Chern-Simons, with a twist

L. Castellani

Univ. Piemonte Orientale
Alessandria - Torino

1. INTRODUCTION

- CHERN-SIMONS (CS) SUPERGRAVITIES : ALTERNATIVE TO STANDARD SUPERGRAVITIES
  TRONCOSO, ZANELLI 1998 ; ZANELLI 2005, 2012

- SUPERSYMMETRY IS REALIZED AS A GAUGE SYMMETRY PART OF A GAUGE SUPERGROUP G, UNDER WHICH THE CS LAGRANGIAN IS INVARIANT UP TO TOTAL DER.
  THE SUPERALGEBRA CLOSES OFF-SHELL BY CONSTRUCTION

- G CONTAINS (ANTI)-DE SITTER SUPERALGEBRA $\Rightarrow$ TRANSLATION INVARIANT THEORY, NO DIMENSIONAL COUPLING CONST.
  GROUP CONTRACTION $\longrightarrow$ POINCARÉ SUPERALGEBRA

- BOTH FEATURES COULD BE RELEVANT FOR QUANTIZATION

- NO MATCHING OF BOSONIC AND FERMIONIC D.O.F. IN SUPERSYMM. CS THEORIES.

IN STANDARD SUPERSYMM THEORIES THE MATCHING RESULTS FROM TWO ASSUMPTIONS:

- SPACETIME SYMMETRY $\rightarrow$ POINCARÉ
- FIELDS IN VECTOR MULTIPLET UNDER SUSY

HERE:
- AdS SYMMETRY
- ALL FIELDS ARE PART OF A CONNECTION $\rightarrow$ BELONG TO ADJOINT REP. OF A SUPERALGEBRA

- CS (SUPER)GRAVITIES EXIST ONLY IN ODD DIM.
  $D = 2m-1$

- CONTAIN:
  - THE USUAL EINSTEIN-HILBERT TERM AND ITS SUPERSYMM.
  - A COSMOLOGICAL TERM AND ITS SUPERSYMM.
  - HIGHER POWERS OF THE CURVATURE 2-FORM (UP TO R^{m-1})

- CS GRAVITIES ARE PARTICULAR EXAMPLES OF LOVELOCK GRAVITIES WITH AT MOST SECOND ORDER FIELD EQS. FOR THE METRIC
  (LOVELOCK 1971)

IN THIS TALK : $\boxed{\text{NONCOMMUTATIVE EXTENSION OF } D=5 \text{ CS SUPERGRAVITY}}$

L.C., hep-th/1305.1566 (JHEP)

- NC CHERN-SIMONS : A.H. CHAMSEDDINE, J. FRÖHLICH (1994)
  S. CACCIATORI, L. MARTUCCI (2002)

- WHY NC CHERN-SIMONS (SUPER)GRAVITIES ?

  - USUAL MOTIVATIONS : • ENCODE QUANTUM EFFECTS IN STRUCTURE OF SPACETIME, OBTAIN HIGHER DER EXTENSIONS OF GRAVITY ($\rightarrow$ COSMOLOGY), ...

  - ALSO : BREAK TRANSLATION INVARIANCE ($\theta^{\mu\nu}$ DIMENSIONFUL) WITHOUT GROUP CONTRACTION ?

  - NOTE : D=5 CS SUPERGRAVITY IS A GAUGE THEORY OF THE SUPERGROUP $SU(2,2|N)$
    ITS NC DEFORMATION REQUIRES $N=4$ SUPERSYMMETRIES $\longrightarrow$ RELATION WITH AdS/CFT ?

2. NONCOMMUTATIVE FIELD THEORIES

- $[x^\mu, x^\nu] = i\theta^{\mu\nu}$ TO ENCODE QUANTUM PROPERTIES IN THE TEXTURE OF SPACETIME

- FIELD THEORIES ON NC SPACETIME REFORMULATED AS :
FIELD TH.S ON ORDINARY SPACETIME WITH $\cdot \longrightarrow *$ (STAR PRODUCT)

- FOR GEOMETRICAL ACTIONS (GRAVITY) NEED A DEFORMED EXTERIOR PRODUCT $\wedge \longrightarrow \wedge_*$

$$\boxed{\tau \wedge_* \tau' \equiv \tau \wedge \tau' + \frac{i}{2} \theta^{AB} (\ell_A \tau) \wedge (\ell_B \tau') + \frac{1}{2!}\left(\frac{i}{2}\right)^2 \theta^{A_1 B_1} \theta^{A_2 B_2} (\ell_{A_1} \ell_{A_2} \tau) \wedge (\ell_{B_1} \ell_{B_2} \tau') + \cdots}$$

BASED ON A TWIST DEFINED BY COMMUTING VECTOR FIELDS
$[X_A, X_B] = 0$, $\ell_A \equiv$ LIE DERIVATIVE ALONG X_A

3. VIERBEIN GRAVITY COUPLED TO FERMIONS

$$\boxed{S = \int \mathrm{Tr}\left[R \wedge V \wedge V (i\gamma_5) - (D\psi\, \bar{\psi} - \psi\, D\bar{\psi}) \wedge V \wedge V \wedge V \gamma_5 \right]}$$

- USUAL EINSTEIN-HILBERT ACTION COUPLED TO SPIN 1/2 FERMIONS

$$R \equiv d\Omega - \Omega \wedge \Omega, \qquad D\psi \equiv d\psi - \Omega\, \psi$$

$$\Omega = \frac{1}{4}\omega^{ab}\gamma_{ab} \qquad V = V^a \gamma_a$$

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- TAKING THE Tr OVER SPINOR INDICES $\longrightarrow$

$$S = \int \underbrace{R^{ab} \wedge V^c \wedge V^d \epsilon_{abcd}}_{R\sqrt{g}\, d^4x} + i \underbrace{(\bar{\psi} \gamma^a D\psi - D\bar{\psi} \gamma^a \psi) \wedge V^b \wedge V^c \wedge V^d \epsilon_{abcd}}_{\bar{\psi} \not{D} \psi \sqrt{g}\, d^4x}$$

$$R^{ab} = d\omega^{ab} - \omega^a{}_c \wedge \omega^{cb}$$

(USE $\mathrm{Tr}(\gamma_{ab}\gamma_c\gamma_d) = -4i\epsilon_{abcd}$, $\gamma_{abc} = i\epsilon_{abcd}\gamma^d\gamma_5$)

$$\boxed{S = \int \mathrm{Tr}\left[R \wedge V \wedge V (i\gamma_5) - (D\psi\, \bar\psi - \psi\, D\bar\psi) \wedge V \wedge V \wedge V \gamma_5 \right]}$$

- USUAL EINSTEIN - HILBERT ACTION COUPLED TO SPIN 1/2 FERMIONS

$$R \equiv d\Omega - \Omega \wedge \Omega , \qquad D\psi \equiv d\psi - \Omega\, \psi$$

$$\Omega = \frac{1}{4} \omega^{ab} \gamma_{ab} \qquad V = V^a \gamma_a$$

- SYMMETRIES :

– DIFF. INVARIANCE

– LORENTZ GAUGE INV. :

$$\delta_\epsilon \Omega = d\epsilon - \Omega\, \epsilon + \epsilon\, \Omega \quad \Rightarrow \quad \delta_\epsilon R = - R\, \epsilon + \epsilon\, R$$

$$\delta_\epsilon V = - V \epsilon + \epsilon V$$

$$\delta_\epsilon \psi = \epsilon\, \psi \quad , \quad \delta_\epsilon \bar\psi = - \bar\psi\, \epsilon$$

$$\epsilon = \frac{1}{4} \varepsilon^{ab} \gamma_{ab}$$

$$\boxed{[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{\epsilon_2 \epsilon_1 - \epsilon_1 \epsilon_2}}$$

DUE TO : CYCLICITY OF Tr

AND $[\epsilon, \gamma_5] = 0$

3. NC VIERBEIN GRAVITY COUPLED TO FERMIONS

P. ASCHIERI, L.C. (2009)

$$\boxed{S = \int Tr\left[R \wedge_* V \wedge_* V (i\gamma_5) - (D\psi * \bar{\psi} - \psi * D\bar{\psi}) \wedge_* V \wedge_* V \wedge_* V \gamma_5\right]}$$

- NC EINSTEIN-HILBERT ACTION COUPLED TO SPIN 1/2 FERMIONS

$$R \equiv d\Omega - \Omega \wedge_* \Omega, \qquad D\psi \equiv d\psi - \Omega * \psi$$

$$\Omega = \frac{1}{4}\omega^{ab}\gamma_{ab} + i\omega \mathbb{1} + \tilde{\omega}\gamma_5 \qquad V = V^a\gamma_a + \tilde{V}^a\gamma_a\gamma_5$$

- SYMMETRIES :

– DIFF. INVARIANCE

– LORENTZ GAUGE INV. :

$$\delta_\epsilon \Omega = d\epsilon - \Omega * \epsilon + \epsilon * \Omega \Rightarrow \delta_\epsilon R = -R * \epsilon + \epsilon * R$$

$$\delta_\epsilon V = -V * \epsilon + \epsilon * V$$

$$\delta_\epsilon \psi = \epsilon * \psi, \quad \delta_\epsilon \bar{\psi} = -\bar{\psi} * \epsilon$$

$$\epsilon = \frac{1}{4}\varepsilon^{ab}\gamma_{ab} + i\varepsilon \mathbb{1} + \tilde{\varepsilon}\gamma_5$$

$$\boxed{[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{\epsilon_2 * \epsilon_1 - \epsilon_1 * \epsilon_2}}$$

DUE TO : CYCLICITY OF Tr, GRADED CYCLICITY OF $\int$ AND $[\epsilon, \gamma_5] = 0$

4. GEOMETRICAL SEIBERG-WITTEN MAP FOR ABELIAN TWISTS

- RELATES NC GAUGE FIELD $\hat{\Omega}$ TO ORDINARY (CLASSICAL) Ω AND NC GAUGE PARAMETER $\hat{\epsilon}$ TO ϵ AND Ω SO THAT

$$\boxed{\hat{\Omega}(\Omega) + \hat{\delta}_{\hat{\epsilon}}\hat{\Omega}(\Omega) = \hat{\Omega}(\Omega + \delta_\epsilon \Omega)}$$

$$\delta_\epsilon \Omega = d\epsilon - \Omega\epsilon + \epsilon\Omega$$
$$\hat{\delta}_{\hat{\epsilon}}\hat{\Omega} = d\hat{\epsilon} - \Omega * \epsilon + \epsilon * \Omega$$

THUS: ORDINARY GAUGE TR. OF Ω INDUCE $*$-GAUGE TR. OF $\hat{\Omega}$

- CAN BE SOLVED ORDER BY ORDER IN θ

$$\hat{\Omega} = \Omega + \Omega^1(\Omega) + \Omega^2(\Omega) + \cdots$$
$$\hat{\epsilon} = \epsilon + \epsilon^1(\epsilon, \Omega) + \epsilon^2(\epsilon, \Omega) + \cdots$$

- RECURSIVE SOLUTION : P. ASCHIERI , L.C. (2011)

$$\Omega^{n+1} = \frac{i}{4(n+1)} \theta^{AB} \{ \hat{\Omega}_A \overset{*}{,} \ell_B \hat{\Omega} + \hat{R}_B \}^n$$

$$\epsilon^{n+1} = \frac{i}{4(n+1)} \theta^{AB} \{ \hat{\Omega}_A \overset{*}{,} \ell_B \hat{\epsilon} \}^n$$

(GENERALIZE ULKER 2008)

- $S_{NC}[\hat{\Omega}] \xrightarrow{SW} S[\Omega, \theta] = S[\Omega] + S^1[\Omega] + S^2[\Omega] + \cdots$

INVARIANT UNDER $\star$-GAUGE TR. ON $\hat{\Omega}$

INVARIANT UNDER ORDINARY GAUGE TR. ON Ω

- TWO ADVANTAGES
  - → CLASSICAL FIELDS (NO PROLIFERATION)
  - → ORDINARY GAUGE SYMMETRY (e.g. LORENTZ)

- EXAMPLE : NC VIERBEIN GRAVITY

P. ASCHIERI, LC, M. DIMITRIJEVIC (2012)
E. DI GREZIA, G. ESPOSITO, M. FIGLIOLIA, P. VITALE (2012)

$$\underbrace{S_{NC}[\hat{V},\hat{\Omega}]}_{V^a,\ \tilde{V}^a,\ \omega^{ab},\ \omega,\ \tilde{\omega}} = \underbrace{S[V,\Omega]}_{V^a,\ \omega^{ab}} + S^1[V,\Omega] + S^2[V,\Omega] + \cdots$$

$*$-LORENTZ INVARIANT
(2 EXTRA GENERATORS)

LORENTZ INVARIANT

- RESULT :

$$S^1 = 0$$

$$S^2 = \frac{1}{32}\theta^{AB}\theta^{CD}\int -\frac{1}{2}((R^{ab}_C R^{cd}_D R^{ef})_{AB} - \frac{1}{2}R^{ef}_{CD}(R^{ab}R^{cd})_{AB})V^g V^h(\varepsilon_{abcd}\delta^{gh}_{ef} + \varepsilon_{efgh}\delta^{cd}_{ab})$$
$$+ (2(L_C R^{ea} L_D R^{eb})_{AB} V^c V^d - (R^{ab}R^{cd})_{AB} L_C V^e L_D V^e$$
$$- 8R^{df}R^{ab}_{AC} L_B V^c L_D V^f - 4R^{ab}(L_A L_C V^c)(L_B L_D V^d))\varepsilon_{abcd}\ .$$

5. CHERN-SIMONS LAGRANGIANS

- $L_{CS}^{(2m-1)}$: 2m-1 FORM CONTAINING Ω, R

$$\boxed{dL_{CS}^{(2m-1)} = STr(R^m)}$$ ← GAUGE INVARIANT

- THUS : $\delta_\epsilon dL_{CS} = d(\delta_\epsilon L_{CS}) = 0$

$\Rightarrow$ $\delta_\epsilon L_{CS} = d\alpha\ (\Omega, R, \epsilon)$ LOCALLY

$\Rightarrow$ $\boxed{\delta_\epsilon \int L_{CS} = 0}$ FOR APPROPRIATE BDY COND.

$\Rightarrow$ THE CS ACTION $\int L_{CS}$ IS GAUGE INVARIANT

- A FORMULA FOR $L_{CS}^{(2m-1)}$:

$$\boxed{L_{CS}^{(2m-1)} = \int_0^1 t^{m-1}\, \mathrm{STr}\left[\Omega\left(R+(1-t)\Omega^2\right)^{m-1}\right] dt}$$

- EXAMPLES : $L_{CS}^{(3)} = \mathrm{STr}\left[R\Omega + \frac{1}{3}\Omega^3\right]$

$$L_{CS}^{(5)} = \mathrm{STr}\left[R^2\Omega + \frac{1}{2}R\Omega^3 + \frac{1}{10}\Omega^5\right]$$

$$L_{CS}^{(7)} = \mathrm{STr}\left[R^3\Omega + \frac{2}{5}R^2\Omega^3 + \frac{1}{5}R\Omega^2R\Omega + \frac{1}{5}R\Omega^5 + \frac{1}{35}\Omega^7\right]$$

- TO CHECK $dL_{CS}^{(2m-1)} = \mathrm{STr}(R^m)$:

USE $d\Omega = R + \Omega^2$ (DEF. OF R)

$dR = \Omega R - R\Omega$ (BIANCHI IDENTITY)

- A FORMULA FOR $\delta_\epsilon L_{CS}$:

$$\boxed{\delta_\epsilon L_{CS} = d\left(\jmath_\epsilon L_{CS}\right)}$$

WHERE $\jmath_\epsilon$ IS A CONTRACTION ACTING SELECTIVELY ON Ω

$$\jmath_\epsilon \Omega = \epsilon \quad , \quad \jmath_\epsilon R = 0$$

WITH A GRADED LEIBNIZ RULE

$$\jmath_\epsilon(\Omega\Omega) = \jmath_\epsilon(\Omega)\Omega - \Omega\jmath_\epsilon(\Omega) = \epsilon\Omega - \Omega\epsilon$$

- EXAMPLE :

$$L^{(5)}_{CS} = \mathrm{STr}\left[R^2\Omega + \tfrac{1}{2}R\Omega^3 + \tfrac{1}{10}\Omega^5\right]$$

$$\delta_\epsilon L^{(5)}_{CS} = d\ \mathrm{STr}\left[R^2\epsilon + \tfrac{1}{2}R\left(\epsilon\Omega^2 - \Omega\epsilon\Omega + \Omega^2\epsilon\right) + \tfrac{1}{2}\epsilon\Omega^4\right]$$

6. NONCOMMUTATIVE CS ACTIONS

THE PRECEDING DISCUSSION HOLDS FOR A GENERIC SUPERGROUP CONNECTION Ω, AND RELIES ONLY ON THE (GRADED) CYCLICITY OF STr

IT CAN BE APPLIED TO THE $\star$-DEFORMED CASE, WITH ONE CAVEAT: THE STr IS NOT CYCLIC FOR TWISTED PRODUCTS. WHAT IS (GRADED) CYCLIC IS THE INTEGRATED SUPERTRACE:

$$\int STr\,(\tau \wedge_\star \tau') = (-1)^{\deg(\tau)\deg(\tau')} \int STr\,(\tau' \wedge_\star \tau) + \text{BDY TERMS}$$

- THIS IS SUFFICIENT TO PROVE THE $\star$-GAUGE INVARIANCE OF THE NC CHERN-SIMONS ACTION

$$\int L_{CS\star}$$

OBTAINED BY REPLACING EXT PRODUCTS $\wedge \longrightarrow \wedge_\star$

- INDEED THE VARIATION FORMULA $\delta_\epsilon L_{cs} = d(j_\epsilon L_{cs})$ HOLDS UNDER INTEGRATION ALSO IN THE $*$- CASE :

$$\delta_\epsilon^* \int L_{cs*} = \int d(j_\epsilon L_{cs*}) = 0$$

FOR SUITABLE BDY COND.

WITH :

$$\delta_\epsilon^* \Omega = d\epsilon - \Omega * \epsilon + \epsilon * \Omega \Rightarrow \delta_\epsilon^* R = -R * \epsilon + \epsilon * R$$

- EXAMPLE : THE D=5 NC CHERN-SIMONS ACTION

$$\int L^{(5)}_{cs*} = \int STr\left[R \wedge_* R \wedge_* \Omega + \frac{1}{2} R \wedge_* \Omega \wedge_* \Omega \wedge_* \Omega + \frac{1}{10} \Omega \wedge_* \Omega \wedge_* \Omega \wedge_* \Omega \wedge_* \Omega\right]$$

IS $*$- GAUGE INVARIANT.

7. D=5 NC CHERN-SIMONS SUPERGRAVITY

- RELEVANT SUPERGROUP $SU(2,2|N)$
  (GROUP GEOMETRIC CONSTRUCTION OF STANDARD D=5 SG : SEE FOR EX. LC, D'AURIA, FRÉ)

- ALL FIELDS CONTAINED IN THE SUPERCONNECTION

$$\Omega = \begin{pmatrix} \Omega^{\alpha}{}_{\beta} & \psi^{\alpha}_{j} \\ -\bar{\psi}^{i}_{\beta} & A^{i}{}_{j} \end{pmatrix}$$

$\alpha, \beta : 1, \dots 4$
$i, j : 1, \dots N$
$a, b : 1, \dots 5$

$$\Omega^{\alpha}{}_{\beta} = \frac{1}{4}\omega^{ab}\gamma_{ab} - \frac{i}{2}V^{a}\gamma_{a} + \frac{i}{4}b\,\mathbb{1}, \quad A^{i}{}_{j} = \frac{i}{N}b\,\delta^{i}_{j} + a^{i}{}_{j}$$

- THE D=5 GAMMA MATRICES γ_a, γ_{ab} AND THE IDENTITY $\mathbb{1}$ SPAN A 4×4 REP. OF THE $U(2,2)$ LIE ALGEBRA

$$\left.\begin{matrix} \omega^{ab} \\ V^a \\ b \end{matrix}\right\} \quad \begin{matrix} U(2,2) \\ \text{GAUGE} \\ \text{FIELDS} \end{matrix} \qquad a^i{}_j \quad \begin{matrix} SU(N) \\ \text{GAUGE} \\ \text{FIELDS} \end{matrix} \qquad \psi_j \quad \begin{matrix} \text{GAUGE} \\ \text{THE} \\ N \text{ SUPERSYMM.} \end{matrix}$$

- THE NC CONNECTION COINCIDES WITH THE COMMUTATIVE ONE, NO EXTRA FIELDS ARE NEEDED BECAUSE $\gamma_a, \gamma_{ab}, \mathbb{1}$ SPAN A COMPLETE BASIS FOR 4×4 MATRICES.

- THE CORRESPONDING CURVATURE 2-FORM IS :

$$\mathbb{R} = d\Omega - \Omega \wedge_* \Omega \equiv \begin{pmatrix} R + \psi_i \wedge_* \psi^i & \Sigma_j \\ -\bar{\Sigma}^i & F^i{}_j + \bar{\psi}^i \wedge_* \psi_j \end{pmatrix}$$

with

$$
\begin{aligned}
R &= d\Omega - \Omega \wedge_\star \Omega \equiv \boxed{\frac{1}{4}R^{ab}\gamma_{ab} - \frac{i}{2}R^a\gamma_a + \frac{i}{4}rI} \\
\Sigma_j &= d\psi_j - \Omega \wedge_\star \psi_j - \psi_k \wedge_\star A^k{}_j \equiv D\psi_j \\
\overline{\Sigma}^i &= d\bar{\psi}^i - \bar{\psi}^i \wedge_\star \Omega - A^i{}_k \wedge_\star \bar{\psi}^k \equiv D\bar{\psi}^i \\
F^i{}_j &= dA^i{}_j - A^i{}_k \wedge_\star A^k{}_j
\end{aligned}
$$

Immediate algebra yields the components of the $U(2,2)$ curvature R:

$$
\boxed{
\begin{aligned}
R^{ab} &= d\omega^{ab} - \frac{1}{2}\omega_c{}^{[a} \wedge_\star \omega^{b]c} + \frac{1}{2}V^{[a} \wedge_\star V^{b]} \\
&\quad + \frac{i}{4}\varepsilon_{cde}{}^{ab}(\omega^{cd} \wedge_\star V^e + V^e \wedge_\star \omega^{cd}) - \frac{i}{4}(\omega^{ab} \wedge_\star b + b \wedge_\star \omega^{ab}) \\
R^a &= dV^a - \frac{1}{2}(\omega^a{}_b \wedge_\star V^b - V^b \wedge_\star \omega^a{}_b) \\
&\quad + \frac{i}{8}\varepsilon^a{}_{bcde}\omega^{bc} \wedge_\star \omega^{de} - \frac{i}{4}(V^a \wedge_\star b + b \wedge_\star V^a) \\
r &= db - \frac{i}{2}\omega^{ab} \wedge_\star \omega_{ab} - iV^a \wedge_\star V_a - \frac{i}{4}b \wedge_\star b
\end{aligned}
}
$$

$\star$-SU(2,2|N) GAUGE TRANSFORMATIONS

$\star$- GAUGE ALGEBRA : $[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{\epsilon_2 \star \epsilon_1 - \epsilon_1 \star \epsilon_2}$

$$\epsilon = \begin{pmatrix} \varepsilon^\alpha{}_\beta & \epsilon^\alpha{}_j \\ -\bar{\epsilon}^i{}_\beta & \eta^i{}_j \end{pmatrix}, \quad \begin{aligned} \varepsilon^\alpha{}_\beta &\equiv \left(\tfrac{1}{4}\varepsilon^{ab}\gamma_{ab} - \tfrac{i}{2}\varepsilon^a\gamma_a + \tfrac{i}{4}\varepsilon \mathbb{1}\right)^\alpha{}_\beta \\ \eta^i{}_j &\equiv \tfrac{i}{N}\varepsilon\delta^i_j + \varepsilon^i{}_j \end{aligned}$$

THE $\star$- GAUGE VARIATIONS ON THE BLOCK ENTRIES OF Ω :

$$\delta\Omega = d\varepsilon - \Omega \star \varepsilon + \varepsilon \star \Omega + \psi_i \star \bar{\epsilon}^i + \epsilon_i \star \bar{\psi}^i$$

$$\delta\psi_i = d\epsilon_i - \Omega \star \epsilon_i + \epsilon_j \star A^j{}_i - \psi_j \star \eta^j{}_i + \varepsilon \star \psi_i$$

$$\delta A^i{}_j = d\eta^i{}_j - A^i{}_k \star \eta^k{}_j + \eta^i{}_k \star A^k{}_j + \bar{\psi}^i \star \epsilon_j - \bar{\epsilon}^i \star \psi_j$$

- NB δb IS CONTAINED BOTH IN $\delta\Omega$ AND $\delta A^i{}_j$; UNDER U(1) →

$\delta\Omega \rightarrow \delta b = d\varepsilon + \frac{i}{4}[\varepsilon \overset{\star}{,} b]$ $\qquad$ $\delta A^i{}_j \rightarrow \delta b = d\varepsilon + \frac{i}{N}[\varepsilon \overset{\star}{,} b]$

$\Rightarrow$ NC DEFORMATION OF CHERN-SIMONS $D=5$ SUPERGRAVITY EXISTS ONLY FOR $N=4$

IN THIS CASE THE SUPERGROUP $SU(2,2|N)$ IS NOT SIMPLE AND THE $U(1)$ GAUGED BY THE b FIELD BECOMES A CENTRAL EXTENSION

THE $*$- ACTION

SUBSTITUTING R AND Ω INTO THE CS ACTION

$$\int L^{(5)}_{CS*} = \int \mathrm{STr}\left[R\wedge_* R\wedge_*\Omega + \frac{1}{2}R\wedge_*\Omega\wedge_*\Omega\wedge_*\Omega + \frac{1}{10}\Omega\wedge_*\Omega\wedge_*\Omega\wedge_*\Omega\wedge_*\Omega\right]$$

YIELDS

$$\boxed{\int L^{(5)}_{CS} = \int L_{U(2,2)} + L_A + L_{FERMI}}$$

EH - TERM COSM. TERM

$$L_{U(2,2)} = \mathrm{Tr}\left[R\wedge_* R\wedge_*\Omega + \boxed{\frac{1}{2}R\wedge_*\Omega^{*3}} + \boxed{\frac{1}{10}\Omega^{*5}}\right]$$

$$L_A = -\mathrm{Tr}\left[F\wedge_* F\wedge_* A + \frac{1}{2}F\wedge_* A^{*3} + \frac{1}{10}A^{*5}\right]$$

$$L_{FERMI} = \frac{3}{2}\bar{\psi}\wedge_*(R\wedge_*\Sigma + \Sigma\wedge_* F) + \frac{3}{2}\bar{\Sigma}\wedge_*(R\wedge_*\psi + \psi\wedge_* F)$$
$$+ \bar{\psi}\wedge_*\psi\wedge_*(\bar{\psi}\wedge_*\Sigma + \bar{\Sigma}\wedge_*\psi)$$

8. CONCLUSIONS

- CONSTRUCTED A NONCOMM. VERSION OF CHERN-SIMONS SUPERGRAVITY IN $D=5$. THE THEORY IS INVARIANT UNDER THE $\star$-GAUGE TRANSFORMATIONS OF THE SUPERGROUP

$$SU(2,2|4)$$

THESE INCLUDE THE 4 $\star$-SUPERSYMMETRIES

- IN PROGRESS: SW-MAP AND θ-EXPANSION OF $\int L^{(5)}_{CS}$

Thank you !