Quarks and Leptons from Extra dimensions:

from general properties to numbers.



J.-M. Frère

Based on work with Maxim Libanov, Serguey Troitsky, Emin Nugaev, Ling Fu-Sin, Simon Mollet (more details on neutrinos in his talk) latest ref **JHEP 1308 (2013) 078**



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Neutrino-less double beta decay controlled by weighted sum of masses (with phases/signs of mixings entering)





Fermions in 6-D

Basically, a 8-component Dirac spinor, Which can be split into 2 (4-components spinors)

$$\psi = \frac{1 + \Gamma_7}{2}\psi_+ + \frac{1 - \Gamma_7}{2}\psi_-$$

$$\psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

If one thinks already in terms of (3+1) +2 dim, one can further show where the chiral L and R fermions of 4D would appear

There are NO Majorana particles in 6D, and scalar (or mass) terms always involve both + and - « 6-parities »

Still, we can generate 4D Majorana's !!!

We can indeed write 2 main types of 6D scalars:

$$\Phi \quad \overline{\psi}\psi \to \Phi \quad (\overline{\psi_{+L}}\psi_{-R} + \overline{\psi_{+R}}\psi_{-L} + \overline{\psi_{-R}}\psi_{+L} + \overline{\psi_{-L}}\psi_{+R})$$

$$Will load to " Direc" of the logical term of the second s$$

Will lead to « Dirac » style 4D particles

$$\Phi \quad \overline{\psi^C}\psi \to \Phi \quad (\psi_{+R} \bullet \psi_{-R} + \psi_{+L} \bullet \psi_{-L} + \psi_{-L} \bullet \psi_{+L} + \psi_{-R} \bullet \psi_{+R})$$

Will lead to « Majorana » style 4D particles





What are Zero Modes ?

$$i\partial_A \gamma^A \Psi = \Phi \Psi$$

For 2 compact extra dim

Dirac equation for fermion coupled to scalar field

Use of dimensional reduction obtain 3+1-dim chiral spinors (L) : use of topological singularities in the extra dimensions to get zero modes, break LR symmetry.



Vortex with winding number n localizes n chiral massless fermion modes in 3+1







In fact, we work now with the 2 extra dimensions on a sphere (of radius R) , The field profiles are pretty much the same. Compared to the plane, we don't need an extra mechanism to confine the gauge fields close to the vortex...





3 families from one in 5+1 dim



we assume a background scalar field Φ providing a vortex in the 2 extra dimensions; It vanishes at the origin– where we live!

R

For some reason, n=3 !!!

The 3 fermion modes have different shapes, and different winding properties in the extra dimension variable ϕ





The transition from 6-D to the zero- modes of 4D dramatically affects

Start from 4 * 2. components $\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-L} \end{pmatrix}$

4 4-54

Each « zero » (massless) mode only has 2-spinor degrees of freedom: For instance,

 $L \sim \sum_{n} \begin{pmatrix} 0 \\ f_{3-n}(r) \ e^{i(3-n)\phi} \psi_{Ln}(x^{\mu}) \\ f_{n-1}[r) \ e^{i(1-n)\phi} \psi_{Ln}(x^{\mu}) \\ 0 \end{pmatrix}$ 4-d chini











Turn to physics .. Introduce masses,

through the Standard Model Scalar H (Brout-Englert-Higgs field)

We also locate it on the vortex, with its proper shape:











Field Content

	Field	Notation	$U(1)_g$	$U(1)_Y$	$SU(2)_w$	$SU(3)_c$
	vortex scalar	Φ	+1	0	1	1
1	BEH boson	H	0	+1/2	2	1
	auxiliary scalar		+1	0	1	1
	quark $SU(2)_W$ doublet	(Q_+, Q)	(3, 0)	1/6	2	3
	up-type quark $SU(2)_W$ singlet	(U_{+}, U_{-})	(0, 3)	2/3	1	3
	down-type quark $SU(2)_{\rm W}$ singlet	(D_+, D)	(0, 3)	-1/3	1	3
	lepton $SU(2)_{\rm W}$ doublet	(L_{+}, L_{-})	(3, 0)	-1/2	2	1
	charged lepton $SU(2)_{W}$ singlet	(E_{+}, E_{-})	(0, 3)	-1	1	1
	sterile neutrino singlet	(N_{+}, N_{-})	(0, 0)	0	1	1











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Additional couplings involving the vortex field, with winding $e^{i\phi}$ can give the small Cabibbo mixings ϵ

The scheme is very constrained, as the profiles are dictated by the equations, instead of being imposed by hand, like in multilocalisation





Neutrinos ARE different

In the same context (0th order in Cabibbo mixing), we will get indeed (see later):

$$M_{\nu} \sim \left(\begin{array}{ccc} \cdot & \cdot & m \\ \cdot & \mu & \cdot \\ m & \cdot & \cdot \end{array}\right)$$

```
Where m >> \mu
```

After 45° 1-3 rotation and 23 permutation, this leads to an inverted hierarchy, (minute solar mass difference found between the heavier neutrinos)

$$M_{\nu} \sim \left(\begin{array}{ccc} m & \cdot & \cdot \\ \cdot & -m & \cdot \\ \cdot & \cdot & \mu \end{array}\right)$$

The – sign may be absorbed in the mixing matrix, *but contributes destructively to the effective mass for neutrinoless double beta decay* (Pseudo-Dirac structure when full Cabibbo-like mixing is introduced)





WHY the difference? --- return in more detail to the 6D spinors,

Zero modes

 $\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-L} \end{pmatrix}$

4 \$ 1-57 Y J & 1+ [7 J $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ => (R_)



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WHY the difference? --- return in more detail to the 6D spinors,

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

In each case, the massless mode only has 2-spinor degrees of freedom: For instance, $f = \frac{1}{2} \frac{1}$





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WHY the difference? --- return in more detail to the 6D spinors, $\Psi = \begin{bmatrix} \psi_{+L} \\ \psi_{-L} \end{bmatrix}$ For the charged spinors, we have both L and R spinors bound to the vortex. $L \sim \sum_{n} \begin{pmatrix} 0 \\ f_{3-n}(r) \ e^{i(3-n)\phi}\psi_{Ln}(x^{\mu}) \\ f_{n-1}[r) \ e^{i(1-n)\phi}\psi_{Ln}(x^{\mu}) \end{pmatrix} \xrightarrow{R \sim \sum_{n}} \begin{pmatrix} f_{n-1}[r) \ e^{i(1-n)\phi}\chi_{Rn}(x^{\mu}) \\ 0 \\ f_{n-1}[r) \ e^{i(3-n)\phi} \psi_{Ln}(x^{\mu}) \\ f_{n-1}[r) \ e^{i(3-n)\phi} \psi_{Ln}(x^{\mu}) \end{pmatrix}$ Effective Lagrangian : integrate over r and ϕ , undame JMF- Hellenic Workshop, Corfou, 2013 19

Interactions







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We can then introduce extra terms carrying winding, to generate the Cabibbo-like mixings,

In the present formulation, it amounts to factors (ΦX) , where Φ carries winding 1 and X none



Fermion-vortex couplings				
g_L^Q	$0.020 \ \Lambda_{ m V}^{-5}$			
g_R^U	$0.005~\Lambda_{ m V}^{-5}$			
g_R^D	$0.063 \ \Lambda_{ m V}^{-5}$			
g_L^L	$0.013~\Lambda_{ m V}^{-5}$			
g_R^E	$0.013~\Lambda_{ m V}^{-5}$			
"Yukawa" fermion-scalars couplings				
$H\bar{Q}_+D$	7.00 $\Lambda_{\rm V}^{-1}$			
$HX^{*}\Phi\bar{Q}_{+}D_{-}$	14.0 $\Lambda_{\rm V}^{-5}$			
$\tilde{H}\bar{Q}_+U$	850 $\Lambda_{\rm V}^{-1}$			
$\tilde{H}X^*\Phi\bar{Q}_+U$	$255 \Lambda_V^{-5}$			
$HX^*\Phi \bar{L}_+E$	56 $\Lambda_{ m V}^{-5}$			
$HX\Phi^*\bar{L}_+E$	$-160 \Lambda_{\rm V}^{-5}$			
$\tilde{H}\bar{L}_{-}N_{+}$	$0.165 \ \Lambda_{ m V}^{-1}$			
$\tilde{H}X\Phi^*\bar{L}N_+$	$(0.91 + 1.04i) \cdot 10^{-2} \Lambda_{\rm V}^{-5}$			
$\tilde{H}(X^*)^4 \Phi \bar{L}_+ N$	$1.00 \ \Lambda_{ m V}^{-11}$			

(the last coupling is needed, but can be set to 1, as its value is redundant with others..)





Quark masses at Z scale				
m_d	$0.01{ m GeV}$	$(0.00282 \pm 0.00048) \mathrm{GeV}$		
m_s	$0.051{ m GeV}$	$(0.057^{+0.018}_{-0.012}){ m GeV}$		
m_b	$2.86\mathrm{GeV}$	$2.86^{+0.16}_{-0.06}{ m GeV}$		
m_u	$0.023{ m GeV}$	$0.00138^{+0.00042}_{-0.00041}{ m GeV}$		
m_c	$0.72{ m GeV}$	$0.638^{+0.043}_{-0.084}{ m GeV}$		
m_t	$172{ m GeV}$	$172.1\pm1.2{\rm GeV}$		
Quark mixing matrix				
$ U_{\rm CKM} $	$\begin{pmatrix} 0.979 & 0.207 & 0.0015 \\ 0.206 & 0.9730 & 0.046 \\ 0.011 & 0.049 & 0.999 \end{pmatrix}$	$ \begin{pmatrix} 0.97427 \pm 0.00015 \ 0.22534 \pm 0.00065 \ 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 \ 0.97344 \pm 0.00016 \ 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} \ 0.0404^{+0.0011}_{-0.0005} \ 0.999146^{+0.000021}_{-0.00046} \end{pmatrix} $		
Charged-lepton masses				
m_e	$0.00061{ m GeV}$	$0.0004866\mathrm{GeV}$		
m_{μ}	$0.089{ m GeV}$	$0.1027\mathrm{GeV}$		
$m_{ au}$	$1.74{ m GeV}$	$1.746\mathrm{GeV}$		





	Neutrino masses			
	m_1	$5.46\cdot 10^{-2}\mathrm{eV}$		
	m_2	$5.53\cdot 10^{-2}\mathrm{eV}$		
	m_3	$4.17\cdot 10^{-5}\mathrm{eV}$		
	Δm_{21}^2	$7.96 \cdot 10^{-5} \text{ eV}^2$	$(7.50 \pm 0.185) \cdot 10^{-5} \text{ eV}^2$	
	Δm^2_{13}	$2.98\cdot 10^{-3}~\mathrm{eV^2}$	$(2.47^{+0.069}_{-0.067}) \cdot 10^{-3} \text{ eV}^2$	
	Lepton mixing matrix			
Note that CP phase is needed for lepton fit (see Simon Mollet's	$ U_{\rm PMNS} $	$\begin{pmatrix} 0.76 & 0.63 & 0.13 \\ 0.39 & 0.58 & 0.72 \\ 0.52 & 0.52 & 0.68 \end{pmatrix}$	$\simeq \begin{pmatrix} 0.795 - 0.846 & 0.513 - 0.585 & 0.126 - 0.178 \\ 0.205 - 0.543 & 0.416 - 0.730 & 0.579 - 0.808 \\ 0.215 - 0.548 & 0.409 - 0.725 & 0.567 - 0.800 \end{pmatrix}$	
talk)	$\langle m_{etaeta} angle$	$0.013\mathrm{eV}$	$\lesssim 0.3 \mathrm{eV}$ [31]	
	J	0.019	$\lesssim 0.036$	
	θ_{12}	39.7°	$\simeq (31.09^{\circ} - 35.89^{\circ})$	
	θ_{23}	46.5°	$\simeq (35.8^{\circ} - 54.8^{\circ})$	
	θ_{13}	7.2°	$\simeq (7.19^{\circ} - 9.96^{\circ})$	



Note a non-vanishing θ_{13} was predicted (in previous version) *before observation*







Some other developments :

- phenomenological implications of the excited modes..

- constraint on B-E-H boson (Libanov and Nugaev: LIGHT) (the tuning of the vev simultaneously leads to a light Standard Model Scalar)

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 $\begin{array}{l} \textbf{IMPORTANT}: & (n) is approximatively \\ \textbf{conserved}! & - e^{in\phi} \text{ plays somewhat like a U(1) horizontal symmetry} \end{array}$

2 extra dim : → II gauge bosons, possess 2 types of Kaluza- Klein excitations in particular, Z and Gluons

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« family number » (n) is approximatively conserved $!\,$ - somewhat like U(1) horizontal symmetry $e^{i\varphi}$

$K_L \rightarrow \mu^- e^+ \text{ or } \mu^+ e^- B.R. < 10^{-12}$

Expect thus typical mass scale $M_{Z1} / \kappa > (10^{12})^{1/4} M_Z$ $M(Z_1) > \kappa 100 \text{ TeV}$

In fact, the small overlap of wave functions implies some suppression of the coupling; K < 1In our calculations κ is κ modest $\sim (0.5) \rightarrow$ inaccessible to LHC

But this may be an example of a broader class of models, with different geometry.... So, explore the possibility in « model independent » way Take κ from .01 to 0.5 \rightarrow Plot for M(Z₁)>1TeV--

are ONE ORDER below at LHC,due to quark content of protons

Fig. 1. Number of events for $(\mu^+ e^-)$ pairs production as a function of the vector bosons mass M, with $\kappa = M/(100 \text{TeV})$. (also s left in underlying event)

See.JETP Lett.79:598-601,2004, Pisma Zh.Eksp.Teor.Fiz.79:734-737,2004. JMF, M Libanov, S Troitsky, E Nugaev hep-ph/0404139

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LHC thus has the potential (in a specific model, of course) to beat even the very sensitive fixed-target K $\rightarrow \mu e$ limit!

t + c or $\overline{b} + s$ are similarly produced by the **gluon excitations**,

Expect a **few 1000's events** --- but must consider background!

Fundamen

Interactions

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Returning to our 6D model, most striking result: Neutrino masses ARE different, and we get inverted hierarchy, pseudo-Dirac, and large mixings as automatic features

In a nutshell:

One family in 6D and proper boundary conditions \rightarrow 3 families in 6D •At lowest order in Cabibbo mixing Charged fermion masses are

- Diagonal
- Strongly hierarchical

At same order, we get 4D Majorana neutrinos with Antidiagonal mass matrix
This yields, in a generic way: Large mixings in the neutrino sector

Inverted Hierarchy

Pseudo- Dirac structure (further suppression of neutrinoless

double beta decay)

•Not as automatic, but typical : measurable Θ_{13}

Canonical model beyond LHC reach, but variants could result in std model like Z' and later more exotic signals (Z' $\pm \rightarrow \mu^+ e^- \gg \mu^- e^+$)

