb{1}_{n^h_1} \otimes \mathbb{1}_{n^h_2} \otimes \Gamma^h_{3,i} \otimes \mathbb{1}_{p^h} \end{pmatrix}$$

- By plugging these configurations into the MM action (2),

$$S_b = -2\beta\mathcal{N}N_4N_5\sum_{l=1}^{3}\sum_{a=1}^{h}n_1^a n_2^a n_3^a p^a \cos\left(2\pi\left(\frac{s_l}{N_l}+\frac{m_l^a}{n_l^a}\right)\right)$$

- Using the relation of the Morita equivalence,

$$\frac{s_l}{N_l}+\frac{m_l^a}{n_l^a}=\frac{q_l^a}{N_l n_l^a}=-\frac{1}{2r}\left(\frac{1}{N_l}-\frac{1}{n_l^a}\right)$$

$\Rightarrow$ The classical action takes the minimum value if and only if

$$q_l^a = 0 \Leftrightarrow n_l^a = N_l$$

for $\forall a$ and $\forall l$.

Small fluctuations around the minimum

- Configurations with $|q_l^a| \ll N_l$
- The classical action is approximated as

$$\Delta S_b \simeq 4\pi^2 \beta \mathcal{N}^2 \sum_{l=1}^{3} \frac{1}{(N_l)^4} \sum_{a=1}^{h} p^a (q_l^a)^2 \tag{3}$$

$\sim$ **Instanton action**

- **Comparing the IIB MM (1) and the unitary MM (2)**
  by using correspondence between Hermitian and unitary matrices:

$$\mathcal{V}_M \sim \exp\left(2\pi i \frac{A_M}{\epsilon N_l}\right)$$

one finds a relation between the coupling constants

$$\frac{1}{2}\beta\mathcal{N}\left(\frac{2\pi}{\epsilon N_l}\right)^4 = \frac{1}{g^2_{\mathrm{IIBMM}}}$$

⇒ The instanton action (3) becomes

$$\Delta S_b = \frac{A}{2\pi^2} \sum_{l=1}^{3} \sum_{a=1}^{h} p^a (q_l^a)^2 \ .$$

with

$$A = \frac{\epsilon^4 \mathcal{N}}{g_{\mathrm{IIBMM}}^2}$$

⇒ Scaling limits of fixing $g_{\mathrm{IIBMM}}^2 \mathcal{N}^\alpha / \epsilon^4$ with

(1) $\alpha < -1 \ \Rightarrow \ A = 0$
(2) $\alpha = -1 \ \Rightarrow \ A =$ finite
(3) $\alpha > -1 \ \Rightarrow \ A = \infty$

How to take large-N limits and probability disribution over the string vacuum space

(1) $g^2_{\mathrm{IIBMM}}\mathcal{N}^{\alpha}/\epsilon^4$ with $\alpha < -1$

- All the topological sectors appear with equal probabilities.
- The estimation for the probability distribution over the string vacuum space reduces to number counting of the classical solutions.

(2) $\alpha = -1$

- All the topological sectors appear, but with different probabilities $\sim e^{-\Delta S_b}$.

(3) $\alpha > -1$

- Only a single topological sector survives.

The instanton action in each phenomenological model

- Each solution in each phenomenological model specifies T^2 fluxes q_l^{ab}.
- Since q_l^a are detemined by

$$q_l^{ab} = q_l^a - q_l^b \ ,$$

q_l^a have an arbitrarilness of integer shifts q_l.

- The mimimum value of the instanton actions within this arbitrarilness q_l

$$\Delta S_b = \frac{A}{2\pi^2} \sum_{l=1}^{3} \sum_{a=1}^{h} p^a (q_l^a)^2$$

gauge group	integral q_l	fractional q_l
U(8)	0	0
$\mathrm{U}(4) \times \mathrm{U}_L(2) \times \mathrm{U}_R(2)$	28	24
	44	40
	36	32
	36	32
$\mathrm{U}_c(3) \times \mathrm{U}(1) \times \mathrm{U}_L(2) \times \mathrm{U}_R(2)$	44	43
	40	39
	36	36
	28	27
$\mathrm{U}(4) \times \mathrm{U}_L(2) \times \mathrm{U}(1)^2$	28	25
	40	37
	32	29
	36	32
	44	40
	40	37
	36	36
	40	37
	36	33

- Rather small instanton actions for the phenomenologcal models
- No substantial differences among the models

§5 Conclusions and Discussions

Summary

- We have considered the situations where
  the IIB MM is compactified on a torus with magnetic fluxes,

- exhausted all the matrix configurations
  that yield all the models whose gauge group is a subgroup of U(8),

- semiclassically estimated probability distribution of their appearance.

Discussions

- Higgs field:
  - Gauge fields in the extra dimensions give scalar fields on our spacetime

$$V_i = \begin{pmatrix} o & o & o \\ & o & \text{Higgs} \\ & & o \end{pmatrix}$$

  - It is difficult to keep scalar fields massless against quantum corrections.
    the naturalness/hierarchy problem
    * How to realize SUSY and how to break it.
    * Other resolutions of the naturalness/hierarchy problem
  - How the electroweak symmetry breaking occurs.
  - Values of the Yukawa couplings and the flavor structures

- Anomaly cancellations
  - Extra U(1) gauge groups $\Rightarrow$Anomaly
  - May be canceled via the Green-Schwarz mechanism by exchange of the RR-fields
- Extend the matrix configurations
  - by introducing the Wilson lines and tilting the tori
- Compactifications
  - Other compactifications than tori
  - Compactifications in MM:
    Matter and gravitons must propagate in the compactified space.

- **Ultimately,
  we hope to analyze full dynamics in the MM,
  and survey the probability distribution over the whole of the landscape!**

Thank you