Flavour in the Higgs Era

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based on collaboration with
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and earlier work with W. Grimus, L. Lavoura

Some of the Open Questions in Flavour Physics.

- Why is there "Flavour"?
  i.e. What is the Origin of Family replication?
- How to understand the observed spectrum of Fermion Masses and Mixing?
  Why is Leptonic Mixing Large, in contrast to Small Quark Mixing?
- Why is there Flavour Alignment in the Quark Sector?

- Does the Scalar Sector play a non-trivial rôle in the question of Flavour? If so, it is likely that Nature chooses a richer scalar sector. Simplest possibility: more than one Higgs doublet

Question: How to avoid Flavour changing neutral currents?

- What is the Origin of CP Violation?
- How to solve the Strong CP problem?

Peccei - Quinn provide an elegant solution, but no Axions have been found!

The various manifestations of CP violation:

- CP violation in the quark sector
- CP violation in the lepton sector
- CP violation needed to generate BAU

A Common Origin?

- What about the Strong CP problem?

CP Violation is closely related to the Question of Flavour

An open question :

- Is CP broken explicitly through complex Yukawa couplings like in the SM

or (T.D. Lee)

- Spontaneously : Lagrangian is invariant under CP but the vacuum violates CP?

At present, there is solid experimental evidence for a complex V^{CKM}.

Does this imply complex Yukawa couplings?

NO!!

There are realistic models of spontaneous CP violation where the vacuum phase generates a complex V^{CKM}.

Can one have geometrical CP violation?

For definiteness, let us consider an extension of the SM where n $SU(2) \times U(1)$ scalar doublets are introduced. In order to include the possible existence of "family" symmetries of the Lagrangian under which the scalar doublets transform non-trivially, one has to consider the most general CP transformation which leaves invariant the kinetic energy terms of the scalar doublets:

$$\boxed{CP\, \phi_i\, (CP)^\dagger = \sum_{j=1}^{n} U_{ij}\, \phi_j^*}$$

Let us assume that the vacuum is CP invariant, meaning that:

$$CP|0\rangle = |0\rangle$$

One can then derive, the following relation:

$$\boxed{\sum_{j=1}^{n} U_{ij} \langle 0|\Phi_j|0\rangle^* = \langle 0|\phi_i|0\rangle}$$

If the vacuum is such that none of the symmetries allowed by the Lagrangian satisfy the above equation, then this means that the vacuum is not CP invariant and we say that CP is spontaneously broken.

There are various examples of multi-Higgs models with "family" symmetries, where in a certain region of parameter space, the minimum of the scalar potential corresponds to fixed values of the vacuum phases, which do not depend on the specific values of the parameters of the potential.

$$Z_2 \text{ symmetry} \longrightarrow \langle \phi_1^0 \rangle = v_1 \;;\; \langle \phi_2^0 \rangle = v_2 e^{i\pi/2}$$

$$S_3 \text{ symmetry} \longrightarrow \langle \phi_1^0 \rangle = v \;;\; \langle \phi_2^0 \rangle = v e^{i2\pi/3} \;;\; \phi_3 = v e^{i4\pi/3}$$

It has been shown by G.B, Gérard, J.M., Grimus, W that, contrary to naïve expectation all these vaccua are CP conserving.

However, an example was found, based on the group $\Delta(27)$ leading to genuine CP and T geometrical valuation.

Recently, interesting other examples have been found:

I. M. Varzielas, D. E. Costa, P. Leser Phys. Lett (2012)

I. P. Ivanov, L. Lavoura, Eur. Phys. (2013)

Realistic model?

Generation of the Baryon Asymmetry of the Universe (BAU)

The ingredients to dynamically generate BAU from an initial state with zero B. A. , were formulated by Sakharov (1967)

(i) Baryon number violation

(ii) C and CP Violation

(iii) Departure from thermal equilibrium

All these ingredients exist in the SM, 50
but it has been established that in the SM, one cannot generate the observed BAU:

$$\boxed{\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.20 \pm .15) \times 10^{-10}}$$

$n_B, n_{\bar{B}}, n_\gamma$ number densities of baryons, antibaryons and photons at present time.

Reasons why the SM cannot generate sufficient BAU:

(i) CP violation in the SM is too small

$$\boxed{\frac{\mathrm{tr}\left[H_u, H_d\right]^3}{T_{EW}^{12}} \approx 10^{-20}}$$

(ii) Successful baryogenesis requires a strongly first order phase transition which would require a light Higgs mass

$$m_H \lesssim 70\,\mathrm{GeV}$$

Weak-basis invariants are a useful to study Flavour.

Example :

J. Bernabeu, G.C.B., M. Gronau

$$\mathrm{tr}\,[H_u, H_d]^3 = 6i\,(m_c^2 - m_u^2)(m_t^2 - m_c^2)(m_t^2 - m_u^2) \times$$
$$\times (m_s^2 - m_d^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2) \times$$
$$\mathrm{Im}\, Q$$

$\propto \det [H_u, H_d]$ for 3 generations (C. Jarlskog)

$Q \rightarrow$ invariant quartet of V_{CKM}

One may also study invariants under Higg basis transformations

G.C.B., M.N. Rebelo, Silva-Marcos ; H. Haber, F. Gunion
S. Davidson, H. Haber

3 Flavour Dogmas introduced in the 70's, soon after the creation and early success of the S.M.

Dogma 1 – Neutrinos are strictly massless!

Dogma 2 – No Z-mediated FCNC at tree level

Dogma 3 – No Higgs mediated FCNC at tree level.

Question: We all know the "Fate" of Dogma N°1
Will the other two Dogmas have the same "Fate" as Dogma 1.

Can one violate these two dogmas in reasonable extensions of the SM? Yes!

"Reasonable" means that FCNC should be naturally suppressed, without fine-tuning.

. In the gauge sector, the Dogma can be violated through the introduction of a $Q = 1/3$ and/or $Q = 2/3$ vector-like quark.

$\Downarrow$

Naturally small violations of 3×3 unitarity of V^{CKM}

$\Downarrow$

Z-mediated, Naturally suppressed FCNC at tree level

Yukawa interactions in the General two Higgs doublet Model

$$-\mathcal{L}_Y = \overline{Q}^0_L \Gamma_1 \phi_1 d^0_R + \overline{Q}^0_L \Gamma_2 \phi_2 d^0_R + \overline{Q}^0 \Delta_1 \tilde{\phi}_1 u^0_R + \overline{Q}_L \Delta_2 \tilde{\phi}_2 u^0_R$$
$$+ h.c.$$

Quark mass matrices:

$$M_d = \frac{1}{\sqrt{2}} \left(v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2 \right) ; \quad M_u = \frac{1}{\sqrt{2}} \left(v_1 \Delta_1 + v_2 e^{-i\alpha} \Delta_2 \right)$$

Diagonalized by:

$$(U_d)^\dagger_L M_d (U_d)_R = D_d = \mathrm{diag}\,(m_d, m_s, m_b)$$

$$(U_u)^\dagger_L M_u U_{uR} = D_d = \mathrm{diag}\,(m_u, m_c, m_t)$$

Expanding around the vacuum:

$$\langle \Phi_j^0 \rangle = e^{i\alpha_j} \frac{1}{\sqrt{2}} \left(v_j + \rho_j + i\eta_j \right) \qquad j = 1,2$$

It is useful to introduce new fields:

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = O \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} \; ; \; \begin{pmatrix} G^0 \\ I \end{pmatrix} = O \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \; ; \; \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = O \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

$$O = \frac{1}{v} \begin{bmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{bmatrix} \; ; \; v = \sqrt{v_1^2 + v_2^2} \cong 246 \text{ GeV}$$

$H^0 \rightarrow$ has couplings to quarks proportional to mass matrices

$G^0 \rightarrow$ neutral pseudo Goldstone boson

$H^\pm \rightarrow$ charged pseudo-Goldstone bosons

Neutral and charged Higgs interactions in the quark sector

$$-\mathcal{L}_Y = \bar{d}^0_L \frac{1}{v}\left(M_d H^0 + N^0_d R + i N^0_d I\right) d^0_R + $$
$$+ \bar{u}^0_L \frac{1}{v}\left(M_u H^0 + N^0_u R + i N^0_u I\right) u^0_R +$$
$$+ \frac{\sqrt{2}}{v} H^+ \left(\bar{u}^0_L N^0_d d^0_R - \bar{u}^0_R N^{0\dagger}_u d^0_L\right) + h.c.$$

$$N^0_d = \frac{1}{\sqrt{2}}\left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2\right) \; ; \; N^0_u = \frac{1}{\sqrt{2}}\left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2\right).$$

In the quark mass eigenstate basis N_d, N_u are not flavour diagonal.

Physical neutral Higgs are combinations of H^0, R, I

Yukawa Couplings in terms of quark mass eigenstates:

$$\mathcal{L}_Y = \frac{\sqrt{2}\,H^+}{v}\,\bar{u}\left[-V N_d \gamma_R + N_u^\dagger V \gamma_L\right] d + \text{h.c.} -$$

$$-\frac{H^0}{v}\left[\bar{u}\, D_u\, u + \bar{d}\, D_d\, d\right] -$$

$$-\frac{R}{v}\left[\bar{u}\left(N_u \gamma_R + N_u^\dagger \gamma_L\right) u + \bar{d}\left(N_d \gamma_R + N_d^\dagger \gamma_L\right) d\right]$$

$$+\frac{I}{v}\left[\bar{u}\left(N_u \gamma_R - N_u^\dagger \gamma_L\right) u - \bar{d}\left(N_d \gamma_R - N_d^\dagger \gamma_L\right) d\right]$$

$\gamma_L = \frac{1}{2}(1-\gamma_5)$; $\gamma_R = \frac{1}{2}(1+\gamma_5)$; V is the CKM matrix

Flavour changing neutral currents are controlled by:

$$N_d = \frac{1}{\sqrt{2}} U_{dL}^{\dagger} \left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2 \right) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{uL}^{\dagger} \left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2 \right) U_{uR}$$

For generic two Higgs doublet models N_u, N_d are non-diagonal, arbitrary matrices.

$\Downarrow$

too large Higgs mediated Flavour changing neutral currents, unless a suppression mechanism is introduced

G.C.B, Grimus, W and Lavoura, L (BGL) have shown that it is possible to find a symmetry which when imposed on a 2-Higgs doublet extension of the SM, leads to a structure of the Yukawa couplings such that there are FCNC at tree level, with strength completely controlled by V_{CKM}, i.e., N_d, N_u only depend on V_{CKM} and on $\frac{v_1}{v_2}$.

Possible choice of the symmetry S:

$$Q^0_{L3} \to \exp(i\alpha)\, Q^0_{L3} \quad ; \quad u^0_{R3} \to \exp(i2\alpha)\, u^0_{R3}$$

$$\phi_2 \to \exp(i\alpha)\, \phi_2 \,, \quad \text{where } \alpha \neq 0, \pi$$

all other fields transform trivially under S.

This leads to the Yukawa couplings:

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}$$

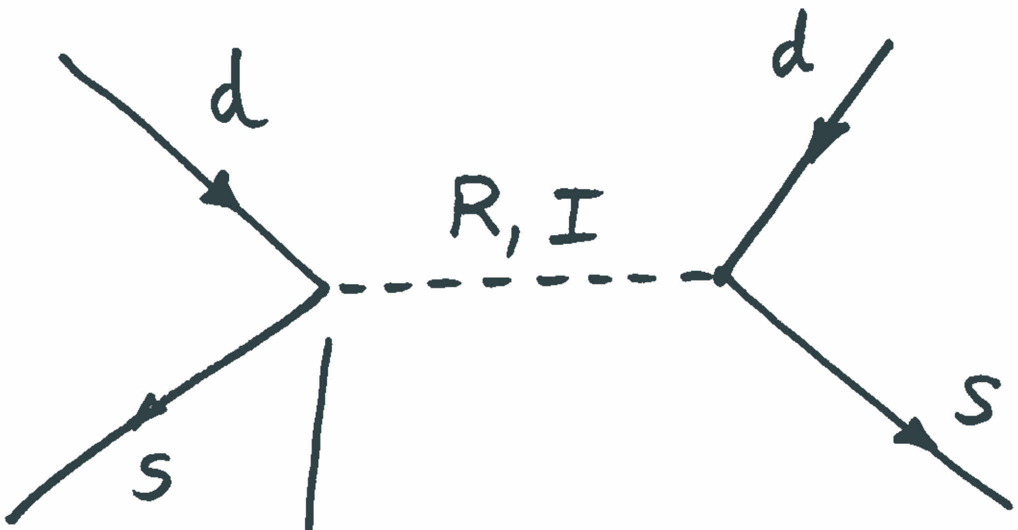
These Yukawa structures lead to:

$$(N_d)_{ij} = \frac{v_2}{v_1}(D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)\left[V_{CKM}\right]_{3j}\left[V^*_{CKM}\right]_{3i}(D_d)_{jj}$$

$$(N_u) = -\frac{v_1}{v_2}\,\mathrm{diag.}(0, 0, m_t) + \frac{v_2}{v_1}\,\mathrm{diag.}(m_u, m_c, 0)$$

In this particular BGL model there are FCNC only in the down sector, but there is a strong, natural suppression of the most "dangerous" processes

$K^0 - \overline{K^0}$ mixing



$V_{td} V_{ts}^* \sim \lambda^5$

λ^{10} suppression!!

There are 6 different BGL models in the quark sector

Can one have a model where gauge mediated FCNC exist at tree level, but are naturally suppressed?

Answer : Yes!!

A Minimal Model

Consider an extension of the SM, where the following new fields are introduced:

- A vectorial quark D^0, with both D^0_L and D^0_R are $SU(2)_L$ singlets with change $Q = -1/3$ (or $Q = 2/3$)
- 3 right-handed neutrinos ν^0_{Rj}
- A neutral complex singlet S

- Since we want to have Spontaneous CP violation, we impose CP invariance at the Lagrangian level: All couplings real.
- Introduce a Z_4 symmetry on the Lagrangian, under which:

$$\psi_\ell^0 \to i\,\psi_\ell^0 \;;\quad e_{R_j}^0 \to i\,e_{R_j}^0 \;;\quad \nu_{R_j}^0 \to i\,\nu_{R_j}^0$$

$$D^0 \to -D^0 \;;\quad S \to -S$$

The Z_4 symmetry is crucial to obtain a solution of the Strong CP problem and Leptogenesis

Scalar Potential

The Scalar potential contains various terms which do not have phase dependence, but there are terms with phase dependence.

$$V_{phase} = \left[\mu^2 + \lambda_1 S^* S + \lambda_2 \phi^\dagger \phi\right](S^2 + S^{*2}) + \lambda_3 (S^4 + S^{*4})$$

There is a range of the parameters of the Higgs potential, where the minimum is at:

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}} \quad ; \quad \langle S \rangle = \frac{V}{\sqrt{2}} e^{i\theta}$$

Most general $SU(2)_L \times U(1) \times SU(3)_c \times Z_4$ invariant Yukawa couplings in the quark sector :

$$\mathcal{L}_Y = -(\bar{u}^0 \; \bar{d}^0)_{L_i} \left[g_{ij} \, d^0_{R_j} + h_{ij} \, \tilde{\phi} \, u^0_{R_j} \right] - \bar{M} \left(\bar{D}^0_L D^0_R \right)$$

$$- \left(f_i S + f' S^* \right) \bar{D}^0 \, d^0_{R_i} + h.c.$$

Quark mass matrix for down-type quarks :

$$\left(\bar{d}^0_{1L} \; \bar{d}^0_{2L} \; \bar{d}^0_{3L} \; \bar{D}^0_L \right) \left[\begin{array}{c:c} \overset{3\times3,\ \text{real}}{m_d} & 0 \\ \hdashline M_1 \; M_2 \; M_3 & \bar{M} \end{array} \right] \begin{bmatrix} d^0_{1R} \\ d^0_{2R} \\ d^0_{3R} \\ D^0_R \end{bmatrix}$$

"zero" due to Z_4 symmetry

$$M_j = f_j V e^{i\theta} + f'_j V e^{-i\theta}$$

$$\mathcal{M} \equiv \left(\begin{array}{c|c} m_d & 0 \\ \hline M_1, M_2, M_3, & \overline{M} \end{array}\right);$$

$$U_L^\dagger \left(\mathcal{M}\mathcal{M}^\dagger\right) U_L = \text{diag.}\left(d^2, D^2\right)$$

$\mathcal{U} = \begin{bmatrix} K & R \\ S & T \end{bmatrix}$; One can easily derive:

$$K^{-1}\left[m_d m_d^\dagger - \frac{m_d \overbrace{M^\dagger M}^{\text{complex}} m_d^\dagger}{\left(MM^\dagger + \overline{M}^2\right)}\right] K = d^2$$

A remarkable feature of the Model:

The phase θ arising from $\langle S \rangle$, generates a non-trivial CKM phase, provided $|M_j|$ and $\bar{M}$ are of the same order of magnitude (This is "natural")

$$K^{-1} \, m_{eff} m_{eff}^{\dagger} \, K = \text{diag.} \left(m_d^2, m_s^2, m_b^2 \right)$$

$$m_{eff} m_{eff}^{\dagger} = m_d m_d^{\dagger} - \frac{m_d M^{\dagger} M m_d^{\dagger}}{M M^{\dagger} + \bar{M}^2}$$

$$M_j = \left(f_j V e^{i\theta} + f_j' V e^{-i\theta} \right)$$

Naturally small deviations of 3x3 unitarity

Naturally Small Flavour-Changing Neutral Currents

For definiteness, consider the case of one isosinglet $Q=-1/3$ quark

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\bar{u}\ \bar{c}\ \bar{t})_L \gamma^\mu \big[\underset{\text{3x3 VCKM}}{K}\ \ R\big] \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} W_\mu^+$$

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W} \Big\{ (\bar{u}\ \bar{c}\ \bar{t})_L \gamma^\mu \begin{bmatrix} u \\ c \\ t \end{bmatrix}_L - \big[\bar{d}\ \bar{s}\ \bar{b}\ \bar{D}\big] \begin{bmatrix} K^\dagger K & K^\dagger R \\ R^\dagger K & R^\dagger R \end{bmatrix} \gamma^\mu \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix}$$

$$- \sin^2\theta_W J^\mu_{em} \Big\} Z_\mu$$

Why deviations of 3×3 unitarity are naturally small:

$$\mathcal{U}_L^\dagger \mathcal{M}\mathcal{M}^\dagger \mathcal{U}_L = \mathrm{diag.}(m_d^2, m_s^2, m_b^2, M_D^2)$$

$$\mathcal{U}_L = \begin{bmatrix} K & R \\ S & T \end{bmatrix} \quad ; \quad K^\dagger K + S^\dagger S = 1$$

but $\quad S \approx -\dfrac{M\, m_d^\dagger K}{\bar{M}^2} \rightarrow O(m/M)$;

$K^\dagger K = 1 - O(m^2/M^2)$. Note that there is nothing strange about violations of 3×3 unitarity.

The PMNS matrix is not unitary in the framework of seesaw mechanism, type I.

A possible solution to the Strong CP problem:

$$\bar{\Theta} = \Theta_{QCD} + \Theta_{QFD} \; ;$$

$$\mathcal{L}_\Theta = \Theta_{QCD} \frac{g_s^2}{32\pi^2} F\tilde{F} \; ; \quad \Theta_{QFD} = \arg\det(M_u M_d)$$

Θ_{QCD} $= 0$ due to CP invariance of the Lagrangian

Θ_{QFD} $= 0$ due to the Z_4 symmetry

Leptonic Sector and Leptogenesis

The Z_4 symmetry forbids the inclusion in the Lagrangian of terms of the type:

$$\nu_R^{0T} C M \nu_R^0 \rightarrow \text{not invariant under } Z_4$$

But allows the following couplings:

$$f_\nu \nu_R^{0T} C S \nu_R^0 + f_\nu' \nu_R^{0T} C S^* \nu_R^0$$

After S acquires a vev: $\langle S \rangle = V e^{i\alpha}$

a mass term is generated:

$$M = V(f_\nu^+ \cos\alpha + i f_\nu^- \sin\alpha); \qquad f_\pm^\nu = f_\nu \pm f_\nu'$$

6×6 neutrino mass matrix:

real

$$\mathcal{M} = \begin{bmatrix} 0 & m_D \\ m_D^T & M \end{bmatrix}$$

Complex

Leptogenis is viable

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Conclusions

- We are entering the **Higgs Era** in Particle Physics.
- A crucial question is: Will Higgs particle(s) play an important rôle in Flavour?
- My guess: Yes, it will not be just **Vanilla Higgs**.
- It may take some time until Higgs Flavour be tasted experimentally. We have to be patient and remember **Neutrinos**
- We can safely violate the two Flavour Dogmas.