Emergent geometry ideas at large N

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Plan for the talk

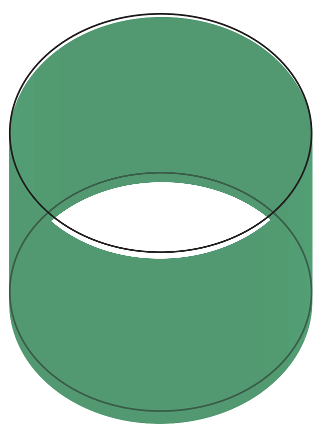
- Introduction
- Half BPS states in N=4 SYM: fermion droplets
- Strings stretching between giant gravitons and coherent states
- Black holes
- Fermion dynamics

Introduction

The gauge/gravity duality has told us that field theory can be equivalent to geometry in higher dimensions.

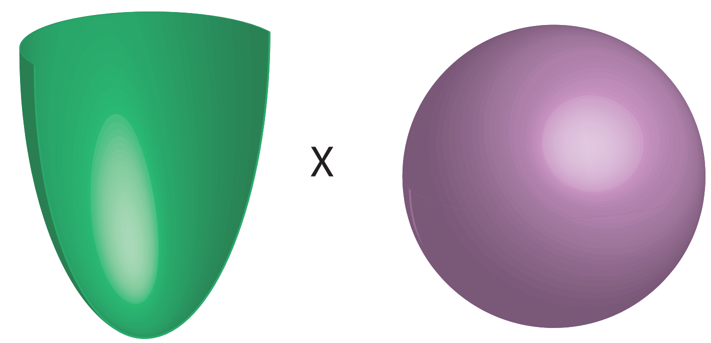
Simplest example of gauge gravity duality

N=4 SYM on a sphere



Type IIB Superstring on

$AdS_5 \times S^5$



Maldacena `hep-th/9711200`

Questions

- Why and when does the large N dynamics organize itself into geometry?
- What can we say about the many geometries that might be related to a single dynamical system? (Black holes versus smooth supergravity solutions)
- How does geometry break down?

Any such geometry in higher dimensions is to be considered an emergent phenomenon. It's not part of the original description of the theory.

Starting with N=4 SYM

The generic state is too complicated.

Consider special states (very supersymmetric).

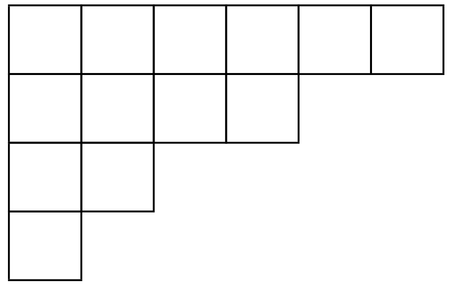
Half BPS states

They are gauge invariant operators built out of multi-traces of Z (one of the complex scalars of N=4 SYM in an N=1 superfield notation)

A good basis (complete and orthogonal)

Corley, Jevicki, Ramgoolam, hep-th/0111222

$$\chi_R(Z)$$



Schur polynomials

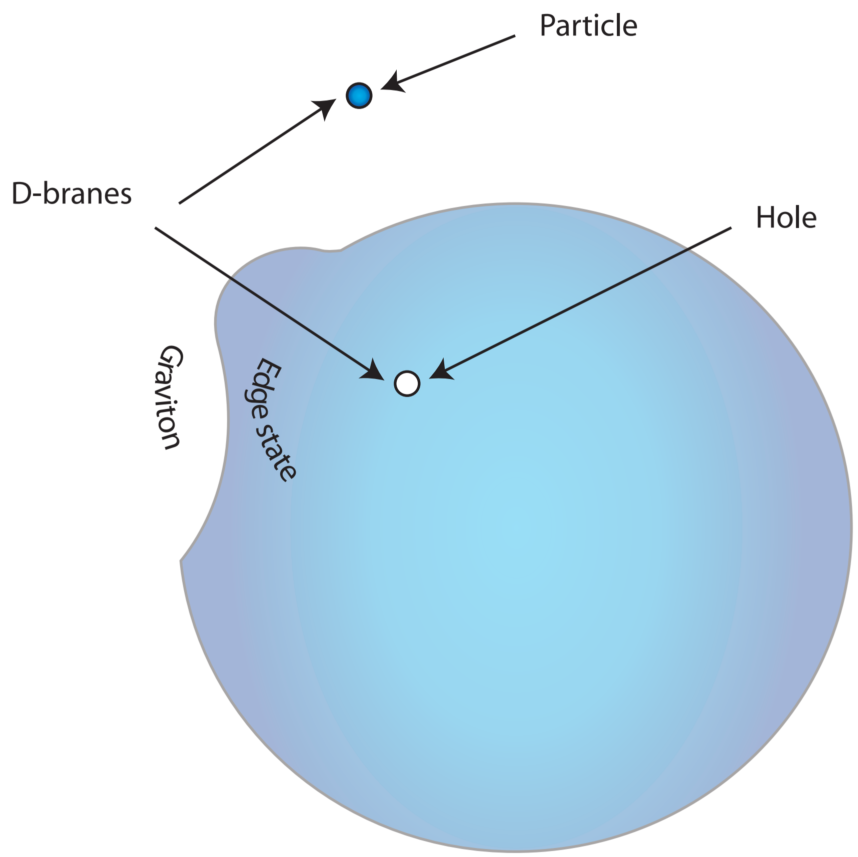
The system can be solved in terms of free fermions on a magnetic plane: lowest Landau level physics

D.B. hep-th/0403110

An adaptation of a standard 1-matrix model trick of

Brezin, Itzykson, Parisi, Zuber

The system is a copy of the quantum hall droplet.



The ‘particle’ and ‘hole’
end up identified as D-branes.

These are giant gravitons.

WHAT ARE GIANT GRAVITONS?

Gravitons: half BPS states of AdS

Point particles moving on a diameter of sphere and sitting at origin of AdS

Preserve SO(4)x SO(4) symmetry

There are also D-brane (D3-branes) states that respect the same symmetry and leave half the SUSY invariant.

SO(4) x SO(4) invariance implies

$\boxed{\text{Branes wrap a 3-sphere of 5-sphere at origin of AdS (moving in time)}}$

McGreevy, Susskind, Toumbas, hep-th/000307

OR

Branes wrap a 3-sphere of AdS, at a point on diameter of 5- sphere

What’s new?

We can now extract an effective action for the giant gravitons from gauge theory.

D.B. arXiv:1301.3519

Need a notion of collective coordinates for a (special) collection of states of some quantum system.

$$|\lambda\rangle = \sum c_n(\lambda)|n\rangle$$

Def: λ is a (set of) collective coordinate (s) if

$$\langle\lambda|\tilde{\lambda}\rangle << 1 \text{ If } |\lambda - \tilde{\lambda}| > A$$ (Orthogonality)

$$\langle\lambda|H|\tilde{\lambda}\rangle << |\langle\lambda|H|\lambda\rangle - \langle\tilde{\lambda}|H|\tilde{\lambda}\rangle| \text{ If } |\lambda - \tilde{\lambda}| > A$$ (Local)

Similar for all other symmetry quantum numbers

Effective action

$$S_{eff} = \int dt\,\left(i\langle\lambda(t)|\partial_t|\lambda(t)\rangle - \langle H\rangle\right)$$

Berry phase

$$\int pdq$$

In principle we can then go and compare to some other system with a particular action.

Collective coordinate for giant gravitons

Consider

$$\det(Z-\lambda) = \sum_{\ell=0}^{N} (-\lambda)^{N-\ell} \det_\ell(Z)$$

This is a linear combination of states with different R-charge, depends on a complex parameter.

Can compute norm of state

$$\langle \det(\bar{Z}-\tilde{\lambda}^*)\det(Z-\lambda)\rangle = \sum_{\ell=0}^{N}(\lambda\tilde{\lambda}^*)^{N-\ell}\frac{N!}{(N-\ell)!} = N!\sum_{\ell=0}^{N}(\lambda\tilde{\lambda}^*)^{\ell}\frac{1}{(\ell)!}$$

can be well approximated by

$$\langle \det(\bar{Z}-\tilde{\lambda}^*)\det(Z-\lambda)\rangle \simeq N!\exp(\lambda\tilde{\lambda}^*)$$

For

$$|\lambda| < \sqrt{N}$$

The parameter belongs to a disk

After plugging in

We get an inverted harmonic oscillator in a first order formulation.

$$S_{eff} = \int dt \left[\frac{i}{2}(\lambda^* \dot{\lambda} - \dot{\lambda}^* \lambda) - (N - \lambda \lambda^*) \right]$$

Approximation breaks down exactly when Energy goes to 0

This is very similar to what happens in gravity

If we rescale the disk to be of radius one, we get

$$S_{eff} = N \int dt \left[\frac{i}{2} (\xi^* \dot{\xi} - \dot{\xi}^* \xi) - (1 - \xi \xi^*) \right]$$

The factor of N in planar counting suggests that this object can be interpreted as a D-brane

Matches exactly with the fermion droplet picture of half BPS states

D. B. hep-th/0403110

Lin, Lunin, Maldacena, hep-th/0409174

Attaching strings

The relevant operators for maximal giant are

$$\epsilon\epsilon(Z, \ldots Z, W^1, \ldots W^k)$$

Balasubramanian, Huang, Levi and Naqvi, hep-th/0204196

These can be obtained from expanding

$$\det(Z + \sum \xi_i W^i)$$

And taking derivatives with respect to parameters

Main idea: for general giant replace Z by Z-λ in the expansion

$$\det(Z + \sum \xi_i W^i) = \det(Z) \exp(\mathrm{Tr} \log(1 + Z^{-a} \sum_i \xi_i W^i Z^{-b}))$$

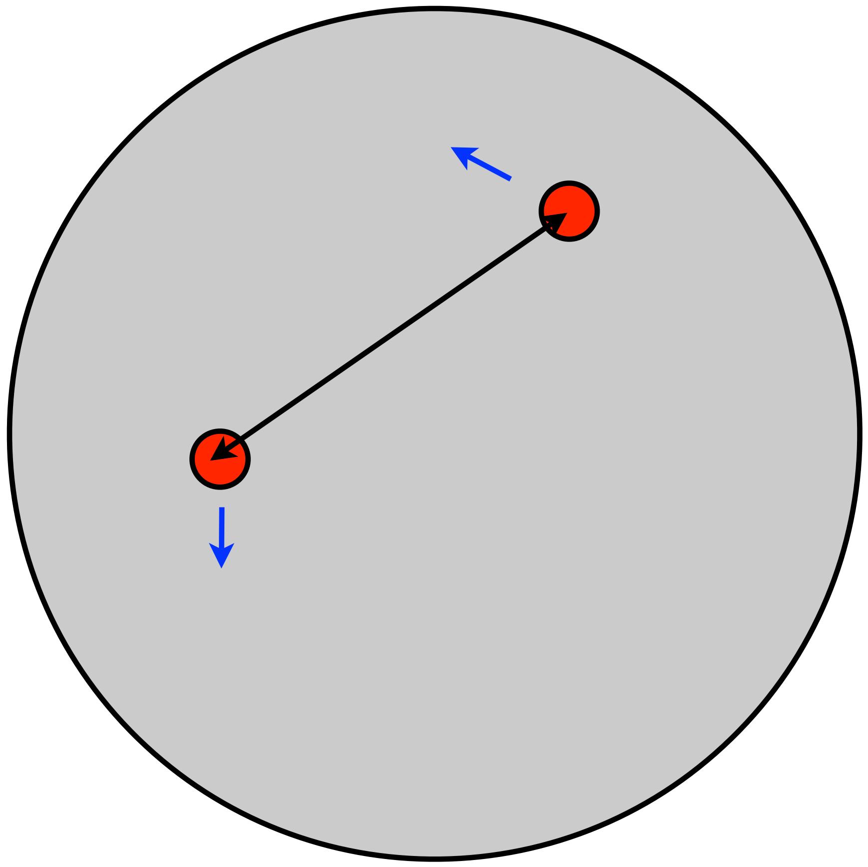
One loop anomalous dimensions = masses of strings

Want to compute effective Hamiltonian of strings stretched between two giants.

$$\det(Z-\lambda_1)\det(Z-\lambda_2)\mathrm{Tr}((Z-\lambda_1)^{-1}Y(Z-\lambda_2)^{-1}X)$$

I have not done full combinatorics of 2 giants on same group, rather use orbifold trick to simplify algebra

End result in pictures



$$m_{od}^2 \simeq g_{YM}^2 |\lambda - \tilde{\lambda}|^2$$

$$E \simeq m_{od}^2 \simeq g_{YM}^2 |\lambda - \tilde{\lambda}|^2$$
$$\simeq g_{YM}^2 N |\xi - \tilde{\xi}|^2$$

Result is local in collective coordinates (terms that could change collective parameters are exponentially suppressed)

Mass proportional to distance is interpreted as Higgs mechanism for emergent gauge theory.

More general states: open Spin chains

$$Y \to Y^n$$

Need to be careful about planar versus non-planar diagrams.

$$\lambda \simeq N^{1/2}$$

Simplest open chains

$$\det(Z-\lambda)\mathrm{Tr}\left(\frac{1}{Z-\lambda}YZ^{n_1}Y\ldots Z^{n_k}Y\right)$$

Just replace the W by n copies of Y: Z can jump in and out at edges. So we need to keep arbitrary Z.

Again, to simplify combinatorics between two giants, go to orbifold

$$\det(Z-\lambda)\det(\tilde{Z}-\tilde{\lambda})\mathrm{Tr}\left(\frac{1}{Z-\lambda}Y_{12}\tilde{Z}^{n_1}Y_{21}Z^{n_2}Y\tilde{Z}^{n_3}\ldots Z^{n_k}Y_{12}\frac{1}{\tilde{Z}-\tilde{\lambda}}X_{21}\right)$$

Choose the following labeling for the basis

$$|n_1, n_2, n_3 \ldots\rangle \simeq |\uparrow,\downarrow^{\otimes n_1}, \uparrow,\downarrow^{\otimes n_2}, \uparrow,\downarrow^{\otimes n_3}, \ldots\rangle$$

One loop Hamiltonian in bulk

$$H_{eff}|n_1, n_2, n_3 \ldots, n_k\rangle \;=\; g_{YM}^2 N \sum_{i=1}^{k} 2|\ldots, n_{i-1}, n_i, n_{i+1}\ldots\rangle \qquad (7)$$
$$-|\ldots, n_{i-1}+1, n_i-1, n_{i+1}\ldots\rangle - |\ldots, n_{i-1}, n_i-1, n_{i+1}+1\ldots\rangle$$

Can be written conveniently if we introduce a Cuntz oscillator for each site

$$aa^\dagger = 1$$

After some work at one loop...and including boundary

We get a sum of squares

$$H_{eff} \simeq g_{YM}^2 N \left[\left(\frac{\lambda}{\sqrt{N}} - a_1^\dagger \right) \left(\frac{\lambda^*}{\sqrt{N}} - a_1 \right) + (a_1^\dagger - a_2^\dagger)(a_1 - a_2) + \dots \right]$$

We need to try to solve the operator equations
to find a minimum.

We can try converting them to c-number equations
if we introduce generalized coherent states

$$a|z\rangle = z|z\rangle$$

Solving the equations then becomes trivial.

$$|z\rangle = \sum_{k=0}^{\infty} z^k |k\rangle$$

$$\langle z'|z\rangle = \frac{1}{1-\bar{z}'z}$$

$$\langle z|z\rangle = \frac{1}{1-|z|^2}$$

Again, we can think of z as a collective coordinate for a site on the chain. $|z|<1$ for convergence.

To find ground state, coherent state ansatz

Berenstein and Dzienkowski, arxiv:1305.2394

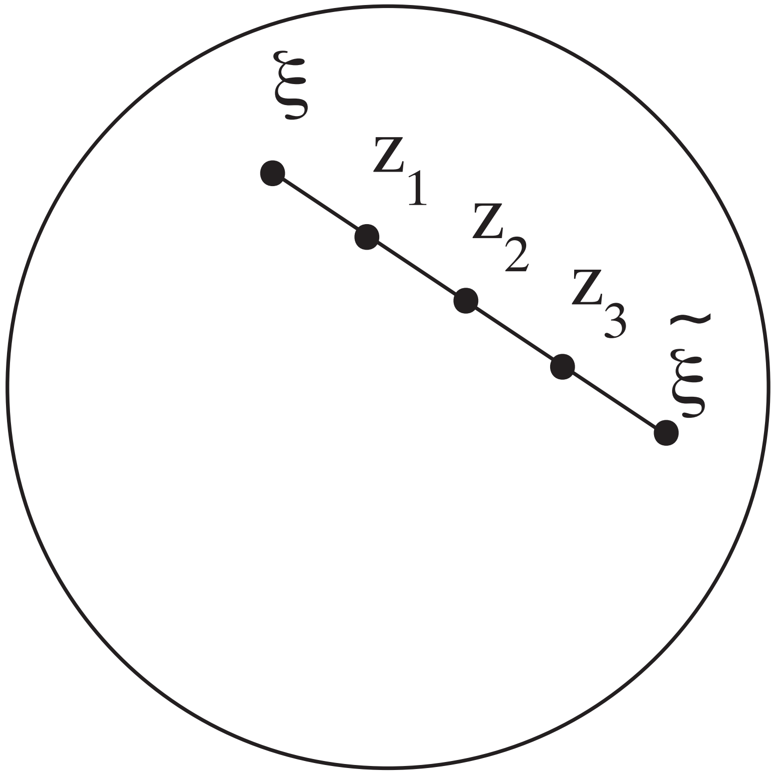
$$\langle z_1, \dots z_k | H_{\text{spin chain}} | z_1, \dots z_k \rangle = g_{YM}^2 N \left[\left| \frac{\lambda^*}{\sqrt{N}} - z_1 \right|^2 + \sum |z_i - z_{i+1}|^2 + \left| \frac{\tilde{\lambda}^*}{\sqrt{N}} - z_k \right|^2 \right]$$

and minimize

$$\frac{\lambda^*}{\sqrt{N}} - z_1 = z_1 - z_2 = \cdots = z_i - z_{i+1} = \cdots = z_k - \frac{\tilde{\lambda}^*}{\sqrt{N}}$$

$\xi = \lambda^* N^{-1/2}$

$\tilde{\xi} = \tilde{\lambda}^* N^{-1/2}$



These can be pictured on the "free fermion disk"

The z coordinates also have a geometric interpretation!

$$E(z_0, \ldots, z_{k+1}) \simeq g_{YM}^2 N \sum |z_{i+1} - z_i|^2$$

$\xi \& z$ describe the same geometry, but for different objects

D-branes and strings see things differently: they wrap different dimensions.

End result:

Full calculation for open spin ground state should give

$$E_n \simeq n + n^{-1} g_{YM}^2 |\lambda - \tilde{\lambda}|^2 \simeq \sqrt{n^2 + g_{YM}^2 |\lambda - \tilde{\lambda}|^2}$$

Starts showing an emergent Lorentz invariance for massive W particles in the worldsheet fluctuations of giant gravitons. We showed this to two loop order.

Moral

- Collective coordinates can appear in more than one way, depending on the object under study.
- We seem to be able to see emergent Lorentz invariance by studying ground states of open spin chains.
- This has an explanation in terms of central charge extensions of the open spin chain (assuming bulk integrability of the spin chain).

Non-ground states: what about black holes?

Want to simulate generic time-dependent states in a strongly coupled QFT.

Building a black hole

For technical reasons, BFSS matrix model black hole is ideal for simulations. Can also take large N.

(Finite number of degrees of freedom in matrices)

BFSS matrix model

Dimensional reduction of U(N) SYM in d=9+1 to 0+1

$$S_{BFSS} = \frac{1}{2g^2}\int dt\,\left((D_t X^I)^2 + \frac{1}{2}[X^I, X^J]^2\right) + \text{fermions}$$

Banks, Fischler, Shenker, Susskind ‘96

There are 9 dynamical matrices and one matrix constraint.

Once you have a typical matrix configuration, how do you probe it geometrically?

Add a D0 brane probe (extra eigenvalue)

The probe lives in $\mathbb{R}^9$

We can ask questions about the probe and locate information relative to this flat geometry.

Distance from probe to configuration

Compute masses of off-diagonal strings connecting probe to configuration.

Better with fermions.

Need to diagonalize 'instantaneous' effective Hamiltonian for o.d. fermions.

$$H_{eff} = \sum (X^i - x^i \otimes 1) \otimes \gamma^i$$

Define (spectral) distance as minimum eigenvalue (in absolute value): this is the 'shortest string energy'

work in progress w. E. Dzienkowski

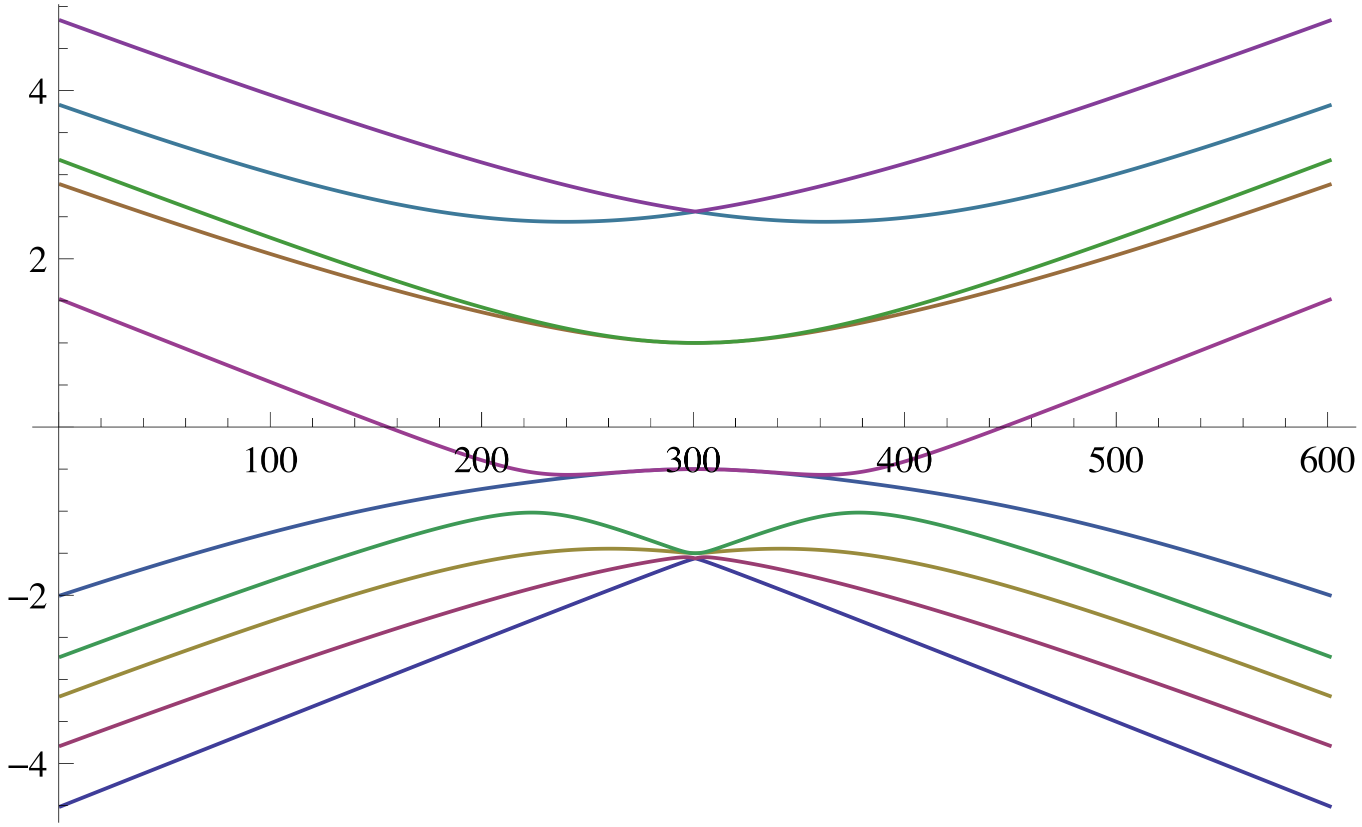
Toy model: in 3d rather than 9

$$\sum_i (X^i - x^i) \otimes \sigma^i$$

We only have 3 matrices and the Gamma matrices are Pauli Matrices.

w. E. Dzienkowski, `arXiv:1204.2788`

We can check on some simple configurations (deformed fuzzy spheres)



We see eigenvalues can cross zero.

Suggest we should count those crossings

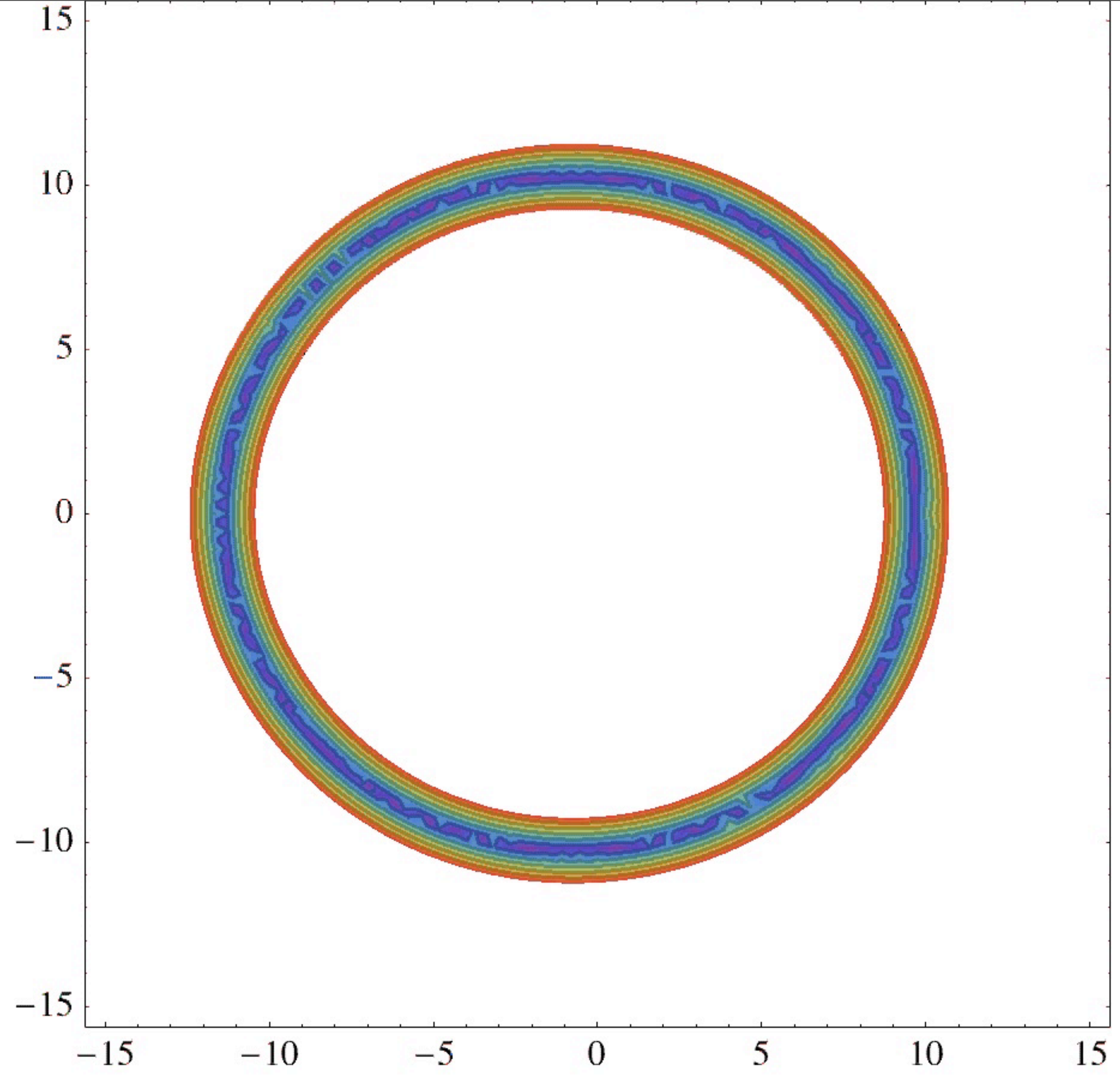
$$I(x) \simeq \frac{\dim(V+) - \dim(V-)}{2}$$

Locally constant: counts how many layers one has to cross to get out.

The locus where index changes are surfaces: the best notion of the geometric embedding of the matrices.

Generalizes to all odd dimensions.

We get even dimensional branes at zero locus.
(They're probably always there)

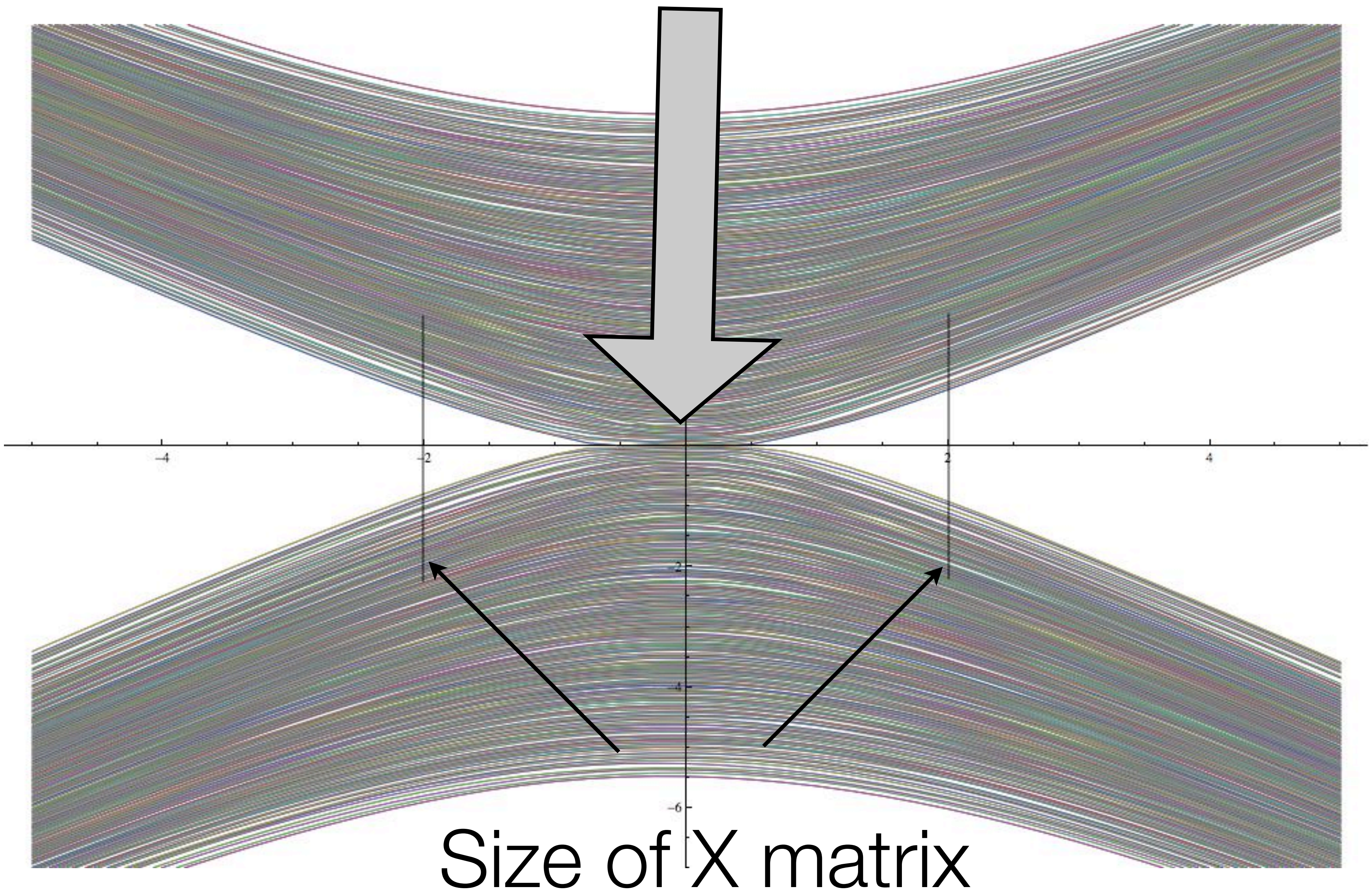


Thermalization in BMN matrix model

21 x 21 matrices

Scan over a 1 parameter set at fixed time

Fermions are gapless in a region



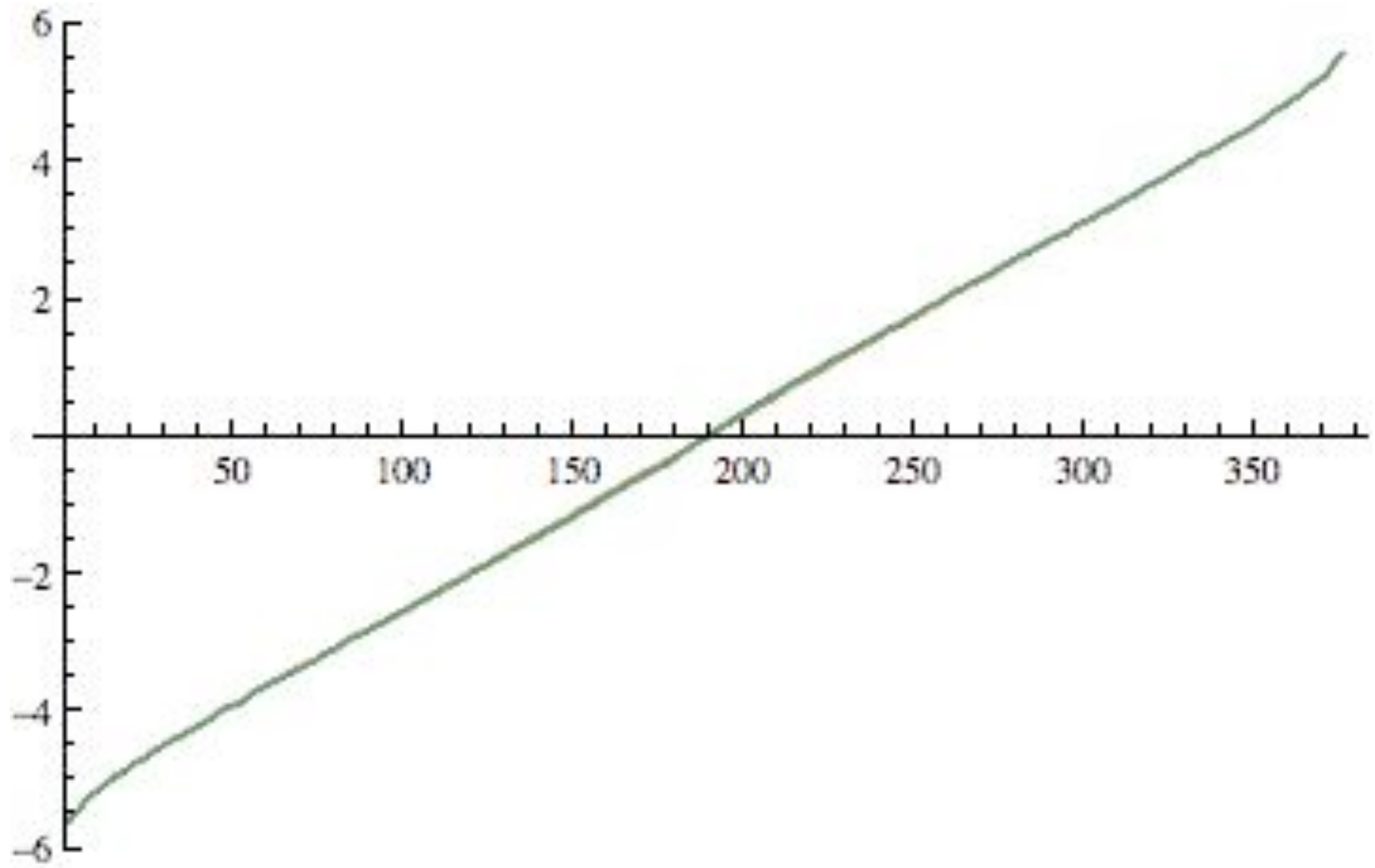
Fix position of probe inside gapless region

Define spectral dimension using density of states near zero

$$\frac{dn}{dE}\Big|_{E\simeq 0} \simeq E^{\gamma-1}$$

$$\text{spectral dim} = \gamma$$

Same density of degrees of freedom as field theory in $\gamma + 1$ dimensions



Spectral dimension = 1

Effective 1+1 field theory

Can not be both space filling and one-dimensional

$$1 \neq 9$$

Physics can not be local in that region.

Interpretation (speculation)

Gapless region is ‘inside the black hole’

EFT breaks down as we get near black hole:
gap becomes smaller than naive distance (could be interpreted as redshift)

Two possibilities:

The dynamics of the BH singularity?

Firewalls?