Reduction of couplings in the MSSM

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based on

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Outline

- Introduction.
- General Method of Reduction.
- Application in the MSSM.
- Summary.

Introduction

• To reduce the number of independent couplings of a theory one can impose a symmetry.

• Also we can adopt a more general approach.

• We reduce this number by imposing relations between the couplings.

• The relations between the dimensionless couplings are such that renormalizability is preserved and are independent of the renormalization point.

• The method is developed for the reduction from n+1 coupling parameters $g_0, g_1, ..., g_n$ to a description in terms of g_0 only. (W. Zimermann, 1985)

General Method of Reduction

Our aim is to express $g_1, g_2, ..., g_n$ as functions of g_0 so that a model involving a single coupling parameter λ_0 is obtained which is again invariant under the renormalization group. They can be written as

$$g_j = g_j(g_0) \qquad j = 1, ..., n \tag{1}$$

Invariance of the Green's functions of the original system under renormalization group implies

$$\left(M\frac{\partial}{\partial M} + \sum_{j=0}^{n} \beta_j \frac{\partial}{\partial g_j} + \gamma \right) G(p_i, M, g_0, g_1, ..., g_n) = 0$$

where M, β_j, γ are the renormalization mass, the beta functions and the anomalous dimension correspondingly.

And for the reduced system

$$\left(M\frac{\partial}{\partial M}+\beta'\frac{\partial}{\partial g_0}+\gamma'\right)G'(p_i,M,g_0,g_1(g_0),...,g_n(g_0))=0$$

We can see that G' is obtained from G by substituting the functions (1)

$$G'=G(g_0,g_1(g_0),...,g_n(g_0))$$

Differentiating with respect to g_0

$$\frac{dG'}{dg_0}=\frac{\partial G}{\partial g_0}+\sum_{j=1}^{n}\frac{\partial G}{\partial g_j}\frac{dg_j}{dg_0}$$

From the above equations we have

$$\beta'=\beta_0,\gamma'=\gamma,\beta'\frac{dg_j}{dg_0}=\beta_j$$

So the functions (1) must satisfy the following differential equations, the Reduction Equations

$$\beta_j=\beta_0\frac{dg_j}{dg_0}$$

For simplicity we assume that the original system has two coupling parameters, g_0 and g_1. The beta-functions can be written

$$\beta_0 = b_0 g_0^2 + \ldots.$$

$$\beta_1 = c_1 g_1^2 + c_2 g_0 g_1 + c_3 g_0^2 \ldots.$$

The reduction equation is

$$\beta_1 = \beta_0 \frac{dg_1}{dg_0}$$

Assuming power series solution:

$$g_1 = p_0^{(1)} g_0 + \sum_{n=1} p_n^{(1)} g_0^{(n+1)}$$

at lowest order we end up with a quadratic equation:

$$c_1 p_0^2 + (c_2 - b_0) p_0 + c_3 = 0$$

Application in the MSSM

- Assuming that α_2 gauge coupling can be related with the α_1 gauge coupling ($\alpha_i = \frac{g_i^2}{4\pi}$) we have the following reduction equation

$$\beta_2 = \beta_1 \frac{d\alpha_2}{d\alpha_1} \tag{2}$$

where

$$\beta_2 \equiv \frac{d\alpha_2}{dt} = \frac{b_2}{2\pi}\alpha_2^2, \qquad \beta_1 \equiv \frac{d\alpha_1}{dt} = \frac{b_1}{2\pi}\alpha_1^2, \qquad t = logE$$

and

$$b_2 = 1, b_1 = 11$$

We can write α_2 in lowest order in perturbation theory as

$$\alpha_2 = c_0 \alpha_1$$

Substituting this relation to the reduction equation (2)

$$c_0 = \frac{\beta_2}{\beta_1} = \frac{b_2 \alpha_2^2}{b_1 \alpha_1^2} = \frac{b_2 c_0^2 \alpha_1^2}{b_1 \alpha_1^2} \Rightarrow$$

$$c_0 = 11$$

Hence α_2 can be written as

$$\alpha_2 = 11 \alpha_1$$

We can check now if this result is compatible with the experimental values.

$$\frac{1}{\alpha_{em}} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \Rightarrow$$

$$\alpha_{em} = \frac{11}{12}\alpha_1$$

We know that

$$\sin^2\theta_w = \frac{\alpha_{em}}{\alpha_2} \Rightarrow$$

$$\sin^2\theta_w = \frac{11}{12}\frac{\alpha_1}{11\alpha_1} = \frac{1}{12} = 0.08333$$

which is unacceptable because

$$\sin^2\theta_w^{exp} = 0.23146 \pm 0.00017$$

• Following the same procedure we assume that α_{top} Yukawa coupling can be related with the α_{bottom} Yukawa coupling, so they must satisfy the reduction equation

$$\beta_{top} = \beta_{bottom} \frac{d\alpha_{top}}{d\alpha_{bottom}} \Rightarrow$$

$$\frac{d\alpha_{top}}{d\alpha_{bottom}} = \frac{\beta_t}{\beta_b} = \frac{\alpha_t(6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}{\alpha_b(6\alpha_b + \alpha_t + \alpha_\tau - \frac{7}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}$$

We can for simplicity neglect the contribution from the τ and the small difference between $\frac{13}{15}$ and $\frac{7}{15}$, so

$$\frac{\beta_t}{\beta_b} = \frac{\alpha_t(6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}{\alpha_b(6\alpha_b + \alpha_t - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)} \tag{3}$$

Assuming again power series solution of the reduction equation we can write

$$\alpha_t = d_0 \alpha_b$$

The derivative of the ratio of the two Yukawa couplings must be zero

$$\frac{d}{dt}\left(\frac{\alpha_t}{\alpha_b}\right) = 0 \Rightarrow$$

$$\frac{1}{\alpha_t^2}(\alpha_b \beta_t - \alpha_t \beta_b) = 0 \Rightarrow$$

$$\frac{\alpha_t}{\alpha_b} = \frac{\beta_t}{\beta_b}$$

Eqn. (3) becomes

$$\frac{\alpha_t}{\alpha_b} = \frac{\alpha_t(6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)}{\alpha_b(6\alpha_b + \alpha_t - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3)} \Rightarrow$$

$$6\alpha_t + \alpha_b - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3 = 6\alpha_b + \alpha_t - \frac{13}{15}\alpha_1 - 3\alpha_2 + \frac{16}{3}\alpha_3 \Rightarrow$$

$$\alpha_t = \alpha_b$$

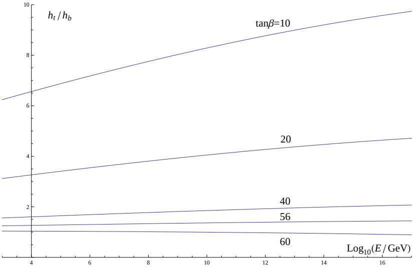
•The next thing to do is to solve numerically the one-loop coupled differential equations of top and bottom Yukawa couplings taken account the τ contribution and the difference between the numerical factors, to see if such a relation like the previous one can exist.

• First, we solve the differential equations for the gauge and Yukawa couplings in the SM. And then at M_{SUSY} we impose the next boundary conditions for some values of $\tan\beta$

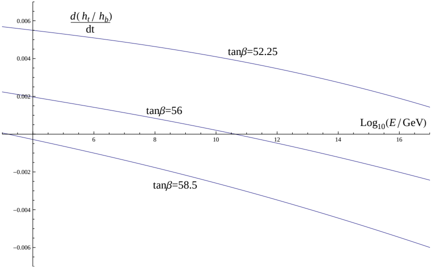
$$\alpha_t(SM) = \alpha_t(MSSM)\sin^2\beta$$

$$\alpha_b(SM) = \alpha_b(MSSM)\cos^2\beta$$

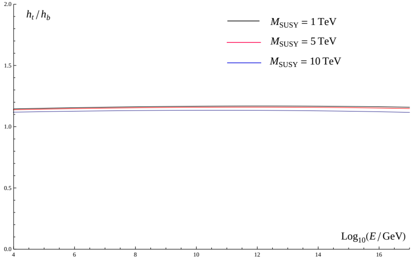
$$\alpha_\tau(SM) = \alpha_\tau(MSSM)\cos^2\beta$$

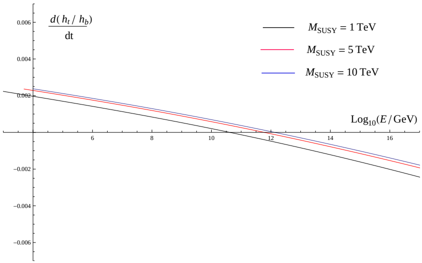


h_t / h_b
tanβ=10
20
40
56
60
$\mathrm{Log}_{10}(E/\mathrm{GeV})$



$\frac{d(h_t/h_b)}{dt}$
tanβ=52.25
tanβ=56
tanβ=58.5
$\mathrm{Log}_{10}(E/\mathrm{GeV})$


h_t / h_b
$M_{SUSY} = 1$ TeV
$M_{SUSY} = 5$ TeV
$M_{SUSY} = 10$ TeV
$\text{Log}_{10}(E/\text{GeV})$


$\frac{d(h_t/h_b)}{dt}$
$M_{SUSY} = 1$ TeV
$M_{SUSY} = 5$ TeV
$M_{SUSY} = 10$ TeV
$\text{Log}_{10}(E/\text{GeV})$

Summary

- No possible reduction between the gauge couplings.

- Assuming $\tan\beta = 56$ we can relate top and bottom Yukawa couplings.

- The idea of reduction of couplings in a field theory is very appealing, since it increases its predictive power. This method has led to Finite Unified Theories with succesfull calculation of top quark mass.

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