

# The Electroweak Fit Revisited

Marco Ciuchini



- Introduction
- Theoretical status (" $R_b$  or not  $R_b$ ")
- SM fit & uncertainties
- EWPO beyond the SM
- Conclusions & Outlook

based on M.C., E. Franco,  
S. Mishima, L. Silvestrini,  
[hep-ph/1306.4644](https://arxiv.org/abs/hep-ph/1306.4644)

Corfu Summer Institute

13th Hellenic School and Workshops on Elementary Particle Physics and Gravity  
Corfu, Greece 2013



# Introduction

The Standard Model works beautifully up to hundreds GeV, but it is bound to be an effective theory valid up to a scale  $\Lambda \leq M_{\text{Planck}}$ :

$$\mathcal{L}_{\text{SM}}(M_W) = \mathcal{L}_{\text{SM}}^4 - \frac{1}{\Lambda} \mathcal{L}_{\text{SM}}^5 + \frac{1}{\Lambda^2} \mathcal{L}_{\text{SM}}^6 + \dots$$

EW scale

Violates in general tree-level relations and accidental symmetries

Spontaneously broken renormalizable theory:  
tree-level relations, accidental symmetries

Spontaneous breaking of  $SU(2)_L \otimes U(1)_Y$  via  
Higgs vev & renormalizability imply:

1) tree-level relations in EW sector

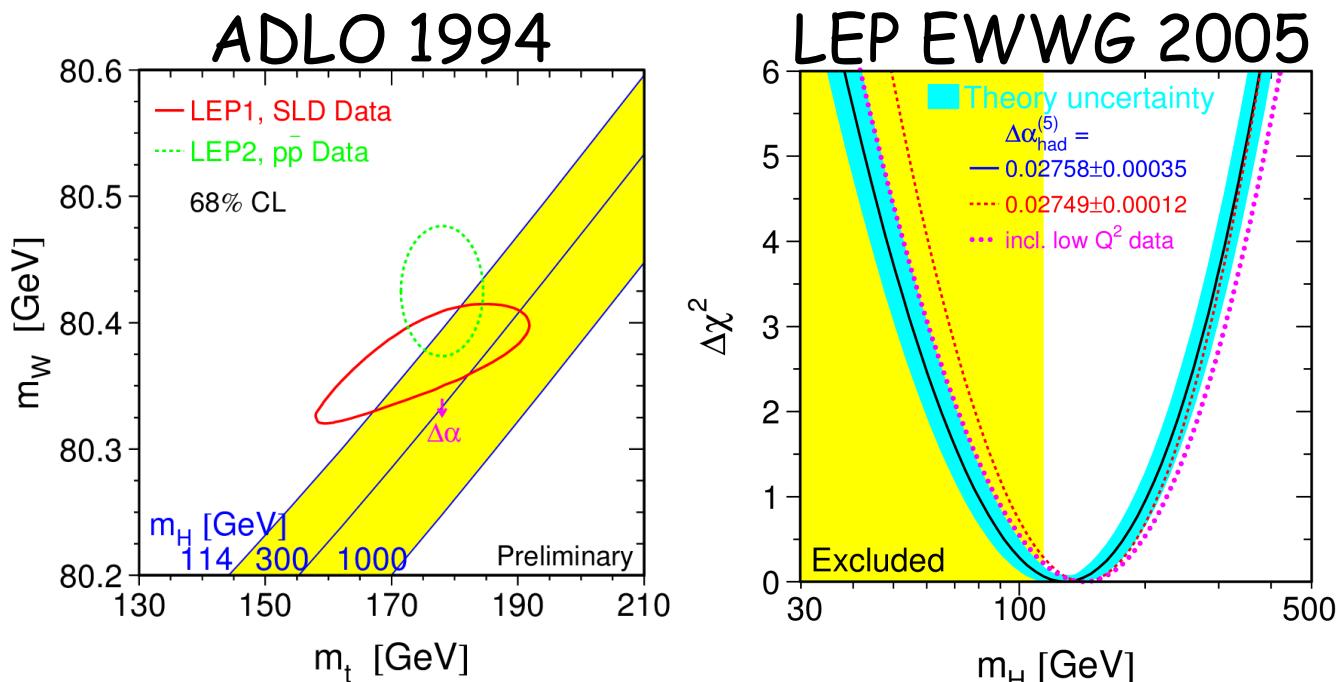
(for example  $M_W = M_Z \cos\theta_W$ )

2) calculable loop corrections

⇒ EWPO potentially very sensitive to NP!!

⇒ actual NP sensitivity depends on the  
exp. precision and th. uncertainty

# EWPO & SM FIT



Qualitative change in the EW fit with the expt.  
observation of the Higgs boson

\* Before:

- top mass & indirect evidence of a light Higgs
- Higgs and NP effects entangled in EWPO

\* After:

- SM predictions fully computable
- constraints on NP (including Higgs couplings)

# The 7 SM Input Parameters

- ▶  $G_\mu = 1.1663787 \cdot 10^{-5} \text{ GeV}^{-2}$ ,  $\alpha = 1/137.035999074$  (PDG)
- ▶  $M_Z = 91.1875 \pm 0.0021 \text{ GeV}$  (LEP)
- ▶  $m_h = (125.6 \pm 0.3) \text{ GeV}$  (naïve average of Atlas & CMS)
- ▶  $\alpha_s(M_Z^2) = 0.1184 \pm 0.0006$  (PDG excluding EWPO)
- ▶  $\Delta\alpha_s^5(M_Z^2) = 0.02750 \pm 0.00033$  (Burkhardt & Pietrzyk;  
see also Davier et al, Hagiwara et al, Jegerlehner)
- ▶  $m_t = 173.2 \pm 0.9 \text{ GeV}$  (TeVatron average; LHC has  
 $m_t = 173.3 \pm 1.4 \text{ GeV}$ )

Pseudo-observables can be written in terms of  $\Delta r$  and effective Zff couplings

$$M_W^2 = \frac{M_Z^2}{2} \left( 1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2} (1 + \Delta r)} \right)$$

LEP EWWG;ZITTER;  
Chetyrkin et al;Baikov et al;  
Czarknecki&Kuhn;  
Harlander et al.Bardin et al

$$\begin{aligned}\mathcal{L} &= \frac{e}{2s_W c_W} Z_\mu \sum_f \bar{f} \left( \textcolor{red}{g_V^f} \gamma_\mu - \textcolor{red}{g_A^f} \gamma_\mu \gamma_5 \right) f, \\ &= \frac{e}{2s_W c_W} Z_\mu \sum_f \bar{f} \left[ \textcolor{red}{g_R^f} \gamma_\mu (1 + \gamma_5) + \textcolor{red}{g_L^f} \gamma_\mu (1 - \gamma_5) \right] f, \\ &= \frac{e}{2s_W c_W} \sqrt{\rho_Z^f} Z_\mu \sum_f \bar{f} \left[ (I_3^f - 2Q_f \kappa_Z^f s_W^2) \gamma^\mu - I_3^f \gamma^\mu \gamma_5 \right] f\end{aligned}$$

$$A_{\text{LR}}^{0,f} = \mathcal{A}_f = \frac{2 \operatorname{Re} \left( g_V^f / g_A^f \right)}{1 + \left[ \operatorname{Re} \left( g_V^f / g_A^f \right) \right]^2} \quad A_{\text{FB}}^{0,f} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f \quad (f = \ell, c, b)$$

$$P_\tau^{\text{pol}} = \mathcal{A}_\tau \qquad \qquad \sin^2 \theta_{\text{eff}}^{\text{lept}} = \text{Re}(\kappa_Z^\ell) s_W^2$$

$$\Gamma_f = \Gamma(Z \rightarrow f\bar{f}) \propto |\rho_Z^f| \left[ \left| \frac{g_V^f}{g_A^f} \right|^2 R_V^f + R_A^f \right] \rightarrow \Gamma_Z, \sigma_h^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_h}{\Gamma_Z^2}, R_\ell^0 = \frac{\Gamma_h}{\Gamma_\ell}, R_{c,b}^0 = \frac{\Gamma_{c,b}}{\Gamma_h}$$

# SM corrections: state of the art

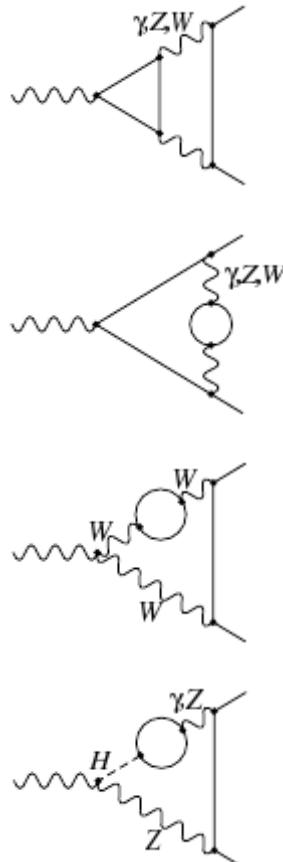
## Available calculations:

- $\Delta r$ : numerical expression including  $O(\alpha)$ ,  
 $O(\alpha\alpha_s)$ ,  $O(G_\mu \alpha_s^2 m_t^2 (1 + m_t^2/M_Z^2 + m_t^4/M_Z^4))$ ,  
 $O(\alpha^2)$ ,  $O(G_\mu^2 \alpha_s m_t^4)$ ,  $O(G_\mu^3 m_t^6)$   
th. err. on  $M_W < 4$  MeV (neglected)
- $\kappa_Z^f$ : numerical expression including  $O(\alpha^2)$ ,  
 $O(G_\mu^2 \alpha_s m_t^4)$ ,  $O(G_\mu^3 m_t^6)$  (bosonic 2-loop  
missing for  $f=b$ ); th. err.  $< 2 \cdot 10^{-4}$  (neglected)

Sirlin; Marciano&Sirlin; Djouadi&Verzegnassi; Djouadi; Kniehl; Halzen&Kniehl; Kniehl&Sirlin; Djouadi&Gambino;  
Avdeev et al.; Chetyrkin et al.; Barbieri et al.; Fleischer et al.; Degrassi et al.; Freitas et al.; Awramik&Czakon;  
Onishchenko&Veretin; Van der Bijl et al.; Faisst et al.; Awramik et al.

# $R_b$ or not $R_b$ : that is the question

A problem with  $\rho_Z^f$ :



- complete two-loop corrections still missing;
- two-loop fermionic corrections to  $R_b^0 = \Gamma_b / \Gamma_h$  recently computed by Freitas & Huang;
- result much larger than expected:  
 $0.21576 \rightarrow 0.21493$  (as large as one-loop!)
- results only available for  $\Gamma_u / \Gamma_b$  and  $\Gamma_d / \Gamma_b$ , cannot compute all  $\rho_Z^f$  at two-loops, only  $R_{b,c}^0$
- two-loop computation of all  $\rho_Z^f$  needed!!!

- inconsistent to use  $\Gamma_u/\Gamma_b$  and  $\Gamma_d/\Gamma_b$  from Freitas & Huang and “old” formulae for all other  $p_z^f$ -related observables ( $R_1^0, \Gamma_Z, \sigma_{had}^0$ )!  
see e.g. Gfitter...
- expect large two-loop corrections to all  $p_z^f$ : introduce additional large th. uncertainty
- we repeat our analysis in two scenarios:
  - “Old  $R_b$ ”: use  $O(\alpha^2 m_t^4/M_W^4)$  &  $O(\alpha^2 m_t^2/M_W^2)$
  - “New  $R_b$ ”: use Freitas & Huang result plus 3 new parameters  $\delta p_z^{b,l,v}$  to account for missing 2-loop corrections, with  $|\delta p_z^{b,l,v}| < 5 \times 10^{-3}$

# Fit Procedure

- Bayesian analysis, exp. likelihood used as priors for all input parameters
- MCMC implemented using the BAT library
- New code for EWPO written from scratch and validated against ZFITTER (thanks to the ZFITTER authors!)
- Part of a larger HEP model fitting project
- Official release of the code to appear (soon)  
recent frequentist analyses: Gfitter, Erler, ...  
*Caldwell et al.*

# SM Fit Results - "old $R_b$ "

	Data	Fit	Indirect	Pull
$\alpha_s(M_Z^2)$	$0.1184 \pm 0.0006$	$0.1184 \pm 0.0006$	$0.1193 \pm 0.0027$	+0.3
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	$0.02750 \pm 0.00033$	$0.02740 \pm 0.00026$	$0.02725 \pm 0.00042$	-0.5
LEP $M_Z$ [GeV]	$91.1875 \pm 0.0021$	$91.1878 \pm 0.0021$	$91.197 \pm 0.012$	+0.8
TeV $m_t$ [GeV]	$173.2 \pm 0.9$	$173.5 \pm 0.8$	$176.3 \pm 2.5$	+1.1
LHC $m_h$ [GeV]	$125.6 \pm 0.3$	$125.6 \pm 0.3$	$97.3 \pm 26.9$	-0.9
$M_W$ [GeV]	$80.385 \pm 0.015$	$80.367 \pm 0.007$	$80.362 \pm 0.007$	-1.4
$\Gamma_W$ [GeV]	$2.085 \pm 0.042$	$2.0891 \pm 0.0006$	$2.0891 \pm 0.0006$	+0.1
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	$2.4953 \pm 0.0004$	$2.4953 \pm 0.0004$	+0.0
$\sigma_h^0$ [nb]	$41.540 \pm 0.037$	$41.484 \pm 0.004$	$41.484 \pm 0.004$	-1.5
$\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{\text{FB}}^{\text{had}})$	$0.2324 \pm 0.0012$	$0.23145 \pm 0.00009$	$0.23144 \pm 0.00009$	-0.8
$P_{\tau}^{\text{pol}}$	$0.1465 \pm 0.0033$	$0.1476 \pm 0.0007$	$0.1477 \pm 0.0007$	+0.3
$\mathcal{A}_\ell$ (SLD)	$0.1513 \pm 0.0021$	$0.1476 \pm 0.0007$	$0.1471 \pm 0.0008$	-1.9
$\mathcal{A}_c$	$0.670 \pm 0.027$	$0.6682 \pm 0.0003$	$0.6682 \pm 0.0003$	-0.1
$\mathcal{A}_b$	$0.923 \pm 0.020$	$0.93466 \pm 0.00006$	$0.93466 \pm 0.00006$	+0.6
$A_{\text{FB}}^{0,\ell}$	$0.0171 \pm 0.0010$	$0.0163 \pm 0.0002$	$0.0163 \pm 0.0002$	-0.8
$A_{\text{FB}}^{0,c}$	$0.0707 \pm 0.0035$	$0.0740 \pm 0.0004$	$0.0740 \pm 0.0004$	+0.9
$A_{\text{FB}}^{0,b}$	$0.0992 \pm 0.0016$	$0.1035 \pm 0.0005$	$0.1039 \pm 0.0005$	+2.8 <span style="color: red;">← large deviation!</span>
$R_\ell^0$	$20.767 \pm 0.025$	$20.735 \pm 0.004$	$20.734 \pm 0.004$	-1.3
$R_c^0$	$0.1721 \pm 0.0030$	$0.17223 \pm 0.00002$	$0.17223 \pm 0.00002$	+0.0
$R_b^0$	$0.21629 \pm 0.00066$	$0.21575 \pm 0.00003$	$0.21575 \pm 0.00003$	-0.8 <span style="color: red;">← old <math>R_b</math></span>

Fit: our fit results

Indirect: determined w/o using the corresponding experimental information

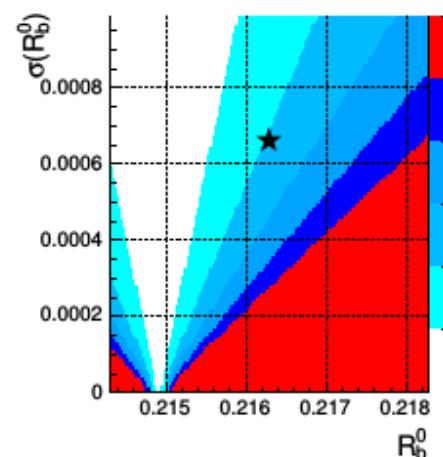
Pull: in units of standard deviations evaluated from the p.d.f's of "Data" and "Indirect"

# SM Fit Results - "new $R_b$ "

	Data	Fit	Indirect	Pull
$\alpha_s(M_Z^2)$	$0.1184 \pm 0.0006$	$0.1184 \pm 0.0006$	$0.078 \pm 0.024$	-1.9
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	$0.02750 \pm 0.00033$	$0.02742 \pm 0.00026$	$0.02728 \pm 0.00043$	-0.4
LEP $M_Z$ [GeV]	$91.1875 \pm 0.0021$	$91.1878 \pm 0.0021$	$91.204 \pm 0.013$	+1.2
TeV $m_t$ [GeV]	$173.2 \pm 0.9$	$173.5 \pm 0.8$	$175.7 \pm 2.6$	+0.9
LHC $m_h$ [GeV]	$125.6 \pm 0.3$	$125.6 \pm 0.3$	$98.5 \pm 27.7$	-0.8
$\delta\rho_Z^\nu$	—	$-0.0052 \pm 0.0031$	—	—
$\delta\rho_Z^\ell$	—	$-0.0002 \pm 0.0010$	—	—
$\delta\rho_Z^b$	—	$-0.0021 \pm 0.0011$	—	—
$M_W$ [GeV]	$80.385 \pm 0.015$	$80.366 \pm 0.007$	$80.361 \pm 0.007$	-1.4
$\Gamma_W$ [GeV]	$2.085 \pm 0.042$	$2.0890 \pm 0.0006$	$2.0890 \pm 0.0006$	+0.1
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	$2.4952 \pm 0.0023$	—	—
$\sigma_h^0$ [nb]	$41.540 \pm 0.037$	$41.539 \pm 0.037$	—	—
$\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{\text{FB}}^{\text{had}})$	$0.2324 \pm 0.0012$	$0.23145 \pm 0.00009$	$0.23145 \pm 0.00009$	-0.8
$P_\tau^{\text{pol}}$	$0.1465 \pm 0.0033$	$0.1476 \pm 0.0007$	$0.1476 \pm 0.0007$	+0.3
$\mathcal{A}_e$ (SLD)	$0.1513 \pm 0.0021$	$0.1476 \pm 0.0007$	$0.1470 \pm 0.0008$	-1.9
$\mathcal{A}_c$	$0.670 \pm 0.027$	$0.6681 \pm 0.0003$	$0.6681 \pm 0.0003$	-0.1
$\mathcal{A}_b$	$0.923 \pm 0.020$	$0.93466 \pm 0.00006$	$0.93466 \pm 0.00006$	+0.6
$A_{\text{FB}}^{0,\ell}$	$0.0171 \pm 0.0010$	$0.0163 \pm 0.0002$	$0.0163 \pm 0.0002$	-0.8
$A_{\text{FB}}^{0,c}$	$0.0707 \pm 0.0035$	$0.0739 \pm 0.0004$	$0.0740 \pm 0.0004$	+0.9
$A_{\text{FB}}^{0,b}$	$0.0992 \pm 0.0016$	$0.1034 \pm 0.0005$	$0.1038 \pm 0.0005$	+2.7
$R_\ell^0$	$20.767 \pm 0.025$	$20.768 \pm 0.025$	—	—
$R_c^0$	$0.1721 \pm 0.0030$	$0.17247 \pm 0.00002$	$0.17247 \pm 0.00002$	+0.1
$R_b^0$	$0.21629 \pm 0.00066$	$0.21492 \pm 0.00003$	$0.21492 \pm 0.00003$	-2.1

← not precise!

consistent with  
Freitas & Huang



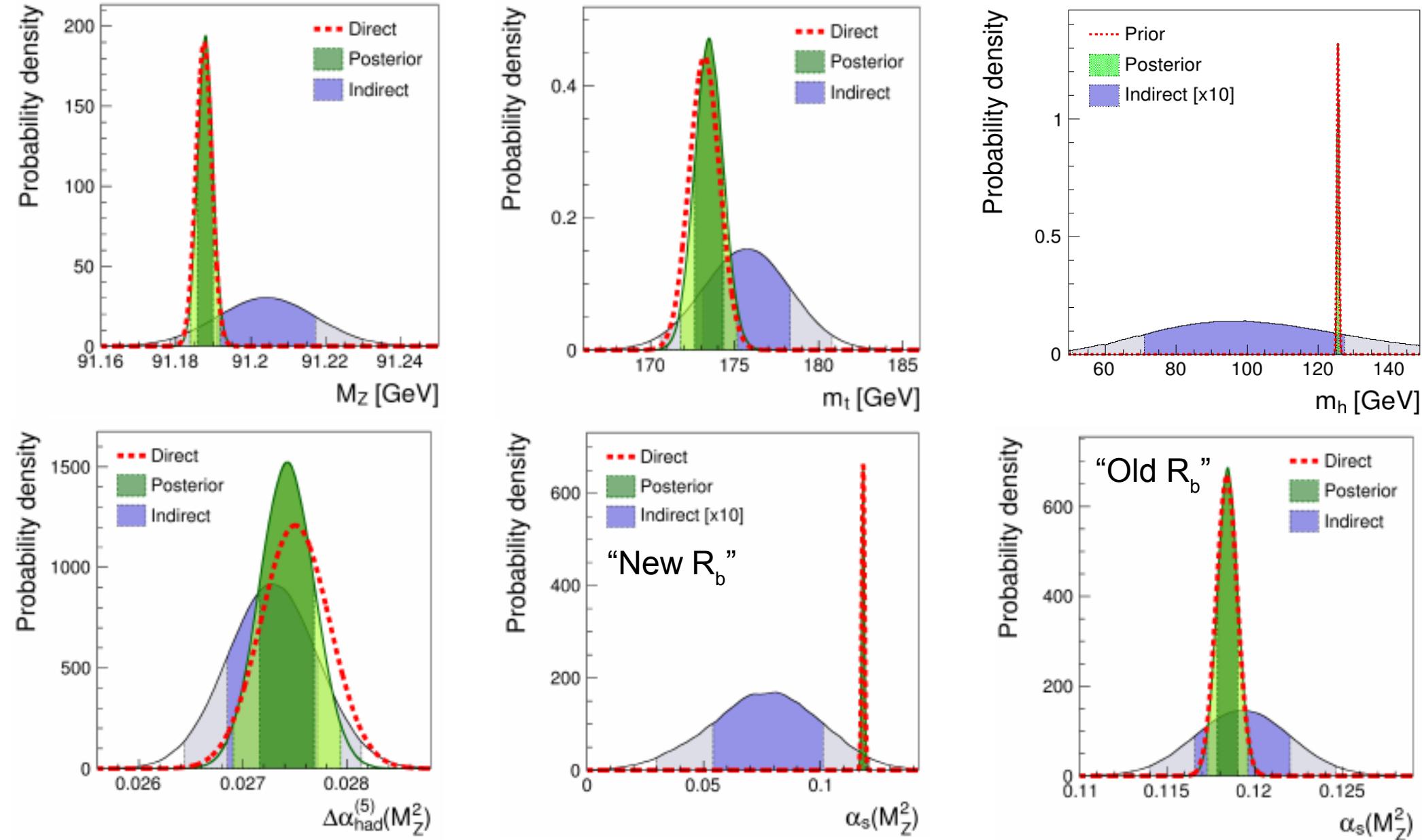
← new  $R_b$

# Predictions & Uncertainties

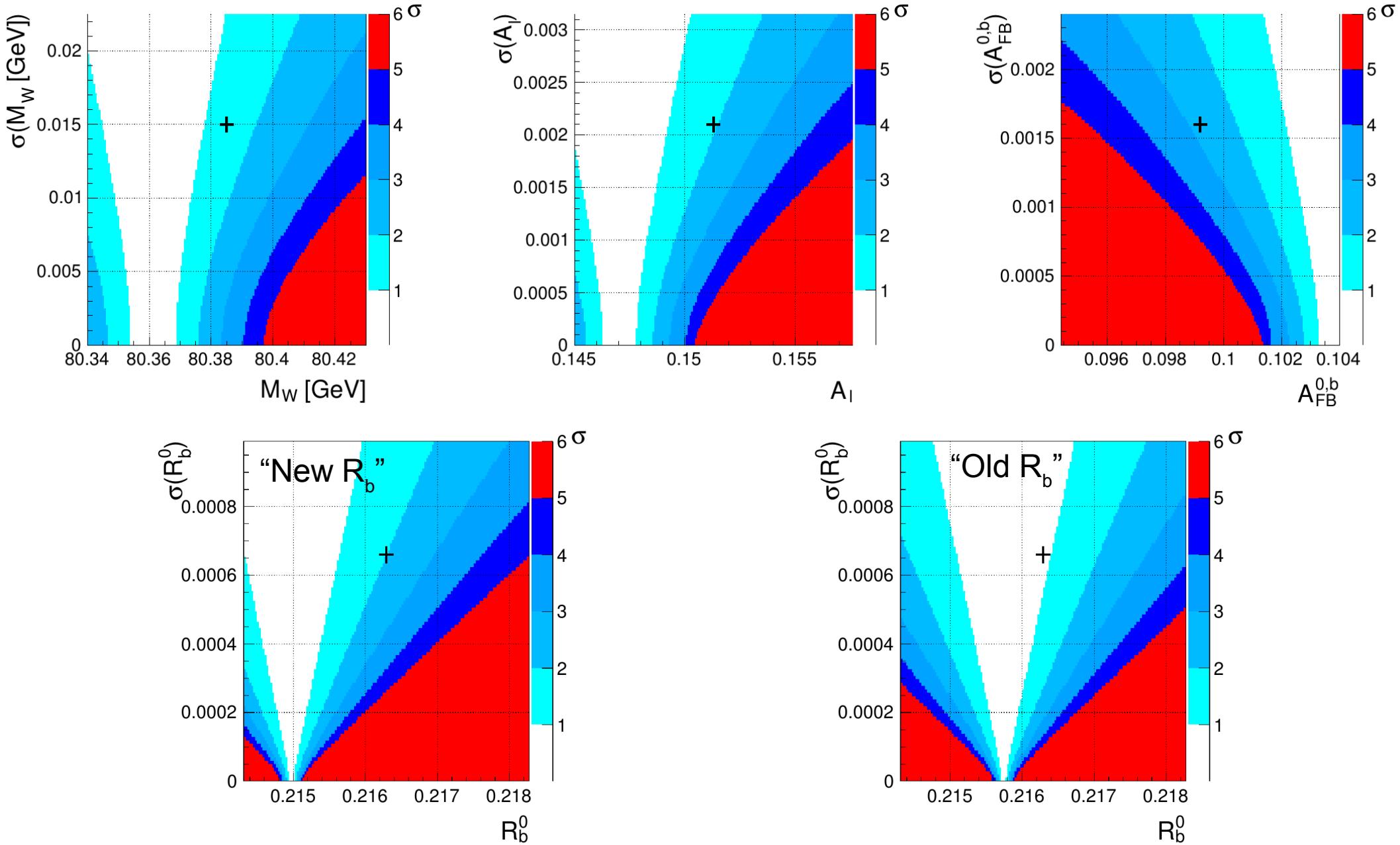
	Prediction	$\alpha_s$	$\Delta\alpha_{\text{had}}^{(5)}$	$M_Z$	$m_t$
$M_W$ [GeV]	$80.3625 \pm 0.0085$	$\pm 0.0004$	$\pm 0.0060$	$\pm 0.0026$	$\pm 0.0054$
$\Gamma_W$ [GeV]	$2.0888 \pm 0.0007$	$\pm 0.0002$	$\pm 0.0005$	$\pm 0.0002$	$\pm 0.0004$
$\Gamma_Z$ [GeV]	$2.5014 \pm 0.0060^*$	$\pm 0.0004$	$\pm 0.0003$	$\pm 0.0001$	$\pm 0.0002$
$\sigma_h^0$ [nb]	$41.425 \pm 0.123^*$	$\pm 0.004$	$\pm 0.000$	$\pm 0.001$	$\pm 0.000$
$\sin^2 \theta_{\text{eff}}^{\text{lept}}(Q_{\text{FB}}^{\text{had}})$	$0.23149 \pm 0.00012$	$\pm 0.00000$	$\pm 0.00012$	$\pm 0.00001$	$\pm 0.00003$
$P_\tau^{\text{pol}} = \mathcal{A}_\ell$	$0.14725 \pm 0.00094$	$\pm 0.00002$	$\pm 0.00091$	$\pm 0.00012$	$\pm 0.00022$
$\mathcal{A}_c$	$0.6680 \pm 0.0004$	$\pm 0.0000$	$\pm 0.0004$	$\pm 0.0001$	$\pm 0.0001$
$\mathcal{A}_b$	$0.9346 \pm 0.0001$	$\pm 0.0000$	$\pm 0.0001$	$\pm 0.0000$	$\pm 0.0000$
$A_{\text{FB}}^{0,\ell}$	$0.01626 \pm 0.00021$	$\pm 0.00000$	$\pm 0.00020$	$\pm 0.00003$	$\pm 0.00005$
$A_{\text{FB}}^{0,c}$	$0.07377 \pm 0.00052$	$\pm 0.00001$	$\pm 0.00050$	$\pm 0.00006$	$\pm 0.00012$
$A_{\text{FB}}^{0,b}$	$0.10322 \pm 0.00067$	$\pm 0.00001$	$\pm 0.00064$	$\pm 0.00008$	$\pm 0.00016$
$R_\ell^0$	$20.808 \pm 0.090^*$	$\pm 0.004$	$\pm 0.002$	$\pm 0.001$	$\pm 0.000$
$R_c^0$	$0.17242 \pm 0.00002$	$\pm 0.00001$	$\pm 0.00001$	$\pm 0.00000$	$\pm 0.00001$
$R_b^0$	$0.214991 \pm 0.000036$	$\pm 0.000009$	$\pm 0.000004$	$\pm 0.000001$	$\pm 0.000035$

\* completely dominated by unknown corrections to  $p_Z^{\text{f!}}$  ("new  $R_b$ ")

# SM Parameters



# COMPATIBILITY PLOTS



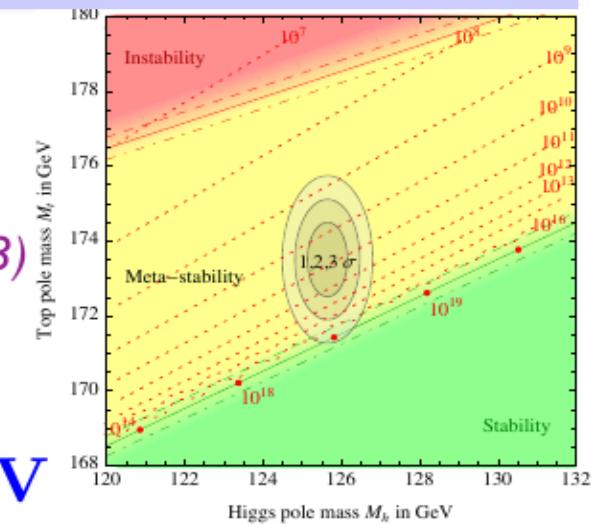
# $m_t$ & SM Vacuum Stability

The measurement of the top mass is crucial for testing the stability of the SM vacuum. *Degrassi et al.(12); Buttazzo et al.(13)*

$$m_t^{\text{pole}} < (171.36 \pm 0.46) \text{ GeV}$$

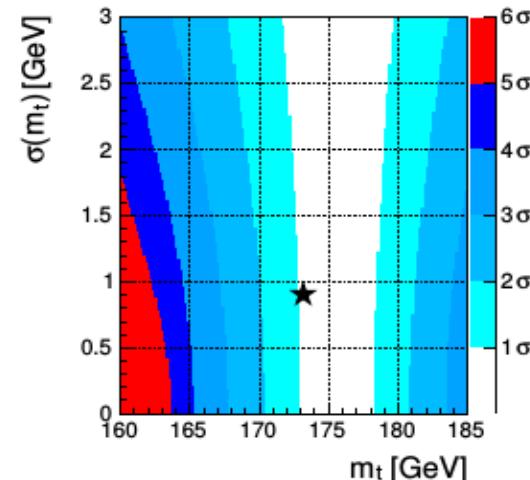
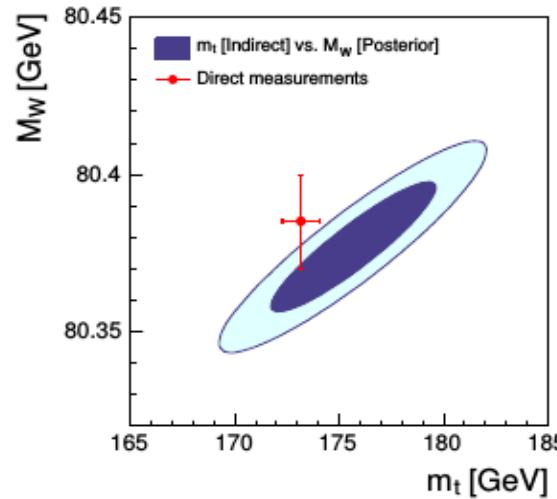
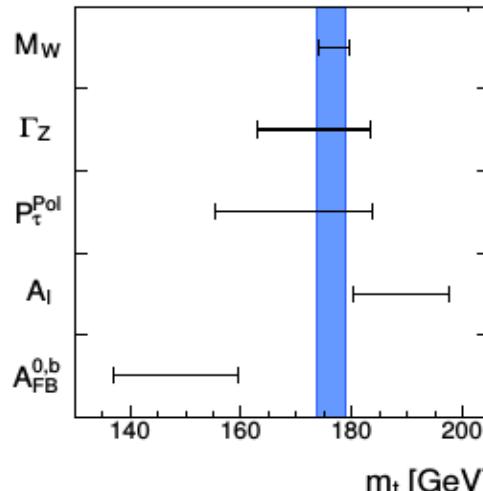
Tevatron pole(?) mass:  **$173.2 \pm 0.9$  GeV**

Pole from MSbar:  **$173.3 \pm 2.8$  GeV**



do not miss Degrassi talk!  
(hopefully before the phase transition)

Indirect determination from EW fit:  **$175.7 \pm 2.6$  GeV**



# EWPO Beyond The SM

Several NP scenarios:

1. Oblique: NP contributes mainly to gauge-boson self-energies;
2. Modified Zbb couplings;
3. Non-standard (composite) Higgs: modified Higgs couplings
4. Effective Lagrangian for EWPO: generic NP contributions to EWPO from dim.-6 operators

# 1. Oblique NP

Suppose that dominant NP effects appear in the vacuum polarizations of the gauge bosons:

$$S = -16\pi\Pi'_{30}(0) = 16\pi \left[ \Pi_{33}^{\text{NP}'}(0) - \Pi_{3Q}^{\text{NP}'}(0) \right]$$

$$T = \frac{4\pi}{s_W^2 c_W^2 M_Z^2} \left[ \Pi_{11}^{\text{NP}}(0) - \Pi_{33}^{\text{NP}}(0) \right]$$

$$U = 16\pi \left[ \Pi_{11}^{\text{NP}'}(0) - \Pi_{33}^{\text{NP}'}(0) \right]$$

*Kennedy & Lynn (89);  
Peskin & Takeuchi (90,92)*

EWPO depend on the combinations:

$$\delta M_W, \delta \Gamma_W \propto -\textcolor{red}{S} + 2c_W^2 \textcolor{red}{T} + \frac{(c_W^2 - s_W^2) \textcolor{red}{U}}{2s_W^2}$$

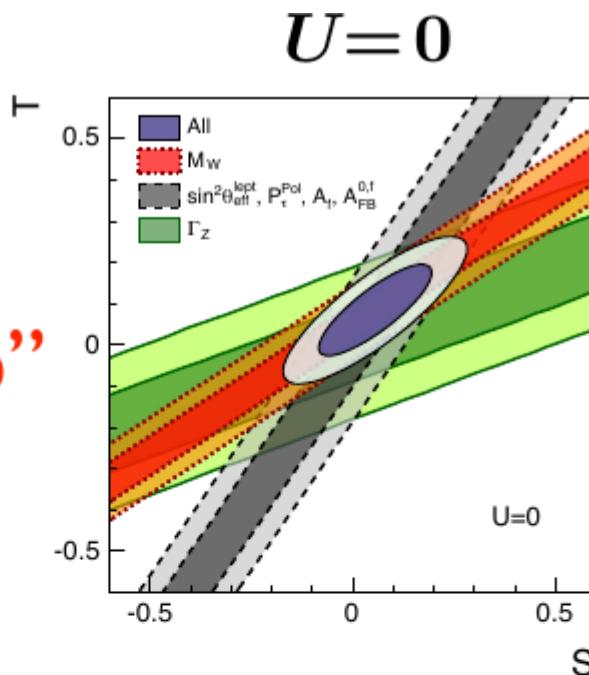
$$\delta \Gamma_Z \propto -10(3 - 8s_W^2) \textcolor{red}{S} + (63 - 126s_W^2 - 40s_W^4) \textcolor{red}{T}$$

$$\text{others} \propto \textcolor{red}{S} - 4c_W^2 s_W^2 \textcolor{red}{T}$$

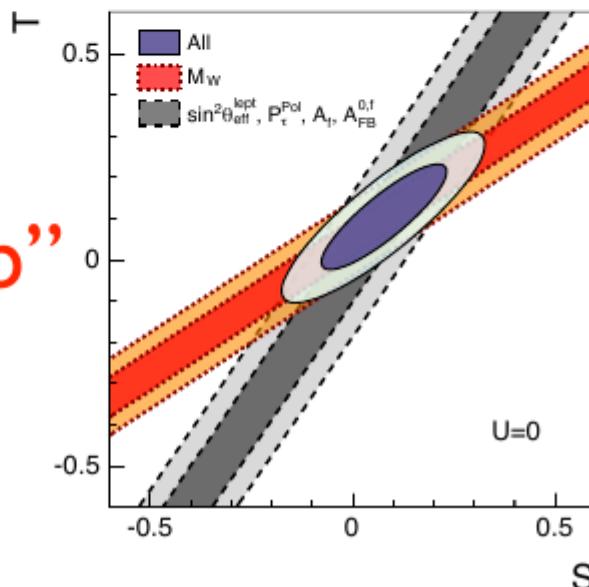
Cannot use  $\Gamma_Z$  with the new Rb.

# Constraints On $S, T, U$

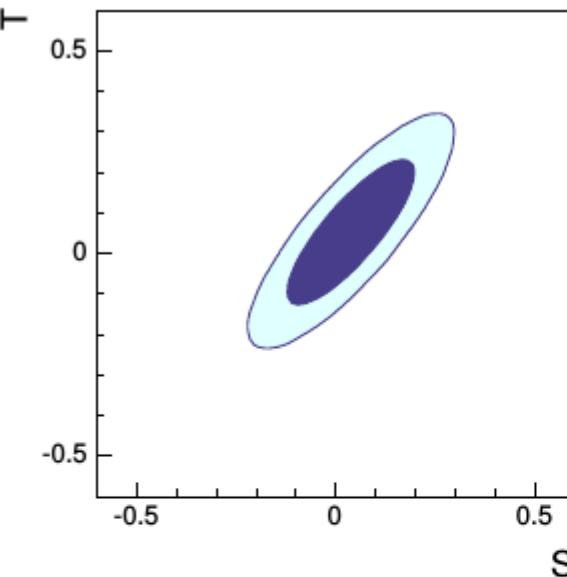
“old Rb”



“new Rb”



$U \neq 0$



	“old Rb” $U \neq 0$	“old Rb” $U = 0$	“new Rb” $U = 0$
$S$	$0.04 \pm 0.10$	$0.06 \pm 0.09$	$0.08 \pm 0.10$
$T$	$0.05 \pm 0.12$	$0.08 \pm 0.07$	$0.10 \pm 0.08$
$U$	$0.03 \pm 0.09$	—	—

see also Gfitter, Erler, ...

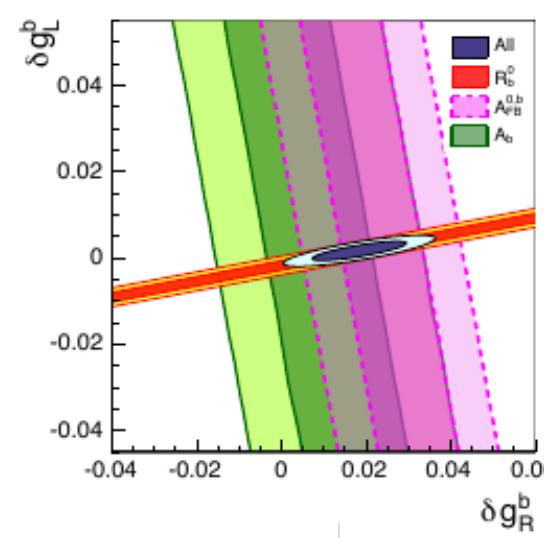
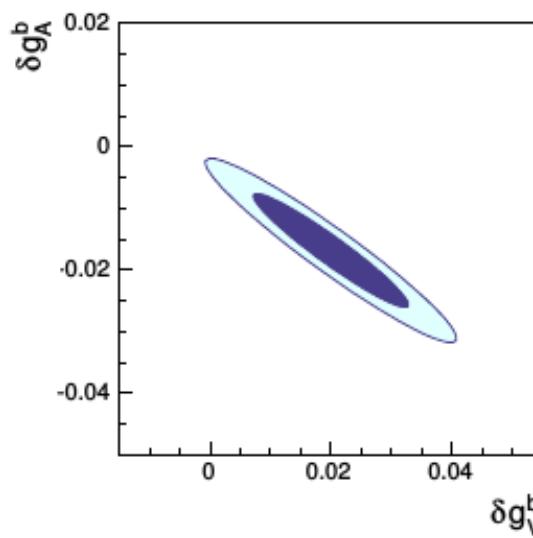
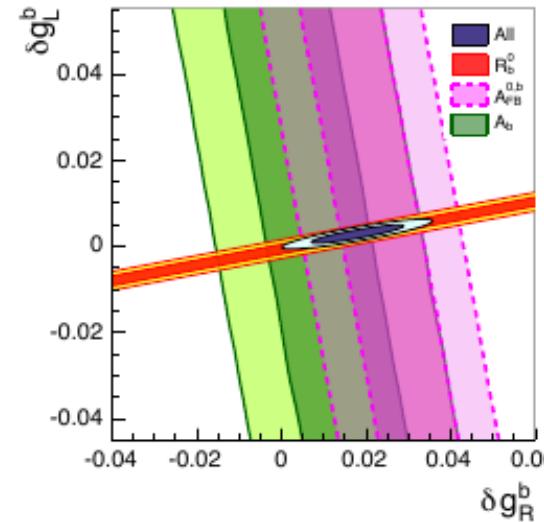
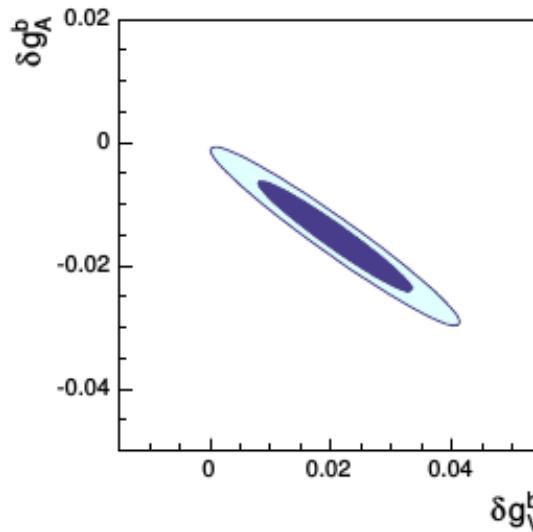
## 2. Modified Zbb Couplings

- The long-standing pull in  $A_{FB}^{0,b}$  and the more recent one in  $R_b^0$  (if confirmed) may be due to a non-standard Zbb vertex
- NP may couple mainly to the third generation
- Parametrize possible NP contributions as

$$g_L^b = (g_L^b)_{\text{SM}} + \delta g_L^b, \quad g_R^b = (g_R^b)_{\text{SM}} + \delta g_R^b$$
$$(\delta g_V^b = \delta g_L^b + \delta g_R^b, \quad \delta g_A^b = \delta g_L^b - \delta g_R^b)$$

Bamert et al.; Haber&Logan; Choudhury et al.; Kumar et al.; Batell, Gori & Wang;...

# Constraints On $\delta g_V^b$ , $\delta g_A^b$ ( $\delta g_R^b$ , $\delta g_L^b$ )



$$A_b \sim \frac{|\delta g_R^b|^2 - |\delta g_L^b|^2}{|\delta g_R^b|^2 + |\delta g_L^b|^2}$$

$$A_{FB}^{0,b} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_b$$

$$\Gamma_b \sim |\delta g_R^b|^2 + |\delta g_L^b|^2$$

$$|(g_R^b)_{SM}| \ll |(g_L^b)_{SM}|$$

	“old Rb”	“new Rb”
$\delta g_R^b$	$0.018 \pm 0.007$	$0.019 \pm 0.007$
$\delta g_L^b$	$0.0028 \pm 0.0014$	$0.0016 \pm 0.0015$
$\delta g_V^b$	$0.021 \pm 0.008$	$0.020 \pm 0.008$
$\delta g_A^b$	$-0.015 \pm 0.006$	$-0.017 \pm 0.006$

See also Batell et al. (13)

tensions in  $A_{FB}^{0,b}$  and in  $R_b^0$  removed

# The Nature Of The Higgs

Consider an extension of the SM in which:

Giudice et al;Contino et al;Azatov et al;Contino et al

- the only new light state below the cutoff is the Higgs boson
- there is a custodial symmetry
- there is no new source of flavour violation

$$\mathcal{L} = \frac{v^2}{4} \text{Tr}(D_\mu \Sigma^\dagger D^\mu \Sigma) \left( 1 + 2\textcolor{red}{a} \frac{h}{v} + \dots \right) + \dots$$

SM :  $\textcolor{red}{a} = 1$

$$S = \frac{1}{12\pi} (1 - \textcolor{red}{a}^2) \ln \left( \frac{\Lambda^2}{m_h^2} \right)$$

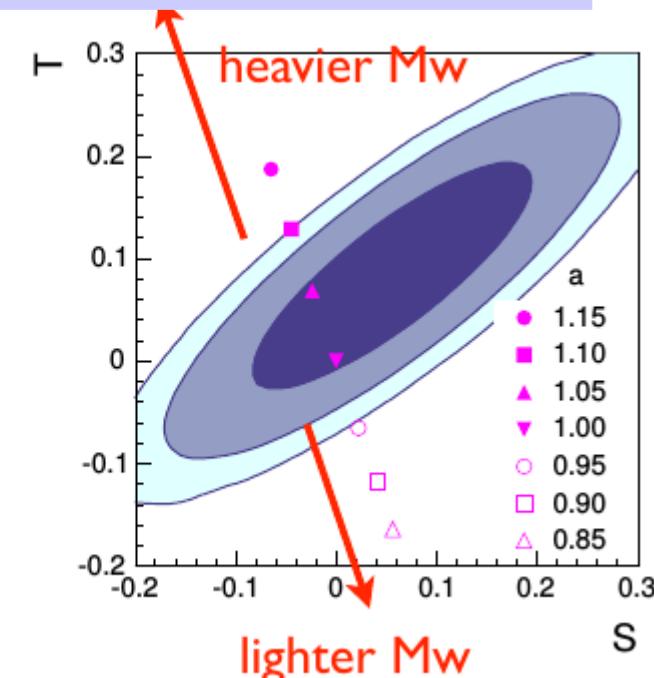
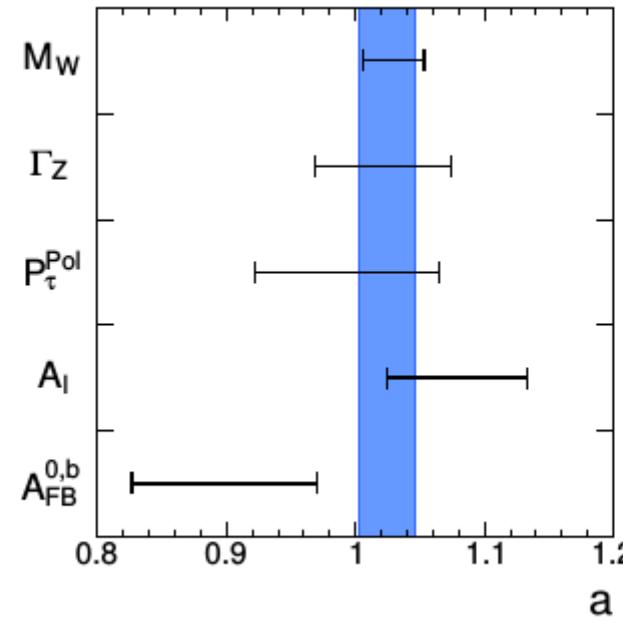
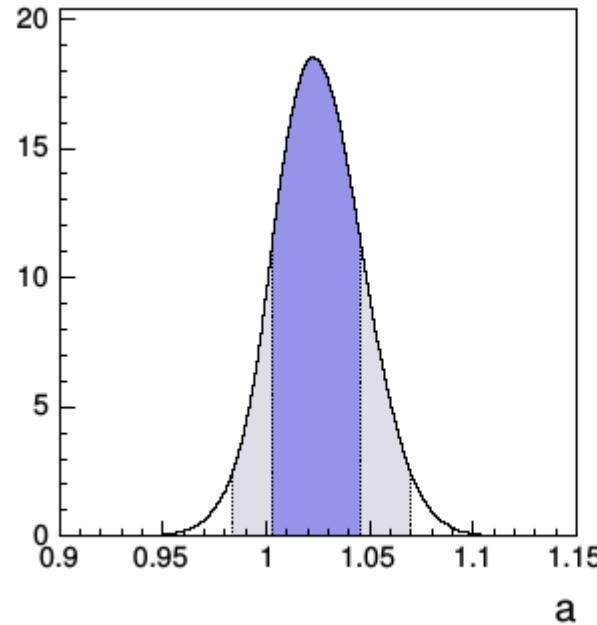
$$\Lambda = 4\pi v / \sqrt{|1 - a^2|}$$

$$T = -\frac{3}{16\pi c_W^2} (1 - \textcolor{red}{a}^2) \ln \left( \frac{\Lambda^2}{m_h^2} \right)$$

Barberi, Bellazzini, Rychkov & Varagnolo (07)

# Constraints On $a$ And $\Lambda$

Probability density



- $a = 1.024 \pm 0.021 ([0.98, 1.07])$  see also: Falkowski, Riva & Urbano; Contino et al.; Pich et al.
- composite Higgs models typically generate  $a < 1$  Falkowski, Rychkov & Urbano
- for  $a < 1$ ,  $\Lambda > 17$  TeV
- need additional light states to fix EW fit! Barbieri et al

# EWPO And The Scale Of NP

Generic effective EW Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i}{\Lambda^2} \mathcal{O}_i$$

Buchmuller & Wyler;  
Grzadkowski et al;  
Aguilar-Saavedra;  
del Aguila et al;  
Barbieri & Strumia;  
del Aguila & de Blas;  
Contino et al

Most general set of dimension-6  
operators relevant to EWPO:

$$\begin{aligned}\mathcal{O}_{WB} &= (H^\dagger \tau^a H) W_{\mu\nu}^a B^{\mu\nu}, & \mathcal{O}_H &= |H^\dagger D_\mu H|^2, \\ \mathcal{O}_{LL} &= \frac{1}{2} (\bar{L} \gamma_\mu \tau^a L)^2, & \mathcal{O}'_{HL} &= i(H^\dagger D_\mu \tau^a H)(\bar{L} \gamma^\mu \tau^a L), \\ \mathcal{O}'_{HQ} &= i(H^\dagger D_\mu \tau^a H)(\bar{Q} \gamma^\mu \tau^a Q), & \mathcal{O}_{HL} &= i(H^\dagger D_\mu H)(\bar{L} \gamma^\mu L), \\ \mathcal{O}_{HQ} &= i(H^\dagger D_\mu H)(\bar{Q} \gamma^\mu Q), & \mathcal{O}_{HE} &= i(H^\dagger D_\mu H)(\bar{E} \gamma^\mu E), \\ \mathcal{O}_{HU} &= i(H^\dagger D_\mu H)(\bar{U} \gamma^\mu U), & \mathcal{O}_{HD} &= i(H^\dagger D_\mu H)(\bar{D} \gamma^\mu D).\end{aligned}$$

Barbieri & Strumia (99)

# Constraints On $c_i/\Lambda$

Switching on one operator at a time (+ LFU):

Coefficient	“old Rb”		“new Rb”	
	$C_i/\Lambda^2$ [TeV $^{-2}$ ] at 95%	$\Lambda$ [TeV] $C_i = -1$ $C_i = 1$	$C_i/\Lambda^2$ [TeV $^{-2}$ ] at 95%	$\Lambda$ [TeV] $C_i = -1$ $C_i = 1$
$C_{WB}$	[-0.0096, 0.0042]	10.2      15.4	[-0.0095, 0.0045]	10.3      15.0
$C_H$	[-0.030, 0.007]	5.8      12.1	[-0.031, 0.008]	5.7      11.5
$C_{LL}$	[-0.011, 0.019]	9.5      7.2	[-0.016, 0.023]	8.0      6.6
$C'_{HL}$	[-0.012, 0.005]	9.2      14.1	[-0.017, 0.009]	7.6      10.8
$C'_{HQ}$	[-0.010, 0.015]	10.2      8.2	[-0.40, 0.20]	1.6      2.2
$C_{HL}$	[-0.007, 0.010]	12.2      10.0	[-0.034, 0.022]	5.5      6.7
$C_{HQ}$	[-0.023, 0.046]	6.6      4.7	[-0.01, 0.11]	11.7      3.0
$C_{HE}$	[-0.014, 0.008]	8.4      11.1	[-0.029, 0.019]	5.9      7.2
$C_{HU}$	[-0.061, 0.087]	4.0      3.4	[-0.37, 0.08]	1.6      3.5
$C_{HD}$	[-0.15, 0.05]	2.6      4.6	[-1.1, -0.2]	1.0      —

NP scale for  $c_i=1$  beyond the LHC reach,  
while TeV NP possible for perturbative  $c_i$

# Fit multiple operators

In NP models,  
contributions to  
different operators  
may be correlated:  
cancellation!!

In some cases,  
determining the  
single coefficient is  
not possible. One  
obtains only the  
combinations in the  
observables

	Large- $m_t$ expansion	Using ref. [16, 83]
Coefficient	$C_i/\Lambda^2 \text{ [TeV}^{-2}\text{] at 95\%}$	$C_i/\Lambda^2 \text{ [TeV}^{-2}\text{] at 95\%}$
Oblique	$C_{WB}$	[−0.009, 0.018]
	$C_H$	[−0.058, 0.015]
	$C'_{HL}$	[−0.026, 0.008]
	$C'_{HQ}$	[−0.18, 0.00]
	$C_{HL}$	[−0.013, 0.020]
	$C_{HQ}$	[−0.11, 0.07]
	$C_{HE}$	[−0.022, 0.018]
	$C_{HU}$	[−0.22, 0.41]
Non-oblique, LFU	$C_{HD}$	[−1.2, −0.2]
	$C[\mathcal{A}_\ell]$	—
	$C'_{HL}$	[−0.026, 0.008]
	$C_{HL}$	[−0.013, 0.020]
	$C_{HE}$	[−0.022, 0.018]
	$C_{HU_2}$	[−0.22, 0.45]
	$C_{HD_3}$	[−1.2, −0.2]
	$C'_{HQ_1} + C'_{HQ_2}$	[−0.59, 0.51]
Non-oblique, no LFU	$C'_{HQ_2} - C_{HQ_2}$	[−0.30, 0.17]
	$C'_{HQ_3} + C_{HQ_3}$	[−0.22, −0.01]
	$C[\mathcal{A}_\ell]$	—
	$C[\Gamma_{uds}]$	[−0.039, 0.044]
	$C[\mathcal{A}_\ell]$	[−0.0021, 0.0050]
	$C[\Gamma_{uds}]$	[−0.42, 0.43]

# Conclusions

- Indirect searches for NP as important as ever: EW fit is alive and kicking even more now that the Higgs is there and measured!
- Overall consistency of the SM fit is very good, but fermionic two-loop contributions to  $p_Z^f$  need to be confirmed and completed
- EWPO very effective in constraining EW-related NP (updated results on oblique NP, Zbb couplings, composite Higgs, EFT analysis)

# Outlook

Indirect searches have always had a twofold role:  
i) probing inaccessible high scales and ii) providing  
clues on the nature of new particles

In the case of the EW fit, an order-of-magnitude improvement would allow to constrain the Lagrangian of new TeV particles and to probe NP scales up to  $\sim 100$  TeV

Strong motivation for new  $e^+e^-$  facilities (ILC, TLEP, ...) and for theorists to go on improving the SM calculations of EWPO

# BACKUP SLIDES

# CORRELATION MATRIX FOR SM

Parameter	$\alpha_s$	$\Delta\alpha_{\text{had}}^{(5)}$	$M_Z$	$m_t$	$m_h$	$\delta\rho_Z^\nu$	$\delta\rho_Z^\ell$	$\delta\rho_Z^b$
$\alpha_s$	1.00							
$\Delta\alpha_{\text{had}}^{(5)}$	-0.01	1.00						
$M_Z$	0.00	0.08	1.00					
$m_t$	0.01	0.18	-0.05	1.00				
$m_h$	0.00	-0.01	0.00	0.00	1.00			
$\delta\rho_Z^\nu$	0.00	-0.01	-0.05	-0.02	0.00	1.00		
$\delta\rho_Z^\ell$	0.00	0.02	-0.11	-0.07	0.00	0.49	1.00	
$\delta\rho_Z^b$	-0.18	0.10	-0.03	-0.06	0.00	-0.28	0.38	1.00

# PREDICTIONS FOR “old $R_b$ ”

	Prediction	$\alpha_s$	$\Delta\alpha_{\text{had}}^{(5)}$	$M_Z$	$m_t$
$M_W$ [GeV]	$80.3625 \pm 0.0085$	$\pm 0.0004$	$\pm 0.0060$	$\pm 0.0026$	$\pm 0.0054$
$\Gamma_W$ [GeV]	$2.0889 \pm 0.0007$	$\pm 0.0002$	$\pm 0.0005$	$\pm 0.0002$	$\pm 0.0004$
$\Gamma_Z$ [GeV]	$2.4951 \pm 0.0052$	$\pm 0.0003$	$\pm 0.0003$	$\pm 0.0002$	$\pm 0.0002$
$\sigma_h^0$ [nb]	$41.484 \pm 0.004$	$\pm 0.003$	$\pm 0.000$	$\pm 0.002$	$\pm 0.001$
$\sin^2 \theta_{\text{eff}}^{\text{lept}}(Q_{\text{FB}}^{\text{had}})$	$0.23149 \pm 0.00012$	$\pm 0.00000$	$\pm 0.00012$	$\pm 0.00001$	$\pm 0.00003$
$P_\tau^{\text{pol}} = \mathcal{A}_\ell$	$0.14725 \pm 0.00094$	$\pm 0.00002$	$\pm 0.00091$	$\pm 0.00012$	$\pm 0.00022$
$\mathcal{A}_c$	$0.6680 \pm 0.0004$	$\pm 0.0000$	$\pm 0.0004$	$\pm 0.0001$	$\pm 0.0001$
$\mathcal{A}_b$	$0.9346 \pm 0.0001$	$\pm 0.0000$	$\pm 0.0001$	$\pm 0.0000$	$\pm 0.0000$
$A_{\text{FB}}^{0,\ell}$	$0.01626 \pm 0.00021$	$\pm 0.00000$	$\pm 0.00020$	$\pm 0.00003$	$\pm 0.00005$
$A_{\text{FB}}^{0,c}$	$0.07377 \pm 0.00052$	$\pm 0.00001$	$\pm 0.00050$	$\pm 0.00006$	$\pm 0.00012$
$A_{\text{FB}}^{0,b}$	$0.10322 \pm 0.00067$	$\pm 0.00001$	$\pm 0.00064$	$\pm 0.00008$	$\pm 0.00016$
$R_\ell^0$	$20.734 \pm 0.044$	$\pm 0.004$	$\pm 0.002$	$\pm 0.000$	$\pm 0.000$
$R_c^0$	$0.17222 \pm 0.00002$	$\pm 0.00001$	$\pm 0.00001$	$\pm 0.00000$	$\pm 0.00001$
$R_b^0$	$0.215762 \pm 0.000033$	$\pm 0.000002$	$\pm 0.000004$	$\pm 0.000007$	$\pm 0.000032$

# Comparison to ZFITTER

- For a given set of the input parameters,

	ZFITTER	OURS	$\frac{\text{OURS} - \text{ZFITTER}}{\text{ZFITTER}} * 100$	Exp uncertainty
$M_W$	80.362216	80.362499	0.00035 %	0.02 %
$\Gamma_W$	2.0906748	2.0887391	-0.093 %	2.0 %
$\Gamma_Z$	2.4953142	2.4951814	-0.0053 %	0.09 %
$\sigma_h^0$	41.479103	41.483516	0.011 %	0.09 %
$\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{\text{FB}}^{\text{had}})$	0.23149326	0.23149297	-0.00012 %	0.52 %
$P_\tau^{\text{Pol}}$	0.14724705	0.14724926	0.0015 %	2.2 %
$A_\ell$	0.14724705	0.14724926	0.0015 %	1.4 %
$A_c$	0.66797088	0.66799358	0.0034 %	4.0 %
$A_b$	0.93460981	0.93464051	0.0033 %	2.2 %
$A_{\text{FB}}^{0,\ell}$	0.016261269	0.016261758	0.0030 %	5.5 %
$A_{\text{FB}}^{0,c}$	0.073767554	0.073771169	0.0049 %	5.0 %
$A_{\text{FB}}^{0,b}$	0.10321390	0.10321884	0.0048 %	1.6 %
$R_\ell^0$	20.739702	20.735130	-0.022 %	0.12 %
$R_c^0$	0.17224054	0.17222362	-0.0098 %	1.7 %
$R_b^0$	0.21579927	0.21578277	-0.0077 %	0.31 %

Our results are in agreement with ZFITTER v6.43.

# Impact of parametric uncertainties

	Prediction	$\alpha_s$	$\Delta\alpha_{\text{had}}^{(5)}$	$M_Z$	$m_t$
$M_W$ [GeV]	$80.362 \pm 0.008$	$\pm 0.000$	$\pm 0.006$	$\pm 0.003$	$\pm 0.005$
$\Gamma_Z$ [GeV]	$2.4951 \pm 0.0005$	$\pm 0.0003$	$\pm 0.0003$	$\pm 0.0002$	$\pm 0.0002$
$P_\tau^{\text{pol}} = \mathcal{A}_\ell$	$0.1472 \pm 0.0009$	$\pm 0.0000$	$\pm 0.0009$	$\pm 0.0001$	$\pm 0.0002$
$A_{\text{FB}}^{0,b}$	$0.1032 \pm 0.0007$	$\pm 0.0000$	$\pm 0.0006$	$\pm 0.0001$	$\pm 0.0002$
$R_b^0$	$0.21493 \pm 0.00004$	$\pm 0.00001$	$\pm 0.00000$	$\pm 0.00000$	$\pm 0.00003$

- $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$  and  $m_t$  are the most important sources of parametric uncertainty.
- The theoretical uncertainty from missing higher-order corrections has been estimated as  $\delta M_W^{\text{theo}} \sim 4$  MeV.

*Awramik et al. (04)*

# Individual constraints on the Higgs mass

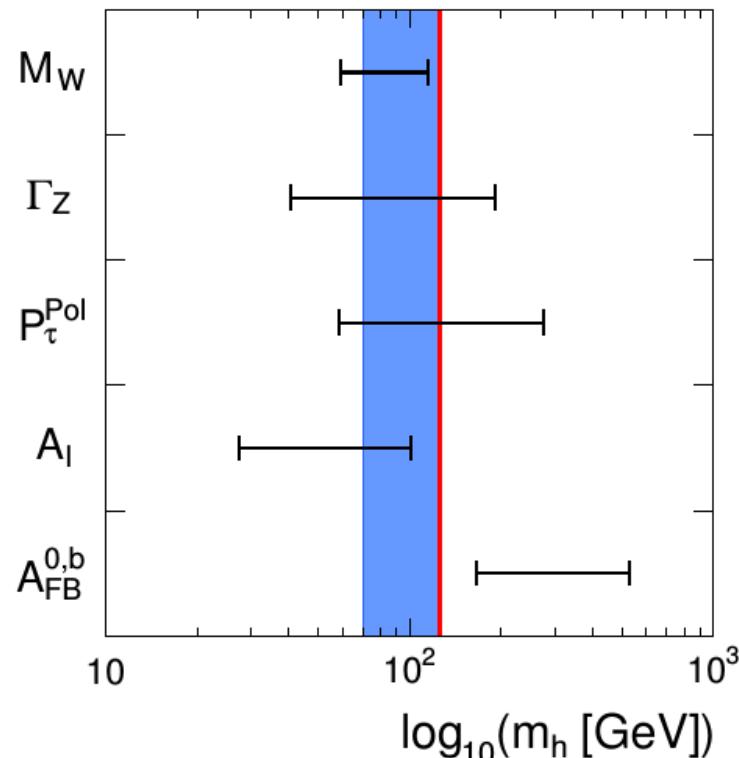
indirect determination from the EW fit:

$$m_h = 97.3 \pm 26.9 \text{ GeV}$$



direct measurement at LHC (ATLAS & CMS):

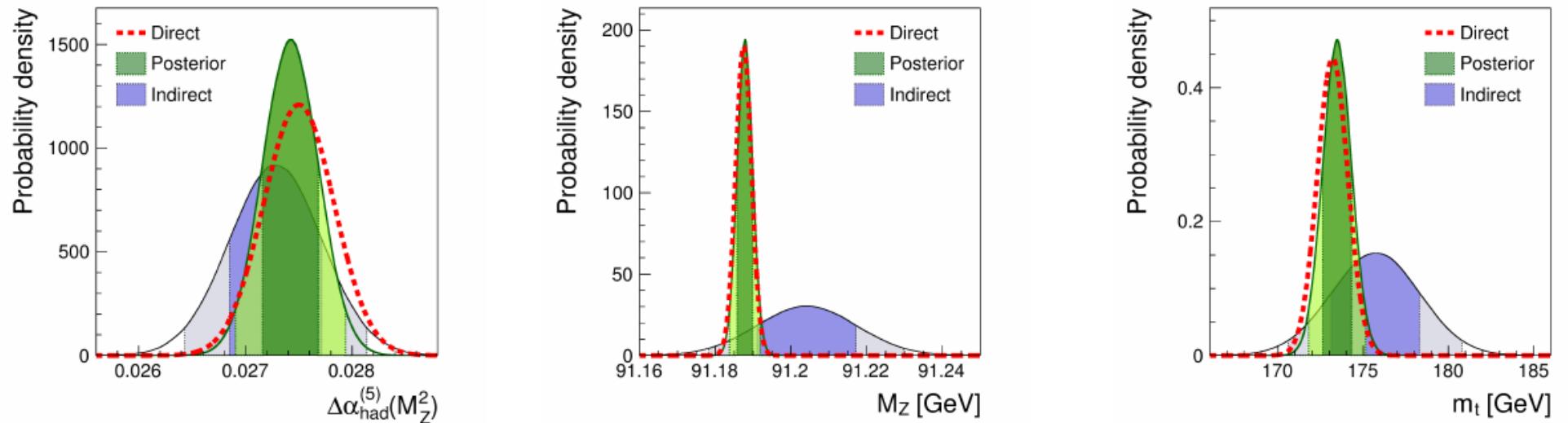
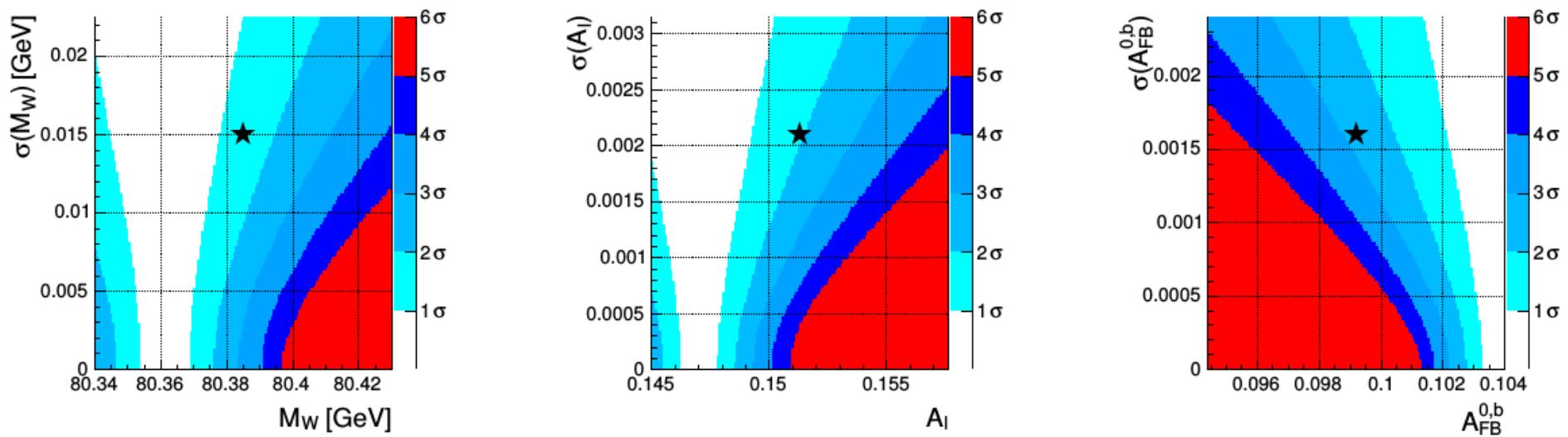
$$m_h = 125.6 \pm 0.3 \text{ GeV}$$



“old Rb”

- $M_W$  gives the most stringent constraint.
- Tension between  $A_I(\text{SLD})$  and  $A_{\text{FB}}^b$ .

# Other plots in the SM fit



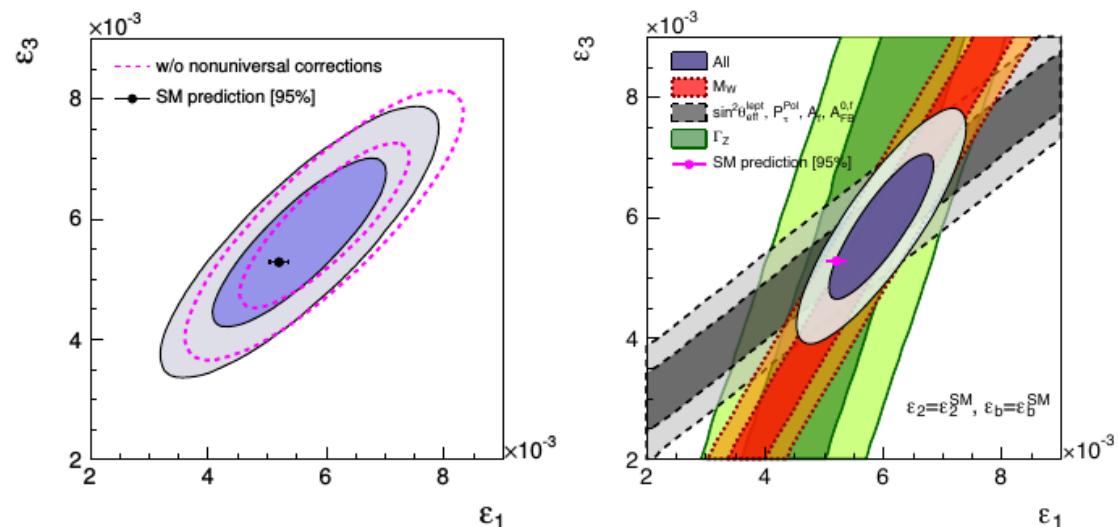
# Constraints on the epsilon parameters

- Unlike STU, the epsilon parameters involve SM contributions.

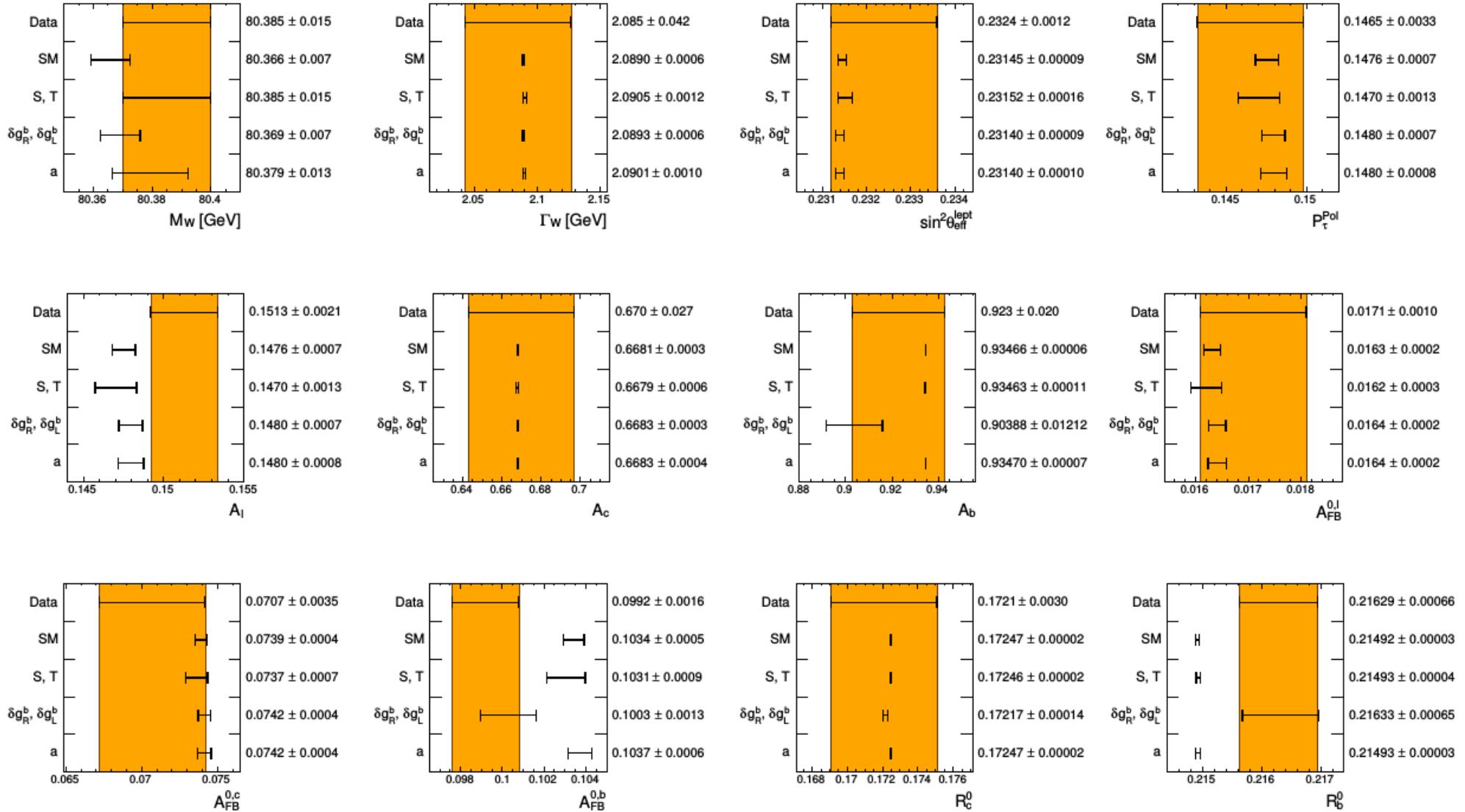
Altarelli et al. (91,92,93)

Flavour non-universal VCs in the SM have to be taken into account.

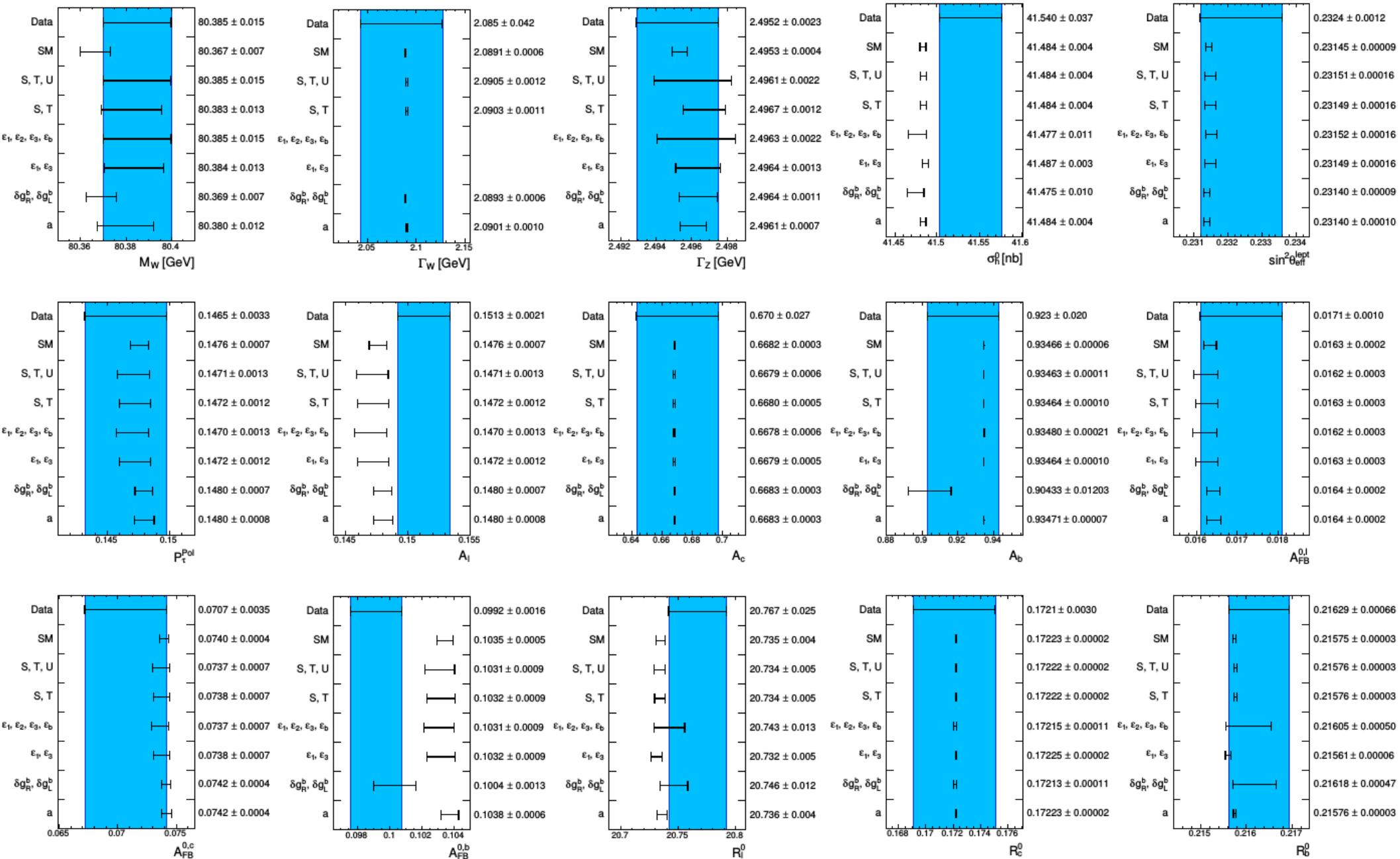
Parameter	“old Rb”	
	$\epsilon_{1,2,3,b}$ fit	$\epsilon_{1,3}$ fit
$\epsilon_1 [10^{-3}]$	$5.6 \pm 1.0$	$6.0 \pm 0.6$
$\epsilon_2 [10^{-3}]$	$-7.8 \pm 0.9$	—
$\epsilon_3 [10^{-3}]$	$5.6 \pm 0.9$	$5.9 \pm 0.8$
$\epsilon_b [10^{-3}]$	$-5.8 \pm 1.3$	—



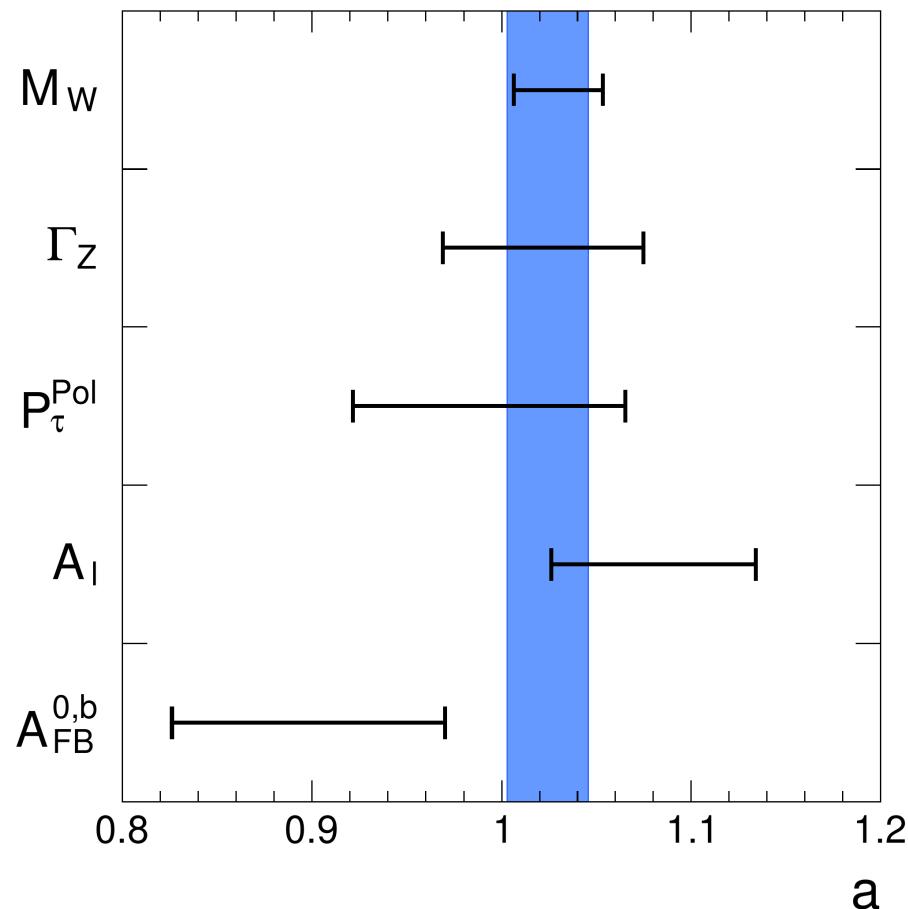
# Summary of fit results with new Rb



# Summary of fit results w/o new Rb

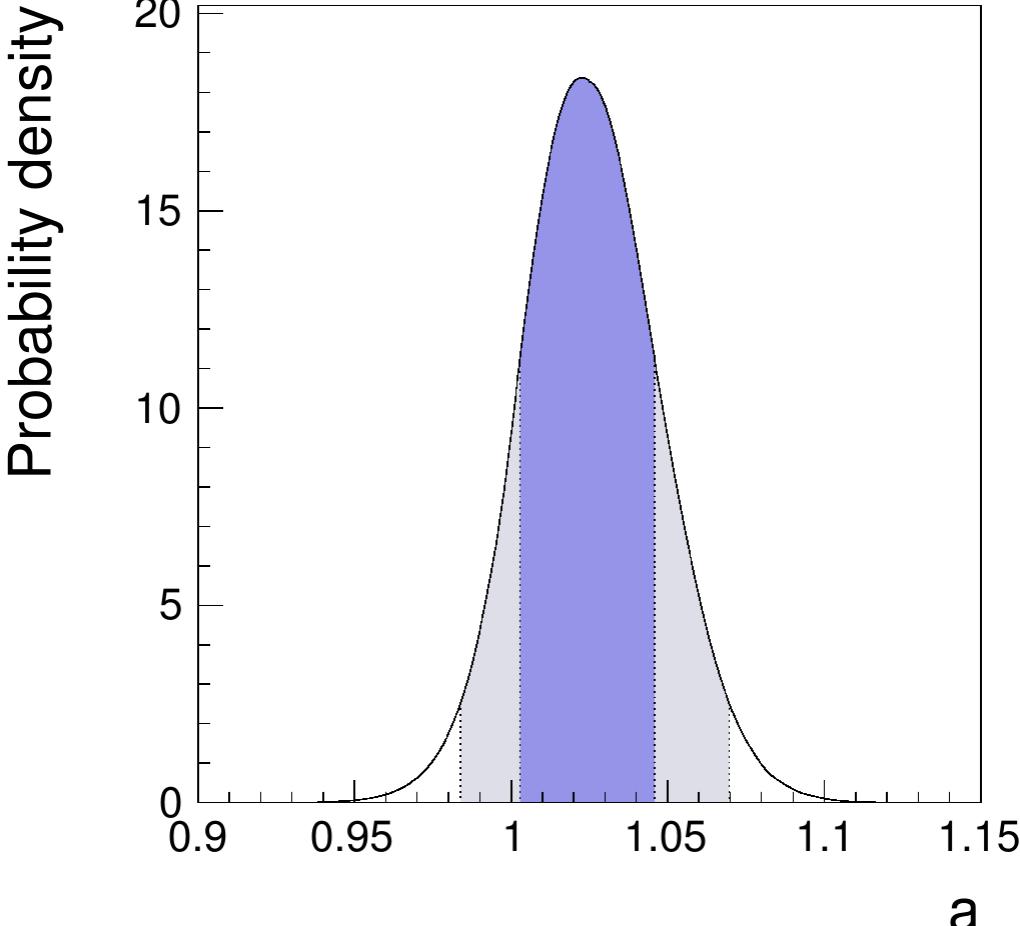


# WHAT CONSTRAINTS $a$ ?

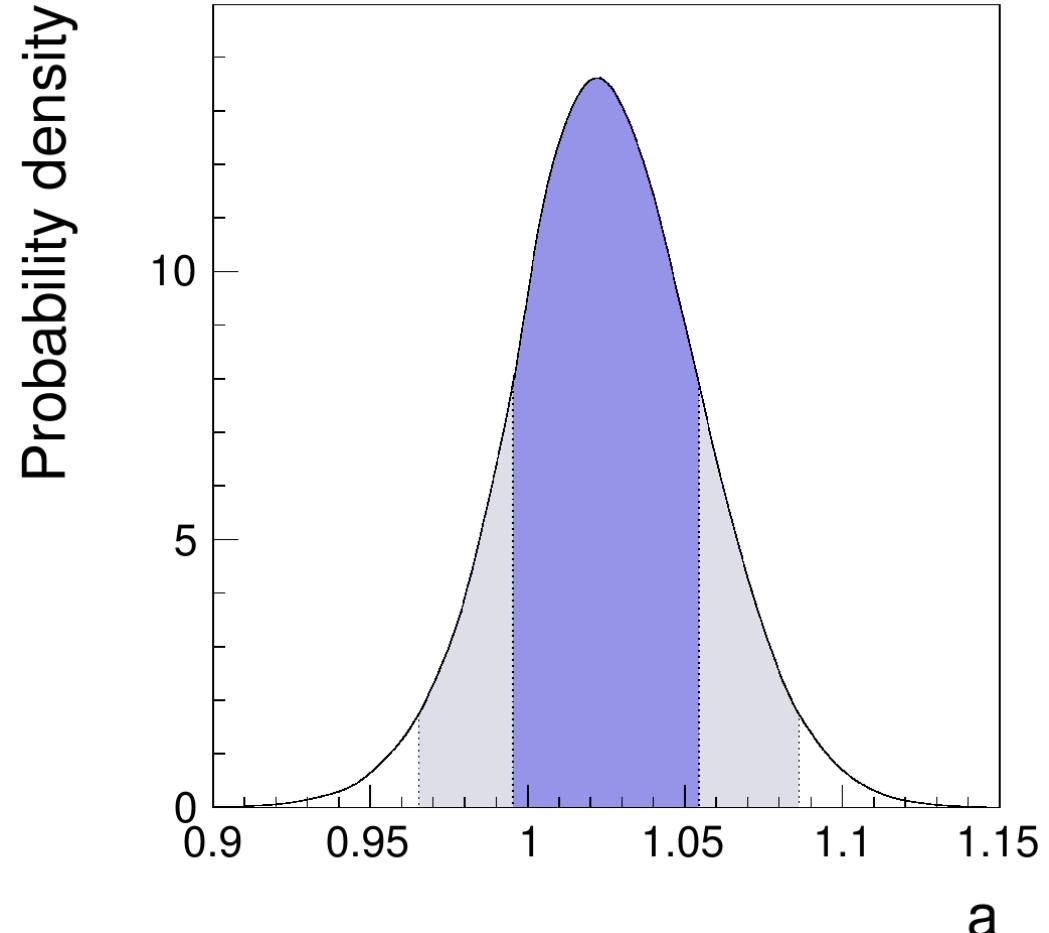


- Apply one of the constraints  $M_W$ ,  $\Gamma_Z$ ,  $P_\tau^{\text{Pol}}$ ,  $A_I$  and  $A_{\text{FB}}^{0,b}$  at a time, and compare with the full fit (blue band)

# STABILITY OF $a$ VS $m_+$



$$m_+ = 173.2 \pm 0.9 \quad a \in [0.98, 1.07]$$



$$m_+ = 173.3 \pm 2.8 \quad a \in [0.97, 1.09]$$

$$\begin{aligned}
M_{W,\text{NP}} &= -\frac{\alpha(M_Z^2) c_W M_Z}{4(c_W^2 - s_W^2)} \left( S - 2 c_W^2 T - \frac{(c_W^2 - s_W^2) U}{2 s_W^2} \right), \\
\Gamma_{W,\text{NP}} &= -\frac{3 \alpha^2(M_Z^2) c_W M_Z}{8 s_W^2 (c_W^2 - s_W^2)} \left( S - 2 c_W^2 T - \frac{(c_W^2 - s_W^2) U}{2 s_W^2} \right), \\
\Gamma_{Z,\text{NP}} &= \frac{\alpha^2(M_Z^2) M_Z}{72 c_W^2 s_W^2 (c_W^2 - s_W^2)} \left[ -10(3 - 8 s_W^2) S + (63 - 126 s_W^2 - 40 s_W^4) T \right], \\
\sigma_{h,\text{NP}}^0 &= -\frac{72\pi\alpha(M_Z^2) (729 - 4788 s_W^2 + 8352 s_W^4 - 6176 s_W^6 + 640 s_W^8)}{M_Z^2 (63 - 120 s_W^2 + 160 s_W^4)^3 (c_W^2 - s_W^2)} (S - 4 c_W^2 s_W^2 T), \\
\sin^2 \theta_{\text{eff,NP}}^{\text{lept}} &= \frac{\alpha(M_Z^2)}{4(c_W^2 - s_W^2)} (S - 4 c_W^2 s_W^2 T), \\
P_{\tau,\text{NP}}^{\text{pol}} &= -\frac{4 \alpha(M_Z^2) s_W^2}{(1 - 4 s_W^2 + 8 s_W^4)^2} (S - 4 c_W^2 s_W^2 T), \\
\mathcal{A}_{\ell,\text{NP}} &= -\frac{4 \alpha(M_Z^2) s_W^2}{(1 - 4 s_W^2 + 8 s_W^4)^2} (S - 4 c_W^2 s_W^2 T), \\
\mathcal{A}_{c,\text{NP}} &= -\frac{48 \alpha(M_Z^2) s_W^2 (3 - 4 s_W^2)}{(9 - 24 s_W^2 + 32 s_W^4)^2 (c_W^2 - s_W^2)} (S - 4 c_W^2 s_W^2 T), \\
\mathcal{A}_{b,\text{NP}} &= -\frac{12 \alpha(M_Z^2) s_W^2 (3 - 2 s_W^2)}{(9 - 12 s_W^2 + 8 s_W^4)^2 (c_W^2 - s_W^2)} (S - 4 c_W^2 s_W^2 T), \\
A_{\text{FB},\text{NP}}^{0,\ell} &= -\frac{6 \alpha(M_Z^2) s_W^2 (1 - 4 s_W^2)}{(1 - 4 s_W^2 + 8 s_W^4)^3} (S - 4 c_W^2 s_W^2 T), \\
A_{\text{FB},\text{NP}}^{0,c} &= -\frac{9 \alpha(M_Z^2) s_W^2 (39 - 310 s_W^2 + 992 s_W^4 - 1600 s_W^6 + 1024 s_W^8)}{(1 - 4 s_W^2 + 8 s_W^4)^2 (9 - 24 s_W^2 + 32 s_W^4)^2 (c_W^2 - s_W^2)} (S - 4 c_W^2 s_W^2 T), \\
A_{\text{FB},\text{NP}}^{0,b} &= -\frac{18 \alpha(M_Z^2) s_W^2 (15 - 76 s_W^2 + 152 s_W^4 - 160 s_W^6 + 64 s_W^8)}{(1 - 4 s_W^2 + 8 s_W^4)^2 (9 - 12 s_W^2 + 8 s_W^4)^2 (c_W^2 - s_W^2)} (S - 4 c_W^2 s_W^2 T), \\
R_{\ell,\text{NP}}^0 &= \frac{8 \alpha(M_Z^2) (3 - 2 s_W^2)(1 - 5 s_W^2)}{3(1 - 4 s_W^2 + 8 s_W^4)^2 (c_W^2 - s_W^2)} (S - 4 c_W^2 s_W^2 T), \\
R_{c,\text{NP}}^0 &= -\frac{9 \alpha(M_Z^2) (9 - 36 s_W^2 + 16 s_W^4)}{(45 - 84 s_W^2 + 88 s_W^4)^2 (c_W^2 - s_W^2)} (S - 4 c_W^2 s_W^2 T), \\
R_{b,\text{NP}}^0 &= \frac{6 \alpha(M_Z^2) (9 - 36 s_W^2 + 16 s_W^4)}{(45 - 84 s_W^2 + 88 s_W^4)^2 (c_W^2 - s_W^2)} (S - 4 c_W^2 s_W^2 T),
\end{aligned} \tag{B.1}$$

where  $s_W^2$  and  $c_W^2$  denote their SM values, and  $c_W = \sqrt{c_W^2}$ .

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = \sin^2 \theta_{\text{eff}, \text{SM}}^{\text{lept}} - \frac{1}{4} \delta \left( \frac{g_V^e}{g_A^e} \right),$$

$$\mathcal{A}_f = \mathcal{A}_{f, \text{SM}} - \frac{2[(g_V^f)^2 - (g_A^f)^2]}{G_f^2} \delta \left( \frac{g_V^f}{g_A^f} \right), \quad (355)$$

$$A_{\text{FB}}^{0,f} = A_{\text{FB}, \text{SM}}^{0,f} - \frac{3 g_V^f g_A^f [(g_V^e)^2 - (g_A^e)^2]}{G_f G_e^2} \delta \left( \frac{g_V^e}{g_A^e} \right) - \frac{3 g_V^e g_A^e [(g_V^f)^2 - (g_A^f)^2]}{G_e G_f^2} \delta \left( \frac{g_V^f}{g_A^f} \right), \quad (356)$$

$$\Gamma_Z = \Gamma_{Z, \text{SM}} + \frac{\alpha(M_Z^2) M_Z}{12 s_W^2 c_W^2} \sum_f N_c^f \delta G_f, \quad (357)$$

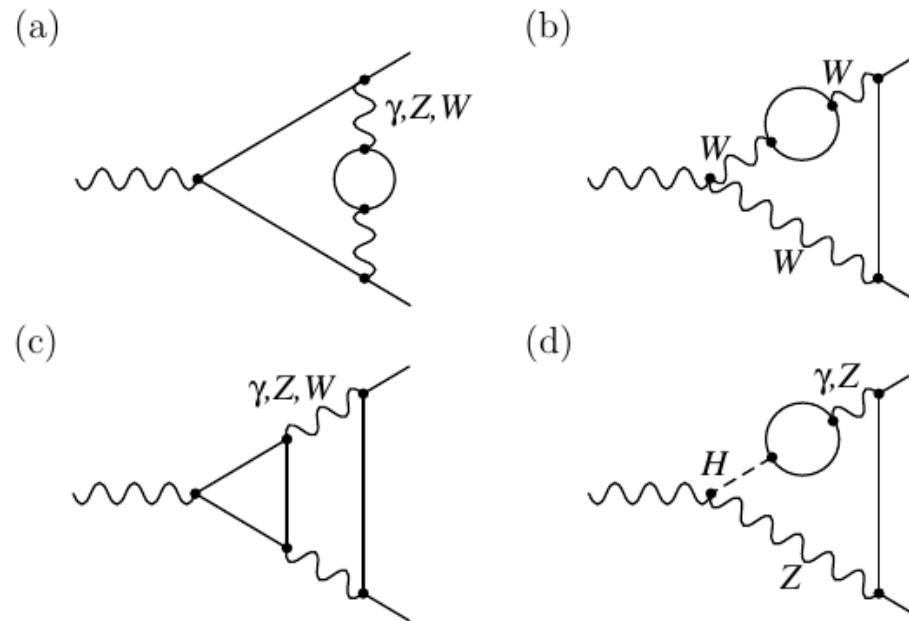
$$\sigma_h^0 = \sigma_{h, \text{SM}}^0 + \frac{12\pi}{M_Z^2} \frac{G_e \left( \sum_q N_c^q G_q \right)}{\left( \sum_f N_c^f G_f \right)^2} \left( \frac{\delta G_e}{G_e} + \frac{\sum_q \delta G_q}{\sum_q G_q} - \frac{2 \sum_f N_c^f \delta G_f}{\sum_f N_c^f G_f} \right), \quad (358)$$

$$R_\ell^0 = R_{\ell, \text{SM}}^0 + \frac{\sum_q N_c^q \delta G_q}{G_\ell} - \frac{(\sum_q N_c^q G_q) \delta G_\ell}{G_\ell^2}, \quad (359)$$

$$R_c^0 = R_{c, \text{SM}}^0 + \frac{\delta G_c}{\sum_q G_q} - \frac{G_c \sum_q \delta G_q}{\left( \sum_q G_q \right)^2}, \quad (360)$$

$$R_b^0 = R_{b, \text{SM}}^0 + \frac{\delta G_b}{\sum_q G_q} - \frac{G_b \sum_q \delta G_q}{\left( \sum_q G_q \right)^2}, \quad G_f \equiv (g_V^f)^2 + (g_A^f)^2, \quad (361)$$

# FREITAS & HUANG



$M_H$ [GeV]	$\mathcal{O}(\alpha) + \text{FSR}_{\alpha, \alpha_s, \alpha_s^2}$ $[10^{-3}]$	$\mathcal{O}(\alpha_{\text{ferm}}^2)$ $[10^{-4}]$	$\mathcal{O}(\alpha_{\text{ferm}}^2) + \text{FSR}_{\alpha_s^3, \alpha \alpha_s, m_b^2 \alpha_s, m_b^4}$ $[10^{-4}]$	$\mathcal{O}(\alpha \alpha_s, \alpha \alpha_s^2)$ $[10^{-4}]$
100	-3.566	-6.583	-8.214	-0.404
200	-3.585	-6.587	-8.216	-0.404
400	-3.609	-6.595	-8.220	-0.404
600	-3.624	-6.594	-8.216	-0.404
1000	-3.645	-6.582	-8.201	-0.404

# No LFU

Coefficient	Large- $m_t$ expansion		Using ref. [16, 83]	
	$C_i/\Lambda^2$ [TeV $^{-2}$ ] at 95%	$\Lambda$ [TeV] $C_i = -1$ $C_i = 1$	$C_i/\Lambda^2$ [TeV $^{-2}$ ] at 95%	$\Lambda$ [TeV] $C_i = -1$ $C_i = 1$
$C'_{HQ_1}$	[-0.026, 0.034]	6.2      5.4	[-0.19, 0.01]	2.3      11.9
$C'_{HQ_2}$	[-0.026, 0.034]	6.2      5.4	[-0.20, 0.01]	2.3      10.8
$C'_{HQ_3}, C_{HQ_3}$	[-0.025, 0.053]	6.3      4.3	[0.00, 0.10]	15.6      3.1
$C_{HQ_1}$	[-0.26, 0.34]	2.0      1.7	[-1.9, 0.1]	0.7      3.9
$C_{HQ_2}$	[-0.16, 0.18]	2.5      2.4	[-0.25, 0.15]	2.0      2.6
$C_{HU_1}$	[-0.13, 0.17]	2.8      2.4	[-0.97, 0.03]	1.0      5.6
$C_{HU_2}$	[-0.11, 0.17]	3.0      2.4	[-0.39, 0.21]	1.6      2.2
$C_{HD_1}, C_{HD_2}$	[-0.34, 0.26]	1.7      2.0	[-0.1, 1.9]	3.8      0.7
$C_{HD_3}$	[-0.38, 0.03]	1.6      6.3	[-0.66, -0.13]	1.2      —