Minimal adjoint-SU$(5) \times$ Z$_4$

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In collaboration with

Catarina Simões, Mariam Tórtola, arXiv:1303:5699 (JHEP)
Catarina Simões, Phys. Rev. D 85 (2012) 016003
Gustavo Branco, Catarina Simões, Phys. Lett. B 690 (2010) 62

School and Workshop on the SM and Beyond
Κερκυρα, 6th September 2013

13th Hellenic School and Workshops on Elementary Particle Physics and Gravity

Outline

- Introduction to Texture Zeroes
- NNI structure and the Fritzsch Ansatz
- NNI texture from a Discrete Flavour Symmetry (Z_4)

- Introduction to GUT models
- Consistent-$SU(5) \times Z_4$ Unified Model
- Adjoint-$SU(5) \times Z_4$ Unified Model

- Neutrino phenomenology

- Conclusions

Texture Zeroes in the Fermion Mass Matrices

[Weinberg; Fritzsch; Ramond,Robert,Ross
Frampton,Glashow,Marfatia
Nishiura,Matsuda,Fukuyama
Branco,D.E.C.,González Felipe]

- Reduce Free Parameters (predictions)
- Possible Origin of texture zeroes:
  - Discrete or Continuous symmetries [exact zeroes]
  - Froggatt-Nielsen Mechanism [suppressed entries]

$$M_{ij} \sim \left(\frac{v}{\Lambda}\right)^{Q_i+Q_j}$$

- Texture Zeroes
  - Can be some zeroes due to Weak Basis transformations?
  - Can we make also M_i also Hermitian?
  - Such zeroes have no physical meaning

Weak Basis Transformations

- General Transf. Leaving Charged Currents Invariant

  - **Quarks**

$$m_u \longrightarrow m'_u = W_q^\dagger \, m_u \, W_{uR}$$
$$m_d \longrightarrow m'_d = W_q^\dagger \, m_d \, W_{dR}$$

  - **Leptons**

$$m_\ell \longrightarrow m'_\ell = W_\ell^\dagger \, m_\ell \, W_{eR}$$
$$m_\nu \longrightarrow m'_\nu = W_\ell^\top \, m_\nu \, W_\ell$$

- In the case of Hermitian Mass Matrices

$W_{uR} = W_q \,, \quad W_{uR} = W_q$ and $W_{eR} = W_\ell$

Nearest-Neighbour-Interaction Basis

[Branco, Lavoura, Mota]

- Obtained through Weak Basis Transformations
- 8 Zeroes in Non-Hermitian Mass Matrices
- No further Physical Consequences, just a weak basis!
- Completely factorisable $V = O_u^\top \, K \, O_d$

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A'_u & 0 & B_u \\ 0 & B'_u & C_u \end{pmatrix} \qquad M_d = \begin{pmatrix} 0 & A_d & 0 \\ A'_d & 0 & B_d \\ 0 & B'_d & C_d \end{pmatrix}$$

- or equivalently

$$(H_u)_{12} = 0 \qquad (H_d)_{12} = 0$$

- with $H_{u,d} = M_{u,d} M_{u,d}^\dagger$
- Can one have Hermiticity in addition?

  $\Longrightarrow$ Fritzsch Ansatz [Physical Implications]

Fritzsch Ansatz

- NNI plus Hermiticity
- The orthogonal matrices $O^\top H O = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$ are just function of mass ratios [exact]

$$O = \begin{pmatrix} \sqrt{\frac{m_2 m_3(m_3+m_2)}{(m_2-m_1)(m_3+m_2+m_1)(m_3-m_1)}} & -\sqrt{\frac{m_1 m_3(m_3+m_1)}{(m_1-m_2)(m_3+m_2+m_1)(m_3-m_2)}} & \sqrt{\frac{m_1 m_2(m_1-m_2)}{(m_3-m_1)(m_3-m_2+m_1)(m_3-m_2)}} \\ \sqrt{\frac{m_1(m_3+m_2)}{(m_1-m_2)(m_3-m_1)}} & \sqrt{\frac{m_2(m_3+m_1)}{(m_2-m_1)(m_3-m_2)}} & \sqrt{\frac{m_3(m_2+m_1)}{(m_3-m_1)(m_2-m_3)}} \\ -\sqrt{\frac{m_1(m_2+m_1)(m_3+m_1)}{(m_2-m_1)(m_3+m_2+m_1)(m_3-m_1)}} & -\sqrt{\frac{m_2(m_2+m_1)(m_3+m_2)}{(m_2-m_1)(m_3+m_2+m_1)(m_3-m_2)}} & \sqrt{\frac{m_3(m_3+m_2)(m_3+m_1)}{(m_3-m_1)(m_3+m_2+m_1)(m_2-m_3)}} \end{pmatrix}$$

- If one takes into account the quark masss Hierarchy and neglet the up Sector contribution

$$V \simeq O_d \simeq \begin{pmatrix} 1 & -\sqrt{\frac{m_d}{m_s}} & \frac{m_s}{m_b}\sqrt{\frac{m_d}{m_b}} \\ \sqrt{\frac{m_d}{m_s}} & 1 & \sqrt{\frac{m_s}{m_b}} \\ -\sqrt{\frac{m_d}{m_b}} & -\sqrt{\frac{m_s}{m_b}} & 1 \end{pmatrix}$$

- Ruled out: m_t is too small and V_{cb} inadequate
- Deviations of the Fritzsch Ansatz

Minimal Deviations from Hermiticity

- Working with $H = MM^\dagger$

$$H = \begin{pmatrix} A^2 & 0 & A\,B' \\ 0 & A'^2 + B^2 & B\,C \\ A\,B' & B\,C & B'^2 + C^2 \end{pmatrix}$$

- Deviation from Hermiticity

$$A \equiv \overline{A}(1-\epsilon_a) \qquad A' \equiv \overline{A}(1+\epsilon_a)$$
$$B \equiv \overline{B}(1-\epsilon_b) \qquad B' \equiv \overline{B}(1+\epsilon_b)$$

- $\epsilon_a\,,\epsilon_b << 1$ but not zero
- $\epsilon_a = 0 = \epsilon_b$ is the Fritzsch ansatz, ruled out by the data

$$O \simeq \begin{pmatrix} 1 & -\sqrt{\frac{m1}{m_2}}\left(1-\epsilon_a-\frac{m_2}{m_3}\epsilon_b\right) & \sqrt{\frac{m_1 m_2^2}{m_3^3}}\,(1+\epsilon_b-\epsilon_a) \\ \sqrt{\frac{m_1}{m_2}}\left(1-\epsilon_a-\frac{m_1}{m_3}\epsilon_b\right) & 1 & \sqrt{\frac{m_2}{m_3}}\,(1-\epsilon_b) \\ -\sqrt{\frac{m_1}{m_3}}\,(1-\epsilon_a-\epsilon_b) & -\sqrt{\frac{m_2}{m_3}}\left(1-\epsilon_b+\frac{m_1}{m_2}\epsilon_a\right) & 1 \end{pmatrix}$$

- Global deviation: $\varepsilon \equiv \sqrt{(\epsilon_a^u)^2 + (\epsilon_b^u)^2 + (\epsilon_a^d)^2 + (\epsilon_b^d)^2/2}$

Experimental Quark Masses and Mixings at Mz scale

[G. Rodrigo et al, Phys. Lett. B 313 (1993) 441
Y. Koide et al, Phys. Rev. D 57 (1998) 3986
Z. z. Xing at al, Phys. Rev. D 77 (2008) 113016]

- Runnig quark masses from PDG'12 to M_Z in the $\overline{MS}$ scheme using RGE for QCD @ 4 loops:

$$m_u = 1.3^{+0.4}_{-0.3}\,\text{MeV} \quad m_c = 0.63 \pm 0.02\,\text{GeV} \quad m_t = 171.8 \pm 0.7\,\text{GeV}$$

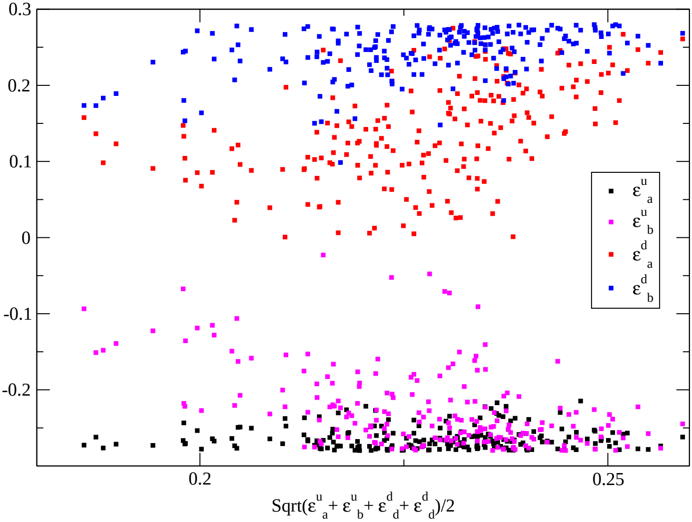
$$m_d = 2.8^{+0.4}_{-0.2}\,\text{MeV} \quad m_s = 55 \pm 3\,\text{MeV} \quad m_b = 2.86 \pm 0.04\,\text{GeV}$$

- CKM constructed from $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$ and either $\sin 2\beta$ or γ constrained by J

$$\beta \equiv \arg(-V_{cd}V^*_{cb}V^*_{td}V_{tb}) \qquad \gamma = \arg(-V_{ud}V^*_{ub}V^*_{cd}V_{cb}) \qquad J \equiv \text{Im}(V_{us}V^*_{ub}V^*_{cs}V_{cb})$$

$$|V_{us}| = 0.22534 \pm 0.00065 \quad |V_{ub}| = (3.51^{+0.15}_{-0.14}) \times 10^{-3} \quad |V_{cb}| = (41.2^{+0.11}_{-0.05}) \times 10^{-3}$$

$$\sin 2\beta = 0.679 \pm 0.020 \qquad \gamma = \left(73^{+10}_{-11}\right)^\circ \qquad J = \left(2.96^{+0.20}_{-0.16}\right) \times 10^{-5}$$



$$\epsilon \gtrsim 0.188$$

Nearest-Neighbour Interaction

- What is the minimal scenario for M_u and M_d be NNI form as a result of an Abelian family symmetry in a multi-Higgs extension in the context of the SM?

Discrete Flavour Symmetries

- NNI quark mass matrix implemented through the introduction of an Abelian family symmetry at the Lagrangian level.

@ least two Higgs doublets are needed: Φ_1, Φ_2

Under $\mathbf{Z_n}$ symmetry: $\Psi_j \longrightarrow \Psi'_j = e^{i\,\frac{2\pi}{n}\,\mathcal{Q}(\Psi_j)}\,\Psi_j$

$$\mathcal{Q}(\Phi_i) \equiv \phi_i \qquad \mathcal{Q}(Q_{Li}) \equiv q_i \qquad \mathcal{Q}(u_{Ri}) \equiv u_i \qquad \mathcal{Q}(d_{Ri}) \equiv d_i$$

The quark fields transformations:

$$(q_1, q_2) = (q_3 + \phi_1 - \phi_2, q_3 - \phi_1 + \phi_2)$$
$$(u_1,\, u_2,\, u_3) = (q_3 - \phi_1 + 2\phi_2, q_3 + \phi_1, q_3 + \phi_2)$$
$$(d_1,\, d_2,\, d_3) = (q_3 - 2\phi_1 + \phi_2, q_3 - \phi_2, q_3 - \phi_1)$$

Discrete Flavour Symmetries

- Allowed bilineares:

$$\mathcal{Q}(\overline{Q}_{L_i} u_{Rj}) = \begin{pmatrix} -2\phi_1 + 3\phi_2 & \phi_2 & -\phi_1 + 2\phi_2 \\ \phi_2 & 2\phi_1 - \phi_2 & \phi_1 \\ -\phi_1 + 2\phi_2 & \phi_1 & \phi_2 \end{pmatrix}$$

$$\mathcal{Q}(\overline{Q}_{L_i} d_{Rj}) = \begin{pmatrix} -3\phi_1 + 2\phi_2 & -\phi_1 & -2\phi_1 + \phi_2 \\ -\phi_1 & \phi_1 - 2\phi_2 & -\phi_2 \\ -2\phi_1 + \phi_2 & -\phi_2 & -\phi_1 \end{pmatrix}$$

- Neither Z_2 nor Z_3 works:

$$-2\phi_1 + 3\phi_2 = \phi_2 \pmod 2$$
$$-2\phi_1 + 3\phi_2 = \phi_1 \pmod 3$$

Minimal symmetry $\rightarrow \mathbf{Z_4}$

Discrete Flavour Symmetries

- The most general Yukawa couplings:

$$
\begin{aligned}
-\mathcal{L}_Y = &\Gamma_u^1 \overline{Q}_L \widetilde{\Phi}_1 u_R + \Gamma_u^2 \overline{Q}_L \widetilde{\Phi}_2 u_R \\
&+ \Gamma_d^1 \overline{Q}_L \Phi_1 d_R + \Gamma_d^2 \overline{Q}_L \Phi_2 d_R + \text{H.c.}\,,
\end{aligned}
$$

$\widetilde{\Phi}_j \equiv i\sigma_2 \Phi_j^*$

Yukawa matrices $\Gamma_{u,d}^{1,2}$:

$$
\Gamma_u^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_u \\ 0 & b'_u & 0 \end{pmatrix} \quad \Gamma_u^2 = \begin{pmatrix} 0 & a_u & 0 \\ a'_u & 0 & 0 \\ 0 & 0 & c_u \end{pmatrix}
$$

$$
\Gamma_d^1 = \begin{pmatrix} 0 & a_d & 0 \\ a'_d & 0 & 0 \\ 0 & 0 & c_d \end{pmatrix} \quad \Gamma_d^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_d \\ 0 & b'_d & 0 \end{pmatrix}
$$

Discrete Flavour Symmetries

- The most general Yukawa couplings:

$$-\mathcal{L}_Y = \Gamma_u^1 \overline{Q}_L \widetilde{\Phi}_1 u_R + \Gamma_u^2 \overline{Q}_L \widetilde{\Phi}_2 u_R + \Gamma_d^1 \overline{Q}_L \Phi_1 d_R + \Gamma_d^2 \overline{Q}_L \Phi_2 d_R + \text{H.c.}\,,$$

$$\widetilde{\Phi}_j \equiv i\sigma_2 \Phi_j^*$$

- After spontaneous symmetry breaking, Higgs VEV's, $\langle \Phi_1 \rangle \equiv v_1$ and $\langle \Phi_2 \rangle \equiv v_2$ generate NNI mass matrices

$$M_u = \begin{pmatrix} 0 & v_2\, a_u & 0 \\ v_2\, a'_u & 0 & v_1\, b_u \\ 0 & v_1\, b'_u & v_2\, c_u \end{pmatrix} \qquad M_d = \begin{pmatrix} 0 & v_1\, a_d & 0 \\ v_1 a'_d & 0 & v_2\, b_d \\ 0 & v_2\, b'_d & v_1\, c_d \end{pmatrix}$$

Accidental Global $U(1)$ symmetry

- Renormalisability implies that Z_4 has to be imposed on the full Lagrangian
- The most general renormalisable scalar potential consistent with Z_4 and gauge symmetry

$$\mathbf{V} = \mu_1|\Phi_1|^2 + \mu_2|\Phi_2|^2 + \lambda_1|\Phi_1|^4 + \lambda_2|\Phi_2|^4 + \lambda_3\,|\Phi_1|^2\,|\Phi_2|^2 + \lambda_4\,\Phi_1^\dagger\,\Phi_2\,\Phi_2^\dagger\,\Phi_1\,.$$

- Potential acquires a new accidental global symmetry which, upon spontaneous symmetry breaking leads to a massless neutral scalar (tree level)
- Soft-breaking term avoids the problem

$$\mathbf{V}' = \mu_{12}\;\Phi_1^\dagger\,\Phi_2\;+\text{H.c.}$$

- Or by introducing a singlet Higgs field [Z_4 charged]

CP Violation in the Model

- Spontaneous CP Violation is not possible, essentially due to the absence of terms like

$$(\Phi_1^\dagger \Phi_2 \Phi_1^\dagger \Phi_2) + \text{H.c}$$

- scalar vacuum:

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2\, e^{i\,\theta} \end{pmatrix},$$

- Two CP Conserving minima

$$\theta = 0 \qquad \text{for} \qquad \mu_{12} < 0$$

$$\theta = \pi \qquad \text{for} \qquad \mu_{12} > 0$$

- CP Violation via Kobayashi-Maskawa mechanism through Yukawa couplings

Grand Unification Theories

$$\mathrm{G}_{SM} \subset \mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6 \subset \mathrm{E}_8$$

- Unification of gauge groups in an simple Group
- Gauge coupling unification
- Charge quantization: $|Q_p + Q_e| < 10^{-21}$, they also explain the charge assignments
- Additional *family symmetries* relating different generations lead to simple models of fermion masses and mixing angles
- Neutrino oscillations governed by a see-saw mechanism are easily included (leptogenesis)
- Bottom-Tau or Top-Bottom-Tau Yukawa coupling unification is predicted in simple $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$, resp.
- Anomaly freedom automatic in many GUTs
- Gravity is not included!

NNI in Grand Unification

- Minimal SM Fermion Content: SU(5) and SO(10)
- Quark and Leptons are together in higher dimension multiplets
- Flavour symmetry which give rise to NNI in the Quark Sector imposes restrictions on the Leptonic Sector
- SU(5): 5^*, 10 and Right-handed Neutrinos are singlets [minimal field content] and then free Z_4 charges

$$\mathcal{Q}(10) = \mathcal{Q}(q) = \mathcal{Q}(u^c) = \mathcal{Q}(e^c)$$
$$\mathcal{Q}(5^*) = \mathcal{Q}(\ell) = \mathcal{Q}(d^c)$$

- SO(10): all in a unique 16 spinorial representation

$$\mathcal{Q}(16) = \mathcal{Q}(q) = \mathcal{Q}(u^c) = \mathcal{Q}(e^c) = \mathcal{Q}(\ell) = \mathcal{Q}(d^c) = \mathcal{Q}(\nu^c)$$

Consistent $\mathbf{SU(5)} \times Z_4$ model

[Georgi and Glashow, 1974]

- Same fermionic content as the $SU(5)$
- Non-renormalisable model: $M_e \neq M_d^{\mathsf{T}}$
- We require that Z_4 forces NNI form to M_u and M_d

Fermionic sector

- $\mathbf{10}_i = (Q, u^c, e^c)_i$
- $\mathbf{5}_i^* = (L, d^c)_i$
- $\mathbf{1}_i = \nu_i^c$

$$\mathcal{Q}(\mathbf{10}_i) = (3q_3 + \phi_2, -q_3 - \phi_2, q_3)$$

$$\mathcal{Q}(\mathbf{5}_i^*) = (q_3 + 2\phi_2, -3q_3, -q_3 + \phi_2)$$

$$\mathcal{Q}(\mathbf{1}_i) = (n_1, n_2, n_3)$$

Scalar sector

- $\Sigma(24)$ ($\mathcal{Q}(\Sigma) = 0$)
- $H_1(5)$ and $H_2(5)$

$$\mathcal{Q}(H_1) = -2q_3$$

$$\mathcal{Q}(H_2) = \phi_2$$

Proton Decay and Unification

- Nonrenormalisable higher dimension operators

$$\sum_{n=1,2} \frac{\sqrt{2}}{\Lambda'} (\Delta_n)_{ij} \, H^*_{n\,a} \, \mathbf{10}^{ab}_i \, \Sigma^c_b \, \mathbf{5}^*_{jc}$$

$$M_d - M_e^\top = 5 \frac{\sigma}{\Lambda'} (v_1^* \Delta_1 + v_2^* \Delta_2)$$

- Heavy gauge bosons X and Y $[M_V = \frac{25}{8} g_U^2 \sigma^2]$

$\tau(p \to \pi^0 e^+) > 1.4 \times 10^{34}$ y $\implies M_V > (4.9 - 5.1) \times 10^{15}$ GeV

with $\alpha_U^{-1} \approx 26 - 35$

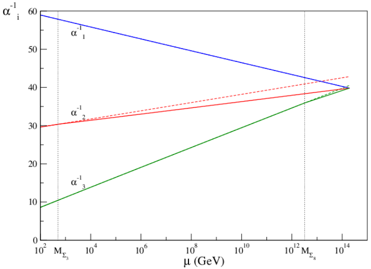
- Coloured Higgs triplets, T_1 and T_2

$$\sum_{n=1,2} \frac{(\Gamma^n_u)_{ij} \, (\Gamma^n_d)_{kl}}{M^2_{T_n}} \left[\frac{1}{2} (Q_i Q_j)(Q_k L_l) + (u^c_i e^c_j)(u^c_k d^c_l) \right]$$

This channel does not induce Proton decay at tree level

Unification of the gauge couplings @ 2 loop level considering the splitting between Σ_3 and Σ_8

- X, Y, T_1, T_2 @ Λ scale and H_1, H_2 @ electroweak scale



$$1.3 \times 10^{14}\,\text{GeV} \leq \Lambda \leq 2.4 \times 10^{14}\,\text{GeV} \text{ [Problematic]}$$

$$M_Z \leq M_{\Sigma_3} \leq 1.8 \times 10^{4}\,\text{GeV}$$

$$5.4 \times 10^{11}\,\text{GeV} \leq M_{\Sigma_8} \leq 1.3 \times 10^{14}\,\text{GeV}$$

`Solution`: 24 fermionic multiplets and 45 bosonic

SU(5) and Neutrinos

- M_e has also NNI form
- Singular Seesaw is not viable
- Effective Neutrino Mass Matrix for $\mathcal{Q}(\Phi_2) = 1$

Charges	$\nu_R = (0,1,3)$	$\nu_R = (1,2,3)$	$\nu_{R\,i} \in \{0,2\}$
$\phi_1=0$, $10=(1,3,0)$, $5^*=(2,0,1)$	$\mathrm{I}_{(123)}$ $\begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}$	$\mathrm{II}_{(12)}$ $\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$	$\mathrm{III}_{(12)}$ $\begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$
$\phi_1=2$, $10=(2,0,3)$, $5^*=(1,3,2)$	II $\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\mathrm{I}_{(13)}$ $\begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & 0 \end{pmatrix}$	III $\begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}$

- In the case of SO(10) does not work for Z_4 [works in SUSY]

Adjoint-$\mathbf{SU(5)} \times Z_4$

[Fileviez Perez]

- More fermionic and bosonic states
- The model is renormalisable
- We require that Z_4 forces NNI form to M_u and M_d
- Abelian discrete flavour symmetry Z_n

Fermionic sector

- $\mathbf{10}_i = (Q, u^c, e^c)_i$
- $\mathbf{5}_i^* = (L, d^c)_i$
- $\mathbf{24}$

Scalar sector

- $\Sigma(24)$ ($\mathcal{Q}(\Sigma) = 0$)
- 5_H and 45_H

$$\mathcal{Q}(\mathbf{10}_i) = (3q_3 + \phi, -q_3 - \phi, q_3)$$

$$\mathcal{Q}(\mathbf{5}_i^*) = (q_3 + 2\phi, -3q_3, -q_3 + \phi)$$

$$\mathcal{Q}(\mathbf{24}_i) = (n_1, n_2, n_3)$$

$$\mathcal{Q}(5_H) = -2\, q_3$$

$$\phi \equiv \mathcal{Q}(45_H)$$

- $45_H = S_{(8,2)_{\frac{3}{10}}} \oplus S_{(6^*,1)_{-\frac{1}{5}}} \oplus S_{(3^*,2)_{-\frac{7}{10}}} \oplus S_{(3^*,1)_{\frac{4}{5}}} \oplus \boldsymbol{\Delta} \oplus T_2 \oplus H_2\,,$

$$
\begin{aligned}
45^{a\,b}_{c} &= \varepsilon^{abd}[S_{(6^*,1)}]_{dc} + \delta^a_c\, T^b_2 - \delta^b_c\, T^a_2 \\
45^{a\,b}_{r} &= \varepsilon^{abc}[S_{(3^*,2)}]_{cr} \\
45^{a\,r}_{b} &= \frac{1}{2} S^{A\,r}_8 [\lambda^A]^a_b + \delta^a_b\, H^r_2 \\
45^{a\,r}_{s} &= \Delta^{a\,r}_s + \delta^r_s\, T^a_2 \\
45^{r\,s}_{a} &= \varepsilon^{rs}[S_{(3^*,1)}]_a \\
45^{r\,s}_{t} &= -3\,(\delta^r_t\, H^s_2 - \delta^s_t\, H^r_2)
\end{aligned}
$$

- New coloured triplets $\boldsymbol{\Delta} \sim (3,3,-1/3)$ [$M_{\boldsymbol{\Delta}} > 3.8 \times 10^{13}$ GeV]

$$
\boldsymbol{\Delta} \equiv \boldsymbol{\Delta}^I \frac{\sigma^I}{2} = \frac{1}{2} \begin{pmatrix} \boldsymbol{\Delta}^{-1/3} & \sqrt{2}\,\boldsymbol{\Delta}^{2/3} \\ \sqrt{2}\,\boldsymbol{\Delta}^{-4/3} & -\boldsymbol{\Delta}^{-1/3} \end{pmatrix}
$$

Adjoint-$\mathbf{SU(5)} \times Z_4$

- The Σ breaks the $\mathbf{SU(5)}$ to the SM
- The doublets H_1, H_2 break SM $\to SU(3)_c \times U(1)_{em}$
- Generate the fermion masses via Yukawa interactions

$$-\mathcal{L}_\mathsf{Y} = \left(\Gamma_u^1\right)_{ij} 10_i 10_j \left(5_H\right) + \left(\Gamma_u^2\right)_{ij} 10_i\, 10_j \left(45_H\right) + \left(\Gamma_d^1\right)_{ij} 10_i\, 5_j^* \left(5_H^*\right)$$
$$+ \left(\Gamma_d^2\right)_{ij} 10_i\, 5_j^* \left(45_H^*\right) + \mathbf{M}_{kl}\, \mathrm{Tr}\left(\rho_k\, \rho_l\right) + \boldsymbol{\lambda}_{kl}\, \mathrm{Tr}\left(\rho_k\, \rho_l\, \Sigma\right)$$
$$+ \left(\Gamma_\nu^1\right)_{ik} 5_i^* \left(\rho_k\right)\left(5_H\right) + \left(\Gamma_\nu^2\right)_{ik} 5_i^* \left(\rho_k\right)\left(45_H\right) + \mathrm{H.c.}\,,$$

- M_u and M_d have NNI form

$$M_u = v_{45} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_u \\ 0 & -b_u & 0 \end{pmatrix} + v_5 \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & 0 \\ 0 & 0 & c_u \end{pmatrix}$$

$$M_d = v_{45}^* \begin{pmatrix} 0 & a_d & 0 \\ a'_d & 0 & 0 \\ 0 & 0 & c_d \end{pmatrix} + v_5^* \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_d \\ 0 & b'_d & 0 \end{pmatrix}$$

Adjoint-$\mathbf{SU(5)} \times Z_4$

- The 45_H allows $M_d - M_e^{\mathsf{T}} = 8\, v_{45}^*\, \Gamma_d^2$
- Since M_d has NNI form $\Longrightarrow$ M_e has also NNI form
- Majorana and Dirac mass matrices

$$\mathbf{M}_0 = \frac{1}{4}\left(\mathbf{M} - \frac{\sigma}{\sqrt{30}}\boldsymbol{\lambda}\right),$$

$$\mathbf{M}_3 = \frac{1}{4}\left(\mathbf{M} - \frac{3\,\sigma}{\sqrt{30}}\boldsymbol{\lambda}\right),$$

$$\mathbf{M}_8 = \frac{1}{4}\left(\mathbf{M} + \frac{2\,\sigma}{\sqrt{30}}\boldsymbol{\lambda}\right).$$

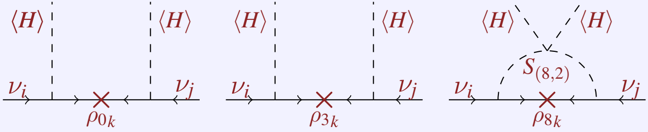
$$m_0^D = \frac{\sqrt{15}v}{2\sqrt{2}}\left(\frac{\cos\alpha}{5}\,\Gamma_\nu^1 + \sin\alpha\,\Gamma_\nu^2\right),$$

$$m_3^D = \frac{v}{\sqrt{2}}\left(\cos\alpha\,\Gamma_\nu^1 - 3\sin\alpha\,\Gamma_\nu^2\right),$$

- Three see-saw contributions

[Fileviez Perez and Wise]

- Type-I, Type-III and Colored-seesaw

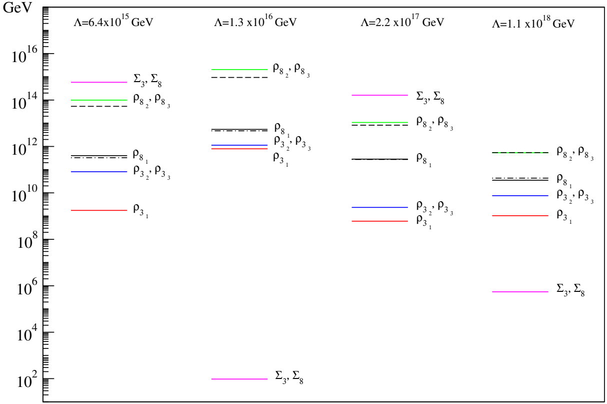


- Seesaw Formula [$F(x) \equiv \frac{x^2-1-\ln x^2}{(1-x^2)^2}$]

$$(m_\nu)_{ij} = -\left(m_0^D \mathbf{M}_0^{-1} m_0^{D^\mathsf{T}}\right)_{ij} - \left(m_3^D \mathbf{M}_3^{-1} m_3^{D^\mathsf{T}}\right)_{ij}$$
$$-\frac{v^2\,\zeta}{8\pi^2}\sum_{k=1}^{n_{24}} \frac{\left(U_8\,\Gamma_\nu^2\right)_{ik}\left(U_8\,\Gamma_\nu^2\right)_{jk}}{m_{\rho_{8k}}}\,F\left[\frac{M_{S_{(8,2)}}}{m_{\rho_{8k}}}\right]$$

Unification

- The presence of the the Adjoint fermions (24) allows Unification of the gauge couplings within a large range of the parameter space



Proton decay

- Via the coloured triplets T_1

$$\frac{\left(\Gamma_u^1\right)_{ij}\left(\Gamma_d^1\right)_{kl}}{M_{T_1}^2}\left[\frac{1}{2}(Q_iQ_j)(Q_kL_l)\;+\;(u_i^{\mathsf{C}}e_j^{\mathsf{C}})(u_k^{\mathsf{C}}d_l^{\mathsf{C}})\right]$$

- Via the coloured triplets T_2

$$\frac{4\left(\Gamma_u^2\right)_{ij}\left(\Gamma_d^2\right)_{kl}}{M_{T_2}^2}(u_i^{\mathsf{C}}e_j^{\mathsf{C}})(u_k^{\mathsf{C}}d_l^{\mathsf{C}})$$

- Via the new coloured triplets $\boldsymbol{\Delta}$

$$\frac{\left(\Gamma_u^2\right)_{ij}\left(\Gamma_d^2\right)_{kl}}{2}\left\{\frac{1}{M_{\boldsymbol{\Delta}^{-1/3}}^2}\left[(u_id_j)(u_ke_l)\;+\;(u_id_j)(d_k\nu_l)\right]\;-\;\frac{1}{M_{\boldsymbol{\Delta}^{2/3}}^2}(d_id_j)(u_k\nu_l)\right\}$$

They do not induce Proton decay at tree level due to Z_4!

Neutrino sector

m_ν	$\mathbf{M}_{0,3,8}$	$\mathcal{Q}(24_i)$	$m^D_{0,3}$	$\mathcal{Q}(5^*)$	$\mathcal{Q}(10_i)$	$\mathcal{Q}(5_H)$	$\mathcal{Q}(45_H)$
$\begin{pmatrix} 0 & \ast & 0 \\ \ast & \ast & \ast \\ 0 & \ast & \ast \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \ast \\ 0 & \ast & 0 \\ \ast & 0 & 0 \end{pmatrix}$	(1,2,3)	$\begin{pmatrix} 0 & 0 & \ast \\ \ast & \ast & 0 \\ 0 & \ast & \ast \end{pmatrix}$	(3,1,0)	(0,2,1)	2	1
			$\begin{pmatrix} \ast & 0 & 0 \\ 0 & \ast & \ast \\ \ast & \ast & 0 \end{pmatrix}$	(1,3,0)	(0,2,3)	2	3
	$\begin{pmatrix} \ast & 0 & 0 \\ 0 & 0 & \ast \\ 0 & \ast & 0 \end{pmatrix}$	(0,1,3)	$\begin{pmatrix} 0 & \ast & 0 \\ \ast & 0 & \ast \\ \ast & \ast & 0 \end{pmatrix}$	(1,3,2)	(2,0,3)	2	1
			$\begin{pmatrix} 0 & 0 & \ast \\ \ast & \ast & 0 \\ \ast & 0 & \ast \end{pmatrix}$	(3,1,2)	(2,0,1)	2	3
$\begin{pmatrix} \ast & \ast & \ast \\ \ast & 0 & 0 \\ \ast & 0 & \ast \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \ast \\ 0 & \ast & 0 \\ \ast & 0 & 0 \end{pmatrix}$	(1,2,3)	$\begin{pmatrix} \ast & \ast & 0 \\ 0 & 0 & \ast \\ 0 & \ast & \ast \end{pmatrix}$	(2,0,1)	(1,3,0)	0	1
			$\begin{pmatrix} 0 & \ast & \ast \\ \ast & 0 & 0 \\ \ast & \ast & 0 \end{pmatrix}$	(2,0,3)	(3,1,0)	0	3
	$\begin{pmatrix} \ast & 0 & 0 \\ 0 & 0 & \ast \\ 0 & \ast & 0 \end{pmatrix}$	(0,1,3)	$\begin{pmatrix} \ast & 0 & \ast \\ 0 & \ast & 0 \\ \ast & \ast & 0 \end{pmatrix}$	(0,2,3)	(3,1,2)	0	1
			$\begin{pmatrix} \ast & \ast & 0 \\ 0 & 0 & \ast \\ \ast & 0 & \ast \end{pmatrix}$	(0,2,1)	(1,3,2)	0	3

Neutrino Mass

$$M_e = K_e \begin{pmatrix} 0 & A_e & 0 \\ A'_e & 0 & B_e \\ 0 & B'_e & C_e \end{pmatrix} \qquad m_\nu^{A_g} = P_g \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu\, e^{i\varphi} \end{pmatrix} P_g^\mathsf{T}$$

Neutrino sector

Neutrino Oscillation data:

$sin^2 2\theta_{13} = 0.092 \pm 0.016$

[DAYA-BAY colaboration, 2012]

[Forero, Tortola, Valle, arXiv:1205.4018]

parameter	best fit	3σ range
Δm^2_{21} $[10^{-5}$ eV$]$	7.62	$7.12 - 8.20$
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}$ eV$]$	2.55 2.43	$2.31 - 2.74$ $2.21 - 2.64$
$\sin^2\theta_{12}$	0.320	$0.27 - 0.37$
$\sin^2\theta_{23}$	0.613 (0.427) 0.600	$0.36 - 0.68$ $0.37 - 0.67$
$\sin^2\theta_{13}$	0.0246 0.0250	$0.017 - 0.033$
δ	0.80π -0.03π	$0 - 2\pi$

Other constraints

Effective Majorana mass: $m_{ee} \equiv \lvert \sum_{i=1}^{3} m_i U_{1i}^{*2} \rvert$

[Pascoli et al, 2003]

$|m_{ee}| \lesssim 0.005$ eV (NH) $\qquad 10^{-2}$ eV $\lesssim |m_{ee}| \lesssim 0.05$ eV (IH)

[Bilenky et al, 2001; Petcov et al, 2005]

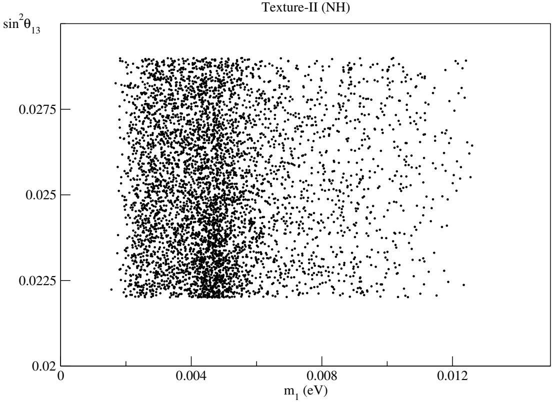
Tritium β decay: $m^2_{\nu_e} \equiv \sum_{i=1}^{3} m_i^2 |U_{1i}|^2 < (2.3\,\text{eV})^2$ at 95% C.L.

[Nakamura et al, 2012(PDG)]

From cosmological and astrophysical data: $\mathcal{T} \equiv \sum_{i=1}^{3} \mathbf{m_i} < \mathbf{0.68}$ eV at 95% C.L.

Results

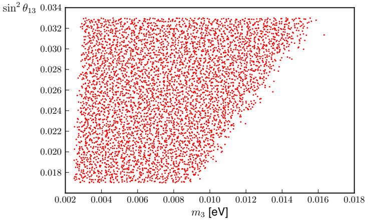
$$m_\nu = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}_{NH}$$

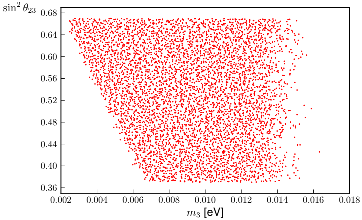


- $0.0015 \leq m_1[\text{eV}] \leq 0.013$
- $0.97 < |m_{ee}|\ [10^{-3}\text{eV}] < 2.1$

Results

$$\mathbf{m}_\nu = \begin{pmatrix} * & * & * \\ * & \mathbf{0} & \mathbf{0} \\ * & \mathbf{0} & * \end{pmatrix}_{\mathbf{IH}}$$

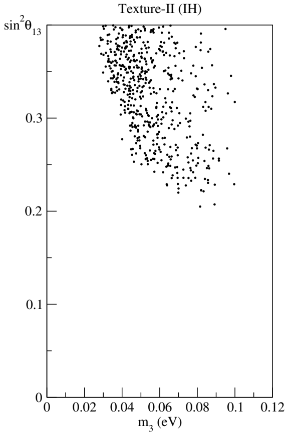


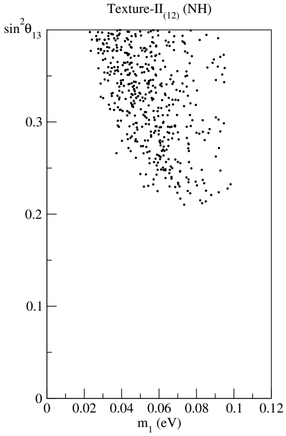


- $0.005 \leq m_3\,[\text{eV}] \leq 0.010$
- $0.015\,\text{eV} < |m_{ee}|\,[\text{eV}] < 0.021$

Results

$$m_\nu = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}_{\text{IH}} \qquad m_\nu = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}_{\text{NH}}$$



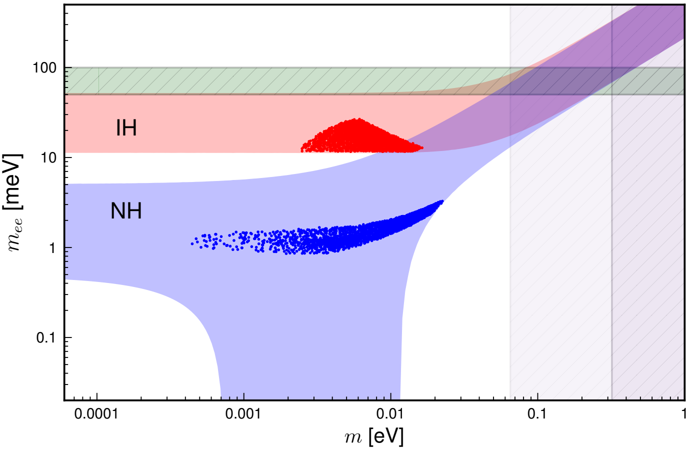


Not viable

Results

$$m_\nu = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}_{NH}$$

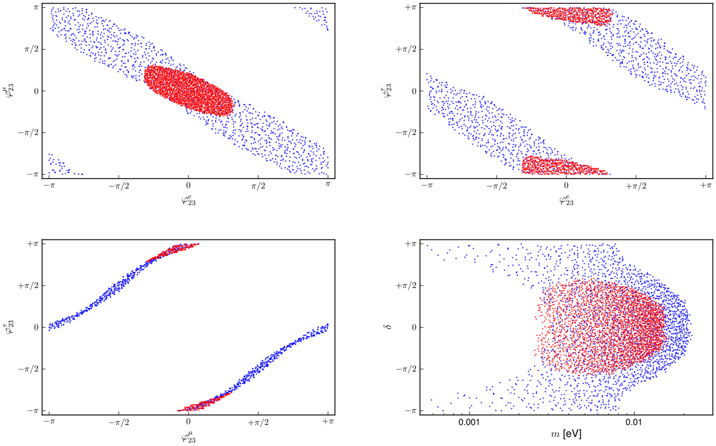
$$\mathbf{m}_\nu = \begin{pmatrix} * & * & * \\ * & \mathbf{0} & \mathbf{0} \\ * & \mathbf{0} & * \end{pmatrix}_{\mathbf{IH}}$$



Majorana- and Dirac-type phases

$$m_\nu = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}_{NH}$$

$$\mathbf{m}_\nu = \begin{pmatrix} * & * & * \\ * & \mathbf{0} & \mathbf{0} \\ * & \mathbf{0} & * \end{pmatrix}_{\mathbf{IH}}$$



$$\varphi^\alpha_{ij} \equiv \arg\left(U_{\alpha i}\, U^*_{\alpha j}\right)$$

Conclusions

- The NNI textures for quark mass matrix obtained through Abelian flavour symmetry in the context of two Higgs doublets. The Minimal realisation is Z_4
- Z_4 is implemented in the Consistente-$SU(5)$ and in Adjoint-$SU(5)$
- The charged lepton mass matrix has also NNI form
- m_ν has two possible patterns
- The renormalisable Adjoint model solves unification and proton decay
- Proton decay through coloured triplet exchange is absent at tree level

Normal Hierarchy	**Inverted Hierarchy**
$\begin{pmatrix} 0 & \star & 0 \\ \star & \star & \star \\ 0 & \star & \star \end{pmatrix}$	$\begin{pmatrix} \star & \star & \star \\ \star & 0 & 0 \\ \star & 0 & \star \end{pmatrix}$
$0.0015\,\text{eV} \leq m_1 \leq 0.013\,\text{eV}$	$0.005\,\text{eV} \leq m_3 \leq 0.010\,\text{eV}$
$0.00097\,\text{eV} < \lvert m_{ee} \rvert < 0.0021\,\text{eV}$	$0.015\,\text{eV} < \lvert m_{ee} \rvert < 0.021\,\text{eV}$


ΒΙΒΛΙΟΧΑΡΤΟΠΩΛΕΙΟ
ΣΥΜΜΕΤΡΙΑ
Ευχαριστώ!