Supersymmetry Breaking and Higher Dimension Operators

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"Structure Of The Scalar Potential In General N=1 Higher Derivative Supergravity In Four-dimensions,"
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Supersymmetry: The chiral multiplet

- The simplest supersymmetric Lagrangian is

$$\mathcal{L}_0 = A\partial^2 \bar{A} + i\partial_a \bar{\chi}\bar{\sigma}^a \chi + F\bar{F} \tag{1}$$

- The supersymmetry transformations are

$$\begin{aligned} \delta_\xi A &= \sqrt{2}\xi\chi \\ \delta_\xi \chi_\beta &= \sqrt{2}\xi_\beta F + i\sqrt{2}\sigma^a_{\beta\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_a A \\ \delta_\xi F &= i\sqrt{2}\bar{\xi}\bar{\sigma}^a \partial_a \chi \end{aligned} \tag{2}$$

- The equations of motion of F read

$$F = 0 \tag{3}$$

No supersymmetry breaking.

Signals of broken SUSY

- A more general supersymmetric Lagrangian is

$$\mathcal{L}_{Int} = A\partial^2\bar{A} + i\partial_a\bar{\chi}\bar{\sigma}^a\chi + F\bar{F} + \text{interactions} \tag{4}$$

When is SUSY broken?

- V.e.v. for the auxiliary field: $< F > \neq 0$
- Existence of a massless fermion G_α: Goldstino
- Supersymmetry becomes a shift for the Goldstino:

$$< \delta G_\alpha > \sim \xi_\alpha < F >$$

Effective descriptions of broken SUSY

- The Volkov-Akulov Lagrangian is

$$\mathcal{L}_{VA} = -f^2 + i\partial_a \bar{G}\bar{\sigma}^a G + \frac{1}{4f^2}\bar{G}^2\partial^2 G^2 - \frac{1}{16f^6} G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2 \quad (5)$$

- It describes broken supersymmetry

$$< \delta G_\alpha > \sim \xi_\alpha f$$

- There is no sgoldstino - sgoldstino is decoupled

Constrained chiral superfields

- Consider the Lagrangian

$$\mathcal{L}_{XNL} = \int d^4\theta\, X_{NL}\bar{X}_{NL} + \sqrt{2}\Lambda^2 \left(\int d^2\theta\, X_{NL} + h.c \right) + \left(\int d^2\theta\, \Psi X_{NL}^2 + h.c \right) \quad (6)$$

- Reproduces the V-A on-shell
- Superspace EOM

$$-\frac{1}{4}\bar{D}^2\bar{X}_{NL} + \sqrt{2}\Lambda^2 + 2\Psi X_{NL} = 0 \quad (7)$$

$$X_{NL}^2 = 0 \quad (8)$$

Duals to chiral multiplets

- Consider the action

$$\mathcal{L}_D = -\int d^4\theta \left(\Sigma\bar{\Sigma} + \Phi\Sigma + \bar{\Phi}\bar{\Sigma}\right) \tag{9}$$

where Φ is chiral and Σ is unconstrained.

- Integrate out Σ to find

$$\mathcal{L}_0 = \int d^4\theta \Phi\bar{\Phi} \tag{10}$$

- Integrate out Φ to find

$$\mathcal{L}_\Sigma = -\int d^4\theta\, \Sigma\bar{\Sigma} \tag{11}$$

with

$$\bar{D}^2\Sigma = 0 \tag{12}$$

Complex linear multiplets

- The kinetic Lagrangian (11) in components reads

$$\mathcal{L}_\Sigma = A\partial^2\bar{A} - F\bar{F} + i\partial_\mu\bar{\psi}\bar{\sigma}^\mu\psi + \frac{1}{2}P_\mu\bar{P}^\mu + \frac{1}{2\sqrt{2}}\left(\chi\lambda + \bar{\chi}\bar{\lambda}\right) \quad (13)$$

- The fermion transformations are

$$\delta\psi_\alpha = \sqrt{2}i\sigma^\mu_{\alpha\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_\mu\bar{A} - \frac{1}{\sqrt{2}}\bar{\xi}^{\dot{\beta}}\bar{P}_{\alpha\dot{\beta}} \quad (14)$$

$$\delta\chi_\alpha = 2i\sigma^\nu_{\alpha\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta}\xi_\beta\partial_\mu\bar{P}_\nu + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}\beta}\xi_\beta\partial_\mu\bar{P}_\nu - 4\xi_\alpha\partial^2\bar{A} + 2i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}\partial_\mu\bar{F} \quad (15)$$

$$\delta\lambda_\alpha = \sqrt{2}\xi_\alpha F - \frac{1}{\sqrt{2}}\bar{\xi}^{\dot{\beta}}P_{\alpha\dot{\beta}} \quad (16)$$

- We see from (16) that supersymmetry can be broken for $< F > \neq 0$, but the auxiliary field λ_α should become the Goldstino (i.e. propagating), can a single term do this?

The HDO in superspace

- We consider the following interaction

$$\mathcal{L}_{HDO} = \int d^4\theta \, \frac{1}{64\,\Lambda^4} \, D^\alpha \Sigma D_\alpha \Sigma \bar{D}_{\dot{\alpha}} \bar{\Sigma} \bar{D}^{\dot{\alpha}} \bar{\Sigma} \tag{17}$$

- This higher dimensional operator does not give rise to any instability
- It may lead to SUSY breaking
- In the broken branch, it gives rise to a kinetic term for λ_α

Lets see how this happens

HDO-Bosonic sector

- The bosonic part of the full Lagrangian turns out to be

$$\begin{aligned}\mathcal{L}^B =& \mathcal{L}^B_\Sigma + \mathcal{L}^B_{HDO} \\ =& - F\bar{F} + A\partial^2\bar{A} + \frac{1}{2}P_\mu\bar{P}^\mu \\ &+ \frac{1}{64\Lambda^4}\left(P^\mu P_\mu\bar{P}^\nu\bar{P}_\nu + 4P_\mu\bar{P}^\mu F\bar{F} + 16F^2\bar{F}^2\right)\end{aligned} \quad (18)$$

- From the equations of motion for the complex auxiliary vector we find that

$$P_\mu = 0 \quad (19)$$

leading to

$$\mathcal{L}^B = -F\bar{F} + A\partial^2\bar{A} + \frac{1}{4\Lambda^4}F^2\bar{F}^2 \quad (20)$$

A SUSY model with two branches

- The equations of motion for the auxiliary scalar turn out to be

$$F\left(1 - \frac{1}{2\Lambda^4} F\bar{F}\right) = 0 \tag{21}$$

- There are now two solutions:

$$(i) \quad F = 0 \tag{22}$$

$$(ii) \quad F\bar{F} = 2\Lambda^4 \tag{23}$$

- Clearly, the first vacuum $F = 0$ is the supersymmetric one, however, the second vacuum, described by the solution (23), explicitly breaks supersymmetry.

Did that auxiliary spinor become propagating?

- Our Lagrangian gives rise to the following coupling for the auxiliary fermion λ

$$\mathcal{L}_{HDO} \supset (\frac{1}{4\Lambda^4} F\bar{F})\ i\partial_\mu \bar{\lambda}\bar{\sigma}^\mu \lambda \tag{24}$$

- In the susy breaking vacuum obtained from (21) we have

$$< F\bar{F} >= 2\Lambda^4 \tag{25}$$

leading to a standard fermionic kinetic term with the correct sign

$$\mathcal{L}_{HDO} \supset \frac{i}{2} \partial_\mu \bar{\lambda}\bar{\sigma}^\mu \lambda \tag{26}$$

Validity of the solution

- We found the solution only looking in the bosonic sector, is this solution valid for the full equations of motion?
- One could proceed and solve the field equations for the auxiliaries. Although this is a formidable task, there is an indirect way to proceed in superspace.
- We will show below that our theory describes a free chiral multiplet and a constrained chiral superfield which describes a Volkov-Akulov mode.

Superspace equations of motion

- In superspace we have

$$\mathcal{L}_\Sigma + \mathcal{L}_{HDO} = -\int d^4\theta\, \Sigma\bar{\Sigma} + \int d^4\theta\, \frac{1}{64\,\Lambda^4}\; D^\alpha\Sigma D_\alpha\Sigma \bar{D}_{\dot{\alpha}}\bar{\Sigma}\bar{D}^{\dot{\alpha}}\bar{\Sigma}$$

- Using standard variational techniques in superspace we find for Σ

$$D_\alpha\Sigma + \frac{1}{32\Lambda^4} D_\alpha \bar{D}_{\dot{\alpha}} \left(D^\beta\Sigma D_\beta\Sigma \bar{D}^{\dot{\alpha}}\bar{\Sigma} \right) = 0 \tag{27}$$

- These equations can equivalently be expressed as

$$\Sigma = -\frac{1}{32\Lambda^4} \bar{D}_{\dot{\alpha}} \left(D^\beta\Sigma D_\beta\Sigma \bar{D}^{\dot{\alpha}}\bar{\Sigma} \right) + \bar{\Phi} \tag{28}$$

where Φ is a chiral superfield.

- Consistency requires

$$\bar{D}^2\bar{\Phi} = 0 \tag{29}$$

which implies that Φ is a free chiral superfield.

Solving the SuperEOM

- Σ may be written as

$$\Sigma = H + \bar{\Phi} \tag{30}$$

where H satisfies the equations of motion

$$H = -\frac{1}{32\Lambda^4}\bar{D}_{\dot{\alpha}}\left(D^{\beta}HD_{\beta}H\bar{D}^{\dot{\alpha}}\bar{H}\right) \tag{31}$$

- It is easy to find two solutions for (31)
  1. The supersymmetric branch

$$H = 0 \tag{32}$$

  2. The broken (non-linear) SUSY branch

$$H = X_{NL} \tag{33}$$

What does our theory describe?

- The superspace EOM are solved for

$$\Sigma = H + \bar{\Phi}$$

where

$$\bar{D}^2 \bar{\Phi} = 0$$

and

$$H = X_{NL}$$

satisfying (7) and (8).

- Thus the broken branch of our theory describes on-shell a free chiral multiplet and a goldstino superfield.

Summary

- Supersymmetry may be broken by higher dimensional operators.
- We have discussed the complex linear multiplet, but there is more examples.
- In all cases there is connection with the non-linear realizations.
- We are now working on the coupling to 4-D minimal supergravity.