NOVEL T-VIOLATION OBSERVABLE OPEN TO ANY DECAY CHANNEL AT MESON FACTORIES

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1 INTRODUCTION

The BABAR Collaboration has recently reported, in the $B^0 - \overline{B}^0$system, the first direct observation of T-violation [1207.5832], in the time evolution of any system with high statistical significance. The measurement is based in a method described in [JHEP 1208,064 (2012)] following the concepts originally proposed in [Phys. Lett. B 464,117(1999),4 and Nucl. Phys. B 590, 19 (2000)].

In the $\left\{ \left| P^0 \right\rangle, \left| \overline{P}^0 \right\rangle \right\}$ system , where P^0 stands for a neutral meson K^0, D^0, B_d^0 or B_s^0 (and $\overline{P}^0$ for the corresponding antimeson) , we define two arbitrary states:

$$
\begin{aligned}
|M_1\rangle &= m_1^0 \left| P^0 \right\rangle + \overline{m}_1^0 \left| \overline{P}^0 \right\rangle \\
|M_2\rangle &= m_2^0 \left| P^0 \right\rangle + \overline{m}_2^0 \left| \overline{P}^0 \right\rangle
\end{aligned}
$$

The T-VIOLATION observable is then given by

$$A_{12}(t) = \frac{\mathsf{Pr}\left[M_1 \rightarrow M_2(t)\right] - \mathsf{Pr}\left[M_2 \rightarrow M_1(t)\right]}{\mathsf{Pr}\left[M_1 \rightarrow M_2(t)\right] + \mathsf{Pr}\left[M_2 \rightarrow M_1(t)\right]}$$

where

$$\mathsf{Pr}\left[M_1 \rightarrow M_2(t)\right] = \left|\langle M_2 | M_1(t)\rangle\right|^2 = \left|\langle M_2 | U(t,0) | M_1\rangle\right|^2$$

is the probability that an initially prepared state M_1, evolving after time t to $M_1(t)$, behave like state M_2.

The BABAR asymmetry is build from $M_1 = B_d^0$, $\overline{B}_d^0$ and $M_2 = B_\pm$ where $B_\pm$ are the B-states tagged or filtered by the decays to CP-eigenstates with definite flavour content. These asymmetries are experimentally independent of CP violation. The Kabir asymmetry, with $M_1, M_2 = K^0, \overline{K}^0$ was measured by the CPLEAR Collaboration [Phys.Lett. B444 (1998) 43-51] with a non-vanishing value near 4 standard deviations.

2 THE REFERENCE TRANSITION AT $C = -$ INITIAL ENTANGLED STATE

The two quantum effects:

- Entanglement between the two neutral mesons produced at a meson factory and
- The filtering measurement induced by the meson decay

are essential ingredients to be incorporated in the analysis for the preparation of initial and final meson states

We want to find out which is the Reference Transition $M_1 \to M_2(t)$ between meson states - some particular combination of P^0 and $\overline{P}^0$- associated to a given pair of decays: f_1 at t_1 and f_2 at $t_2 > t_1$ if any (f_1, f_2).

The entangled state of the two mesons in an antisymmetric combination of individual orthogonal states,

$$\left|\Phi_{(C=-)}\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|P^0\left(\overrightarrow{k}\right)\right\rangle\left|\overline{P}^0\left(\overrightarrow{-k}\right)\right\rangle - \left|\overline{P}^0\left(\overrightarrow{k}\right)\right\rangle\left|P^0\left(-\overrightarrow{k}\right)\right\rangle\right\}$$

tells us that the (still living) meson at time t_1 is tagged as "the state that does not decay into f"

$$\left|P_{\nrightarrow f}\right\rangle = \frac{1}{\sqrt{\left|A_f\right|^2 + \left|\overline{A}_f\right|^2}}\left[\overline{A}_f\left|P^0\right\rangle - A_f\left|\overline{P}^0\right\rangle\right]$$

where $A_f\left(\overline{A}_f\right)$ is the decay amplitude from $P^0\left(\overline{P}^0\right)$ to f:

$$A_f = \langle f|\, W \left|P^0\right\rangle \quad ; \quad \overline{A}_f = \left\langle \overline{f}\right| W \left|P^0\right\rangle$$

to first order in the weak Hamiltonian H_w and to all orders in strong interactions $W = U_S(\infty, 0)\, H_w$. $U_S(\infty, 0)$ is the strong evolution operator and is equal to the identity if we can neglect final sate interactions. (For hadronic decays we will assume transitions with one helicity amplitude: $0 \to 0 + j$)

The corresponding orthogonal state $\left\langle P^{\perp}_{\nrightarrow f} | P_{\nrightarrow f}\right\rangle = 0$ is given by

$$\left|P^{\perp}_{\nrightarrow f}\right\rangle = \frac{1}{\sqrt{\left|A_f\right|^2 + \left|\overline{A}_f\right|^2}} \left[A_f^* \left|P^0\right\rangle + \overline{A}_f^* \left|\overline{P}^0\right\rangle\right]$$

and it is the one filtered by the decay. What we call the "filtering identity" define the precise meaning of this statement:

$$\left|\left\langle P^{\perp}_{\nrightarrow f_2}|M_1\right\rangle\right|^2 = \frac{\left|\langle f_2|\, W\, |M_1\rangle\right|^2}{\left(\left|A_{f_2}\right|^2 + \left|\overline{A}_{f_2}\right|^2\right)}$$

Note that we can also writte

$$\begin{aligned}\left|\Phi_{(C=-)}\right\rangle &= \frac{1}{\sqrt{2}}\left\{\left|P^0\left(\overrightarrow{k}\right)\right\rangle\left|\overline{P}^0\left(\overrightarrow{-k}\right)\right\rangle - \left|\overline{P}^0\left(\overrightarrow{k}\right)\right\rangle\left|P^0\left(-\overrightarrow{k}\right)\right\rangle\right\} = \\ &= \frac{1}{\sqrt{2}}\left\{\left|P^{\perp}_{\nrightarrow f}\left(\overrightarrow{k}\right)\right\rangle\left|P_{\nrightarrow f}\left(\overrightarrow{-k}\right)\right\rangle - \left|P_{\nrightarrow f}\left(\overrightarrow{k}\right)\right\rangle\left|P^{\perp}_{\nrightarrow f}\left(-\overrightarrow{k}\right)\right\rangle\right\}\end{aligned}$$

Experimentally, the Reference Transition $M_1 \rightarrow M_2(t)$ is therefore directly connected to $M_1 = P_{\nrightarrow f_1}$, $M_2 = P^{\perp}_{\nrightarrow f_2}$, i.e.,

$$P_{\nrightarrow f_1}(t_1) \rightarrow P^{\perp}_{\nrightarrow f_2}(t_2)$$

The T transformed transition

$$P^{\perp}_{\nrightarrow f_2}(t_1) \rightarrow P_{\nrightarrow f_1}(t_2)$$

does not correspond to the pair of decays f_2 at t_1 , f_1 at $t_2 > t_1$, neither in the initial nor in the final decays. This is the "orthogonality problem" that prevents taking an arbitrary pair of decay channels.

To connect the T transformed transition with experiment, we need to find a pair of decay channels such that, for each of them,

$$\text{Given } f \rightarrow \exists\ f' \ / \ \left|P_{\nrightarrow f'}\right\rangle = \left|P^{\perp}_{\nrightarrow f}\right\rangle$$

This condition is satisfied by either CP conjugate decay channels $\left(P^0, \overline{P}^0\right)$ or CP-eigenstates of opposite sign with the same flavour content (P_+, P_-) and no direct CP violation. Hence the exceptionality of the transitions between semileptonic and

CP eigenstate decays. The method has also been applied to $K^0 - \overline{K}^0$ at a Φ-factory
.

As a consequence, the orthogonality condition limits the pair of decay channels suitable for T-symmetry tests if we start, as a Reference, from the "experimental" transition $(P_{\not\to f_1}(t_1) \to P^{\perp}_{\not\to f_2}(t_2))$. This limitations is the prize to be paid to obtain a genuine T-violation asymmetry:

given f what is the decay channel f' such that the P state not going to f' coincides with the orthogonal state to the P state not going to f ? $\left(\lambda_{f'}\lambda_f^* = -\left|q/p\right|^2 \sim -1\right)$

The scientific community was recently interested in extending the T-symmetry tests to additional pairs of decay channels, in a program similar to the one developed for CP violation studies. How to do it ?

Having identified the orthogonality problem, we give a bypass consisting in having an alternative reference transition once we fix the two channels f_1, f_2.

We make the following replacements

$$\left\{ \begin{array}{cccc} & \text{OLD} & & \text{NEW} \\ \begin{array}{c}\text{REFERENCE}\\ \text{TRANSITION}\end{array} & P_{\nrightarrow f_1}(t_1) \rightarrow P^{\perp}_{\nrightarrow f_2}(t_2) & \Longrightarrow & P_{\nrightarrow f_1}(t_1) \rightarrow P_{\nrightarrow f_2}(t_2) \\ \begin{array}{c}\text{T TRANSFORMED}\\ \text{TRANSITION}\end{array} & P^{\perp}_{\nrightarrow f_2}(t_1) \rightarrow P_{\nrightarrow f_1}(t_2) & \Longrightarrow & P_{\nrightarrow f_2}(t_1) \rightarrow P_{\nrightarrow f_1}(t_2) \end{array} \right\}$$

Whereas now the two initial P-states are directly connected to experiment, the price to be paid now is that the two final state are not. One has to work out what is the connection of the novel genuine theoretical T-asymmetry observable to experimental measurements.

3 THE NEW T-VIOLATING ASYMMETRY

As stated, the novel asymmetry proposed for a T-symmetry test is then

$$A(f_1, f_2; t) \equiv \frac{\text{Pr}\left[P_{\nrightarrow f_1} \rightarrow P_{\nrightarrow f_2}(t)\right] - \text{Pr}\left[P_{\nrightarrow f_2} \rightarrow P_{\nrightarrow f_1}(t)\right]}{\text{Pr}\left[P_{\nrightarrow f_1} \rightarrow P_{\nrightarrow f_2}(t)\right] + \text{Pr}\left[P_{\nrightarrow f_2} \rightarrow P_{\nrightarrow f_1}(t)\right]}$$

where $\text{Pr}\left[P_{\nrightarrow f_1} \rightarrow P_{\nrightarrow f_2}(t)\right]$ is the probability that an initially prepared $\left|P_{\nrightarrow f_1}\right\rangle$ becomes after a time t an $\left|P_{\nrightarrow f_2}\right\rangle$ state. If $\left|P_{\nrightarrow f_1}(t)\right\rangle$ is the evolved state at time t of a $\left|P_{\nrightarrow f_1}\right\rangle$ at time $t = 0$, then this probability will be:

$$\text{Pr}\left[P_{\nrightarrow f_1} \rightarrow P_{\nrightarrow f_2}(t)\right] = \left|\left\langle P_{\nrightarrow f_2} \middle| P_{\nrightarrow f_1}(t)\right\rangle\right|^2$$

This probability can be rewritten, using closure in the two-dimensional space of the meson system as

$$\begin{aligned}\mathsf{Pr}\left[P_{\nrightarrow f_1} \to P_{\nrightarrow f_2}(t)\right] &= \left\langle P_{\nrightarrow f_1}(t) | P_{\nrightarrow f_2}\right\rangle \left\langle P_{\nrightarrow f_2} | P_{\nrightarrow f_1}(t)\right\rangle = \\ &= \left\langle P_{\nrightarrow f_1}(t)\right| \left[I - \left|P^{\perp}_{\nrightarrow f_2}\right\rangle \left\langle P^{\perp}_{\nrightarrow f_2}\right|\right] \left|P_{\nrightarrow f_1}(t)\right\rangle = \\ &= \left\langle P_{\nrightarrow f_1}(t) | P_{\nrightarrow f_1}(t)\right\rangle - \left|\left\langle P^{\perp}_{\nrightarrow f_2} | P_{\nrightarrow f_1}(t)\right\rangle\right|^2\end{aligned}$$

with the second term directly connected to the probability for the decay of a tagged $\left|P_{\nrightarrow f_1}\right\rangle$ state at t_1 to f_2 after a time $t = t_2 - t_1$ using the "filtering identity"

$$\mathsf{Pr}\left[P_{\nrightarrow f_1} \to P_{\nrightarrow f_2}(t)\right] = \left\langle P_{\nrightarrow f_1}(t) | P_{\nrightarrow f_1}(t)\right\rangle - \frac{\left|\langle f_2| W \left|P_{\nrightarrow f_1}(t)\right\rangle\right|^2}{\left(\left|A_{f_2}\right|^2 + \left|\overline{A}_{f_2}\right|^2\right)}$$

Note that also the first term in the right-hand-side $\left\langle P_{\nrightarrow f_1}(t) | P_{\nrightarrow f_1}(t)\right\rangle$ is a well defined and measurable quantity: the ratio between the number of mesons that have not decayed after a time t and the initial number tagged at t_1 as $\left|P_{\nrightarrow f_1}\right\rangle$ by the observation of the first decay f_1. We may then call this term the "total survival probability".

The other piece $\left|\langle f_2|\, W \left|P_{\nrightarrow f_1}(t)\right\rangle\right|^2 / \left(\left|A_{f_2}\right|^2 + \left|\overline{A}_{f_2}\right|^2\right)$ is the "normalized" usual double rate at a meson factory with the decaying states f_1 and f_2 ordered in time $\left|\langle f_2|\, W \left|P_{\nrightarrow f_1}(t)\right\rangle\right|^2 / \left(\left|A_{f_2}\right|^2 + \left|\overline{A}_{f_2}\right|^2\right)$

Similarly, for the T transformed transition associated to the Motion Reversal Asymmetry. Therefore we emphasize that this new observable becomes entirely measurable in a 1^{--} meson factory for any pair of decay channels f_1, f_2.

We have not imposed any particular condition to the pair of decay channels f_1, f_2, so one is not forced to use flavour specific or CP eigenstate decay channels. We will discuss later whether the measurable $A(f_1, f_2; t)$ becomes a genuine Time-Reversal-Violation Asymmetry for any pair of decays .

4 THE THEORETICAL EXPRESSIONS

Using the time evolution imposed by Quantum Mechanics we know the time dependent structure of the needed measurable quantities in terms of Δm and $\Delta\Gamma$, determined by the eigenvalues of the entire Hamiltonian for the $\left(P^0, \overline{P}^0\right)$ system

$$\left\langle P_{\nrightarrow f}(t) \,|P_{\nrightarrow f}(t)\right\rangle = e^{-\Gamma t} \left\{ \begin{array}{c} \mathcal{C}_h[f] \cosh\left(\frac{\Delta\Gamma t}{2}\right) + \mathcal{C}_c[f] \cos(\Delta m t) + \\ +\mathcal{S}_h[f] \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \mathcal{S}_c[f] \sin(\Delta m t) \end{array} \right\}$$

$$\frac{\left|\langle g|\, W \left|P_{\nrightarrow f}(t)\right\rangle\right|^2}{\left(|A_g|^2 + \left|\overline{A}_g\right|^2\right)} = e^{-\Gamma t} \left\{ \begin{array}{c} \mathfrak{C}_h[f,g] \cosh\left(\frac{\Delta\Gamma t}{2}\right) + \mathfrak{C}_c[f,g] \cos(\Delta m t) \\ +\mathfrak{S}_h[f,g] \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \mathfrak{S}_c[f,g] \sin(\Delta m t) \end{array} \right\}$$

In other words, each one of the two transitions is determined by the four measurable parameters $\mathcal{C}_h[f]$, $\mathcal{C}_c[f]$, $\mathcal{S}_h[f]$, $\mathcal{S}_c[f]$ and $\mathfrak{C}_h[f,g]$, $\mathfrak{C}_c[f,g]$, $\mathfrak{S}_h[f,g]$, $\mathfrak{S}_c[f,g]$.

The numerator of the asymmetry $N(f,g;t)$:

$$A(f,g;t) = \frac{\mathsf{Pr}\left[P_{\nrightarrow f} \to P_{\nrightarrow g}(t)\right] - \mathsf{Pr}\left[P_{\nrightarrow g} \to P_{\nrightarrow f}(t)\right]}{\mathsf{Pr}\left[P_{\nrightarrow f} \to P_{\nrightarrow g}(t)\right] + \mathsf{Pr}\left[P_{\nrightarrow g} \to P_{\nrightarrow f}(t)\right]} = \frac{N(f,g;t)}{D(f,g;t)}$$

is then given by three "asymmetry parameters": the non-vanishing value of any of these asymmetry parameters would be a signal of Time-Reversal-Violation.

$$N(f,g;t) = e^{-\Gamma t}\left\{\begin{array}{c} \mathcal{C}^N[f,g]\left(\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \cos(\Delta m t)\right) + \\ +\mathcal{S}_h^N[f,g]\sinh\left(\frac{\Delta\Gamma t}{2}\right) + \mathcal{S}_c^N[f,g]\sin(\Delta m t) \end{array}\right\}$$

The theoretical connection of these measurable parameters to the matrix element of the meson evolution Hamiltonian H_{ij} involved in the Weisskopf-Wigner Approach

(WWA) for the $\left(P^0, \overline{P}^0\right)$ system is

$$\begin{aligned}
\mathcal{C}^N[f,g] &= \delta\left(C_f - C_g\right) - \mathrm{Im}\,(\theta)\left(S_f - S_g\right) \\
\mathcal{S}_h^N[f,g] &= \delta\left(C_f R_g - C_g R_f\right) + \mathrm{Im}\,(\theta)\left(R_f S_g - R_g S_f\right) \\
\mathcal{S}_c^N[f,g] &= \left(C_f S_g - C_g S_f\right)\left\{1 + \delta\left(C_f + C_g\right)\right\} + \\
&\quad + \delta\left(S_f - S_g\right) + \mathrm{Re}\,(\theta)\left(R_f S_g - R_g S_f\right)
\end{aligned}$$

Where we have presented the results at leading order in terms of the usual mixing parameters -see the book of G. Branco et al. - $\delta = \left(1 - |q/p|^2\right) / \left(1 + |q/p|^2\right)$ that is real, T and CP violating and the complex parameter θ that is CPT and CP violating $\theta = (H_{22} - H_{11}) / (\Delta m - i\Delta\Gamma/2)$. We also use the parameter R_i, S_i and C_i defined in terms of the wellknown $\lambda_i = q\overline{A}_{f_i}/pA_{f_i}$

$$C_i = \frac{1-|\lambda_i|^2}{1+|\lambda_i|^2} \quad : \quad S_i = \frac{2\,\mathrm{Im}(\lambda_i)}{1+|\lambda_i|^2} \quad ; \quad R_i = \frac{2\,\mathrm{Re}(\lambda_i)}{1+|\lambda_i|^2}$$

where q/p in the CPT violating case is defined in the usual way $(q/p)^2 = H_{21}/H_{12}$. Note that this parameters are not all independent $C_i^2 + S_i^2 + R_i^2 = 1$.

In order to show how the new T-violating observable really open the T analysis to "all channels" at B-factory we will concentrate in the case where we choose f to be a flavour specific channel corresponding to $\left|P_{\nrightarrow f}\right\rangle = \left|\overline{B}^0\right\rangle \left(\left|P_{\nrightarrow f}\right\rangle = \left|\overline{B}^0\right\rangle\right)$and $\overline{A}_f = 0 \left(A_f = 0\right)$ therefore $\lambda_f = 0 \left(\lambda_f = \infty\right)$

$$C_f = 1\left(C_f = -1\right) \quad ; \quad S_f = 0 \quad ; \quad R_f = 0$$

To avoid complications we also use the approximation $\Delta\Gamma = 0$ and $\delta = 0$. We will use, even if misleading, the notation $f = B^0 \left(f = \overline{B}^0\right)$. Under this conditions we have in any channel g for the numerator of the asymmetry

$$\begin{aligned} N\left(B^0, g; t\right) &= e^{-\Gamma t} S_g \{\sin\left(\Delta m t\right) + \mathrm{Im}\,\theta\left[1 - \cos\left(\Delta m t\right)\right]\} \\ N\left(\overline{B}^0, g; t\right) &= e^{-\Gamma t} S_g \{-\sin\left(\Delta m t\right) + \mathrm{Im}\,\theta\left[1 - \cos\left(\Delta m t\right)\right]\} \end{aligned}$$

For completness we also give the denominator

$$D\left(B^0, g; t\right) = e^{-\Gamma t}\left\{\begin{array}{c}(1 + C_g \cos(\Delta m t)) - R_g \,\mathrm{Re}\,\theta\,(1 - \cos(\Delta m t)) + \\ + \mathrm{Im}\,\theta\,(1 + C_g)\sin(\Delta m t)\end{array}\right\}$$

$$D\left(\overline{B}^0, g; t\right) = e^{-\Gamma t}\left\{\begin{array}{c}(1 - C_g \cos(\Delta m t)) + R_g \,\mathrm{Re}\,\theta\,(1 - \cos(\Delta m t)) - \\ \mathrm{Im}\,\theta\,(1 - C_g)\sin(\Delta m t)\end{array}\right\}$$

Note that for $g = J/\psi K^0_S$ we get an asymmetry of the same order that the one measured by BABAR.

$$A\left(B^0, J/\psi K^0_S; t\right) \sim S_{J/\psi K^0_S} \sin(\Delta m t) \sim \sin(2\beta)\sin(\Delta m t)$$

But of course experimentally is completly different: here there is no $J/\psi K^0_L$. It is interesting to note that also

$$\left\langle P_{\nrightarrow f}(t) \middle| P_{\nrightarrow f}(t)\right\rangle = e^{-\Gamma t}\left\{1 + \mathcal{O}(\delta) + \mathcal{O}(\theta) + R_f \mathcal{O}\left(\frac{\Delta\Gamma t}{2}\right)\right\}$$

5 SELECTING A FEW CHANNELS

A suferficial inspection of the numerator of our asymmetry would indicate that the existence of a non-vanishing factor S_g is, by itself, a signal of T violation. This conclusion would be wrong in general. The problem can be traced back to antiunitary character of the operator U_T implementing the T symmetry in the space of physical states. The principle of T-invariance or microreversibility estrictly means that

$$\left|\left\langle \underline{U_T P_{\nrightarrow f}} \right| U(t,0) \left| \underline{U_T P_{\nrightarrow g}} \right\rangle\right|^2 = \left|\left\langle P_{\nrightarrow g} \right| U(t,0) \left| P_{\nrightarrow f} \right\rangle\right|^2$$

but not in general

$$\left|\left\langle \underline{P_{\nrightarrow f}} \right| U(t,0) \left| \underline{P_{\nrightarrow g}} \right\rangle\right|^2 = \left|\left\langle P_{\nrightarrow g} \right| U(t,0) \left| P_{\nrightarrow f} \right\rangle\right|^2$$

being $U_T = \widehat{U}_T K$ where K is the complex conjugation operator and $\widehat{U}_T$ is a unitary operator almost irrelevant in the meson case. The possible difficulty in asigning a genüine character of T violation to the numerator of our asymmetry is concentrated on K. Sufficient conditions to satisfy

$$U_T \left| P_{\nrightarrow f} \right\rangle = e^{i\varphi} \left| P_{\nrightarrow f} \right\rangle$$

in the "T invariant limit" will be presented in a future publication. Here we mention an incomplete list of types of decay channels that can be used with the effect of U_T being a global phase factor:

1. Flavour Specific (FS) channels.

2. CP eigenstates without direct CP violation. The absence of direct CP violation in the decay to CP eigenstates makes irrelevant the presence of the complex conjugation operator, in the sense that it changes the global phase of $\left| P_{\nrightarrow f_i} \right\rangle$ and therefore $S_g \neq 0$ is a genüine signal of T violation.

3. For arbitrary states, including non CP eigenstates, without CP violation in the decay one is able to prove

$$\lambda_g \lambda_{\overline{g}} = e^{-2i\xi} \left(\frac{q}{p} \right)^2$$

where ξ is the arbitrary phase of the CP operator $U_{CP} \left| P^0 \right\rangle = e^{i\xi} \left| \overline{P}^0 \right\rangle$ in such a way that $\lambda_g \lambda_{\overline{g}} \neq 1$ is signal of T violation ($(q/p)^2 = e^{2i\xi}$ in the T invariant limit). As a consequence the genüine signal of T violation is given by the condition $S_g + S_{\overline{g}} \neq 0$.

4. A decay product that is an eigenstates of the Strong Scattering Matrix. One case of possible interest is that of the $K^0, \overline{K}^0 \rightarrow (\pi\pi)_I$ channels with welldefined Isospin I at a Φ-factory.

It is worth noticing that the asymmetries constructed with the combination of semileptonic and CP eigenstate decay channels are now experimentally different from those already measured by BABAR Collaboration. A case of particular interest is given by the reference transition B^0 or $\overline{B}^0$, leading to $J/\psi K_S^0$ which does not involve in the novel asymmetry $J/\psi K_L^0$. This case can be extended to other charmonium final states and many more channels like $\phi K_S^0, \pi^0 K_S^0, \rho^0 K_S^0, J/\psi \pi^0$ and $(\pi\pi)_{I=2}$ as some interesting examples.

For the decay products which are not CP eigenstates, our results suggest the use of $N\left(B^0, g; t\right)$ and $N\left(B^0, \overline{g}; t\right)$ which can be combined to avoid any possible presence of fake T violation in the individual transitions. The following observable:

$$N\left(B^0, g; t\right)+N\left(B^0, \overline{g}; t\right) = e^{-\Gamma t}\left(S_g + S_{\overline{g}}\right)\{\sin(\Delta m t) + \mathrm{Im}\,\theta\,[1-\cos(\Delta m t)]\}$$

for decay channels $g, \overline{g}$ without direct CP violation is a genuine T violating observable. Among those channels we have, for example, $g = D^*(2010)^+ D^-, \overline{g} =$

$D^* (2010)^- D^+$ where this asymmetry should be almost maximal: $\left(S_g + S_{\overline{g}}\right) = 2 * (-0.73 \pm 0.11)$. Other channels like $D^{(*)\pm}\pi^{\mp}$ or even in $B_s^0 \rightarrow D_s^{\pm} K^{\mp}$ should be of interest.

A final remark concerns the fact that we can combine several asymmetry numerators to avoid the measurement of the total survival probabilities:

$$N(f_1, f_2) + N(f_2, f_3) + N(f_3, f_1)$$

6 CONCLUSIONS

- We have modified the reference transition of BABAR T violating measurement $P_{\nrightarrow f_1}(t_1) \rightarrow P^{\perp}_{\nrightarrow f_2}(t_2)$ - directly connected to decay $\Upsilon(4S) \rightarrow \underset{\rightarrow f_1 \text{ at } t_1}{P} \underset{\rightarrow f_2 \text{ at } t_2}{P}$ - by $P_{\nrightarrow f_1}(t_1) \rightarrow P_{\nrightarrow f_2}(t_2)$.

- With the New Reference we proposed in addition to the Babar and CPLEAR asymmetries

$$A_{12}(t) \equiv \frac{\text{Pr}\left[P_{\nrightarrow f_1} \rightarrow P^{\perp}_{\nrightarrow f_2}(t)\right] - \text{Pr}\left[P^{\perp}_{\nrightarrow f_2} \rightarrow P_{\nrightarrow f_1}(t)\right]}{\text{Pr}\left[P_{\nrightarrow f_1} \rightarrow P^{\perp}_{\nrightarrow f_2}(t)\right] + \text{Pr}\left[P^{\perp}_{\nrightarrow f_2} \rightarrow P_{\nrightarrow f_1}(t)\right]}$$

the new one

$$A\left(f_1, f_2; t\right) \equiv \frac{\mathsf{Pr}\left[P_{\nrightarrow f_1} \rightarrow P_{\nrightarrow f_2}(t)\right] - \mathsf{Pr}\left[P_{\nrightarrow f_2} \rightarrow P_{\nrightarrow f_1}(t)\right]}{\mathsf{Pr}\left[P_{\nrightarrow f_1} \rightarrow P_{\nrightarrow f_2}(t)\right] + \mathsf{Pr}\left[P_{\nrightarrow f_2} \rightarrow P_{\nrightarrow f_1}(t)\right]}$$

- The connection to the experimental observables is given by the survival probability and the normalized double decay rate to f_1 at t_1 and f_2 a later time $t_2 = t + t_1$

$$\begin{aligned} \mathsf{Pr}\left[P_{\nrightarrow f_1} \rightarrow P_{\nrightarrow f_2}(t)\right] &= \left\langle P_{\nrightarrow f_1}(t) \middle| P_{\nrightarrow f_1}(t)\right\rangle - \left|\left\langle P^{\perp}_{\nrightarrow f_2} \middle| P_{\nrightarrow f_1}(t)\right\rangle\right|^2 = \\ &= \left\langle P_{\nrightarrow f_1}(t) \middle| P_{\nrightarrow f_1}(t)\right\rangle - \frac{\left|\langle f_2| W \left| P_{\nrightarrow f_1}(t)\right\rangle\right|^2}{\left(\left|A_{f_2}\right|^2 + \left|\overline{A}_{f_2}\right|^2\right)} \end{aligned}$$

- The proposed asymmetry can be used for decay products that include flavour specific, CP eigenstates and non CP eigenstates. In the last case one has to combine the reference transitions $B^0 \to g$ and $B^0 \to \overline{g}$. In this way we eliminate possible sources of fake T violation.

- With the proposal here discussed, the way is more open to a full experimental program of studies of T violation observables at meson factories.

- $$\hat{U}_T = \begin{pmatrix} e^{i(\nu-\xi)} & 0 \\ 0 & e^{i(\nu+\xi)} \end{pmatrix}$$

  where ν is the phase of the CPT operator $U_{CPT}\left|P^0\right\rangle = e^{i\nu}\left|\overline{P}^0\right\rangle$ and $U_{CP}\left|P^0\right\rangle = e^{i\xi}\left|\overline{P}^0\right\rangle$

- The condition

  $$U_T\left|P_{\nrightarrow f}\right\rangle = e^{i\varphi}\left|P_{\nrightarrow f}\right\rangle$$

  becomes

  $$A_f\overline{A}_f^* = A_f^*\overline{A}_f e^{2i\xi}$$