Matrix theory origins of non-geometric fluxes

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Based on:

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A.C. and Larisa Jonke, 1202.4310 [hep-th] (PRD 85 (2012))
A.C. and Larisa Jonke, 1207.6412 [hep-th]

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Introduction and Motivation

Main Objective

Study properties of string/M theory compactifications beyond low-energy SUGRA.

E.g. unconventional compactifications
(winding modes, dualities, non-geometric fluxes, non-commutative manifolds etc.).

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Frameworks:

- ✔ Doubled formalism - Twisted Doubled Tori
- ✔ Generalized Complex Geometry
- ✔ Double Field Theory
- ✔ CFT - Sigma models
- ✔ Matrix Models

See lectures by Hull,
talks by Lindstrom and Lüst.

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Frameworks:

- ✔ Doubled formalism - Twisted Doubled Tori

  Hull; Hull, Reid-Edwards; Dall'Agata, Prezas, Samtleben, Trigiante
- ✔ Generalized Complex Geometry

  Andriot, Hohm, Larfors, Lüst, Patalong; Berman, Musaev, Thompson
- ✔ Double Field Theory

  Hohm, Hull, Zwiebach; Aldazabal et.al.; Geissbuhler; Grana, Marques; Dibitetto et.al.
- ✔ CFT - Sigma models

  Lüst; Blumenhagen, Plauschinn; Mylonas, Schupp, Szabo
- ✔ Matrix Models

  Lowe, Nastase, Ramgoolam; A.C., Jonke

Why Matrix Models?

I ♡ SUGRA

but...

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For certain aspects Matrix Models appear more advantageous:

- ✔ Matrix Theory: inherently quantum-mechanical (crucial role of phase space).
- ✔ Non-commutative structures.
- ✔ SUGRA excludes stringy winding modes.
- ✔ Flux Quantization.

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Non-perturbative framework, analytical *and* numerical approaches.
What is more, recent progress in

- Particle physics, "matrix model building".
  Aoki '10, A.C., Steinacker, Zoupanos '11
- Early and late time cosmology.
  Kim, Nishimura, Tsuchiya '11-'12

Matrix Models as non-perturbative definitions of string/M theory.

Banks, Fischler, Shenker, Susskind '96, Ishibashi, Kawai, Kitazawa, Tsuchiya '96, ...

Matrix Model Compactifications (MMC) on non-commutative tori.

Connes, Douglas, A. Schwarz '97

Constant background B-field $\longleftrightarrow$ Non-commutative deformation

$$B_{ij} \quad \stackrel{CDS}{\longleftrightarrow} \quad \theta^{ij}$$

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What about fluxes?

- Geometric (related e.g. to nilmanifolds/twisted tori): f
- NSNS (non-constant B-fields): H
- "Non-geometric" (T-duality): Q, R

Q: How can they be traced in Matrix Compactifications?

Main Results

- ✔ MMC on nilmanifolds in diverse dimensions. Analog of geometric flux.
- ✔ MMC with diverse algebraic structures.
  Interpretation as analogs of NSNS and non-geometric fluxes.
- ✔ (Generalized) T-duality operations connecting different flux situations appear as phase space transformations in the MM.
- ✔ Trading of properties between geometric and non-geometric fluxes under position-momentum space exchange.
  $\rightsquigarrow$ relations between non-commutativity and generalized geometry.
- ✔ Resolution of non-associativity among unitary operators $\rightsquigarrow$ flux quantization.
- ✔ Effective actions for non-commutative gauge theories with fluxes.

Overview

Matrix Theory and Compactification

Matrix Theory: suggested as non-perturbative definition of M-theory.

Banks, Fischler, Shenker, Susskind '96

Action:

$$\mathcal{S}_{BFSS} = \frac{1}{2g}\int dt \left[Tr\left(\dot{\mathcal{X}}_a\dot{\mathcal{X}}_a - \frac{1}{2}[\mathcal{X}_a, \mathcal{X}_b]^2\right) + \text{fermions}\right],$$

$\mathcal{X}_a(t)$: 9 time-dependent $N \times N$ Hermitian matrices (large N).

EOM:

$$\ddot{\mathcal{X}}_a + [\mathcal{X}_b, [\mathcal{X}^b, \mathcal{X}_a]] = 0.$$

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Compactification : Restriction of the action functional under certain conditions (same logic for any MM, e.g. type IIB models).

Toroidal T^d:

$$\begin{aligned} \mathcal{X}_i + R_i &= U^i\mathcal{X}_i(U^i)^{-1}, \quad i=1,...,d, \\ \mathcal{X}_a &= U^i\mathcal{X}_a(U^i)^{-1}, \quad a\neq i, \quad a=1,\dots,9, \end{aligned}$$

with U^i unitary and invertible.

Toroidal Compactification

Solutions: Connes, Douglas, Schwarz '97

$$\mathcal{X}_i \;=\; iR_i\hat{\mathcal{D}}_i, \quad \mathcal{X}_m = \mathcal{A}_m, (m = d+1, \ldots, 9), \quad U^i = e^{i\hat{x}^i},$$

with covariant derivatives $\hat{\mathcal{D}}_i = \hat{\partial}_i - i\mathcal{A}_i$.

Phase space of $\hat{x}$ and $\hat{p}$ with algebra:

$$\begin{aligned}
[\hat{x}^i, \hat{x}^j] &= i\theta^{ij}, \\
[\hat{x}^i, \hat{p}_j] &= i\delta^i_j, \\
[\hat{p}_i, \hat{p}_j] &= 0.
\end{aligned}$$

The U-algebra is: $U^i U^j = \lambda^{ij} U^j U^i$ with complex constants $\lambda^{ij} = e^{-i\theta^{ij}}$.
This is a non-commutative torus in Connes' non-commutative geometry.

Substitution back into the action $\rightsquigarrow$ NCSYM theory on a dual NC torus.

Interpretation: Deformation parameters θ are reciprocal to background field in SUGRA, $(\theta^{-1})_{ij} \propto \int dx^i dx^j B_{ij}$.

Twisted Toroidal Compactification

Twisted Tori: twisted fibrations of toroidal fibers over toroidal bases;
the geometry of the fiber changes non-trivially as the base is traversed.

Scherk, Schwarz '79; Kaloper, Myers '99; Kachru et.al. '02; Hull, Reid-Edwards '05; Grana et.al. '06

Described as:

- ✔ Homogeneous spaces constructed out of nilpotent Lie groups (nilmanifolds).
- ✔ T-duals of square tori with H flux.

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MMC: Same logic; restrict the action by imposing conditions corresponding to nilmanifolds.

Lowe, Nastase, Ramgoolam '03; A.C., Jonke '11-'12

Twisted $\tilde{\mathrm{T}}^3$:

$$
\begin{aligned}
U^i \mathcal{X}_i (U^i)^{-1} &= \mathcal{X}_i + 1, \quad i = 1,2,3, \\
U^1 \mathcal{X}_3 (U^1)^{-1} &= \mathcal{X}_3 - N\mathcal{X}_2, \quad U^2 \mathcal{X}_3 (U^2)^{-1} = \mathcal{X}_3 + N\mathcal{X}_1, \\
U^i \mathcal{X}_a (U^i)^{-1} &= \mathcal{X}_a, \quad a \neq i, \quad a = 1,\dots,9, \quad (a,i) \neq \{(3,1),(3,2)\}.
\end{aligned}
$$

Solutions:

$$\mathcal{X}_i \;=\; iR_i\hat{\mathcal{D}}_i, \quad \mathcal{X}_m = \mathcal{A}_m, (m = 4, \ldots, 9), \quad U^i = e^{i\hat{x}^i},$$

with covariant derivatives $\hat{\mathcal{D}}_i = \hat{\partial}_i - i\mathcal{A}_i + Nf_i{}^{jk}\mathcal{A}_j\hat{\partial}_k, \quad f_3{}^{12} = 1.$

Algebra of phase space:

$$\begin{aligned} [\hat{x}^i, \hat{x}^j] &= i\theta^{ij} + iNf^{ij}{}_k\hat{x}^k, \\ [\hat{p}_i, \hat{p}_j] &= 0, \\ [\hat{p}_i, \hat{x}^j] &= -i\delta_i^j - iNf_i{}^{jk}\hat{p}_k. \end{aligned}$$

The U-algebra is now given by: $U^i U^j = e^{-i\theta^{ij} - iNf^{ij}{}_k\hat{x}^k} U^j U^i$.
This is a non-commutative twisted torus.

The effective action is a NC gauge theory on a dual NC twisted torus.

Interpretation: The non-constant deformation is the analog of a geometric flux.

Direct generalization for a large class of higher-D nilmanifolds.

More fluxes?

At hand: geometric flux ${f_{ij}}^k$ (twisted torus).

T-dual to NSNS flux H_{ijk}: $H_{ijk} \xrightarrow{T_k} {f_{ij}}^k$.

Enlarged chain with unconventional fluxes:

$$H_{ijk} \xrightarrow{T_k} {f_{ij}}^k \xrightarrow{T_j} {Q_i}^{jk} \xrightarrow{T_i} R^{ijk}.$$

Q: Matrix Model description?

More fluxes?

At hand: geometric flux ${f_{ij}}^k$ (twisted torus).

T-dual to NSNS flux H_{ijk}: $H_{ijk} \xrightarrow{T_k} {f_{ij}}^k$.

Enlarged chain with unconventional fluxes:

$$H_{ijk} \xrightarrow{T_k} f_{ij}{}^{k} \xrightarrow{T_j} Q_i{}^{jk} \xrightarrow{T_i} R^{ijk}.$$

Q: Matrix Model description?

Observe: Although full phase space operates, $e^{i\hat{p}_i}$ were previously ignored...

Introduce:

$$\mathcal{X}_i = i\hat{\partial}_i + \hat{\mathcal{A}}_i,$$
$$\tilde{\mathcal{X}}^i = (-1)^{c_i}\hat{x}^i + \hat{\tilde{\mathcal{A}}}^i,$$

and:

$$U^i = e^{i\hat{x}^i},$$
$$\tilde{U}_i = e^{(-1)^{c_i}\hat{\partial}_i}.$$

The grading will guarantee correct Heisenberg relation.

Algebraic Building Blocks

The set-up reminds of the doubled formalism $\rightsquigarrow$ Twisted Doubled Tori.

Hull, Reid-Edwards '07, Dall'Agata, Prezas, Samtleben, Trigiante '07

Use TDT formalism to describe MMC, then project to appropriate subsector.

H-block: ($H^{123} = 1$ and $c_i = 0$ for every $i = 1, 2, 3$.)

Compactification Conditions:

$$\begin{aligned} U^i \mathcal{X}_i (U^i)^{-1} &= \mathcal{X}_i + 1, \\ \tilde{U}_i \tilde{\mathcal{X}}^i (\tilde{U}_i)^{-1} &= \tilde{\mathcal{X}}^i + 1, \\ U^i \tilde{\mathcal{X}}^j (U^i)^{-1} &= \tilde{\mathcal{X}}^j + H^{ijk} \mathcal{X}_k, \end{aligned}$$

Phase space algebra:

c.f. Lüst '10

$$\begin{aligned} [\hat{x}^i, \hat{x}^j] &= i H^{ijk} \hat{p}_k, \\ [\hat{p}_i, \hat{p}_j] &= 0, \\ [\hat{p}_i, \hat{x}^j] &= -i \delta_i^j. \end{aligned}$$

The U-algebra is: $U^i U^j = e^{-H^{ijk} \hat{\partial}_k} U^j U^i$, i.e. $\theta^{ij} = H^{ijk} \hat{p}_k$.

The Connes-Douglas-Schwarz correspondence suggests a SUGRA B-field

$$B = x^1 dx^2 \wedge dx^3 + x^2 dx^3 \wedge dx^1 + x^3 dx^1 \wedge dx^2,$$

where x^i are standard toroidal coordinates.

Q-block: ($Q^1_{23} = 1$, while $c_1 = 0$ and $c_2 = c_3 = 1$.)

Compactification Conditions:

$$\begin{aligned} U^i \mathcal{X}_i (U^i)^{-1} &= \mathcal{X}_i + 1, \\ U^1 \mathcal{X}_2 (U^1)^{-1} &= \mathcal{X}_2 + \tilde{\mathcal{X}}^3, \\ U^1 \mathcal{X}_3 (U^1)^{-1} &= \mathcal{X}_3 - \tilde{\mathcal{X}}^2, \end{aligned}$$

and

$$\begin{aligned} \tilde{U}_i \tilde{\mathcal{X}}^i (\tilde{U}_i)^{-1} &= \tilde{\mathcal{X}}^i + 1, \\ \tilde{U}_2 \mathcal{X}_3 (\tilde{U}_2)^{-1} &= \mathcal{X}_3 + \mathcal{X}_1, \\ \tilde{U}_3 \mathcal{X}_2 (\tilde{U}_3)^{-1} &= \mathcal{X}_2 - \mathcal{X}_1, \\ \tilde{U}_2 \tilde{\mathcal{X}}^1 (\tilde{U}_2)^{-1} &= \tilde{\mathcal{X}}^1 - \tilde{\mathcal{X}}^3, \\ \tilde{U}_3 \tilde{\mathcal{X}}^1 (\tilde{U}_3)^{-1} &= \tilde{\mathcal{X}}^1 + \tilde{\mathcal{X}}^2, \end{aligned}$$

Phase space algebra:

$$\begin{aligned} [\hat{x}^i, \hat{x}^j] &= 0, \\ [\hat{p}_i, \hat{p}_j] &= -iQ_{ij}{}^k \hat{p}_k, \\ [\hat{p}_i, \hat{x}^j] &= -i\delta_i^j + iQ_{ik}{}^j \hat{x}^k. \end{aligned}$$

The U-algebra is commutative. But the $\tilde{U}$ one is not: $\tilde{U}_i \tilde{U}_j = e^{Q_{ij}{}^k \hat{\partial}_k} \tilde{U}_j \tilde{U}_i$.

$\tilde{\theta}_{ij} = -Q_{ij}{}^k \hat{p}_k$, expected to account for non-geometric Q flux.

R-block: ($c_1 = 1$ for all $i = 1, 2, 3$.)

Compactification Conditions:

$$\begin{aligned} U^i \mathcal{X}_i (U^i)^{-1} &= \mathcal{X}_i + 1, \\ \tilde{U}_i \tilde{\mathcal{X}}^i (\tilde{U}_i)^{-1} &= \tilde{\mathcal{X}}^i + 1, \\ \tilde{U}_i \mathcal{X}_j (\tilde{U}_i)^{-1} &= \mathcal{X}_j + R_{ijk} \tilde{\mathcal{X}}_k. \end{aligned}$$

Phase space algebra:

$$\begin{aligned} [\hat{x}^i, \hat{x}^j] &= 0, \\ [\hat{p}_i, \hat{p}_j] &= i R_{ijk} \hat{x}^k, \\ [\hat{p}_i, \hat{x}^j] &= -i \delta_i^j. \end{aligned}$$

The Us commute again, unlike the $\tilde{U}$s: $\tilde{U}_i \tilde{U}_j = e^{-iR_{ijk}\hat{x}^k} \tilde{U}_j \tilde{U}_i$.

$\tilde{\theta}_{ij} = R_{ijk} \hat{x}_k$, expected to account for non-geometric R flux.

Block-to-block moves and T-duality

At hand: 4 types of solutions of the compactified Matrix Model.

Q: Which operations take each solution to the other?

$\underline{H \to f}$:

At the level of the phase-space algebra,

$$\hat{x}^3 \to -\hat{p}_3, \quad \hat{p}_3 \to \hat{x}^3.$$

Grading correction,

$$(-1)^{\hat{c}_i}_f = \mathrm{diag}(1,1,1,1,1,-1).$$

May be represented as a matrix $M_{H\to f}$ acting on $\begin{pmatrix} \hat{x}^i \\ \hat{p}_i \end{pmatrix}$.

Block-to-block moves and T-duality

Full Picture:

$$\begin{array}{ccccccc} H & \overset{T_3}{\longleftrightarrow} & f & \overset{T_2}{\longleftrightarrow} & Q & \overset{T_1}{\longleftrightarrow} & R \\ \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\ \theta(\hat{p}) & \overset{M_{H\to f}\cdot(-1)^{\hat{c}_i}_f}{\longleftrightarrow} & \theta(\hat{x}) & \overset{M_{f\to Q}\cdot(-1)^{\hat{c}_i}_Q}{\longleftrightarrow} & \tilde{\theta}(\hat{p}) & \overset{M_{Q\to R}\cdot(-1)^{\hat{c}_i}_R}{\longleftrightarrow} & \tilde{\theta}(\hat{x}) \end{array}$$

with $\theta^{ij} = [\hat{x}^i, \hat{x}^j]$ and $\tilde{\theta}_{ij} = [\hat{p}_i, \hat{p}_j]$.

Finding the correct subsector

Matrix theory does not really possess $\tilde{\mathcal{X}}^i$ as dynamical DoF.
Q: Which is the correct subsector?

For f and H cases, easy: formulate everything just for $\mathcal{X}_i$.

But: for Q and R cases, compactification on $\mathcal{X}_i$-sector is not well-defined.
What is more, for the R case: $[\mathcal{X}_i, \mathcal{X}_j, \mathcal{X}_k] \neq 0$!
But Hermitian matrices cannot be non-associative!

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But Hermitian matrices cannot be non-associative!

Resolution: For Q and R, the correct subsector is the $\tilde{\mathcal{X}}^i$ in the momentum rep.

$\rightsquigarrow$ There is a correspondence:

$$\theta^{ij}|_f \quad \text{or} \quad \theta^{ij}|_H \quad \text{in} \quad \hat{x}\text{-space} \quad \longleftrightarrow \quad \tilde{\theta}_{ij}|_Q \quad \text{or} \quad \tilde{\theta}_{ij}|_R \quad \text{in} \quad \hat{p}\text{-space} \; .$$

Similar result in Generalized Complex Geometry approach...
Andriot, Larfors, Lüst, Patalong '11

Indication: Just as $\theta^{ij} \sim (B_{ij})^{-1}$, also $\tilde{\theta}_{ij} \sim (\beta^{ij})^{-1}$, β: the bivector of GCG.

Non-Associativity and Flux Quantization

All encountered phase space algebras exhibit some non-associativity.

E.g. $[\hat{p}_i, \hat{x}^j, \hat{x}^k] \propto f_i{}^{jk}$ for the f-block, $[\hat{x}^i, \hat{x}^j, \hat{x}^k] \propto H^{ijk}$ for the H-block, etc.

The induced non-associativity of $\mathcal{X}_i$ is resolved as above.

Q: What about the algebraic elements U^i, which define the NC torus?

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H-case: $U^i(U^j U^k) = e^{\frac{i}{2}H^{ijk}}(U^i U^j)U^k$.

$\rightsquigarrow$ 3-cocycle; typical in QM systems with fluxes. Jackiw '85

Resolution: The flux **has to** be quantized,

$$H = 4\pi n, \quad n \in \mathbb{Z}.$$

$\rightsquigarrow$ Flux Quantization.

Similar to DFT, where large gauge transformations associate even when coordinate maps do not. Hohm, Zwiebach '12

Gauge Theories

Effective action for toroidal matrix compactification:

$$\mathcal{S} \propto \int dt \, \mathrm{Tr}(F_{ij}F^{ij} + \text{scalars} + \text{fermions}),$$

with $\mathrm{Tr} \to \int d^3x \, \mathrm{tr}$ and $F_{ij} = \partial_i A_j - \partial_j A_i + iA_i \star A_j - iA_j \star A_i$

Moyal-Weyl $\star$ product: $\quad f \star g = e^{\frac{i}{2}\frac{\partial}{\partial x^i}\hat{\theta}^{ij}\frac{\partial}{\partial y^j}} f(x)g(y)|_{y\to x}.$

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Effective actions with fluxes: additional terms are induced.

$\rightsquigarrow$ diverse non-commutative gauge theories and $\star$ products.

e.g. for the nilmanifold:

$$f \star g = e^{-\frac{i}{2}f^{ij}{}_k x^k \frac{\partial}{\partial y^i}\frac{\partial}{\partial z^j}} f(y)g(z)|_{y,z\to x}\ .$$

Main messages

- ✔ Matrix Models: useful framework for unconventional string compactifications.
- ✔ Fluxes, dualities, non-geometry, non-commutativity.
- ✔ Relations to other frameworks (double field theory, generalized geometry, etc.)

Some prospects

- Analysis of the effective theories with fluxes. in progress, with L. Jonke
- Full study of possible vacua. Coexistence of all types of fluxes.
  in progress, with M. Schmitz
- Phenomenology of unconventional compactifications?
- Non-perturbative dualities?